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INTERPOLATION WITH FUNCTION PARAMETER AND UMD SPACES

Fernando Cobos

División de Matemáticas. Facultad de Ciencias.
Universidad Autónoma. 28049 Madrid. España.

A (real or complex) Banach space E is said to have the unconditionality property for martingale differences (UMD-property, for short) if E-values martingale differences are unconditional in $L_p(E;[0,1])$ [see [2], [3] and the references given there]. The main reason for the interest in this new class of spaces is that the analogues of several classical results on martingales and singular integrals are also true for a Banach space belonging to this class.

There are only a few known examples of concrete spaces with the UMD-property. For instance $\mathbb{R}$, the Lebesgue spaces l_p, $L_p(T)$ where $1 < p < \infty$ and T is the unit circle, and the Schatten classes $S_p(H,K)$ are all UMD spaces. On the contrary, the spaces l_1 and l_∞ fail to have the UMD-property because they are not reflexive.

In this note we are interested in studying whether or not the spaces ocurring in classical harmonic analysis have the UMD-property. We also show some operator spaces with that property.

Let us start by recalling the definition of the (ϕ,q)-method of interpolation (see [6] and the references given there).

Let $\mathcal{B}$ be the class of all functions $\phi : (0,+\infty) \to (0,+\infty)$ continuous, with $\phi(1) = 1$ and such that

$\bar{\phi}(t) = \sup\{\phi(ts)/\phi(s) : s > 0\} < \infty$ for every $t > 0$.

The Boyd indices of the sub-multiplicative function $\bar{\phi}$ are represented by $\alpha_{\bar{\phi}}$ and $\beta_{\bar{\phi}}$.

Let (A_0,A_1) be a compatible couple of Banach spaces, let $1 \leq q \leq \infty$ and $\phi \in \mathcal{B}$ with $0 < \beta_{\bar{\phi}} \leq \alpha_{\bar{\phi}} < 1$. The space $(A_0,A_1)_{\phi,q}$ consists of all $x \in A_0+A_1$ which have a finite norm

$$\|x\|_{\phi,q} = \left(\int_0^\infty (\phi(t)^{-1}K(t,x))^q dt/t\right)^{1/q}$$

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where $K(t,x)$ is the functional of J. Peetre.

Concerning the UMD-property, this interpolation method is stable if $1 < q < \infty$:

Theorem 1. Let (A_0,A_1) *be a couple of UMD spaces, let* $1 < q < \infty$ *and* $\phi \in \mathcal{B}$ *with* $0 < \beta_{\bar\phi} \leq \alpha_{\bar\phi} < 1$. *Then* $(A_0,A_1)_{\phi,q}$ *is a UMD-space.*

The proof is based on the formula

$$(L_q(A_0),L_q(A_1))_{\phi,q} = L_q((A_0,A_1)_{\phi,q})$$

and the characterization of UMD spaces in terms of the vector-values Hilbert transform.

As a consequence we obtain that the Lorentz-Zygmund spaces $L_{p,q}(\log L)^\gamma$ and the Zygmund spaces $L_p(\log L)^\gamma$ (see [1]) are UMD spaces provided that $1 < p < \infty$, $1 < q < \infty$ and $-\infty < \gamma < +\infty$.

On the other hand, if $1 \leq p < \infty$ and $\gamma > 0$, then $L(\log L)^\gamma$, the O'Neil space $K^p(\log^+ K)^\gamma$, and the Zygmund space Z^γ (see [7], [1]) fail to have the UMD-property because they are not reflexive.

Theorem 1 also allows to get some new information on the Lorentz-Marcinkiewicz operator spaces $S_{\phi,q}(H,K)$ [4]: For $0 < \beta_{\bar\phi} \leq \alpha_{\bar\phi} < 1$, the spaces $S_{\phi,q}(H,K)$ have the UMD-property if $1 < q < \infty$, while $S_{\phi,1}(H,K)$ and $S_{\phi,\infty}(H,K)$ fail to have that.

The second part of this statement follows from the fact that the $(\phi,1)$ and (ϕ,∞)-methods are not stable for the UMD-property.

Full details and proofs [5] will appear elsewhere.

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