EXTRACTA MATHEMATICAE 1, n.3, 133-135 (1986)

CLASSIFICATION OF FILIFORM LIE ALGEBRAS IN DIMENSION 8.

By

José María ANCOCHEA BERMUDEZ

and

Michel GOZE

J.M.A.B. Departamento de Geometría y Topología. Facultad de Matemáticas. Universidad Complutense. 28040 MADRID. (SPAIN).

M.G. Université de Haute Alsace. I.S.E.A. 4, rue des Frères Lumière. 68093 MULHOUSE Cédex. (FRANCE).

A.M.S. classification: 17 B 30.

The classification of real and complex filiform Lie algebras is known in dimension less or equal than 7 (cf. [3] for dimension less or equal than 6 and [1] for dimension 7). The set of isomorphism classes has a finite number of points up to dimension 6. In dimension 7 we get a line (real or complex on the case) and 9 points (resp. 8) for the real case (resp. complex). Note that dimension p=7 is the smallest for which it does not exist any rigid filiform law in the algebraic variety N^p of nilpotent Lie algebra laws in dimension p (cf. [1]). In this work we give the classification of complex filiform Lie algebras in dimension 8, and we obtain that the set of isomorphism classes is union of a finite number of lines (only two intersecting) and a finite number of points. In this case, we find a unique rigid filiform law in N^8.

Theorem.- Any filiform Lie algebra in $\mathbb{C}^8$ is isomorphic to one of the following:

* $\mu^1(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^1(X_4,X_7)=X_2$, $\mu^1(X_4,X_8)=X_3+X_2$, $\mu^1(X_5,X_6)=-X_2$, $\mu^1(X_5,X_7)=(-2/5)X_2$, $\mu^1(X_5,X_8)=X_4+(3/5)X_3$, $\mu^1(X_6,X_7)=(-2/5)X_3$, $\mu^1(X_6,X_8)=X_5+(1/5)X_4$, $\mu^1(X_7,X_8)=X_6+(1/5)X_5$.
* $\mu^2(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^2(X_4,X_7)=X_2$, $\mu^2(X_4,X_8)=X_3$, $\mu^2(X_5,X_6)=-X_2$, $\mu^2(X_5,X_8)=X_4$, $\mu^2(X_6,X_7)=X_2$, $\mu^2(X_6,X_8)=X_5+X_3+X_2$, $\mu^2(X_7,X_8)=X_6+X_4+X_3$.
* $\mu^3(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^3(X_4,X_7)=X_2$, $\mu^3(X_4,X_8)=X_3$, $\mu^3(X_5,X_6)=-X_2$, $\mu^3(X_5,X_8)=X_4$, $\mu^3(X_6,X_8)=X_5+X_2$, $\mu^3(X_7,X_8)=X_6+X_3$.

* $\mu^4(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^4(X_4,X_7)=X_2$, $\mu^4(X_4,X_8)=X_3$, $\mu^4(X_5,X_6)=-X_2$, $\mu^4(X_5,X_8)=X_4$, $\mu^4(X_6,X_7)=X_2$, $\mu^4(X_6,X_8)=X_5+X_2$, $\mu^4(X_7,X_8)=X_6+X_3$.
* $\mu^5(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^5(X_4,X_7)=X_2$, $\mu^5(X_5,X_6)=-X_2$, $\mu^5(X_i,X_8)=X_{i-1}$ for $i=4,5,6,7$.
* $\mu^6_z(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^6_z(X_4,X_8)=zX_2$, $\mu^6_z(X_5,X_7)=X_2$, $\mu^6_z(X_5,X_8)=(1+z)X_3+X_2$, $\mu^6_z(X_6,X_7)=X_3$, $\mu^6_z(X_6,X_8)=(2+z)X_4+X_3$, $\mu^6_z(X_7,X_8)=(2+z)X_5+X_4$ with $z\in\mathbb{C}-\{-1\}$.
* $\mu^7_z(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^7_z(X_4,X_8)=zX_2$, $\mu^7_z(X_5,X_7)=X_2$, $\mu^7_z(X_5,X_8)=(1+z)X_3$, $\mu^7_z(X_6,X_7)=X_3$, $\mu^7_z(X_6,X_8)=(2+z)X_4$, $\mu^7_z(X_7,X_8)=$ $=(2+z)X_5$ with $z\in\mathbb{C}$.
* $\mu^8(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^8(X_4,X_8)=-X_2$, $\mu^8(X_5,X_7)=X_2$, $\mu^8(X_6,X_7)=X_3+X_2$, $\mu^8(X_6,X_8)=X_4+X_3$, $\mu^8(X_7,X_8)=X_5+X_4$.
* $\mu^9_z(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^9_z(X_4,X_8)=X_2$, $\mu^9_z(X_5,X_8)=X_3$, $\mu^9_z(X_6,X_7)=X_2$, $\mu^9_z(X_6,X_8)=X_4+X_3+zX_2$, $\mu^9_z(X_7,X_8)=X_5+X_4+zX_3$ with $z\in\mathbb{C}$.
* $\mu^{10}_z(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^{10}_z(X_4,X_8)=X_2$, $\mu^{10}_z(X_5,X_8)=X_3$, $\mu^{10}_z(X_6,X_8)=X_4+X_2$, $\mu^{10}_z(X_7,X_8)=X_5+X_3+zX_2$ with $z\in\mathbb{C}$.
* $\mu^{11}(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^{11}(X_i,X_8)=X_{i-2}$ for $i=4,5,6$, $\mu^{11}(X_7,X_8)=X_5+X_2$.
* $\mu^{12}(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^{12}(X_i,X_8)=X_{i-2}$ for $i=4,5,6,7$.
* $\mu^{13}_z(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^{13}_z(X_5,X_8)=zX_2$, $\mu^{13}_z(X_6,X_7)=X_2$, $\mu^{13}_z(X_6,X_8)=(1+z)X_3+X_2$, $\mu^{13}_z(X_7,X_8)=(1+z)X_4+X_3$ with $z\in\mathbb{C}$.
* $\mu^{14}_z(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^{14}_z(X_5,X_8)=zX_2$, $\mu^{14}_z(X_6,X_7)=X_2$, $\mu^{14}_z(X_6,X_8)=(1+z)X_3$, $\mu^{14}_z(X_7,X_8)=(1+z)X_4$ with $z\in\mathbb{C}$.
* $\mu^{15}_z(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^{15}_z(X_5,X_8)=X_2$, $\mu^{15}_z(X_6,X_8)=X_3+$ $+X_2$, $\mu^{15}_z(X_7,X_8)=X_4+X_3+zX_2$ with $z\in\mathbb{C}$.
* $\mu^{16}(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^{16}(X_5,X_8)=X_2$, $\mu^{16}(X_6,X_8)=X_3$, $\mu^{16}(X_7,X_8)=X_4+X_2$.
* $\mu^{17}(X_1,X_i)=X_{i-1}$ for $i=3,4,5,6,7,8$, $\mu^{17}(X_6,X_8)=X_2$, $\mu^{17}(X_7,X_8)=X_3+$

$+X_2$.

* $\mu^{18}(X_1,X_i)=X_{i-1}$ for i=3,4,5,6,7,8, $\mu^{18}(X_6,X_8)=X_2$, $\mu^{18}(X_7,X_8)=X_3$.
* $\mu^{19}(X_1,X_i)=X_{i-1}$ for i=3,4,5,6,7,8, $\mu^{19}(X_7,X_8)=X_2$.
* $\mu^{20}(X_1,X_i)=X_{i-1}$ for i=3,4,5,6,7,8.

(the other brackets zero).

Moreover, these algebras are not pairwise isomorphic.

Remark.- The law μ^7_{-1} is isomorphic to: $\mu^6_{-1}(X_1,X_i)=X_{i-1}$ for i=3,4,5,6,7,8, $\mu^6_{-1}(X_4,X_8)=-X_2$, $\mu^6_{-1}(X_5,X_7)=X_2$, $\mu^6_{-1}(X_5,X_8)=X_2$, $\mu^6_{-1}(X_6,X_7)=X_3$, $\mu^6_{-1}(X_6,X_8)=X_4+X_3$, $\mu^6_{-1}(X_7,X_8)=X_5+X_4$.

Proposition.- The algebra μ^1 is the only rigid filiform law in N^8. (Recall that the law μ^1 is rigid in N^8 provided that it has an open orbit under the natural action of Gl($8,\mathbb{C}$) on the variety N^8. Or, equivatly, μ^1 is rigid in N^8 if every law in N^8, infinitely close to μ^1, is isomorphic to it).

REFERENCES.

[1] J.M.ANCOCHEA BERMUDEZ and M.GOZE. Sur la classification des algèbres de Lie nilpotentes de dimension 7. C.R.Acad. Sc. Paris, t. 302. p.611-613. (1986).

[2] J.M.ANCOCHEA BERMUDEZ and M.GOZE. Classification des algèbres de Lie filiformes de dimension 8. Preprint Univ. de Mulhouse. (1986).

[3] M.VERGNE. Sur la variété des lois nilpotentes. These 3-ème cycle. Paris. (1966).