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SOME NUMERICAL METHODS IN BIFURCATION THEORY

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Bifurcation theory studies the quantitative and cualitative behaviour of the solutions of nonlinear problems.

General Scheme:

- Nolinear equations
- Partial differential equations
- Sobolev spaces
- Fluid Mechanics
- Elasticity
- Finite elements
- Gradient methods
- Continuation methods

we shall study the following general problem:

(A) $F(\lambda,u)=0$

with λ a real parameter, $u\in V$, V a real Banach space, $F: R\times V \longrightarrow V$ a nolinear continuous map.

In bifurcation theory it is assumed that there exists a continuous curve C_0 in RxV of solutions of (A). C_0 is of the form:

$$C_o = \{ (\lambda(\alpha), u(\alpha)), \alpha\in I\subset R\}$$

We say that $(\lambda(\alpha_0), u(\alpha_0))$ is a point of bifurcation relative to the equation and C_o, if $(\lambda(\alpha_0), u(\alpha_0))$ lies on the curve and every neigborhood of $(\lambda(\alpha_0), u(\alpha_0))$ has a solution of (A) that does not belong to C_o.

Examples:

1.- We study the following situation: In a neighborhood of $(\lambda(\alpha_0), u(\alpha_0))$ the set of solutions of (A) is two curves.

2.- It can be considered also a continuum of points.

Different problems that appear in bifurcation theory.

(i) Existence of points of bifurcation; that is, determine the values of the parameter λ for wich the multiplicity of the set of solutions changes.

(ii) Problems of multiplicity: that is, the multiplicity of the number of solutions in a neighborhood of the point of bifurcation.

(iii) Spectral problems.

With regard to the numerical approximation of the bifurcation problems, we are interested in the approximation of the solution branches containing, or not containing, points of bifurcation. The difficult lies in the fact that, in general, it is not known the structure of the solutions of the continuous problem.

We have to take into account that there are several and different problems depending on whether the problem is industrial or theoretical. We present some algorithms that allow to find the solutions as well as the branches of these solutions. We point out that there are many open problems in this field.

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