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A NEW OPERATOR CHARACTERIZATION OF THE DUNFORD-PETTIS PROPERTY.

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A Banach space E is said to have the Dunford-Pettis property (DPP in short) if every weakly compact operator on E sends weakly convergent sequences into norm convergent ones. This last class of operators are called, by obvious reasons, Dunford-Pettis operators. The DPP was introduced by Grothendieck in his remarkable paper |5| and has been intensively studied (see |3|). The long standing open question of whether the Banach space C(K,E) of all the continuous E-valued functions on the compact Hausdorff space K has the DPP if E has, was answered in the negative by Talagrand, who built in |6| a Banach space H with the DPP and a weakly compact operator from C(|0,1|,H) into c_o which is not Dunford-Pettis. In |1| a class of operators on C(K,E) spaces, more general than the Dunford-Pettis operators are introduced, namely the almost Dunford-Pettis operators. It turns out that the weakly compact, non Dunford-Pettis operator exhibited by Talagrand in |6|, is in fact almost Dunford-Pettis. Hence, the following natural question arises (see |1|): Which are the Banach spaces E such that, regardless the compact Hausdorff space K, every weakly compact operator on C(K,E) is almost Dunford-Pettis?. In this note we prove that such spaces are precisely those with the DPP, giving so a new operator characterization of this property.

For notation and terminology used along the paper, we refer to |4|. Proofs will appeared elsewhere.

Definition. (|1|, def. 1.8). An operator T from C(K,E) into F, whose representing measure has semivariation continuous at ϕ, is called almost Dunford-Pettis if for every weakly null sequence (x_n) in E and every bounded sequence (ϕ_n) im C(K), we have

$$\lim_{n\to\infty} T(\phi_n x_n)=0$$

With this definition at hand, we can state the main result of the paper:

Theorem: Let E be a Banach space. The following assertions are equivalent:

a) For any compact Hausdorff space K, every weakly compact Hausdorff space K, every weakly compact operator on C(K,E) is almost Dunford-Pettis.

b) Every weakly compact operator on C([0,1],E) is almost Dunford-Pettis.

c) Every weakly compact operator from C([0,1],E) into c_o is almost Dunford-Pettis.

d) E has the Dunford-Pettis property.

Recall that the space rcabv (Bo(K),E*) of all regular, countably additive E*-valued measures of bounded variation defined on the Borel-σ-field Bo(K) of K, endowed with the variation norm can be identified with the topological dual of the Banach space C(K,E). One of the main tools in the proof of the proof of the above theorem is the following lemma, that extends a previous result of Bourgain for spaces of Bochner integrable functions (|2|, prop. 10):

Lemma: If B is a weakly conditionally compact subset of the space rcabv (Bo(K),E*) and D is a bounded subset of C(K), then

$$D.B=\{\phi m:\phi \in D,\ m \in D\}$$

is also weakly conditional compact in rcabv (Bo(K),E*).

Finally, it should be mentioned that, by using the theorem above and reasoning as in theorem 3.2 of |1|, one can prove:

Proposition: For a Banach space E, the following conditions are equivalent:

a) For some non dispersed compact Hausdorff space K, every weakly compact operator on C(K,E) is almost Dunford-Pettis.

b) Every weakly compact operator on C([0,1],E) is almost Dunford-Pettis.

c) E has the Dunford-Pettis property.

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