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CHEBYSHEV CENTERS IN ULTRAMETRIC SPACES

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In a recent paper, M.Z.M.C. Soares [2] studied Chebyshev radii and centers in non-archimedean normed spaces. She also posed the main problems in the ultrametric theory of the best simultaneous approximation, which are in decreasing order of generality the following,

PROBLEM I: Let $W\subset E$ be given. Determine if W has the relative Chebyshev center property in E. In particular, determine if E admits Chebyshev centers.

PROBLEM II: Let $W\subset E$ be given. Determine the class $\mathfrak{B}$ of all non-empty bounded sets $B\subset E$ such that the relative Chebyshev center of B, $cent_W(B)$, is non-empty.

PROBLEM III: Let $W\subset E$ be given. Determine if W is proximinal in E, i.e., determine if the class $\mathfrak{B}$ of problem II contains all sets of the form $B=(f)$, $f\in E$.

In this paper we give some results about the preceding questions within the more general context of ultrametric spaces. Proofs of these results will appear in [1].

With the above notation, we will denote by $rad_W(B)$ the relative Chebyshev radius of B with respect to W (i.e., $rad_W(B) = \inf_{w\in W} \sup_{b\in B} d(b,w)$) and by $cent_W(B)$ the set of elements of W where the infimum of $rad_W(B)$ is attained. If we denote by $\delta(B)$ the diameter of B and by $d(B,W)$ the distance between B and W, then the following useful expressions for $rad_W(B)$ and $cent_W(B)$ are obtained,

THEOREM 1: Let E be an ultrametric space, let $W \subset E$ and let B be a non-empty bounded subset of E. Then,

(i) $rad_W(B)=max(\delta(B), d(B,W))$

(ii) For every $b \in B$, $cent_W(B)=W \cap B(b,rad_W(B))$ where $B(b,rad_W(B))$ is the closed ball with center $b \in B$ and radius $rad_W(B)$.

As a consequence of the preceding theorem, we give a partial solution to problem I:

THEOREM 2: Every ultrametric space admits Chebyshev centers.

Related to problem III we have obtained the following result,

THEOREM 3: Let E be an ultrametric space and let $W \subset E$. Then W is proximinal if and only if W verifies the relative Chebyshev center property in E.

Proximinality is closely related to spherical completeness as it is shown in theorem 5. First we give the following definition,

DEFINITION 4: A subset W of an ultrametric space E is called relatively spherically complete in E if every pseudo-Cauchy sequence in W that has a pseudo-limit in E also has one in W.

THEOREM 5: Let E be an ultrametric space and let $W \subset E$. Then, W is proximinal in E if and only if W is relatively spherically complete in E.

In particular, every closed ball and every spherically complete subset of E are proximinal. Also if E is spherically complete, every proximinal subset W of E must be spherically complete.

In [2], Soares studied the Banach spaces $C_0(X,E)$ of all continuous functions from X into a non-archimedean Banach space E which vanish at infinity. For this kind of spaces she obtained some results about Chebyshev radii and centers. By using our theorems 2 and 3 one can easily improve theorems 3.8 to 3.11 in Soares' paper. First the hypothesis of E having Chebyshev centers in 3.9 can be dropped. Also theorems 3.9 and 3.11 are valid not only for non-empty equicontinuous and bounded sets $B \subset C_0(X,E)$ which vanish collectively at infinity but for all non-empty bounded sets.

REFERENCES

1. J.Martínez-Maurica, T.Pellón. *Non-archimedean Chebyshev centers*, Indag. Math., to appear.

2. M.Z.M.C.Soares. *Best approximants from non-archimedean Stone-Weierstrass subspaces*, Comp. Math. 56, (1985), 331-349.