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CONTINUITY OF DERIVATIONS ON A COMPLETE NORMED ALTERNATIVE ALGEBRA

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The problem of the automatic continuity of derivations is a classical topic in the problem of automatic continuity of linear operators on complete normed algebras. Perhaps the most interesting result about this problem is the Johnson-Sinclair theorem (see [2]) stating that every derivation on a semisimple (associative) Banach algebra is continuous.

The problem of the continuity off derivations on complete normed nonassociative algebras has at this time a lamentable situation. Aside from the observation in [4] that derivations of JB^*-algebras are continuous, the result in [1] that derivations on "classical Banach-Lie algebras of operators on Hilbert space" are continuous, and the proof in [3] of continuity of derivations on complete normed nonassociative algebras with the property that the socle of the "full multiplication algebra" separates the points in the algebra, no much more is known.

Here we make a contribution to this nonassociative problem proving the automatic continuity of derivations on a semisimple complete normed alternative algebra.

From now on:

A will be a complete normed alternative algebra, i. e. a complete normed algebra with $a(ab) = a^2b$ and $(ba)a = ba^2$ for all a and b in A.

P will be a primitive ideal of A, i. e. the largest two-sided

ideal of A contained in a maximal modular left ideal, M, of A (see [5; Definition 10.1]).

$\cap_P$ will be the quotient map $A \to A/P$ and D will be a derivation on A, i.e. a linear map $D: A \to A$ with

$D(ab) = D(a)b + aD(b)$ for all a and b in A.

Our method is inspired in [2].

First, for infinite codimensional P, by considering the (automatically associative) Banach algebra A/P and the Banach (left) A/P-module, A/M (in the associative classical sense) we obtain as in [2; Theorem 2.2]

1. THEOREM.

$\cap_P D$ is continuous for infinite codimensional P.

For finite codimensional $P_1, \ldots, P_n$ we show that the homomorphism $a \mapsto (a+P_1, \ldots, a+P_n)$ from A into $\bigoplus_{j=1}^{n} (A/P_j)$ is onto. And from this we obtain

2. THEOREM.

The set of primitive ideals P of A of finite codimension such that $\cap_P D$ is discontinuous, is finite.

Finally, from these theorems we deduce the main theorem

3. THEOREM.

Let A be a semisimple complete normed alternative algebra and D a derivation on A. Then D is continuous.

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