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Absolutely (∞,p) summing and Weakly-p-compact operators in C(K)

Jesus M.F.Castillo y Fernando Sánchez

Departamento de Matematicas. Universidad de Extremadura. Avda de Elvas s/n. 06071 Badajoz. España (Spain).

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In this note we review some results about:

1.Representation of Absolutely (∞,p) summing operators ($\Pi_{\infty,p}$) in C(K,E)

2.Dunford-Pettis properties.

For the basic definitions and especially those of the ideals $\Pi_{\infty,p}$ and $\mathfrak{B}_p$ please consult the preceding report.

1. Since $\Pi_{\infty,p}$ operators are intermediate between the Unconditionally summing operators $\mathfrak{U}$ ($=\Pi_{\infty,1}$) and the Completely continuous operators $\mathfrak{B}$ ($=\Pi_{\infty,\infty}$) it is possible to extend to these ideals the results obtained for $\mathfrak{U}$ and $\mathfrak{B}$ by several authors:Bombal,Cembranos,Rodriguez-Salinas,Saab,... about the relationships among the pertenence of an operator $T:C(K,E)\longrightarrow F$ to $\mathfrak{U},\mathfrak{B}$ and the pertenence of its representing measure to $\mathfrak{U},\mathfrak{B}$.

Lemma 1.Let $T:C(K,E)\longrightarrow F$ be in $\Pi_{\infty,p}$. If μ denotes its representing measure:

a) μ has semivariation continuous at $\emptyset$

b) For any Borel set $A\subseteq K$, $\mu(A):E\longrightarrow F$ is in $\Pi_{\infty,p}$.

The converse is false:[B1] shows an operator $T:C([0,1],c_0)\longrightarrow c_0$ not in $\Pi_{\infty,1}$ such that its representing measure has continuous semivariation at $\emptyset$ and for any Borel set $A\subseteq K$, $m(A):c_0\longrightarrow c_0$ is compact. Under certain assumptions the converse of 1 can be onbtained: following [BR]:

Lemma 2. Let $T:C(K,E)\longrightarrow F$ be an operator such that its representing measure μ satisfies a) and b) of Lemma 1. If μ admits a discrete control measure λ, then T is in $\Pi_{\infty,p}$.

Lemma 2. especially applies when K is a dispersed compact set. When something more can be said, for example that $id(E)\in\Pi_{\infty,p}$ then not only a) and b) implies that $T\in\Pi_{\infty,p}$, but we have in fact an equivalence. Following [S]:

Lemma 3. The following are equivalent:

a) $Id(E)\in\Pi_{\infty,p}$

b) For any compact K and any Banach space F, $T:C(K,E)\longrightarrow F$ is in $\Pi_{\infty,p}$ if and only if its representing measure μ has continuous semivariation at $\emptyset$ and for any Borel set $A\subseteq K$, $\mu(A):E\longrightarrow F$ is in $\Pi_{\infty,p}$.

2. The classical Dunford-Pettis property in a Banach space X is defined via the contention $\mathfrak{W}\subseteq\mathfrak{V}$ for operators defined on X.

Definition. We shall say that X has the Dunford-Pettisproperty of order p (in short DPP_p), $1\leqslant p\leqslant+\infty$, if $\mathfrak{W}(X,Y)\subseteq\Pi_{\infty,p}(X,Y)$ for any Banach space Y.

All Banach spaces have DPP_1, and the clasical DPP is DPP_∞. DPP_p are weaker properties than classical DPP, so that reflexive spaces may have it. For example, ℓ_r has DPP_p for $p<r^*$, and $L_r(0,1)$ has DPP_p for $p<\min(2,r^*)$.

The definition of DPP_p properties and the following characterization result appear in [C1]:

Proposition. The following are equivalent:

1.X has DPP_p, $1\leqslant p\leqslant+\infty$.

2.If (x_n) is a weakly-p-summable sequence in X and (x_n^*) is weakly null in X^*, then $\lim\langle x_n^*,x_n\rangle=0$.

3.Any weakly compact operator $T:X\longrightarrow Y$ transforms weakly-p-compact sets into compact sets.

Concerning the relationships between the DPP_p in X and in C(K,X) we have: A simple modification of Talagrand's example [T] shows that a Banach space $\mathcal{T}_p$ exists such that it has DPP but $C(K,\mathcal{T}_p)$ has not DPP_p. On the other hand:

Lemma 4. If $Id(E)\in\Pi_{\infty,p}$, then C(K,E) has DPP_p

And following [BR]:

Lemma 5. If K is a dispersed compact set, then they are equivalent: a) E has DPP_p and b) C(K,E) has DPP_p

For the general case the notion of Almost-Dunford-Pettis operator introduced in [BR] and the subsequent characterization of Bombal [B] still works. We shall say that an operator $T:C(K,E) \longrightarrow F$ is almost-$\Pi_{\infty,p}$ if it transforms sequences in C(K,E) of the form $(f_n x_n)$ (where $f_n:K \longrightarrow \mathbb{K}$ is a bounded sequence and (x_n) a weakly-p-summable sequence in E) into norm null sequences. Finally we say that C(K,E) has the almost-DPP_p if weakly compact operators $T:C(K,E) \longrightarrow F$ are almost-$\Pi_{\infty,p}$. We have:

Proposition. The following are equivalent:

a) E has the DPP_p

b) C(K,E) has the almost-DPP_p

Results for weakly-p-compact operators are being developed [F].

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