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A Non Iterative Method for the Skeleton Estimation

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In this paper we give two theorems, the first one is based in a previous theorem on thinning, it provide a method to obtain the skeleton directly in three steps. The second result insure a perfect recontructibility of image from this skeleton.

Two points in $\mathbb{Z}^2$ are said to be 8–adjacent if they are distinct and each coordinate of one differs from the corresponding of the other by at most 1, two points in $\mathbb{Z}^2$ are said to be 4–adjacent if they are 8–adjacent and differ in at most one of their coordinates. A straight line segment that joins two adjacent points p and q is called an adjacency defined by p and q.

DEFINITION 1. ([2]) A 2–dimensional binary digital picture is a quadruple $(\mathbb{Z}^2,m,n,B)$, where $B\subset\mathbb{Z}^2$ is called the set of black points, the points of $\mathbb{Z}^2-B$ are the white points, and where $(m,n)=(8,4)$, (4.8) (or $(6,6)$). In this paper B will always a finite set.

Two black points in a digital picture $\mathscr{P}=(\mathbb{Z}^2,m,n,B)$ are said to be adjacent if they are m–adjacent, and two white points or a white point and a black point are said to be adjacent if they are n–adjacent. Given a point p in a digital picture, N_p denotes the set consisting of p and those points that are 8–adjacent to p, N_p is called the 8–neighborhood of p or simply the neighborhood of p.

DEFINITION 2. ([2]) Two points $p,q\in B$ (respectively $p,q\in\mathbb{Z}^2-B$) are called connectable if there is $n\in\mathbb{N}$ and a set $\alpha=\{x_1,\dots,x_n\}\subset B$ (respectively $\{x_1,\dots,x_n\}\subset\mathbb{Z}^2-B$) such that for every i, $1\leqslant i\leqslant n$, x_i and x_{i+1} are m–adjacent (analogous n–adjacent) where $p=x_1$ and $q=x_n$. We say that α join p and q. We say that a set A, of black or white points in a digital picture is connected if: $\forall p\in A$, $A_p=A$, where $A_p=\{x\in A : x \text{ is connectable to } p\}$. The components of a set A are $A_p \cap A$, $\forall p \in A$.

A point $p \in B$ is called a border point if it is n-adjacent to some white point. The set of all border point δ is called the boundary of B.

Let $\mathscr{P} = (\mathbb{Z}^2, m, n, B)$ a digital picture and $D \subset B$, the digital picture $\mathscr{P}' = (\mathbb{Z}^2, m, n, D)$ is called obtained from $\mathscr{P}$ by deleting the points in D.

A special case of deletion on digital picture is the so called image thinning, see [1]. It is an image processing operation in which the set of black point is reduced with some requirements. The most important requirement is that deletion preserve the topology. Another condition can be expresed saying that this thinning must leave "2 by n rectangles" unchanged. The topological aspects of a digital picture are given by the connectivity, actually by the connected component of the black and white points. A black point p is called a simple point if its deletion preserves topology. There are some useful characterizations of simple points [6], [4], [1].

It is known that the simultaneous deletion of simple points do not preserve the toplogy. It is needed a sequential deletion with simple points, Ronse [5]. In order to obtain algorithms which deletes points, there are important theorems, applicable to border sequential algorithms and to border parallel algorithms, and other. We show the following theorem, which is applicable to algorithms that deletes points from more than one side in parallel.

THEOREM 1. ([2]) *Let P be a (m,n) digital picture, where $(m,n) = (4,8)$, $(8,4)$ or $(6,6)$. Let T be the subset of δ such that:*

1) $\forall p \in T$ is $N_p \cap (B - \delta) \neq \emptyset$ and connected.

2) $\forall q \in (N_p - \{p\}) \cap \delta$ if q is adjacent to p then $\exists r$, $r \in N_p \cap (B - \delta)$ and r is adjacent to q.

Then deletion of points preserves topology.

It is easy to see that all points in T are simple.

In a digital picture $\mathscr{P}$, for each black point p we define the level of p, $L(p)$, as the minimum lenght of a path α connecting p with the boundary of P. There are algorithms to obtaining the level of every point in parallel as well as in sequential way [7].

DEFINITION 3. A point $p \in B$ is called a first base point if it does not verify some of the following conditions:

1) In N_p there are at less a point q such that $L(q) > L(p)$ and the set $\{q \in N_p : L(q) > L(p)\}$ is connected.

2) For every point r with $L(r) = L(p)$, r is adjacent to the set of points q having $L(r) < L(q)$.

We call second base points those black p points which do not are first base points and they lie in a neighborhood of a first base point, and p not verify the following condition: for every point r with $L(r) \leqslant L(p)$, r is adjacent to the set of point having $L(r) < L(q)$.

DEFINITION 4. We call base points to the points that are either first or second base points.

THEOREM 2. *Deletion of points not belonging to the set of base points preserve topology. Moreover, the base points are the skeleton of digital picture* $\mathcal{P}$ (*see* [7]).

THEOREM 3. *Let* A *be the set of first base points. Let* $S = \{(p, L(p)) : p \in A\}$, *then* S *contains all the information needed to codify the digital picture* (*see* [7]).

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