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Output subsidies and quotas under uncertainty and firm heterogeneity

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RESUMEN

En este artículo se estudia la eficiencia relativa de dos clases de regulaciones: restricciones a la cantidad (cuotas) y subsidios a la producción, en un mercado de competencia imperfecta bajo la existencia de dos tipos de incertidumbre: incertidumbre en costes e incertidumbre en demanda. El resultado obtenido indica que cuando las dos fuentes de incertidumbre están independientemente distribuidas, los subsidios a la producción tienen ventaja comparativa frente a las restricciones a la cantidad. Sin embargo, si tenemos en cuenta la posibilidad de correlación entre los componentes aleatorios y entre los costes marginales de las empresas, encontramos que una correlación positiva (negativa) favorece al instrumento de cantidades (subsidios). Finalmente, mostramos que cuando la correlación es positiva, es posible encontrar situaciones en las cuales el instrumento de cantidades tiene ventaja comparativa sobre los subsidios a la producción.

Palabras clave: Incertidumbre en costes, incertidumbre en demanda, heterogeneidad de empresas, subsidios a la producción, cuotas.

ABSTRACT

This paper studies the relative efficiency of two kinds of regulations, quantity restrictions (quotas) and output subsidies, in an imperfectly competitive market under the existence of two sources of uncertainty: uncertainty in both costs and prices. We find that when the two sources of uncertainty are independently distributed, the output subsidy instrument has comparative advantage over the quantity instrument. However, when we take into account the possibility of correlation between the random components and across firms marginal costs, we find that a positive (negative) correlation tends to favor the quantity (subsidy) instrument. Finally, we show that when the correlation is positive, it is possible to find situations in which the quantity instrument has comparative advantage over the subsidy instrument.

Keywords: Cost uncertainty, demand uncertainty, firm heterogeneity, output subsidy and quantity instruments.
JEL classification: D8, L51

1 Introduction

In this paper we study how to drive the Cournot equilibrium allocations to the optimal ones in an imperfectly competitive market under the presence of firm heterogeneity and uncertainty. We will consider two types of instruments: output subsidies and quantity restrictions (quotas). The question we want to solve is which type of instrument should be used when there are two sources of uncertainty: uncertainty in marginal costs and uncertainty in prices. Under certainty, the two policies, output subsidies and quantity restrictions, yield the same result[1]. However, this is not always the case given the existence of uncertainty in prices (imperfect information about future demand) and/or uncertainty in marginal costs. Also, because of firm heterogeneity, marginal costs may vary across firms. This may create a situation in which output subsidies policy are more efficient for some firms and quantity restrictions are more efficient for others. Finally, when both sources of uncertainty are present, it is necessary to ask whether there is some degree of statistical dependence between them. Additionally, given firm heterogeneity some degree of statistical dependence across firms marginal costs is possible.

Since the seminal work of Weitzman (1974), much research has focused on the role of different policy instruments in a context of uncertainty. In this sense, Weitzman (1974) derives a condition for the relative efficiency between tax and quantities under uncertainty, when firms are price-takers, that is, in the case of non-strategic interaction between firms.

Following Weitzman's (1974) model, several researches have analyzed the design of regulatory policies in the presence of uncertainty in the pollution control literature. This literature mainly studies the use of two instruments: prices and quantities. The question of interest is which type of instrument should be used. Stavins (1996) extends Weitzman's model considering correlation between benefit and cost uncertainty. He

[1]Note that this is true if we assume that the private cost and the social cost of subsidies is the same.

obtains that the correlation effect is likely to overwhelm the usual result that benefit uncertainty is irrelevant for choosing between price and quantity instruments, but that cost uncertainty matters, with the identity of the efficient instrument depending upon the relative slopes of the marginal benefit and cost functions. Choi and Johnson (1987), in a model with price uncertainty, showed the ex ante equivalent variation and the expected equivalent variation are equal for income risk neutral consumers and expected equivalent variation provides a lower bound for ex ante equivalent variation when income risk aversion is presumed. On the other hand, Wu (2000) extends the analysis to the case in which the planner does not know the firms type and considering the existence of input substitution. Blair, Lewis and Sappington (1995) introduce uncertainty in the demand function, showing that the imposition of minimum sales levels and maximum consumption levels can provide significant welfare gains relative to the case where the regulator can dictate only a single price for a single quantity of the regulated product. More recently, Hoel and Karp (2001) have analyzed the case of multiplicative uncertainty.

The effects of uncertainty have also been widely studied in the context of international strategic trade policy. The third-market model developed by Brander and Spencer (1985) have been extended in several directions to account for uncertainty in the demand function. Cooper and Riezman (1989) expand the instrument set considered by Brander and Spencer to include quantity controls and allow for a multiple number of firms in each country. Cooper and Riezman (1989) consider a model in which the countries choose the type of policy (an export subsidy or a strict quantity control) in the first stage and a level for policy in the second stage. The random intercept of demand is then revealed, hereafter firms in each country compete. With small noise, countries choose quantity controls because each country is able to immunize its firms from the profit-shifting policies of rivals. However, with large noise, countries choose export subsidies because firms are given the flexibility to respond. Arvan (1991) considers a similar model and obtains that the country with the relative small number of firms act like a Stackelberg leader while the country with the relatively large number of firms acts like a Stackelberg follower. Hwang and Schulman (1993) extend the previous analyses considering non-intervention as another policy instrument. They show that the subsidy policy is the best response if the two countries have the same number of firms.

In this paper we extend the previous analysis in several directions. First, we consider an imperfectly market with uncertainty in both, marginal costs and prices. Second, we consider that marginal costs are not identical across firms, i.e., firm heterogeneity. Finally, we take into account that the random components of demand and marginal costs may be not independently distributed.

In our model, we assume that the planner does not have firm-level information to implement differential output subsidies or quantity restrictions. In this respect, we compute the social surplus for each instrument, then the comparative advantage of subsidies over quantities is defined as the difference of the expected social surplus. We also assume the existence of a limited degree of uncertainty in order to justify the use of a second order approximation.

The comparative advantage of one instrument over the other depends on the number of firms, the market size, the firms heterogeneity, the degree of demand and cost uncertainty and the correlations between both sources of uncertainty and across firms. Assuming no correlation between perturbations, we obtain that the output subsidy instrument has always comparative advantage over the quantity instrument. The intuition behind this result is the following. In the case of the quantity instrument, the government selects the level of output for each firm and then firms are required to produce this level of output regardless of the state of nature. Therefore, with the quantity instrument, firms have no flexibility in choosing their output. However, in the case of output subsidies, firms can choose their output, given the subsidy and the reaction of the other firms. The subsidy instrument allows firms to adjust their output decisions to any shocks of the demand, taking the subsidy level as given. This asymmetry will lead to a relative advantage of the subsidy instrument over the quantity instrument.

To proceed further, we consider the possibility of correlation between the random components. When both sources of uncertainty are present, some degree of statistical dependence between them may be possible. It can also be possible to find some degree of statistical dependence across firms marginal costs. By incorporating in the analysis the correlation effects, we obtain that a positive (negative) correlation tends to favor the quantity (subsidy) instrument. In fact, it is possible to find situations in which, in the case of a positive correlation, the quantity instrument has comparative advantage over the subsidy instrument. The explanation of this result is as follows: When the correlation is positive, the marginal costs of the firms are lower or higher than the average marginal cost of the industry estimated by the planner. In this case the subsidy instrument will lead to situations of underproduction or overproduction. If the marginal costs of the firms are lower than the estimated average marginal cost, the subsidy instrument will provoke that total output will be higher than the expected optimal one. On the other hand, if the marginal costs of the firms are higher than the estimated average marginal cost, then, the subsidy instrument will lead to a total output lower than the expected optimal one. This will reduce the relative advantage of the subsidy instrument versus the quantity instrument.

The rest of the paper is organized as follows. Section 2 presents the model. In Section 3 we study the comparative advantage of output subsidies over quantity restrictions. We finish with our main conclusions, gathered in Section 4.

2 The model

We concentrate on a quantity-setter model with n firms, which are distinguished by their type (a set of characteristics) θ^i, $i = 1, .., n$. It is assumed that although individual firms know their own characteristics, the planner does not. The planner views each θ^i as a random variable with strictly positive density on the interval $\left[\underline{\theta}, \overline{\theta}\right]$.

There is an homogenous good produced by firms where p is the market price of the good and x is the total quantity produced. The consumer surplus is given by

$$U^0(x,\eta) = V(x,\eta) - px \tag{1}$$

where $V(x,\eta)$ is a strictly increasing function of x and η is a random variable that can be observed by the firms, but not by the planner. As in the case of the firms' type, the planner views η as a random variable with strictly positive density on the interval $\left[\underline{\eta}, \overline{\eta}\right]$.

Firm $i = 1,..,n$ has a cost function denoted by $C(x^i, \theta^i)$ such that $C(0, \theta^i) = 0$ where x^i is the output of firm i and θ^i her type. Let firms i's payoff function be

$$U^i(x^i, x, \theta^i, \eta) = V_1(x,\eta)x^i - C\left(x^i, \theta^i\right) \tag{2}$$

where $V_1(x,\eta)$ is the derivative of $V(x,\eta)$ representing the inverse demand function mapping aggregate output into prices and x is total output.

It is easy to show that the resulting market equilibrium is not optimal. Under homogeneous product, firms underproduce in relation to the optimum and aggregate output is less than the optimal one. In this context, a policy that increases output is generally welfare enhancing. We focus on output subsidies and quantity restrictions policies because these are two simple messages, traditionally employed and frequently contrasted. In a context of perfect certainty there is a formal identity between the use of subsidies and the use of quantities as planning instruments. However, under uncertainty, both instruments have different effects. In the case of the quantity instrument, the government selects the level of output for each firm. Firms are required to produce this level of output regardless of the state of nature. Therefore, with the quantity instrument, firms have no flexibility in choosing their output. In the case of output subsidies, firms can choose their output considering the subsidy and the reaction of the other firms. The subsidy instrument allows firms to adjust their output decisions to any shocks of the demand, taking the subsidy level as given. Because of the asymmetry of information between firms and governments, there is a trade-off between the quantity instrument (which prevents firms from adjusting to demand shocks) and the output subsidy instrument (which allows adjustment).

We assume that the planner knows the distribution function of η and the distribution of θ across firms and can use this information in policy formulation. The planner has to choose the instruments before observing the random components. We also make the following standard assumption in order to ensure the existence of an interior solution.

A.1. For all $i = 1, .., n$ and for all values of θ and η, we have that $V_{11}(x, \eta) + V_{111}(x, \eta)x^i < 0$ and $V_{11}(x, \eta) - C_{11}\left(x^i, \theta^i\right) < 0$.

Since firms are not necessarily identical, marginal costs may vary across firms. However, the planner may not have firm-level information to implement differential output subsidies or differential quantity restrictions. Furthermore, it may also be politically infeasible and technically difficult to apply differential regulations to firms. Therefore, the problem to know the exact value of θ^i, which can be different across firms, will provoke that the level of output subsidies and quantity restrictions to be the same for all firms. We turn next to examine the firms' and the planner's decisions under such uniform instruments.

2.1 Quantities

The optimal quantity instrument under uncertainty and firm heterogeneity is those target outputs, $\widehat{x}^i$, which maximizes expected total surplus, so that

$$Max_{\{\widehat{x}^i\}}\ E\left[V(\widehat{x}, \eta) - \sum_{i=1}^{n} C\left(\widehat{x}^i, \theta^i\right)\right]$$

where $E[.]$ is the expected value operator. The solution $\widehat{x}^i$ must satisfy the following first order conditions

$$E[V_1(\widehat{x}, \eta)] = E\left[C_1\left(\widehat{x}^i, \theta^i\right)\right] \quad \forall i = 1, ..n \tag{3}$$

This implies that the quantity should be set at the level where the mean price equals the mean marginal cost.

2.2 Output subsidy

When a subsidy instrument s is announced, each firm will choose the output to maximize profits

$$Max_{\{x^i\}}\ V_1(x,\eta)x^i - C\left(x^i,\theta^i\right) + sx^i$$

implying

$$V_1(x,\eta) + V_{11}(x,\eta)x^i + s = C_1\left(x^i,\theta^i\right) \tag{4}$$

The planner will choose the level of subsidies s which maximizes the expected total surplus given the reaction functions $x^i(s,\eta,\theta^1,..,\theta^n)$:

$$Max_{\{s\}}E\left[V(x(s,\eta,\theta^1,..,\theta^n),\eta) - \textstyle\sum_{i=1}^n C(x^i(s,\eta,\theta^1,..,\theta^n),\theta^i)\right]$$

The solution must satisfy the first order condition

$$E\left[V_1(x(s,\eta,\theta^1,..,\theta^n),\eta)x_1(s,\eta,\theta^1,..,\theta^n)\right] =$$

$$E\left[\sum_{i=1}^n C_1\left(x^i(s,\eta,\theta^1,..,\theta^n),\theta^i\right)x_1^i(s,\eta,\theta^1,..,\theta^n)\right]$$

where x_1 is defined as $\sum_i \frac{\partial x^i}{\partial s}$ and x_1^i is defined to be $\frac{\partial x^i}{\partial s}$. The above expression can be rewritten as

$$s = -\frac{E\left[V_{11}(x(s,\eta,\theta^1,..,\theta^n),\eta)(\sum_{i=1}^n x^i(s,\eta,\theta^1,..,\theta^n)x_1^i(s,\eta,\theta^1,..,\theta^n)\right]}{E\left[x_1(s,\eta,\theta^1,..,\theta^n)\right]} \tag{5}$$

Corresponding to the optimal ex ante subsidy $\widetilde{s}$ is the ex post profit maximizing outputs $(\widetilde{x}^1,..,\widetilde{x}^n)$ expressed as a function of $(\eta,\theta^1,..,\theta^n)$

$$\widetilde{x}^i(\eta,\theta^1,...,\theta^n) = x^i(\widetilde{s},\eta,\theta^1,...,\theta^n) \tag{6}$$

where $\widetilde{x}(\eta,\theta^1,..,\theta^n) = \sum_{i=1}^n \widetilde{x}^i(\eta,\theta^1,..,\theta^n)$ and $\widetilde{x}^{-i}(\eta,\theta^1,..,\theta^n) = \sum_{j\neq i}\widetilde{x}^j(\eta,\theta^1,..,\theta^n)$.

Following Weitzman (1974) we assume that the amount of uncertainty with respect to the cost functions and the inverse demand function is taken as sufficiently small to justify a second order approximation of $C\left(x^i,\theta^i\right)$ and $V(x,\eta)$ within the small range of $\widetilde{x}(\eta,\theta^1,..,\theta^n)$ as it varies around $\widehat{x}$. Let the symbol "$\cong$" denote an "accurate local approximation" within an appropriate neighborhood of $x=\widehat{x}$:

$$C\left(x^i,\theta^i\right) \cong C(\widehat{x}^i,\theta^i) + (C' + \alpha(\theta^i))(x^i - \widehat{x}^i) + \frac{C''}{2}(x^i - \widehat{x}^i)^2 \tag{7}$$

$$V\left(x,\eta\right) \cong V(\widehat{x},\eta) + (V' + \upsilon(\eta))(x - \widehat{x}) + \frac{V''}{2}(x - \widehat{x})^2 \tag{8}$$

In the above equations, $V(\widehat{x},\eta)$, $C(\widehat{x}^i,\theta^i)$, $\alpha(\theta^i)$ and $\upsilon(\eta)$ are stochastic functions and V', V'', C' and C'' are fixed coefficients. $\alpha(\theta^i)$ is a pure unbiased shift of the marginal cost function whereas $\upsilon(\eta)$ is a shift of the inverse demand function. Note that Assumption 1 means that $V''<0$ and $V''-C''<0$. Without loss of generality, we also make the following assumption:

A.2. $E[\alpha(\theta^i)] = E[\upsilon(\eta)] = 0$ for $i = 1,..,n$.

Differentiating (7) and (8) with respect to x^i and x, yields

$$C_1\left(x^i,\theta^i\right) \cong (C' + \alpha(\theta^i)) + C''(x^i - \widehat{x}^i) \tag{9}$$

$$V_1\left(x,\eta\right) \cong (V' + \upsilon(\eta)) + V''(x - \widehat{x}) \tag{10}$$

and applying the expected value operator we obtain the following expressions for the fixed coefficients of (7) and (8),

$$E[C_1\left(\widehat{x}^i,\theta^i\right)] \cong C'$$

$$E[V_1\left(\widehat{x},\eta\right)] \cong V'$$

$$C_{11}\left(x^i,\theta^i\right)\cong C''$$

$$V_{11}\left(x,\eta\right)\cong V''$$

and from (3) we obtain that

$$V'=C' \tag{11}$$

From (4), (7) and (8) we have that for $i=1,..,n$

$$V'+\upsilon(\eta)+V''(x(\widetilde{s},\eta,\theta^1,..,\theta^n)-\widehat{x})+V''x^i(\widetilde{s},\eta,\theta^1,..,\theta^n)+\widetilde{s}\cong$$

$$C'+\alpha(\theta^i)+C''(x^i(\widetilde{s},\eta,\theta^1,..,\theta^n)-\widehat{x}^i)$$

that can be written as

$$x^i(\widetilde{s},\eta,\theta^1,..,\theta^n)\cong\frac{C'+\alpha(\theta^i)+(V''-C'')\widehat{x}^i-\widetilde{s}-V'-\upsilon(\eta)-V''(x^{-i}(\widetilde{s},\eta,\theta^1,..,\theta^n)-\widehat{x}^{-i})}{2V''-C''} \tag{12}$$

and solving the above system of equations

$$x^i(\widetilde{s},\eta,\theta^1,..,\theta^n)\cong\frac{C'-\widetilde{s}-V'-\upsilon(\eta)}{(n+1)V''-C''}+$$

$$\frac{\alpha(\theta^i)(nV''-C'')-V''\sum_{j\neq i}\alpha(\theta^j)+(V'')^2\widehat{x}^{-i}+[(V'')^2-(n+1)V''C''+(C'')^2]\widehat{x}^i}{((n+1)V''-C'')(V''-C'')} \tag{13}$$

and

$$x(\widetilde{s},\eta,\theta^1,..,\theta^n)\cong\frac{n(C'-\widetilde{s}-V'-\upsilon(\eta))+\sum_{i=1}^n\alpha(\theta^i)+(nV''-C'')\widehat{x}}{(n+1)V''-C''} \tag{14}$$

implying

$$x_1(\widetilde{s},\eta,\theta^1,..,\theta^n)\cong-\frac{n}{(n+1)V''-C''} \tag{15}$$

and

$$x_1^i(\tilde{s},\eta,\theta^1,..,\theta^n) \cong -\frac{1}{(n+1)V''-C''} \tag{16}$$

Substituting from (15) and (16) into (5) and cancelling out $((n+1)V''-C'')$ yields

$$\tilde{s} \cong \frac{1}{n}E[V_{11}(x(\tilde{s},\eta,\theta^1,..,\theta^n))x(\tilde{s},\eta,\theta^1,..,\theta^n)] \tag{17}$$

Next, we obtain the expression for the optimal ex ante level of output subsidy. Replacing x in (8) by the expression for $x(\tilde{s},\eta,\theta^1,..,\theta^n)$ from (11) and (14) and plugging into (17), the following equation is obtained after using A.2,

$$\tilde{s} \cong -\frac{V''\hat{x}}{n} \tag{18}$$

Combining (6), (14) and (18), the ex ante total subsidy output is,

$$\tilde{x} \cong \hat{x} + \frac{\sum_{i=1}^n \alpha(\theta^i) - n\upsilon(\eta)}{(n+1)V''-C''} \tag{19}$$

and combining (6), (13) and (19), the ex ante subsidy output of firm i is:

$$\tilde{x}^i \cong \frac{\hat{x}}{n} + \frac{\alpha(\theta^i)(nV''-C'') - V''\sum_{j\neq i}\alpha(\theta^j) - \upsilon(\eta)(V''-C'')}{((n+1)V''-C'')(V''-C'')} \tag{20}$$

3 Output subsidies versus quantities

Next, we compare the social welfare under the above two instruments: output subsidies and quantities, that is, we compare their relative efficiency in the presence of uncertainty and firm heterogeneity. Following Weitzman (1974), we define the comparative advantage of subsidies over quantities as

$$\Delta \equiv E\left[(V(\tilde{x}(\eta,\theta^1,..,\theta^n),\eta) - \sum_{i=1}^n C\left(\tilde{x}^i(\eta,\theta^1,..,\theta^n),\theta^i\right) - (V(\hat{x},\eta) + \sum_{i=1}^n C\left(\hat{x}^i,\theta^i\right)\right] \tag{21}$$

namely, as the expected net difference in gains obtained under the two instruments. If the above expression is positive, the output subsidies instrument has a comparative advantage over the quantities instrument and vice versa. Alternatively, substituting $x = \widehat{x}$, $x^i = \widehat{x}^i$ and $x = \widetilde{x}(\eta, \theta^1, .., \theta^n)$, $x^i = \widetilde{x}^i(\eta, \theta^1, .., \theta^n)$ from (20) into (7) and (8) and plugging the resulting values into (21) using A.2, and collecting terms, the comparative advantage of output subsidies over quantities in the presence of uncertainty and firm heterogeneity can be written as follows (see the Appendix):

$$\Delta \cong \left[(a_2 - a_1)\sigma_\theta^2 + a_2 n \sigma_\eta^2 + a_1 n \mu_\theta - 2a_2 \mu_{\theta,\eta}\right] \tag{22}$$

where $a_1$and a_2 are functions of V'', C'' and the number of firms,

$$a_1 = \frac{n(n-1)\left[2n + 3 - (n+4)\frac{C''}{V''} + (\frac{C''}{V''})^2\right]}{2V''((n+1) - \frac{C''}{V''})^2(1 - \frac{C''}{V''})^2} \tag{23}$$

$$a_2 = -(\frac{n[(n+2) - \frac{C''}{V''}]}{2(n+1 - \frac{C''}{V''})^2}) \tag{24}$$

σ_θ^2 is the mean square error in marginal costs

$$\sigma_\theta^2 \equiv E\left[(C_1(x^i, \theta^i) - E[C_1(x^i, \theta^i)])^2\right] \cong E[(\alpha(\theta^i))^2] \text{ for } i = 1, ..., n$$

σ_η^2 is the mean square error in prices

$$\sigma_\eta^2 \equiv E\left[(V_1(x, \eta) - E[V_1(x, \eta)])^2\right] \cong E[(\upsilon(\eta))^2]$$

$\mu_\theta = E[\alpha(\theta^i)\alpha(\theta^j)]$, $\forall i, j = 1, ..., n, i \neq j$ is the covariance between marginal costs due to firm heterogeneity. The coefficient of correlation between marginal costs across firms, $\rho_\theta = \mu_\theta / \sigma_\theta^2$, can be positive or negative. Firms heterogeneity can be explained by differences in production technologies. For example, a positive correlation can be generated by a general improvement in technology or a shift in the price of a factor (or a tax paid) by all the firms. A negative correlation can be generated by an improvement in the technology used by one firm that decreases his marginal costs and increases the marginal costs of his competitors. For example, suppose two fishermen, $i = 1, 2$, that catch fish from a common sea. Each firm i's cost function is linear and is given by cx^i. Let us suppose that the per-unit cost of production of firm 1 decreases due to an improvement in technology, $c^1 < c$. It is not unreasonable to assume that $c < c^2$, since the potential catches of firm 2 will usually be an activity in a rush, spurred by firm 1's activity.

$\mu_{\eta,\theta} = E[\upsilon(\eta)\alpha(\theta^i)]$ is the covariance between marginal costs uncertainty and price uncertainty. As we have already pointed out, the coefficient of correlation between marginal costs and price uncertainty, $\rho_{\eta,\theta} = \mu_{\eta,\theta}/\sigma_\theta\sigma_\eta$, can be positive or negative. Substitution effects can generate a negative correlation. This way, an increase in marginal costs in the production of some good, can provoke in the consumer a substitution effect of this good for other similar goods. Therefore, an increase in marginal costs can generate a shift to the left of the inverse demand function and hence a negative correlation. Investments in cleaner technologies, for instance, can generate a shift to the right of the inverse demand functions, representing a positive correlation. It is becoming more demanded by consumers those goods produced by means of a technology respectful with the environment.

From A.1, (7) and (8), we have that $V'' < 0$ and $V'' - C'' < 0$. Therefore, parameter a_2 is always positive whereas the parameter a_1 is always negative.

First, we study the case when there is no correlation effects, that is, $\mu_\theta = \mu_{\eta,\theta} = 0$. In this case we obtain that the output subsidy instrument has comparative advantage over the quantity instrument. The intuition of the above result can be found in the different reaction of firms. Given the uncertainty about the cost function, the planner has to use a uniform level for the instruments, in spite of firm heterogeneity. The quantities instrument does not permit the firms to adjust its output to the rival firms' reaction. However, this will be possible with the use of output subsidies. With this second instrument, the firms have flexibility to respond to the level of the subsidy and to the reaction of the rival firms. In this context, a small miscalculation of the quantity results in a larger deviation from the optimal outcome than with a small miscalculation of the subsidy. Consequently, the uniform output subsidies are the best instrument to maximize expected social welfare in a context of uncertainty. In other words, the error in which incurs the planner is larger with the quantity instrument than with the subsidy instrument. In both cases, the final output is the expected optimal one, but the distribution of the output across the firms is different. Whereas in the first case all firms produce the same quantity, in the second case, the more efficient firms produce more than the less efficient firms.

Next, we consider the possibility of correlation between the random components and across firms marginal costs. Inspection of expression (22) reveals that a positive (negative) correlation favors the quantity (subsidy) instrument. Nonetheless, the question now is whether the correlation effect is really likely to reverse the instrument choice, that is, under what condition a positive correlation makes the quantity instrument to have comparative advantage over the subsidy instrument. We explore the most advantageous case for the quantity instrument, that is, when $\mu_\theta = \sigma_\theta^2$ and $\mu_{\eta,\theta} = \sigma_\theta \sigma_\eta$ (i.e. $\rho_\theta = \rho_{\theta,\eta} = 1$). In this case, expression (22) can be written as,

$$\Delta \cong a_2 \sigma_\theta^2 \left[(n-1)\frac{a_1}{a_2} + 1 + n\left(\frac{\sigma_\eta}{\sigma_\theta}\right)^2 - 2\left(\frac{\sigma_\eta}{\sigma_\theta}\right) \right] \tag{25}$$

Figure 1 plots the simulation of expression (25) for a fixed number of firms, in terms of the ratios C''/V'' and $\sigma_\eta/\sigma_\theta$. We represent the locus $\Delta = 0$. In the region above that locus, $\Delta < 0$, the quantity instrument has relative advantage over the subsidy instrument. In the region below that locus, $\Delta > 0$, that is, the subsidy instrument has relative advantage over the quantity instrument. As we increases the number of firms, the locus $\Delta = 0$ shifts to the right. The relative advantage of subsidies over quantities is increasing in the ratio $\sigma_\eta/\sigma_\theta$, and decreasing in the ratio C''/V'' and the number of firms. Therefore, it is possible find situations in which the quantity instrument has comparative advantage over the subsidy instrument when correlation among firms cost is positive.

The intuition why positive correlation favors the quantity instrument is the following. When the correlation is positive, the marginal costs of the firms are below or upper the marginal cost of the industry estimated by the planner. This will provokes that the level of the subsidy will be higher or lower than the optimal subsidy level, which increases the error in which incurs the planner with the subsidy instrument given that the reaction of the firms will lead to an output level different than the expected optimal one. In fact, if the marginal costs of the firms are below the average marginal cost of the industry estimated by the planner, the firms will overproduce. In this case, the total output will be larger than the expected optimal one, due to an overestimation of the optimal subsidy level. On the contrary, if the marginal costs of the firms are larger than the average cost of the industry estimated by the planner, this will leads to an underestimation of the optimal subsidy level and the, to an underproduction situation.[2]

4 Conclusions

In this paper we have analyzed the effects of the presence of simultaneous uncertainty in both cost functions and demand function and firm heterogeneity over the application of two instruments: output subsidies and quantities.

In general, we obtain that, under only cost uncertainty or under only demand uncertainty or both sources of uncertainty when they are independently distributed, the output subsidy instrument has a comparative advantage over the quantity instrument. The explanation for this result is that the quantity instrument does not allow the firms to adjust their output to the rival firms' reaction, whereas with the subsidy instrument,

[2] A simple numerical example can easily illustrates this fact. Suppose that the inverse demand function is the following: $p(x) = 50 - x$, and that the average marginal cost of the industry estimated by the planner is $\overline{\theta} = 20$. Therefore, the expected optimal output is $\widehat{x} = 50 - \overline{\theta} = 30$. Considering the existence of two firms (named 1 and 2), the output of each firm under the quantity instrument will be $\widehat{x}_1 = \widehat{x}_2 = 15$. Suppose now that the correlation of costs is positive and this leads to a marginal cost of each firm of $\theta_1 = 30$ and $\theta_2 = 25$, that is, the marginal costs of both firms are larger than the estimated average cost. Using expression (20) in the text, the output produced by each firm under the subsidy instrument will be $\widetilde{x}_1 = 15 - 5/3$ and $\widetilde{x}_2 = 15$, that is, a total output lower than the expected optimal one.

the firms have flexibility to respond to the level of the subsidy and to the reaction of the rival firms. In this context, a small miscalculation of the quantity results in a larger deviation from the optimal than with a small miscalculation of the subsidy.

However, when we allow for correlation due to both, uncertainty and heterogeneity, this correlation effect is likely to reverse the instrument choice. We obtain that a positive (negative) correlation tends to favor the quantity (subsidy) instrument. In particular, it is possible to find situations in which the quantity instrument has comparative advantage over the subsidy instrument. Therefore, this analysis shows that in identifying the efficient policy instrument, we have to pay attention to the correlation effects.

5 Appendix

The expected difference in gains under the two instruments is given by

$$\Delta \equiv E\left[V(\widetilde{x}(\eta,\theta^1,..,\theta^n),\eta) - V(\widehat{x},\eta)\right] - \sum_{i=1}^{n} E\left[C\left(\widetilde{x}^i(\eta,\theta^1,..,\theta^n),\theta^i\right) - C\left(\widehat{x}^i,\theta^i\right)\right]$$

We now analyze each of the above two terms.

(i) $E\left[V(\widetilde{x}(\eta,\theta^1,..,\theta^n),\eta) - V(\widehat{x},\eta)\right]$.

From (8) we get

$$E\left[V(\widetilde{x}(\eta,\theta^1,..,\theta^n),\eta) - V(\widehat{x},\eta)\right] = E\left[V'(\widetilde{x}-\widehat{x}) + \frac{V''}{2}(\widetilde{x}-\widehat{x})^2\right]$$

and plugging (20) in the two terms of the above equation and using A.2 we obtain

$$E\left[(V' + \upsilon(\eta))(\widetilde{x}-\widehat{x})\right] = \frac{-n\sigma_\eta^2 + n\rho_{\eta,\theta}}{(n+1)V'' - C''}$$

and

$$E\left[\frac{V''}{2}(\widetilde{x}-\widehat{x})^2\right] = \frac{V''}{2((n+1)V''-C'')^2}(n\sigma_\theta^2 + n^2\sigma_\eta^2 + n(n-1)\rho_\theta - 2n^2\rho_{\eta,\theta})$$

(ii) $E\left[C\left(\widetilde{x}^i(\eta,\theta^1,..,\theta^n),\theta^i\right) - C\left(\widehat{x}^i,\theta^i\right)\right]$.

From (7) we get

$$E\left[C\left(\widetilde{x}^i(\eta,\theta^i,\theta^j),\theta^i\right)-C\left(\widehat{x}^i,\theta^i\right)\right]=E\left[(C'+\alpha(\theta^i))(\widetilde{x}^i-\widehat{x}^i)+\tfrac{C''}{2}(\widetilde{x}^i-\widehat{x}^i)^2\right]$$

and plugging (20) in the two terms of the above equation and using A.2 we obtain

$$\sum_{i=1}^{n}E\left[(C'+\alpha(\theta^i))(\widetilde{x}^i-\widehat{x}^i)\right]=$$

$$\frac{n}{((n+1)V''-C'')(V''-C'')}\left[(nV''-C'')\sigma_\theta^2-(n-1)V''\rho_\theta-(V''-C'')\rho_{\eta,\theta}\right]$$

and

$$\sum_{i=1}^{n}E\left[\frac{C''}{2}(\widetilde{x}^i-\widehat{x}^i)^2\right]=$$

$$\frac{nC''((n^2+n-1)(V'')^2-2nV''C''+(C'')^2)}{2((n+1)V''-C'')^2(V''-C'')^2}\sigma_\theta^2+\frac{nC''}{2((n+1)V''-C'')^2}\sigma_\eta^2$$

$$-\frac{(V'')^2(n-1)(n-2)-2(n-1)(nV''-C'')V''}{2((n+1)V''-C'')^2(V''-C'')^2}\rho_\theta-\frac{nC''}{((n+1)V''-C'')^2}\rho_{\eta,\theta}$$

Finally, collecting terms, we obtain the following expression,

$$\Delta\cong n(a_1\mu_\theta+a_2\sigma_\eta^2+a_3\sigma_\theta^2+a_4\mu_{\eta,\theta})$$

where a_1, a_2, a_3, and a_4 are functions of V'', C'' and the number of firms

$$a_1=\frac{n(n-1)V''\left[(2n+3)(V'')^2-(n+4)V''C''+(C'')^2\right]}{2((n+1)V''-C'')^2(V''-C'')^2}$$

$$a_2=-\left(\frac{n[(n+2)V''-C'']}{2((n+1)V''-C'')^2}\right)$$

$$a_3=-\left(\frac{(2n^2+2n-1)(V'')^3-(n^2+5n+1)(V'')^2C''+(3+2n)V''(C'')^2-(C'')^3}{2((n+1)V''-C'')^2(V''-C'')^2}\right)$$

$$a_4=\frac{(n+1)V''+(V''-C'')}{((n+1)V''-C'')^2}$$

Given that $a_3=\frac{a_2-a_1}{n}$ and $a_2=-\frac{n}{2}a_4$, we obtain expression (22) in the text.

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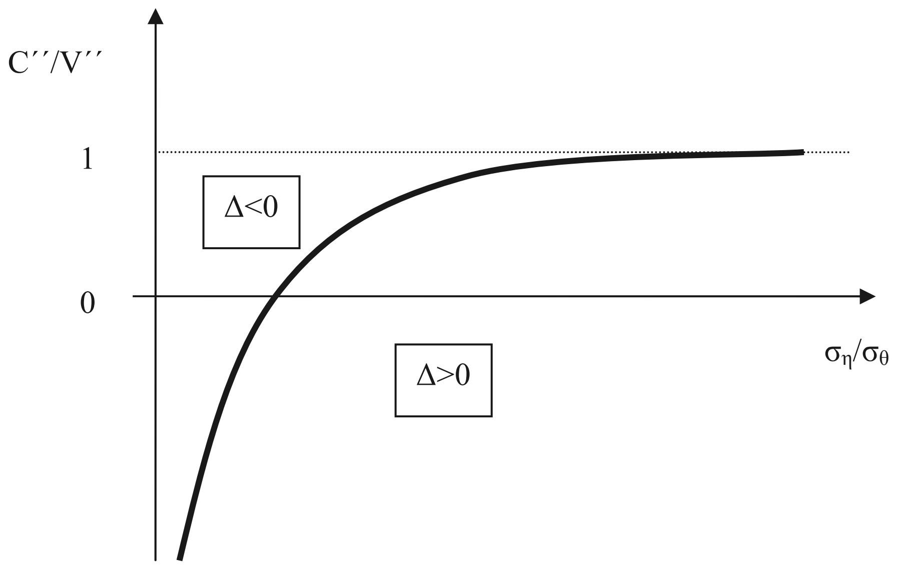

Figure 1: Relative advantage of the subsidy instrument over the quantity instrument