

Banco Central de Chile  
Documentos de Trabajo

Central Bank of Chile  
Working Papers

N° 341

Diciembre 2005

## **LARGE TERM STRUCTURE MOVEMENTS IN A FACTOR MODEL FRAMEWORK**

Marcelo Reyes

---

La serie de Documentos de Trabajo en versión PDF puede obtenerse gratis en la dirección electrónica: <http://www.bcentral.cl/esp/estpub/estudios/dtbc>. Existe la posibilidad de solicitar una copia impresa con un costo de \$500 si es dentro de Chile y US\$12 si es para fuera de Chile. Las solicitudes se pueden hacer por fax: (56-2) 6702231 o a través de correo electrónico: [bcch@bcentral.cl](mailto:bcch@bcentral.cl).

Working Papers in PDF format can be downloaded free of charge from: <http://www.bcentral.cl/eng/stdpub/studies/workingpaper>. Printed versions can be ordered individually for US\$12 per copy (for orders inside Chile the charge is Ch\$500.) Orders can be placed by fax: (56-2) 6702231 or e-mail: [bcch@bcentral.cl](mailto:bcch@bcentral.cl).



**BANCO CENTRAL DE CHILE**

**CENTRAL BANK OF CHILE**

La serie Documentos de Trabajo es una publicación del Banco Central de Chile que divulga los trabajos de investigación económica realizados por profesionales de esta institución o encargados por ella a terceros. El objetivo de la serie es aportar al debate temas relevantes y presentar nuevos enfoques en el análisis de los mismos. La difusión de los Documentos de Trabajo sólo intenta facilitar el intercambio de ideas y dar a conocer investigaciones, con carácter preliminar, para su discusión y comentarios.

La publicación de los Documentos de Trabajo no está sujeta a la aprobación previa de los miembros del Consejo del Banco Central de Chile. Tanto el contenido de los Documentos de Trabajo como también los análisis y conclusiones que de ellos se deriven, son de exclusiva responsabilidad de su o sus autores y no reflejan necesariamente la opinión del Banco Central de Chile o de sus Consejeros.

The Working Papers series of the Central Bank of Chile disseminates economic research conducted by Central Bank staff or third parties under the sponsorship of the Bank. The purpose of the series is to contribute to the discussion of relevant issues and develop new analytical or empirical approaches in their analyses. The only aim of the Working Papers is to disseminate preliminary research for its discussion and comments.

Publication of Working Papers is not subject to previous approval by the members of the Board of the Central Bank. The views and conclusions presented in the papers are exclusively those of the author(s) and do not necessarily reflect the position of the Central Bank of Chile or of the Board members.

Documentos de Trabajo del Banco Central de Chile  
Working Papers of the Central Bank of Chile  
Agustinas 1180  
Teléfono: (56-2) 6702475; Fax: (56-2) 6702231

## **LARGE TERM STRUCTURE MOVEMENTS IN A FACTOR MODEL FRAMEWORK**

Marcelo Reyes

Gerencia de Inversiones Internacionales  
Banco Central de Chile

### **Resumen**

Este trabajo analiza los cambios en la estructura de tasas norteamericana utilizando modelos de factores basados en el análisis de componentes principales y el modelo de Diebold y Li. El análisis de las series de factores sugiere que no es posible rechazar la hipótesis de normalidad en los cambios de los primeros dos factores que representan el efecto nivel y pendiente. Esto posibilita asumir que los factores siguen procesos de Ornstein-Uhlenbeck correlacionados, y luego construir elipses de confianza al 95% permitiéndonos identificar movimientos importantes en la curva los cuales son interpretados como movimientos no anticipados por los agentes en el mercado. Los resultados sugieren la importancia de los comunicados de la autoridad monetaria respecto al escenario económico esperado, y la capacidad por parte de los agentes para anticipar las acciones de la reserva federal durante el período estudiado 1997:01 2005:04

### **Abstract**

This paper analyzes US Term Structure changes using linear factor models based on principal components analysis and the model of Diebold and Li. The analysis of factors time series could not reject the hypothesis of normality for changes in the first two factors that accounts for level and slope effects. This enables the assumption that factors follow correlated Ornstein-Uhlenbeck processes, and then construct 95% confidence ellipses that allow us to identify large movements that are interpreted as unanticipated by market participants. The results suggest the importance of the economic assessment released by the monetary authority, and the ability of agents to anticipate Fed's actions over the sample period 1997:01 2005:04.

---

I thank Alejandro Corvalan, Leonardo Luna, Alvaro Rojas, Mauricio Saez and Esteban Jadresic from Central Bank of Chile, Jeffrey Greco from The University of Chicago, Jaime Casassus from Pontificia Universidad Católica de Chile and an anonymous referee for their useful comments. All errors are my own.

E-mail: [mreyes@bcentral.cl](mailto:mreyes@bcentral.cl).

## 1. INTRODUCTION

The term structure of interest rates is a function that assigns a relevant interest rate to each maturity at a given point in time. This characterization allows, on the one hand, to price any string of payments or measurable quantities at specific points in time, and on the other hand, to obtain an implicit measure concerning what will interest rates look like in the future, information that is usually considered as market expectations. In addition to that, the term structure of interest rates constitutes the basis for the calibration of derivative pricing models and risk management.

The mechanism of assigning a relevant interest rate to each maturity is usually described as a conditional expectation which is continuously updated with the arrival of new information. In this regard, it seems natural to expect that this function changes over time as agents revise their expectations according to some set of rules. Two questions then arise from this: first, what information is relevant and drives these changes and, second, how large this changes can be. To the best of our knowledge these two questions have been addressed in the literature assuming that yields can be described by some probability distribution, and then the problem reduces to test which variables cause big responses in yields, where big is a concept related to their standard variability. The basic assumption behind these ideas is that prices in financial markets react to some extent to the arrival of new relevant information. In this regard, evidence indicates a relation between macroeconomic data and yield dynamics. Ang and Piazzesi [1] analyze the reaction of monthly data to surprises in unemployment and inflation, Fleming and Remolona [6], Balduzzi, Elton and Green [2], Li and Engle [12] and Furfine [7] find that the largest yield movements are generated by macroeconomic announcements. Johannes [9] using a jump-diffusion model finds that large movements in interest rates occurs only when announcements contains significant unexpected components.

This paper rather concentrates on the second question, along the lines of the work of Johannes [9], and studies large changes in the US term structure using linear factor models based on principal components analysis and the model of Diebold and Li [5]. Thus, at each point in time, the term structure is approximated as a linear combination of variables that only depend on time, called factors, and functions which only depend on maturity, denominated loadings. Formally, the term structure corresponds to an element of the function space completely described by its coordinates in an arbitrary functional basis.

In order to analyze large movements in the term structure, we assume that factors follow correlated Ornstein-Uhlenbeck processes, and then construct 95% confidence ellipses that allow us to identify large or unexpected market movements. The results using principal components and the model of Diebold and Li suggest some ability of agents to anticipate Fed's actions

over the sample period. That is, the moves outside confidence ellipses correspond mostly to events not anticipated for market participants such as inter-meeting fed funds moves and changes in the economic assessment. Indeed, over the sample period Federal Reserve moved Fed Funds rate in 29 occasions, and five of them were considered as unexpected events.

In general, zero coupon interest rates are not observable in the market, and then it is necessary to estimate them from other available quantities, such as coupon bonds and derivatives. In this paper we estimate zero coupon rates using cubic exponential splines of Vasicek and Fong [17] for monthly cross section prices of the Merrill Lynch US treasury bullet index over the period 1997:01 2005:4. Each sample was filtered such that only bonds near par were considered, in other words, the bonds considered are mostly new issues, and otherwise, bonds with coupon rates close enough to the relevant market yield. These zero rates are then used to estimate the factor model.

The paper is organized as follows. Section two, summarizes a general representation that is applied to the case of principal components and the parametrization of Diebold and Li. Section three, compares and analyzes the results of the calibration of both models. Section four introduces the dynamics of factors and relates historical events to the extreme events signaled by the model. Finally, section five summarizes the results and discusses further directions to explore.

## 2. FACTOR REPRESENTATION

Using principal components analysis, we will utilize as in Rebonato [16] and Balasanov [3] a factor representation of the term structure which we compare with the factor model of Diebold and Li.

Assume we have  $N$  observations for  $n$  centered zero rates grouped in a matrix  $r(t, \tau)$ ,  $t = 1, \dots, N$  and  $\tau = 1, \dots, n$ , and we want to find an approximation that reproduces an arbitrary fraction of their variability. Specifically, the  $\tau$ -maturity zero rate at time  $t$  is given by

$$(1) \quad r(t, \tau) = \sum_{i=1}^k L_i(\tau) \times f_i(t)$$

where

$$\begin{aligned} L_i(\tau) &: i\text{-th estimated } \tau\text{-maturity loading.} \\ f_i(t) &: i\text{-th estimated factor associated to loading } i. \end{aligned}$$

thus, at a given point in time  $t$ , the  $\tau$ -maturity zero coupon rate corresponds to the sum of  $k$  variables called *factors* that only depend on time, weighted by coefficients denominated *loadings* that only depend on maturity.

The difference between model (1) and a traditional regression one is that not only the parameters are not observable, but also the regressors, and thus they must be estimated from sample observations. The procedure that simultaneously estimates factors and loadings from the sample covariance

matrix is known as principal components analysis.<sup>2</sup> Specifically, the matrix of  $n$  principal components  $F = \{f_1, \dots, f_n\}$  is defined as

$$(2) \quad F = r \cdot L$$

where  $L = \{l_1, \dots, l_n\}$  are the eigenvectors of the covariance matrix  $S$  of the  $n$  zero coupon rates. We will also assume that eigenvectors were previously ordered by the size of their corresponding eigenvalues  $\Lambda = \{\lambda_1, \dots, \lambda_n\}$ .

Ideally, we would like to find a  $k_0 \ll n$ , such that the whole term structure is approximated by a few variables. Using the fact that  $Tr(S) = Tr(\Lambda)$ , it turns out that if we want to reproduce an arbitrary fraction  $\alpha$  of the term structure variability, then we need a  $k_0$  such that  $\alpha \leq \left(\frac{1}{Tr(S)}\right) \cdot \sum_{i=1}^{k_0} \lambda_i$ , and model (1) can be written as

$$(3) \quad r(t, \tau) = \sum_{i=1}^{k_0} L_i(\tau) \times f_i(t) + \varepsilon(t, \tau)$$

where  $\varepsilon(t, \tau)$  is assumed to be small.<sup>3</sup>

In the following section we calibrate model (3) using a parametric functional form for loadings introduced by Cheyette [4], which is compared to that of Diebold and Li [5].

### 3. CALIBRATION OF THE MODEL

This section shows the calibration and compares two special cases of model (3) that can be obtained when we choose arbitrary functions as loadings. In other words, we are going to parameterize the estimated loadings as if they correspond to observations of unknown continuous functions. The first case is obtained following Cheyette [4] such that the  $i$ -th loading is given by

$$(4) \quad L_i(\tau) = \sum_{j=1}^{m_i} b_{ij} \frac{1 - e^{-a_{ij}\tau}}{a_{ij}\tau}$$

where  $b_{ij}$ ,  $a_{ij}$  and  $m_i$  are constants. Equation (4) can satisfactorily reproduce the shape of the loadings by means of combinations of exponential functions.

Alternatively, the second case is the parametrization of Diebold and Li [5] which corresponds to a dynamic redefinition of Nelson and Siegel [13] term structure such that the time  $t$  zero rate associated to maturity  $\tau$  is given by

$$(5) \quad r(t, \tau) = \beta_{1t} + \beta_{2t} \left[ \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} \right] + \beta_{3t} \left[ \frac{1 - e^{-\lambda_t \tau}}{\lambda_t \tau} - e^{-\lambda_t \tau} \right] + \varepsilon(t, \tau)$$

<sup>2</sup>See [10] for further details.

<sup>3</sup>In general it is customary to choose a  $k_0$  such that  $\alpha \geq 90\%$ .

$k_0$	1	2	3	4	5	6	7	8	9	10	11
$\alpha$	0.9508	0.0475	0.0061	0.0022	0.0003	3.7e-6	2.0e-6	1.0e-7	1.2e-8	1.4e-9	5.5e-10

TABLE 1. Explained Fraction of Term Structure Variability

Formally, we can state the following proposition

**Proposition 1.** *Fix  $\lambda_t = \lambda$ . Then equation (5) admits the following factor representation*

$$(6) \quad r(t, \tau) = \sum_{i=1}^3 L_i(\tau) \times f_i(t) + \varepsilon(t, \tau)$$

*Proof.* See Appendix. □

In what follows we will refer to model (3) and (6), as Factor Model and Diebold and Li Model, respectively.

Even though the Factor Model and Diebold and Li Model share the same linear representation, their interpretation in terms of factors and loadings is not the same. In the first case, loadings are regarded as modes of move of the term structure. Indeed, Litterman and Scheinkman [11] named the first three components as level, slope and convexity according to how shocks to these factors affect the yield curve. Thus, at a given point in time, the term structure is determined by adding the states of the three modes, and consequently, each change in the term structure along time corresponds to the summation of changes in the three modes. In our work we will follow other authors using the first three principal components to approximate zero rates, that is, we will use  $k_0 = 3$ . Table 1 presents the percentage of variance explained by each factor. As can be seen using  $k_0 = 3$  we will capture approximately 99% of the term structure variability. In the second case though, loadings are interpreted as components affecting sectors (maturities) of the term structure, i.e., the long term and short term rates.<sup>4</sup> In this regard, it is useful to analyze the limiting cases. For instance, in the Diebold and Li model when time to maturity  $\tau$  tends to infinity only loading one remains which is regarded as the implicit (estimated) long term rate. Conversely, when  $\tau$  tends to zero we are left with loading one and two which account for the implicit short term or instantaneous rate. On the other hand, when analyzing the Factor Model only the instantaneous rate can be recovered. That is when  $\tau$  tends to infinity it tends to zero, and as  $\tau$  goes to zero we obtain a weighted sum of the factors. Table 2 summarizes the limits for each case.

---

<sup>4</sup>Nelson and Siegel also identify a third component associated to medium term rates.

Limit	Diebold and Li	Factor Model
$\lim_{\tau \rightarrow \infty} r(t, \tau)$	$\beta_1(t)$	0
$\lim_{\tau \rightarrow 0} r(t, \tau)$	$\beta_1(t) + \beta_2(t)$	$f_1(t) \sum_{j=1}^{m_1} b_{1j} + f_2(t) \sum_{j=1}^{m_2} b_{2j} + f_3(t) \sum_{j=1}^{m_3} b_{3j}$

TABLE 2. Limiting Cases

Parameter	$L_1$	$L_2$	$L_3$
$b_{i1}$	-0.0076	0.8905	0.0356
$b_{i2}$	0.2448	1.4646	0.0120
$b_{i3}$	-0.7421	-2.7186	-0.0753
$a_{i1}$	-0.0854	0.0463	0.4347
$a_{i2}$	0.8819	0.7847	-0.2394
$a_{i3}$	0.3636	0.4945	-0.1432

TABLE 3. Estimated Parameters

Having analyzed their interpretation, in what follows we will show the results of the calibration of both models and also compare their ability to approximate yield to maturities in terms of their in-sample Root Mean Squared Error.

First of all, we need to calibrate the loadings of both models and then estimate their associated factors at each point in time. For model (3) we use non-linear least squares to find the parameters of equation (4) for  $i = 1, 2, 3$ . Table 3 shows the estimated constants.

On the other hand, calibration of model (6) is trivial since we only need to fix  $\lambda_t = \lambda_0$  as mentioned in proposition 1. In our case we will use  $\lambda_0 = 0.5$ . These parameters can then be used to find the loading associated to any maturity. Since loadings are assumed to remain constant over the sample period, they can be used to estimate the factor values at each point in time. Formally, the choice of loadings represents the basis that we will use to represent the term structure at a given date. Figure 1 depicts the shape of the fitted loadings and the factor time series for each model.

As shown in figure 1 factor time series captures the variability of the term structure over time. Moreover, they can be interpreted as observations of certain stochastic processes obeying probability distributions.

In order to evaluate the ability of both models to approximate the term structure dynamics, we present an in-sample exercise that recovers the yield for four maturities: 2, 5, 10 and 30 years. Figure 2 shows actual and estimated time series in both cases.

Table 4 summarizes the RMSE in basis points for each maturity. As shown, with the exception of 2 year yield, on average the model of Diebold and Li outperforms the Factor Model specially on the belly of the curve. Indeed, model (6) yields an average error of order 12 basis points, whereas model (3) has a much higher average error of 37 basis points. The poor



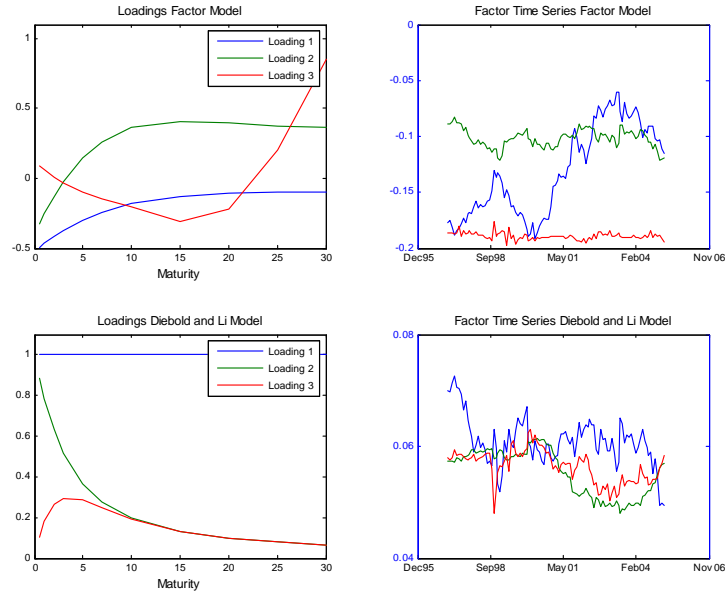


FIGURE 1. Estimated Models

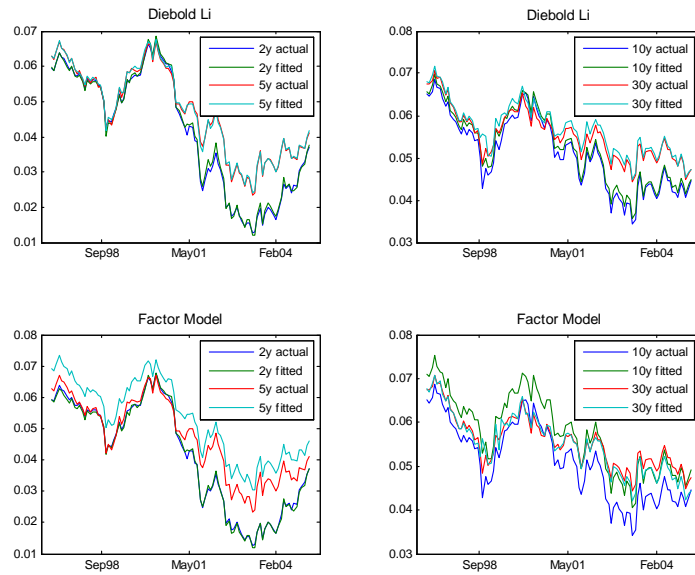


FIGURE 2. Estimated v/s Actual Series

RMSE in basis points		
	Diebold and Li	Factor Model
2y	9.03	7.18
5y	6.02	58.82
10y	15.42	63.88
30y	15.38	16.50

TABLE 4. Root Mean Squared Error

performance of the Factor Model can be attributed to the small number of factors included  $k = 3$ . Although the model performance can be improved by choosing a larger  $k$ , the dimension of the problem also gets larger and then we would be departing from our initial goal. Nevertheless, level errors might not be important if we are interested in describing the variability of the term structure, i.e., even though the approximation is poor for yield levels it can be appropriate for yield variability.

#### 4. FACTOR DYNAMICS

Following Balasanov [3] in this section we introduce the assumption that factors are driven by stochastic processes. Specifically, assume that the  $i$ -th factor is the solution of the following stochastic differential equation

$$(7) \quad df_i(t) = -\mu_i f_i(t)dt + \sigma_i dW_i(t); i = 1, 2, 3.$$

where  $W_i(t)$  is a standard Brownian motion, with  $\mu_i$ , and  $\sigma_i$  constants, and  $d\langle W_i, W_j \rangle_t = \rho_{ij}dt$  for  $i \neq j$ . Process (7) is known as Ornstein-Uhlenbeck and its main characteristic is the mean reversion.<sup>5</sup> Moreover, it is well known that process (7) is jointly normally distributed. In this work we will only use the first two factors of each model, i.e., level and slope, and the short and long term rate.

At this point it is important to verify whether normality is a sensible choice to describe factor dynamics. Figure 3 shows the qq-plot of estimated factors for both models. As shown in the graphs with the exception of the instantaneous rate in model (6), the data seems to fit relatively well to the normality assumption. This goes in line with Piazzesi [15] who suggests some reversion to normality on bond yields data for the period from 1990:1 2001:12.

Alternatively, Table 5 summarizes the results of Jarque-Bera test and the Kolmogorov-Smirnov counterpart known as the Lilliefors test, under the null hypothesis of normality for changes in level and slope, and in long and short rate.

<sup>5</sup>This process has a discrete counterpart commonly known as autorregressive process of order one.

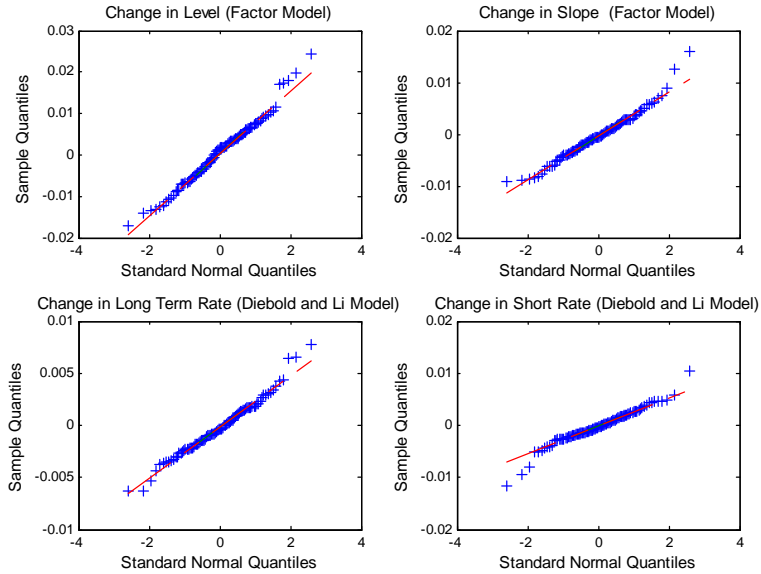


FIGURE 3. qq-plot of Factor Changes

	Jarque-Bera	Lilliefors
Level	1.6552	0.0523
Slope	11.6434*	0.0656
Long rate	4.5892	0.0573
Short rate	23.0889*	0.0950*

\* rejection of null hypothesis at 95%

TABLE 5. Normality Test

It is interesting to note that regardless the model, changes in the factor that accounts for low-frequency variability, i.e., the long rate in the case of Diebold and Li model, and the level in the factor model seem to obey a normal law. Also, when using the Lilliefors test we were not able to reject normality of changes in level and slope in the case of the factor model, whereas in the case of Diebold and Li model normality is rejected for changes in the short rate. Specifically, using Jarque-Bera test, only changes in level and long rate cannot be rejected at 95% confidence level. Since Jarque-Bera is an asymptotic test we decided to include the results of the Lilliefors test which is more appropriate for small data with unknown mean and variance. The results, as suggested by the qq-plots, show that the hypothesis of normality is rejected only for the short rate.

These findings support our assumption that factor changes in the factor model are normally distributed. Even though we cannot conclude the same

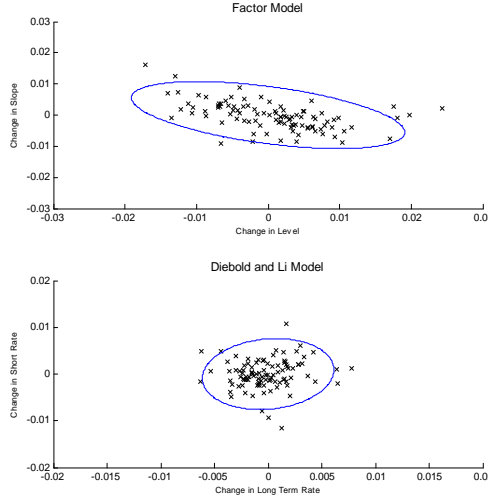


FIGURE 4. Confidence Ellipses for factor changes

for the Diebold and Li model, we will do so as a mean to gain more insight in our analysis. This fact allows us to restate the analysis of changes in the term structure in the factor space, and categorize their magnitudes in probabilistic terms. In this way we are able to recognize when we are in presence of an important movement versus typical ones.

**Definition 1.** *Let the pair  $\{x_1, x_2\}$  be the centered changes in level and slope, and in short and long rate, for the Factor Model and Diebold and Li model, respectively. We will say that a pair  $\{x_1, x_2\}$  corresponds to a large term structure movement with probability  $\exp(-\zeta^2)$  if  $\{x_1, x_2\} \notin \Phi(\zeta)$ .*

where

$$\Phi(\zeta) = \left\{ x_1, x_2 : \frac{1}{2(1-\rho^2)} \left( \frac{x_1^2}{\sigma_1^2} - 2\rho \frac{x_1 x_2}{\sigma_1 \sigma_2} + \frac{x_2^2}{\sigma_2^2} \right) = \zeta^2 \right\}$$

is a concentration ellipse for some  $\zeta$  containing all pairs  $\{x_1, x_2\}$  with probability  $1 - \exp(-\zeta^2)$ .

Definition 1 sets the framework that we will utilize to categorize any term structure change. Thus if we observe a term structure movement and we are interested in evaluate how large is it, we only need to verify whether it belongs to the concentration ellipse associated to a certain confidence level. Figure 4 shows the confidence ellipses for the Factor Model and Diebold and Li model at a 95% confidence level.

Even though both models have different interpretations, intuitively we would expect that extreme events coincide in both cases, since the object under analysis (the term structure) is the same in two different basis. As

Date	Diebold and Li	Factor Model	Event Description
Oct-98	x	x	Inter-meeting Fed Funds move
Mar-99	x	x	Fed minutes states inflation risk
Feb-00	x	x	FF up 25bp + assessment: "Inflation"
Jan-01	x	x	Inter-meeting Fed Funds move
Oct-01	x	x	FF cut 50bp + September 11 effect
Aug-02	x	x	Balance risk from neutral to weakness
Aug-03	x	x	Fed statement: deflation to disinflation
Jun-04	x		Beginning of tightening cycle

TABLE 6. Extreme Events at 95 percent

shown in the graphs, with only one exception, all the abnormal events coincide.

Additionally, it would be interesting to verify whether these events corresponds to exogenous shocks such as changes in policy rates and market expectations. This issue has been largely addressed in the past using weekly and daily data on single maturity instruments. Flemming and Remolona [6], Li and Engle [12], among others, find evidence of correlation among macroeconomic events and large yield movements. Johannes [9] represents these macroeconomic surprises as a jump component in the interest rate dynamics.

Table 6 summarizes the events signaled using definition 1. As can be seen these events correspond mostly to shocks not anticipated for market participants. These changes in expectations are due to inter-meeting fed funds moves and also to changes in the economic assessment. This might suggest the ability of agents to anticipate monetary policy and also the commitment of policy makers to transparency, i.e., markets expectations are rapidly incorporated into the term structure and thus when policy actions are materialized the curve does not move too much. On the contrary, any information not incorporated in financial prices will tend to yield large term structure swings. In fact, over the sample period Federal Reserve moved Fed Funds rate in 29 occasions, and only five of them were considered as unexpected events. These results coincide with that of Johannes [9] who points out that what matters is the unanticipated component of announcements.

The findings also suggest the separation of announcements and assessments. This has been already stressed in Gurkaynak et al. [8] who consider Fed's actions and assessments separately, and then utilize proxies to capture the effect of Fed's announcements and statements on asset prices.

Figure 5 depicts the extreme events in the original interest rate space. As shown, both models coincide in all events with the exception of the beginning of the last tightening cycle signaled by the model of Diebold and Li. The latter must be taken with caution due to the questionable validity

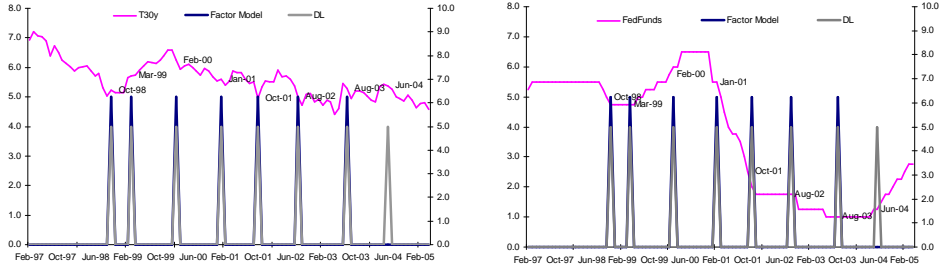


FIGURE 5. Unanticipated Events in the Interest Rate Space

of the imposed distributional assumption to this model, yielding eventually spurious signals.

## 5. FINAL REMARKS

This paper studies large changes in the US term structure using linear factor models that characterize interest rates as the linear sum of stochastic factors weighted by simple functions associated to the maturity of each interest rate. Depending on the choice of the functional basis as loadings, it is possible to obtain a Factor Model and the model of Diebold and Li that differ in their interpretation and limiting cases.

The analysis of factor time series could not reject the hypothesis of normality of changes in level and slope effects, allowing the identification of extreme events in the term structure associated to shocks not anticipated by market participants. These shocks are mostly related to unusual policy actions in the form of statements and inter-meeting policy rate moves. These findings, might be attributed to the ability of agents to incorporate new information into the term structure so as when policy actions are materialized the curve does not move too much, and only information not incorporated will tend to yield big term structure swings. Indeed, over the sample period Federal Reserve moved Fed Funds rate in 29 occasions, and only five of them corresponded to unexpected events.

A natural extension to explore in the future would be to relax the distributional assumptions of factors allowing, for instance, stochastic volatility and jumps in factor dynamics, and then jointly analyze level and variability of interest rates.

## 6. APPENDIX

*Proof.* Equation (5) for  $\lambda_t$  fixed corresponds to a linear sum of functions of time weighted by functions of maturity.

In particular, by choosing

$$L_1(\tau) = 1, L_2(\tau) = \frac{1 - e^{-\lambda\tau}}{\lambda\tau}, L_3(\tau) = \frac{1 - e^{-\lambda\tau}}{\lambda\tau} - e^{-\lambda\tau}$$

as functions of maturity, and

$$f_1(t) = \beta_1(t), f_2(t) = \beta_2(t), f_3(t) = \beta_3(t)$$

as functions of time, we get

$$r(t, \tau) = \sum_{i=1}^3 L_i(\tau) \times f_i(t)$$

□

#### REFERENCES

- [1] Ang, A., Piazzesi, M., "A no-arbitrage vector autoregression of term structure dynamics with macroeconomic and latent variables", *Journal of Monetary Economics*, 2003, 50:4, 745-787.
- [2] Balduzzi, P., Elton E., Green, T. C., "Economic news and bond prices: Evidence from the U.S. treasury market", *Journal of Financial and Quantitative Analysis*, 2001, 36, 523-543.
- [3] Balasanov, Y., "Statistical Model of Interest Rates", *Financial Mathematics Lectures Notes*, The University of Chicago, 1999.
- [4] Cheyette, O., "Markov representation of the Heath-Jarrow-Morton Model", BARRA Inc. Working Paper, 1995.
- [5] Diebold, F.X. Li, C., "Forecasting the Term Structure of Government Bond Yields", *Journal of Econometrics*, forthcoming, 2005.
- [6] Flemming, M., Remolona E., "What moves the bond market?", *Federal Reserve Bank of New York, Economic Policy Review*, December 1997, 31-50.
- [7] Furfine, C., "Do macroeconomic announcements still drive the Treasury market", *BIS Quaterly Review*, June 2001, 49-57.
- [8] Gurkaynak, R., Sack, B., Swanson, E., "Do Actions Speak Louder than Words? the Response of Asset Prices to Monetary Policy Actions and Statements", *Finance and Economics Discussion Series 66*, 2004.
- [9] Johannes, M., "The Statistical and Economic Role of Jumps in Continuous-Time Interest Rate Models", *Journal of Finance*, 2004, 59, 227-259
- [10] Johnson, R., Wichern D., *Applied Multivariate Statistical Analysis*, 4th Edition. Prentice Hall, 1998
- [11] Litterman, R., Scheinkman J., "Common factors affecting bond returns". *Journal of Fixed Income*, 1991, 1, 54-61
- [12] Li, L., Engle R. F., "Macroeconomic Announcements and Volatility of Treasury Futures", Working Paper 98-27, 1998, UC San Diego.
- [13] Nelson, C., Siegel A., "Parsimonious Modeling of Yield Curves". *Journal of Business*, 1987, 60, 473-489
- [14] Øksendal, B., *Stochastic Differential Equations: an Introduction with Applications*, 5th Edition, Universitext, Springer-Verlag, 2000.
- [15] Piazzesi, M., "Affine term Structure Models", forthcoming in *Handbook of Financial Econometrics*, 2003.
- [16] Rebonato, R., "Interest Rate Option Models", 2nd edition, Wiley, 1998.
- [17] Vasicek, O., Fong H., "Term Structure Modeling Using Exponential Splines". *Journal of Finance*, 1981, 28, 339-348

**Documentos de Trabajo  
Banco Central de Chile**

**Working Papers  
Central Bank of Chile**

NÚMEROS ANTERIORES

PAST ISSUES

La serie de Documentos de Trabajo en versión PDF puede obtenerse gratis en la dirección electrónica: [www.bcentral.cl/esp/estpub/estudios/dtbc](http://www.bcentral.cl/esp/estpub/estudios/dtbc). Existe la posibilidad de solicitar una copia impresa con un costo de \$500 si es dentro de Chile y US\$12 si es para fuera de Chile. Las solicitudes se pueden hacer por fax: (56-2) 6702231 o a través de correo electrónico: [bcch@bcentral.cl](mailto:bcch@bcentral.cl).

Working Papers in PDF format can be downloaded free of charge from: [www.bcentral.cl/eng/stdpub/studies/workingpaper](http://www.bcentral.cl/eng/stdpub/studies/workingpaper). Printed versions can be ordered individually for US\$12 per copy (for orders inside Chile the charge is Ch\$500.) Orders can be placed by fax: (56-2) 6702231 or e-mail: [bcch@bcentral.cl](mailto:bcch@bcentral.cl).

DTBC-340  
**Fiscal and Monetary Policy Rules: the Recent Chilean Experience**  
Macarena García, Pablo García y Bernardita Piedrabuena  
Diciembre 2005

DTBC-339  
**Emerging Market Spreads at the Turn of the Century: a Roller Coaster**  
Sergio Godoy  
Diciembre 2005

DTBC-338  
**Managing the Capital Account**  
Sebastián Edwards  
Diciembre 2005

DTBC-337  
**Una Cuantificación del Efecto Causal de las Metas de Inflación**  
Jorge Lorca  
Diciembre 2005

DTBC-336  
**Well Diversified Efficient Portfolios**  
Alejandro Corvalán  
Noviembre 2005

DTBC-335  
**Monetary Policy and Long-Term Interest Rates in Chile**  
Mauricio Larraín  
Noviembre 2005

DTBC-334  
**Brechas de Ingreso entre Hombres y Mujeres: ¿Perpetuadas o Exacerbadas en la Vejez?**  
Solange Berstein y Andrea Tokman  
Noviembre 2005



DTBC-333	Noviembre 2005
<b>Sovereign Spread in Emerging Markets: A Principal Component Analysis</b>	
Mónica Fuentes y Sergio Godoy	
DTBC-332	Septiembre 2005
<b>Spreads Soberanos: ¿Diferencian los Inversionistas Internacionales entre Economías Emergentes?</b>	
Valentín Délano y Felipe Jaque	
DTBC-331	Septiembre 2005
<b>Business Cycle Dynamics and Shock Resilience in Chile</b>	
Helmut Franken, Guillermo Le Fort, y Eric Parrado	
DTBC-330	Septiembre 2005
<b>Sovereign Debt, Volatility and Insurance</b>	
Kenneth Kletzer	
DTBC-329	Septiembre 2005
<b>Contingent Reserves Management: an Applied Framework</b>	
Ricardo Caballero y Stavros Panageas	
DTBC-328	Septiembre 2005
<b>Why are Capital Flows so much more Volatile in Emerging than In Developed Countries?</b>	
Fernando A. Broner y Roberto Rigobon	
DTBC-327	Septiembre 2005
<b>Probability Of Insolvency</b>	
Marcelo Reyes y Eugenio Saavedra	
DTBC-326	Agosto 2005
<b>Exchange Rate Interventions and Insurance: is “Fear of Floating” a Cause for Concern?</b>	
Francisco Gallego y Geraint Jones	
DTBC-325	Agosto 2005
<b>Is the Fx Derivatives Market Effective and Efficient in Reducing Currency Risk?</b>	
Esteban Jadresic y Jorge Selaive	
DTBC-324	Julio 2005
<b>No Estaba Muerta, ... : la Teoría Cuantitativa y la Relación Entre Dinero e Inflación</b>	
Rómulo Chumacero y Jorge Hermann	