

Banco Central de Chile
Documentos de Trabajo

Central Bank of Chile
Working Papers

N° 382

Diciembre 2006

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Documentos de Trabajo del Banco Central de Chile
Working Papers of the Central Bank of Chile
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FORECASTING CANADIAN TIME SERIES WITH THE NEW-KEYNESIAN MODEL

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Resumen

Este trabajo documenta la precisión de las proyecciones fuera de muestra de un modelo neokeynesiano para Canadá. Estimamos nuestra variante del modelo en series de submuestras móviles, calculando proyecciones fuera de muestra uno a ocho trimestres adelante en cada paso. Comparamos estas proyecciones con las provenientes de modelos VAR simples, utilizando tests econométricos de precisión predictiva. Nuestros resultados muestran que el modelo neokeynesiano se compara favorablemente con los modelos de referencia, en particular a medida que se aumenta el horizonte de proyección. Estos resultados sugieren que el modelo podría constituir una herramienta útil para proyectar series de tiempo de la economía canadiense.

Abstract

This paper documents the out-of-sample forecasting accuracy of the New Keynesian Model for Canada. We estimate our variant of the model on a series of rolling subsamples, computing out-of-sample forecasts one to eight quarters ahead at each step. We compare these forecasts to those arising from simple vector autoregression (VAR) models, using econometric tests for forecasting accuracy. Our results show that the forecasting accuracy of the New Keynesian model compares favorably to that of the benchmarks, particularly as the forecasting horizon is increased. These results suggest that the model could become a useful forecasting tool for Canadian time series.

* Este trabajo fue presentado en el Taller de Modelamiento Macroeconómico de Bancos Centrales, organizado por el Banco Central de Chile, el 28 y 29 de Septiembre de 2006, Santiago, Chile.

The authors wish to thank Takashi Kano, André Kurmann, Lynda Khalaf, Pierre St Amant, Greg Tkacz, Carolyn Wilkins, and seminar participants at Laval University, the Bank of Canada, and the 2005 SCE conference for useful comments and discussions. We would also like to thank two anonymous referees for their constructive criticism and suggestions. The views expressed in this paper are those of the authors. No responsibility for them should be attributed to the Bank of Canada.

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1 Introduction

New-Keynesian models are becoming standard tools in applied macroeconomic analysis.¹ They are used widely to study the impact of shocks on economic activity and inform the decisions of monetary policy-makers in several central banks worldwide. These models are relevant because their optimizing environment coherently determines the time paths of aggregate variables in a framework suitable for monetary policy analysis. Recently, it has become common to estimate the parameters of these models using aggregate time series and standard econometric techniques.² However, the models are less often used to generate out-of-sample forecasts: evidence on the quality of these forecasts thus remains scarce.

To contribute to this evidence, this paper documents the out-of-sample forecasting properties of a New Keynesian model for Canada. Specifically, we develop a variant of the model, estimate it on a series of rolling subsamples, and compute out-of-sample forecasts one to eight quarters ahead at each step. We then compare these forecasts with those arising from vector autoregressions (VARs), using several econometric tests of forecasting accuracy.

We find that the model's forecasting accuracy compares favourably with that of the VAR benchmarks, particularly as the forecasting horizon increases. Specifically, the model can forecast output, interest rates, and money as well as or better than the benchmarks, while for inflation the difference with the benchmarks are less important. Our results also suggest that a combination of the two sets of forecasts may have forecasting power that is superior to each set alone. Overall, our findings indicate that the New Keynesian class of models has the potential to become a useful forecasting tool for Canadian time series.

Using VARs as the benchmarks for comparing forecasts is natural, because DSGE environments like the New Keynesian model can be written as VARs whose parameters are restricted by non-linear constraints linked to the model's structure. Our forecasting experiments thus com-

¹New Keynesian models are dynamic stochastic general-equilibrium (DSGE) environments where monopolistically competitive firms set prices subject to various adjustment costs. They are built around a core that consist of a price-setting equation (the 'New Phillips Curve'), an equation linked to intertemporal consumption smoothing, and a monetary policy rule. Although derived from the Real Business Cycle methodology, their emphasis on nominal rigidities and monetary features makes them well suited to monetary policy analysis. Woodford (2003) provide a synthesis of the model's implications for monetary policy analysis.

²For example, Ireland (1997, 2001a, 2003, 2004), Dib (2003a,b, 2006) and Bouakez et al. (2005) estimate parameters using maximum likelihood; Christiano et al. (2005) do so by minimizing the distance between the model's impulse responses following monetary policy shocks and those computed with VARs; Smets and Wouters (2003) and Del Negro and Schorfheide (2004) employ a Bayesian strategy to compute the posterior distribution for the parameters.

pare the out-of-sample forecasting properties of a restricted model with those of an unrestricted counterpart. Clements and Hendry (1998) discuss conditions under which better forecasting accuracy may be attained by the restricted model. This requires a trade-off between squared inconsistency (how ‘wrong’ the restrictions are) and sampling uncertainty (estimating a large number of parameters lowers precision) to favour the parsimonious specification.³ This situation is more likely when the sample size for estimation is small and the forecasting horizon is high, as in monetary policy practice.

Evidence is emerging about the practical value of DSGE models for forecasting. First, Ingram and Whiteman (1994), Del Negro and Schorfheide (2004), and Del Negro et al. (forthcoming) show that Bayesian VARs whose priors are linked to DSGE models⁴ can often improve the VARs’ forecasting accuracy. Alternatively, Ireland (2004) shows, using US data, that an estimated Real Business Cycle model can have better forecasts than simple VARs⁵, while Boivin and Giannoni (2005) show that casting a New Keynesian model within a forecasting exercise with factor models can also lead to forecasting improvements.

The objectives of the present paper are more closely aligned with those of Korenok and Swanson (2005) and Adolfson et al. (2006), who present rolling estimation and forecasting exercises with New Keynesian models, using American and European aggregate data. These papers report similar results to those presented here for Canadian data: overall, variants of the New Keynesian model forecast well relative to simple time-series benchmarks. Interestingly, Korenok and Swanson (2005) report better performance for output than for inflation, a result that echoes the one reported here. By confronting the model to Canadian data and applying several econometric tests of forecast accuracy, the present paper thus strengthens the emerging evidence about the good forecasting properties of New Keynesian models.

The rest of this paper is organized as follows. Section 2 develops our variant of the New Keynesian model. Section 3 discusses the model’s estimation and provides estimation results for the first subsample. Section 4 describes our forecasting experiment and reports the results. Section 5 offers some conclusions.

³In addition, problems associated with the small-samples properties of the more complex non-linear estimation must be relatively small.

⁴Ingram and Whiteman (1994) derive their priors from the basic Real Business Cycles model, while the priors in Del Negro and Schorfheide (2004) and Del Negro et al. (forthcoming) arise from New Keynesian models.

⁵Dolar and Moran (2002) have verified that Ireland’s results also hold for Canadian data.

2 Model

This section develops our variant of the New-Keynesian class of models. The structure of the model is similar to that in Dib (2006) and Ireland (2003). Time is discrete and one model period represents a quarter. There are two sectors of production. The first produces final goods and is competitive: firms take input prices as given and produce an homogenous good that they sell at flexible prices. Final-good production is divided between consumption and investment. Capital-adjustment costs restrict the accumulation of capital and thus influence investment choices. The firms in the second sector, which produce intermediate goods, operate under monopolistic competition. Each firm produces a distinct good for which it chooses the market price. Changes to the price of these goods are constrained by the Calvo (1983) mechanism, so that these prices are ‘sticky’. Intermediate-good production requires capital and labour services, inputs for which the firms act as price takers. The economy is closed.⁶ The monetary authority’s policy rule manages movements in the short-term nominal interest rate to respond to inflation deviations from its target, as well as deviations of output and money growth from their trends.

2.1 Households

There exist a continuum of identical, infinitely-lived households that derive utility from consumption C_t , detention of real money balances M_t/P_t , and leisure $(1 - h_t)$, where h_t represents hours worked. A representative household’s expected lifetime utility is described as follows:

$$U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(C_t, M_t/P_t, h_t), \quad (1)$$

⁶Although important open-economy features are thus missing from our analysis, we believe our forecasting experiments remain valid, for two reasons. First, the forecasting accuracy of the New Keynesian model is compared to that of VAR benchmarks that have been estimated using the same four (Canadian only) variables. Neither model thus use foreign variables in the estimation or forecasting stages. Second, Dib (2003b) estimates both closed-economy and open-economy versions of a model similar to the present one and shows that the estimates of parameters present in both versions (such as the discount factor β or the parameter governing the severity of nominal rigidities) are not significantly different in the two versions of the model. This suggests that our parameter estimates are not strongly biased by the absence of open-economy features in our version of the New Keynesian model. Nevertheless, introducing open-economy features in our experiments would allow the model to capture important information from foreign variables and will constitute natural and important extensions of this paper.

where $\beta \in (0, 1)$ is the discount factor and the single-period utility function is specified as:

$$u(\cdot) = \frac{\gamma z_t}{\gamma - 1} \log \left(C_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} (M_t/P_t)^{\frac{\gamma-1}{\gamma}} \right) + \zeta \log(1 - h_t), \quad (2)$$

where γ and ζ are positive structural parameters, and z_t and b_t are serially correlated shocks. As McCallum and Nelson (1999) show, the preference shock z_t resembles, in equilibrium, a shock to the IS curve of more traditional Keynesian analysis. On the other hand, b_t is interpreted as a shock to money demand. These shocks follow the first-order autoregressive processes

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{zt}, \quad (3)$$

and

$$\log(b_t) = (1 - \rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}, \quad (4)$$

where $\rho_z, \rho_b \in (-1, 1)$ and the serially uncorrelated innovations ε_{zt} and ε_{bt} are normally distributed with zero mean and standard deviations σ_z and σ_b , respectively.

The representative household enters period t with K_t units of physical capital, M_{t-1} units of nominal money balances, and B_{t-1} units of bonds. During period t , the household supplies labour and capital to the intermediate-good-producing firms, for which it receives total factor payment $R_{kt}K_t + W_t h_t$, where R_{kt} is the rental rate for capital and W_t is the economy-wide wage. Further, the household receives a lump-sum transfer from the monetary authority, X_t , as well as dividend payments D_t from intermediate-good-producing firms.⁷ The household allocates these funds to consumption purchases C_t and investment in capital goods I_t (both priced at P_t), to money holdings M_t and to bond holdings B_t , priced at $1/R_t$, where R_t denotes the gross nominal interest rate between t and $t + 1$. The following budget constraint therefore applies:

$$P_t (C_t + I_t) + M_t + B_t/R_t \leq R_{kt}K_t + W_t h_t + M_{t-1} + B_{t-1} + X_t + D_t. \quad (5)$$

Investment i_t increases the capital stock over time according to

$$K_{t+1} = (1 - \delta)K_t + I_t - \Psi(K_{t+1}, K_t), \quad (6)$$

where $\delta \in (0, 1)$ is the constant capital depreciation rate and $\Psi(\cdot, \cdot)$ is a capital-adjustment cost function specified as $\frac{\psi}{2} \left(\frac{K_{t+1}}{K_t} - \eta \right)^2 K_t$, where $\psi > 0$ is the capital-adjustment cost parameter

⁷The transfer X_t is related to the monetary authority's managements of short-term interest rates through its policy rule (described below).

and $\eta > 1$ is the growth rate of the economy. With this specification both total and marginal costs of adjusting capital are zero in the steady-state equilibrium.

The representative household chooses C_t, M_t, h_t, K_{t+1} and B_t in order to maximize expected lifetime utility (1) subject to the budget constraint (5) and the investment constraint (6). The first-order conditions for this problem are as follows:

$$\frac{z_t C_t^{-\frac{1}{\gamma}}}{C_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} (M_t/P_t)^{\frac{\gamma-1}{\gamma}}} = \Lambda_t; \quad (7)$$

$$\frac{z_t b_t^{\frac{1}{\gamma}} (M_t/P_t)^{-\frac{1}{\gamma}}}{C_t^{\frac{\gamma-1}{\gamma}} + b_t^{\frac{1}{\gamma}} (M_t/P_t)^{\frac{\gamma-1}{\gamma}}} = \Lambda_t - \beta E_t \left(\frac{P_t \Lambda_{t+1}}{P_{t+1}} \right); \quad (8)$$

$$\frac{\zeta}{1 - h_t} = \Lambda_t \frac{W_t}{P_t}; \quad (9)$$

$$\beta E_t \left[\frac{\Lambda_{t+1}}{\Lambda_t} \left(\frac{R_{kt+1}}{P_{t+1}} + 1 - \delta + \psi \left(\frac{K_{t+2}}{K_{t+1}} - \eta \right) \frac{K_{t+2}}{K_{t+1}} \right) \right] = \psi \left(\frac{K_{t+1}}{K_t} - \eta \right) + 1; \quad (10)$$

$$\frac{\Lambda_t}{R_t} = \beta E_t \left[\frac{P_t \Lambda_{t+1}}{P_{t+1}} \right]; \quad (11)$$

where Λ_t is the Lagrange multiplier associated with constraint (5).

As Ireland (1997) shows, combining conditions (7), (8) and (11) yields the following optimization-based money-demand equation:

$$\log(M_t/P_t) \simeq \log(C_t) - \gamma \log(r_t) + \log(b_t), \quad (12)$$

where $r_t = R_t - 1$ denotes the net nominal interest rate between t and $t+1$, γ is the interest rate-elasticity of money demand, and b_t is the serially correlated money-demand shock described above.

2.2 The final goods-producing firm

The final good, Y_t , is produced by assembling a continuum of intermediate goods $y_{jt}, j \in (0, 1)$ that are imperfect substitutes with a constant elasticity of substitution θ . The aggregation function is defined as

$$Y_t \leq \left(\int_0^1 Y_{jt}^{\frac{\theta-1}{\theta}} dj \right)^{\frac{\theta}{\theta-1}}, \quad \theta > 1. \quad (13)$$

Final good-producing firms behave competitively, maximizing profits and taking the market price of the final good P_t as well as the intermediate-good prices $P_{jt}, j \in (0, 1)$ as given. The

maximization problem of a representative, final good-producing firm is therefore

$$\max_{\{Y_{jt}\}_{j=0}^1} \left[P_t Y_t - \int_0^1 P_{jt} Y_{jt} dj \right],$$

subject to (13). The resulting input-demand function for the intermediate good j is

$$y_{jt} = \left(\frac{P_{jt}}{P_t} \right)^{-\theta} Y_t. \quad (14)$$

Equation (14) represents the economy-wide demand for good j as a function of its relative price and of the economy's total output of final good Y_t . Competition in the sector and the constant-returns-to-scale production (13) imply that these firms make zero profits. Imposing the zero-profit condition leads to the following description of the final-good price index, P_t :

$$P_t = \left(\int_0^1 P_{jt}^{1-\theta} dj \right)^{\frac{1}{1-\theta}}. \quad (15)$$

2.3 The intermediate-goods-producing firm

The intermediate good-producing firm j uses capital and labour services, k_{jt} and h_{jt} , respectively, to produce Y_{jt} units of good j , according to the following constant-returns-to-scale technology:

$$Y_{jt} \leq Y_{jt}^\alpha (A_t \eta^t h_{jt})^{1-\alpha}, \quad \alpha \in (0, 1), \quad (16)$$

where $\eta_t > 1$ denotes the gross rate of labour-augmenting technological progress. The presence of such growth implies a balanced growth path, so that output, investment, consumption, the real wage, capital and real money balances all grow at the same rate, η . Thus, these variables must be linearly trended.⁸ A_t describes an aggregate technology shock common to all firms. This shock follows a stationary first-order autoregressive process:

$$\log A_t = (1 - \rho_A) \log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}, \quad (17)$$

where $\rho_A \in (-1, 1)$ is an autoregressive coefficient, $A > 0$ is a constant, and ε_{At} is normally distributed with mean zero and standard deviation σ_A .

Each intermediate-good-producing firm sells its output under monopolistic competition; the economy-wide demand for the good produced by producer j is given by (14). Following

⁸An important extension to consider in future work would be to model technological progress as an I(1) process, which would introduce stochastic trends in these variables.

Calvo (1983), we assume that each firm is only allowed to reoptimize its output price at specific times. Specifically, with probability ϕ , the firm must charge the price that was in effect in the preceding period, indexed by the steady-state rate of inflation, π ; with probability $1 - \phi$, the firm is free to reoptimize and choose an unrestricted new price. On average, each firm therefore reoptimizes every $1/(1 - \phi)$ periods.⁹

At time t , if firm j receives the signal to reoptimize, it chooses a price \tilde{P}_{jt} , as well as contingency plans for h_{jt+k} and k_{jt+k} , for all $k \geq 0$ that maximize its discounted, expected (real) total profit flows for the period where it will not be able to reoptimize. The profit-maximization problem is as follows (θ^k represents the probability that \tilde{P}_{jt} remains in effect at $t + k$):

$$\max_{\{K_{jt}, h_{jt}, \tilde{P}_{jt}\}} E_0 \left[\sum_{k=0}^{\infty} (\beta\phi)^k \Lambda_{t+k} D_{jt+k} / P_{t+k} \right],$$

with D_{jt+k}/P_{t+k} , the real profit flow at time $t + k$ and

$$D_{jt+k} = \tilde{P}_{jt} \pi^k Y_{jt+k} - R_{kt+k} K_{jt+k} - W_{t+k} h_{jt+k}. \quad (18)$$

Profit maximization is subject to the demand for good j (14) and the production function (16) (to which the Lagrange multiplier $\Xi_t > 0$ is associated). The first-order conditions for K_{jt+k} , h_{jt+k} , and \tilde{P}_{jt} are:

$$\frac{R_{kt}}{P_t} = \alpha q_t \frac{Y_{jt}}{K_{jt}}; \quad (19)$$

$$\frac{W_t}{P_t} = (1 - \alpha) q_t \frac{Y_{jt}}{h_{jt}}; \quad (20)$$

$$\tilde{P}_{jt} = \frac{\theta}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} (\beta\phi\pi^{-\theta})^k \Lambda_{t+k} Y_{t+k} q_{t+k} P_{t+k}^{\theta}}{E_t \sum_{k=0}^{\infty} (\beta\phi\pi^{1-\theta})^k \Lambda_{t+k} Y_{t+k} P_{t+k}^{\theta-1}}; \quad (21)$$

where $q_t \equiv \Xi_t/\Lambda_t$ is the real marginal cost of the firm.

The symmetry in the demand for their good implies that all firms allowed to reoptimize choose the same price \tilde{P}_{jt} , which we denote \tilde{P}_t . Considering the definition of the price index in (15) and the fact that, at the economy's level, a fraction $1 - \phi$ of intermediate-good-producing firms reoptimize, the aggregate price index, P_t , evolves according to

$$P_t^{1-\theta} = \phi(\pi P_{t-1})^{1-\theta} + (1 - \phi)(\tilde{P}_t)^{1-\theta}. \quad (22)$$

⁹This specification of the Calvo mechanism follows Yun (1996). Alternatively, Christiano et al. (2005) assume that when the reoptimization signal is not received, the price is increased by the *preceding period's* rate of inflation. Smets and Wouters (2003) implement a flexible specification that nests the two cases.

Equations (19) and (20) state that firms choose production inputs in order for their costs to equal the marginal product times real marginal costs. Equation (21) relates the optimal price to the expected future price of the final good and to expected future marginal costs. Taking a first-order approximation of this condition and of (22), and combining them, gives the model's New Keynesian Phillips curve:

$$\widehat{\pi}_t = \beta \widehat{\pi}_{t+1} + \frac{(1-\phi)(1-\beta\phi)}{\phi} \widehat{q}_t, \quad (23)$$

where a hatted variables denotes its deviation from the steady-state value. This expression relates the current period's inflation rate to its expected future value, as well as to the current marginal costs, an indicator of the strength of economic activity.

2.4 The monetary authority

As in Ireland (2003) and Dib (2006), we assume that the monetary authority manages the short-term nominal interest rate, R_t , to respond to deviations of inflation, $\pi_t \equiv P_t/P_{t-1}$, output, $y_t = Y_t/\eta^t$, and money growth, $\mu_t \equiv M_t/M_{t-1}$, from their steady-state equilibrium values.¹⁰ This monetary policy rule is given by:

$$\log(R_t/R) = \varrho_\pi \log(\pi_t/\pi) + \varrho_y \log(y_t/y) + \varrho_\mu \log(\mu_t/\mu) + \log(v_t), \quad (24)$$

where R , π , y , and μ are the steady-state values of R_t , π_t , y_t , and μ_t , respectively. Further, v_t is a monetary policy shock that evolves according to

$$\log(v_t) = \rho_v \log(v_{t-1}) + \varepsilon_{vt}, \quad (25)$$

where $\rho_v \in [0, 1)$ is an autoregressive coefficient and ε_{vt} is a zero-mean, serially uncorrelated shock with standard deviation σ_v . The monetary authority implements this rule with the appropriate lump-sum injection/withdrawal of money X_t .

The policy coefficients ϱ_π , ϱ_y , and ϱ_μ are chosen by the monetary authorities. When $\varrho_\pi > 0$, $\varrho_y > 0$, and $\varrho_\mu = 0$, monetary policy follows a Taylor (1993) rule, in which nominal interest rates increase in response to deviations of inflation and (detrended) output from their steady-state values.

In contrast, (24) states that monetary policy follows a modified Taylor (1993) rule that adjusts short-term nominal interest rates in response to changes in money-growth as well as to

¹⁰ $y_t = Y_t/\eta^t$ is stationarized (lineary detrended) output.

deviations of inflation and output. In that case, a unique equilibrium exists as long as the sum of ϱ_π and ϱ_μ exceeds one.

Ireland (2003) interprets such a rule as a combination policy that influences a linear combination of the interest rate and the money-growth rate to control inflation. Alternatively, the money-growth rate can be interpreted as an indicator of expected inflation or as a proxy for some omitted variables, such as the exchange rate or financial variables, to which monetary policy responds. Alternatively,

2.5 Symmetric equilibrium

In a symmetric equilibrium, all intermediate-goods-producing firms are identical. They make the same decisions, so $Y_{jt} = Y_t$, $\tilde{P}_{jt} = \tilde{P}_t$, $K_{jt} = K_t$, $h_{jt} = h_t$, $D_{jt} = D_t$. Let $\tilde{r}_{kt} \equiv R_{kt}/P_t$, $\tilde{w}_t \equiv W_t/P_t$, and $\tilde{m}_t \equiv M_t/P_t$ denote the real capital rental rate, the real wage, and real money balances, respectively. A symmetric equilibrium for this economy consists in a sequence of allocations $\{Y_t, C_t, I_t, \tilde{m}_t, h_t, K_t\}_{t=0}^\infty$, a sequence of prices and co-state variables $\{\tilde{w}_t, \tilde{r}_{kt}, R_t, \pi_t, \lambda_t, q_t\}_{t=0}^\infty$, and the stochastic processes for preference, money demand, technology, and monetary policy shocks. These allocations, prices, and shocks are such that (i) households, final-good-producing firms, and intermediate-good-producing firms optimize, (ii) the monetary policy rule (24) is satisfied, and (iii) the following market-clearing conditions are satisfied:

$$K_t = \int_0^1 K_{jt} dj; \quad (26)$$

$$h_t = \int_0^1 h_{jt} dj; \quad (27)$$

$$M_t = M_{t-1} + X_t; \quad (28)$$

$$B_t = 0; \quad (29)$$

$$Y_t = C_t + I_t. \quad (30)$$

Allowing for trend productivity growth in the production process (13) implies that Y_t, C_t, I_t, K_t, w_t , and m_t all grow at the same rate η in equilibrium. This parameter is estimated among the other model's structural parameters. In the equilibrium, most of the model's real variables inherit a deterministic trend, so we transform them by dividing by η^t to induce stationarity.¹¹

¹¹The transformed variables are: $y_t = Y_t/\eta^t$, $c_t = C_t/\eta^t$, $i_t = I_t/\eta^t$, $k_t = K_t/\eta^t$, $r_{kt} = \tilde{r}_{kt}/\eta^t$, $w_t = \tilde{w}_t/\eta^t$, $m_t = \tilde{m}_t/\eta^t$, $\lambda_t = \Lambda_t\eta^t$.

Next, the steady-state of the system is computed, a first-order linear approximation of the equilibrium system around the steady-state values is formed, and Blanchard and Kahn (1980)'s procedure is used to transform this forward-looking model into the following state-space solution:

$$\widehat{s}_{t+1} = \Phi_1 \widehat{s}_t + \Phi_2 \varepsilon_{t+1}, \quad (31)$$

$$\widehat{d}_t = \Phi_3 \widehat{s}_t, \quad (32)$$

where \widehat{s}_t is a vector of state variables that includes predetermined and exogenous variables; \widehat{d}_t is the vector of control variables; and the vector ε_{t+1} contains the random innovations.¹² The elements of matrices Φ_1 , Φ_2 , and Φ_3 depend on the model's structural parameters.

3 Estimation

3.1 Methodology

It is usual in this literature to calibrate the values of some of the model's parameters, before estimating the values of the remaining ones, because the data used contain weak or no information about them. In light of this, we set the weight on leisure in the utility function ζ to 1.35, which implies that households spend around one-third of their non-sleeping time in market activities (work). The share of capital in production, α , and the depreciation rate, δ , are assigned values of 0.33 and 0.025, respectively; these values are commonly used in the literature. The degree of monopoly power in intermediate-goods markets, θ , is equal to 6, which implies a markup of 20% in steady state: this matches values usually used in similar studies. Both Ireland (2001a) and Dib (2003b) remark that the capital adjustment parameter, ψ , is difficult to estimate without data on capital stock. We fix this parameter to 15, as in Dib (2003b).¹³

The remaining 18 parameters are estimated using the maximum-likelihood procedure.¹⁴ This requires that we select a subset of the control variables, \widehat{d}_t , in (32) for which data are available, and select the appropriate rows of Φ_3 . Next, the likelihood of the sample $\{\widehat{d}_t\}_{t=1}^T$

¹²For any stationary variable x_t , $\widehat{x}_t = \log(x_t/x)$ denotes the deviation of x_t from its steady-state value x . In our specification, $\widehat{s}_t = (\widehat{k}_t, \widehat{m}_{t-1}, \widehat{z}_t, \widehat{b}_t, \widehat{A}_t, \widehat{v}_t)'$, $\widehat{d}_t = (\widehat{\lambda}_t, \widehat{q}_t, \widehat{m}_t, \widehat{y}_t, \widehat{R}_t, \widehat{r}_{kt}, \widehat{c}_t, \widehat{i}_t, \widehat{\pi}_t, \widehat{w}_t, \widehat{h}_t, \widehat{\mu}_t)'$, and $\varepsilon_{t+1} = (\varepsilon_{zt+1}, \varepsilon_{bt+1}, \varepsilon_{At+1}, \varepsilon_{vt+1})'$. Appendix A lists the equilibrium conditions of the model, the steps involved in finding the steady-state, and the linearized equations introduced into the Blanchard and Kahn (1980) algorithm.

¹³The calibrated value for ψ does not affect the estimated values of the remaining parameters.

¹⁴These are $\beta, \gamma, \varrho_\pi, \varrho_\mu, \varrho_y, \rho_v, \sigma_v, \phi, A, \rho_A, \sigma_A, b, \rho_b, \sigma_b, \rho_z, \sigma_z, \pi$, and η .

is computed recursively using the Kalman filter (Hamilton, 1994, Chap. 13). The parameter values that maximize the likelihood are found using standard numerical procedures.¹⁵

Since the model is driven by four shocks, we estimate the model using data for four series, to avoid problems of stochastic singularity. We use Canadian data on output, inflation, a short-term interest rate and real money balances. Output is measured by real final domestic demand that includes only personal consumption expenditures and gross private investment. Inflation is the gross rate of increase in the GDP deflator. The nominal interest rate is the rate of the three-month treasury bill. Real money balances are measured by dividing the M2 money stock by the GDP deflator. Output and real money balances are expressed in per-capita terms using the civilian population aged 15 and over.¹⁶

Our procedure directly estimates the parameter η , which describes the growth rate of technology (and thus of output and real money balances) in the balanced-growth steady state. This follows Ireland (1997, 2004) and Smets and Wouters (2005). This trend is not shared by inflation and the nominal interest rate, which we assume to be trendless (stationary). This treatment of trends differs from some previous estimations of DGSE models (Smets and Wouters, 2003; Dib, 2003a) where the authors used previously detrended data to estimate model versions where growth was absent.¹⁷ We believe this strategy is particularly attractive in the context of a forecasting exercise. It enables us to produce forecasts for the log levels of the data directly, rather than forecasts for detrended series that must then be transformed into forecasts for log levels.

3.2 Estimation Results

The model's parameters are estimated using Canadian data from 1981Q1 to 2004Q1.¹⁸ In the course of our rolling forecasting exercise, we estimate the model using several subsamples, from 1981Q3 to 1996Q4, 1981Q4 to 1997Q1, and so on until 1988Q4 to 2003Q4. Table 1 reports the maximum-likelihood estimates of the parameters for the first subsample (1981Q3 to 1996Q4),

¹⁵In addition to Dib (2003a,b, 2006) and Ireland (2003, 2004), this estimation method is used by Bergin (2003), Bouakez et al. (2005) and several others. Ireland (2004) provides some of the details about the estimation procedure. We employ the simplex algorithm, as implemented by *Matlab*.

¹⁶Appendix B provides additional details, notably the mnemonics, about the data.

¹⁷In future work, we plan to explore the consequences, for the forecasting accuracy of the model, of adopting a difference-stationary assumption for technology. Such a specification is adapted to American data by Ireland (2001b), Korenok and Swanson (2005), and Del Negro et al. (forthcoming) as well as to Euro-area data by Adolfson et al. (2006).

¹⁸The sample starts at 1981Q3 to reflect the fact that the Bank of Canada officially abandoned targeting the M1 growth rate in mid-1981.

with their standard deviations and t -statistics. Almost all of the estimated parameters are statistically significant and economically meaningful. The estimate of the discount rate, β , is 0.99, which implies an annual steady-state real interest rate of just over 4 per cent. The estimates of b , determining the steady-state ratio of real balances to consumption, is 0.5, whereas the constant elasticity of substitution between consumption and real balances, γ , is around 0.06, similar to that estimated by Dib (2003a) for the Canadian economy. The estimate of ϕ , the probability of not adjusting prices in the next period, is 0.64. Thus, on average, firms keep their prices unchanged, except for indexation, for about two quarters and a half. This estimate is very close to the one arrived at in Dib (2003b).

The estimates of the monetary policy parameters are statistically significant, with the exception of ϱ_y . Specifically, the responses of monetary policy to inflation, output, and money growth (ϱ_π , ϱ_y , and ϱ_μ) are 0.93, 0.001, and 0.58, respectively.¹⁹ The estimates of ρ_v and σ_v , the persistence coefficient and standard deviation of monetary policy shocks, are 0.24 and 0.007, respectively. Overall, the estimates of monetary policy parameters are similar to the estimates of Dib (2003b, 2006) for the Canadian economy. They indicate that, to achieve its objectives, the Canadian monetary authorities have responded significantly to inflation and money growth, and hardly (if at all) to output deviations from trend.

The autoregressive coefficient estimates indicate that the technology, money-demand, and preference shocks are relatively persistent, with the money-demand shock being the most persistent ($\rho_b = 0.996$). The standard deviation estimates suggest that the aggregate preference shocks are the most volatile.

Over the subsequent estimations that we perform for the other subsamples (1981Q4 to 1997Q1, 1982Q1 to 1997Q2 and so on until 1988Q3 to 2003Q4), the estimated values of the structural parameters (like the discount factor β or the ‘Calvo’ parameter ϕ) do not change significantly, although some small changes in the estimated values of the parameters are apparent. On the other hand, the estimated value of parameters that are more policy-dependent does change. For example, the steady-state rate of inflation π decreases from its first estimate of 4% (on an annualized basis) to a value of 1.5%. These results, showing stability in some parameters but some changes where we expect them, adds to our confidence that our model provides a good laboratory with which to study the likely forecasting performance of the New Keynesian model.²⁰

¹⁹Indeterminate equilibria do not occur as long as $\varrho_\pi + \varrho_\mu > 1$.

²⁰Estimated values for all samples are available from the authors.

3.3 Properties of the Model

To assess our estimation results, we briefly analyze the impulse response functions drawn from the estimated model.²¹ Figures 1 to 4 show the economy’s response to the four types of exogenous shocks, at the estimated parameter values for the first subsample. The response of output is measured as a deviation from its steady-state value, whereas the responses of the other variables are in net (annualized) percentage points.

Figure 1 plots the economy’s response to a monetary policy tightening, i.e. setting the monetary innovation, ε_{vt} , to 0.01, a value close to its estimated standard deviation. Following the tightening, the interest rates increases and returns to steady state moderately fast (recall that the estimated serial correlation in monetary policy shocks, ρ_v , is 0.24.) Output, inflation and money growth by contrast, fall sharply on impact. Output and inflation return gradually to steady state, while money growth overshoots slightly in the following periods, converging back to steady state. This gradual return to steady state reflects the actions of the Calvo (1983) mechanism and the serial correlation of the shock. Notice that the negative, contemporaneous correlation between interest rates and money growth –the liquidity effect– is consistent with the evidence.²²

Figure 2 shows the economy’s responses to a money demand shock (setting the money-demand innovation, ε_{bt} , to 0.01). The shock causes output and inflation to decrease only slightly on impact. Money growth increases sharply, however, to accommodate the increase in demand. Since the rule followed by the monetary authority includes a response to increases in money growth, the nominal interest rate increases slightly, which is at the source of the slight decreased in output. These responses roughly match Poole’s (1970) classic analysis, in which the monetary policy authority changes the short-term nominal interest rate in response to exogenous demand-side disturbances.

Figure 3 shows responses following a shock to technology (an increase in ε_{At} of 0.01). Output jumps on impact, while the nominal interest rate and inflation fall below their steady-state levels. Money growth responds positively to the shock before falling below its steady-state level after two quarters. The deflationary pressure brought about by the shock leads to a

²¹Similar analysis is available elsewhere; see Dib (2003a) for example.

²²Evidence also suggests the responses of inflation and output following monetary policy shocks is characterized by hump-shaped patterns, where the maximum impact on the variables is attained several periods after the shock. Christiano et al. (2005) show that adding several additional features to the model enables it to display these patterns. Because our emphasis is on the out-of-sample forecasting ability of the model and we want to keep the model parsimonious, we do not use such a model in our experiments.

sustained easing of monetary policy; recall the monetary policy rule in (24). This mechanism serves to accommodate the shock and gradually increase output, which peaks three quarters after the shock. Therefore, the monetary authority’s response helps the economy to adjust to the supply-side disturbances.

Figure 4 shows the impulse responses to a 1 per cent increase in the preference shock, an disturbance to households’ marginal utility of consumption. In response to this shock, output, the nominal interest rate, inflation, and money growth jump immediately above their steady-state levels before returning gradually to those levels. Because the estimates of the preference autoregressive coefficient, ρ_z , are relatively large, the computed impulse responses are highly persistent. To control the rises in output and inflation, the monetary authority increases short-term interest rates slightly, but persistently.

The estimation results indicate that our variant of the New Keynesian model can provide a coherent explanation for how several types of shocks affect the economy. Certain aspects of its in-sample performance, like the ability to produce hump-shaped responses in some variables following monetary tightenings, could be improved by adding some additional features. However, the purpose of the present paper is not to provide an extensive in-sample evaluation of various versions of the New Keynesian models, but rather to provide a quantitative assessment of the likely out-of-sample forecasting ability of the general paradigm. The model we use therefore provides a reasonably good laboratory with which we can perform our experiments.²³

4 The Model’s Forecasting Properties

4.1 The Experiment

We compute out-of-sample forecasts for the New Keynesian model (NK), for a simple VAR and for a Bayesian VAR (BVAR). The VAR(2) and BVAR(2) models are used as benchmarks²⁴ and they include separate, linear deterministic trends for output, real money balances, the nominal interest rates, and inflation. The BVAR model imposes the Minnesota prior of Doan et al.

²³Moreover, more complete versions of the New Keynesian model, although better at replicating in-sample features of the data, might not have always and everywhere the better out-of-sample forecasting accuracy. The limited number of parameters needed to estimate our model might be important where sampling uncertainty is a big issue. The results in Korenok and Swanson (2005) validate this conjecture, by showing that a pared-down version of the model performs better than more complete ones in some circumstances.

²⁴The number of lags in the VAR and BVAR benchmarks were chosen using Akaike’s information criterion.

(1984) (univariate random walks for all variables).²⁵

We begin by estimating the models using data from 1981Q3 through 1996Q4. These estimates are used to produce forecasts one- to eight-quarters-ahead, i.e. for 1997Q1 to 1998Q4, for the four variables used. We next use data from 1981Q4 to 1997Q1 to update the estimates, and then produce another set of forecasts for 1997Q2 through 1999Q1. Estimates and forecasts are updated in this manner until the end of the sample, at which point we have time series for one- to eight-quarter-ahead forecasts from 1997Q1 to 2004Q1 (Table 2 summarizes graphically the experiment).

Figure 5 to 8 compare the forecasts with actual data for the period. Figure 5 compares the model's forecasts with actual data, and shows that the model provides what appears to be a relatively good characterization of output fluctuations, for the one-quarter-ahead, four-quarter-ahead, and eight-quarter ahead horizons. The model maintains a reasonably balanced forecast for inflation, although the actual data exhibits some transitory fluctuations that are not well captured by the model. Further, the model is slow to incorporate the interest rate decreases of 2001 in its forecasts. Finally, the model's forecasts for money track are reasonably accurate.

Figures 6 to 8 show the forecasting errors of the model (the solid line) with those arising from the VAR benchmark (the dotted lines) for the case of one-quarter-ahead (Figure 6), four-quarter-ahead (Figure 7), and eight-quarter-ahead forecasts (Figure 8). In Figure 6, the two models appear to give forecasts that are roughly equivalent, except for output, where the VAR benchmark may produce smaller errors. At the four-quarter-ahead horizon (Figure 7), the NK model seems to outperform the benchmark for output interest rates and real money balances, whereas the inflation forecasts appear to be very close. At the eight-quarter horizon (Figure

²⁵Forecasting with the New Keynesian model is conducted with the state space system (31)-(32). For example, one-quarter-ahead forecasts are generated as follows:

$$\begin{aligned}\hat{s}_{T+1|T} &= \hat{\Phi}_1 \hat{s}_T; \\ \hat{d}_{T+1|T} &= \hat{\Phi}_3 \hat{s}_{T+1|T};\end{aligned}$$

where $\hat{\Phi}_1$ and $\hat{\Phi}_3$ arise from the parameters estimated using the subsample ending at time T .

The VAR model is as follows:

$$\mathbf{y}_t = \alpha + \delta t + \Theta_1 \mathbf{y}_{t-1} + \Theta_2 \mathbf{y}_{t-2} + \epsilon_t,$$

where the 4 by 1 vector \mathbf{y} contains the variables used in the estimation of the New Keynesian model: output, inflation, the interest rate and real money balances. The BVAR implements the Doan et al. (1984) prior by setting the own-lag parameters in Θ_1 to 1 and the cross parameters to 0, whereas all parameters in Θ_2 are 0. The strength of the prior controls the departure of the model from pure random walks: we follow the suggestions of Doan et al. (1984) to set it.

8), the model’s forecasting is better than the benchmark for output, interest rates, and money, while the inflation forecasts remain close.

The first column of Table 3 synthesizes the information contained in Figures 6-8. It reports the Mean Square Error (MSE) of the New Keynesian model, relative to that of the VAR benchmark. Values smaller than one suggest that the NK model has superior forecasting accuracy, while values bigger than one favour the VAR benchmark. As suggested earlier, the MSEs tend to favour the NK model, particularly as the forecasting horizon increases. In particular, at the eight-quarter ahead horizon, the model’s MSE for output is only 19% of the VAR benchmark MSE. For real balances and interest rates, the advantage to the NK model is less important but still substantial. For inflation, the NK model does not appear to forecast significantly better, but does not forecast worse either.²⁶ Table 3 also shows that, for very short-term horizons, the advantage for the NK models vanishes for some variables: the VAR benchmark appears to be more accurate in forecasting one-quarter-ahead and two-quarter-ahead output.

4.2 Econometric Tests of Forecasting Accuracy

To test whether these improvements in MSE are statistically significant, we first use Diebold and Mariano (1995)’s test. Let the forecast errors of the New Keynesian model be $\{e_t^M\}_{t=1}^T$ and those from the VAR(2) benchmark $\{e_t^B\}_{t=1}^T$. Further, define a sequence of ‘loss differentials’ $\{l_t\}_{t=1}^T$ where $l_t = (e_t^B)^2 - (e_t^M)^2$. If the NK model is a better forecasting tool, one would expect that, on average, the loss differentials l_t would be positive. Conversely, one would expect negative values if the VAR benchmark is superior. Following this intuition, the Diebold and Mariano (1995) test considers the null hypothesis $H_0 : E[l_t] = 0$; positive values of the statistic suggest that the forecasts from the New Keynesian model have lower mean-squared errors, while negative values favour the VAR benchmark. The test statistic (denoted DM) is asymptotically normal and standard critical values are used.²⁷ Harvey et al. (1997) propose a corrected Diebold and Mariano (1995) statistic, in order to reduce size distortions that might be significant in small samples such as ours. The corrected statistic is compared to a Student’s t distribution with $N - 1$ degrees of freedom, where N is the number of forecasted data.

²⁶In the working paper version of this work, (Dib et al., 2006), we reported less favourable results for inflation when estimation was conducted following a *recursive* scheme in which parameter estimates are updates as more data is included without discarding older data.

²⁷The statistic is computed as $DM = l/\hat{\sigma}(l)$ where l is the sample average of l_t and $\hat{\sigma}(l)$ is a heteroscedastic and autocorrelation(HAC)-consistent estimate of the standard deviation of l .

The last two columns of Table 3 report the Diebold and Mariano (1995) and Harvey et al. (1997) statistics, as well as their p -values in parenthesis. Due to the small number of forecasts available (26 for the one-quarter-ahead forecasts, and 18 for the eight-quarter ahead one), it is not surprising that many test statistics are not significant. Nevertheless, Table 3 shows that the NK model’s forecasting accuracy compares very favourably with that of the VAR(2) benchmark, performing significantly better for output and interest rates at some of the longer-term horizons.²⁸ As indicated above, the New Keynesian model performs less impressively for inflation. This result echoes the one presented in Korenok and Swanson (2005), where a New Keynesian model similar to ours is found to have lesser predictive power for inflation than extensions of the model including more sophisticated inflation indexing as proposed in Smets and Wouters (2003) and Christiano et al. (2005).²⁹ This fact probably arises because as estimated, the model does not allow trends to affect inflation, when ample anecdotal or econometric evidence suggests that structural breaks have affected inflation over the last two decades. The conclusion discusses one possibility for future research on New Keynesian model to tackle this important issue.

A non-parametric alternative to the Diebold and Mariano (1995) test is the Sign test. The test statistic is simply the number of times the VAR benchmark delivers a smaller MSE than the NK counterpart, which can under some assumptions, be modeled as draws from a binomial distribution. A high number of times when the VAR benchmark delivers a lower MSE naturally suggest that the benchmark possesses the better forecasting accuracy.³⁰ Table 4 reports the results from this test. They confirm those in Table 3, in that at longer horizons, the NK model performs better than the VAR benchmark for output and interest rates, whereas for very short horizons, the VAR benchmark forecasts output better than the NK paradigm. For inflation, no significant advantage can be identified.

An alternative measure derived from the New Keynesian forecasts is to ask whether there is *any* information in these forecasts that is not already contained in the VAR forecasts. If so (perhaps because one model forecasts better than the other at specific times over the business cycle), combining both forecasts could reduce forecasting errors.

In this context, Granger and Newbold (1973) define the forecasts from one model as “conditionally efficient” if combining them with those from another model does not lead to an overall

²⁸This favourable performance is also obtained when the New Keynesian model is compared to a VAR with one lag in each variable. These additional results are available from the authors.

²⁹However, the authors also report that for output, the simple model actually performs better.

³⁰The test is discussed in more detail in Diebold and Mariano (1995).

decrease in forecast accuracy. Chong and Hendry (1986) define the same situation as one where the first set of forecasts “encompass” those from the second model: there is no need to keep the second model’s forecasts because the information they contain is encompassed by those of the first model.

To implement the test for forecast encompassing, we follow Harvey et al. (1998), which propose test statistics similar to those in Diebold and Mariano (1995) and its Harvey et al. (1997) correction. The null hypothesis is that the New Keynesian model’s forecasts contain no information that isn’t already contained in those from the VAR.³¹

Table 5 reports the results. The first column shows the test statistic proposed by Diebold and Mariano (1995) and the second the correction proposed by Harvey et al. (1997). Recall that high values of the test statistics reject the hypothesis that no value can be gained from using the NK forecasts when the VAR model is available. Overall, the results in Table 5 decisively reject the hypothesis that the VAR forecasts cannot be improved when combined with those from the NK model, even in the case of inflation. Said otherwise, the test strongly suggest that a combination of the two sets of forecasts would be stronger than those of the VAR benchmark alone. Although the forecast encompassing test is weaker in spirit than the ‘horse race’ context of the Diebold and Mariano (1995) test, it presents important evidence that forecasts arising from New Keynesian models can provide a useful contribution to forecasting Canadian data.

Researchers have often pointed out that imposing the Minnesota prior –all variables follow univariate random walks– on the Bayesian estimation of simple VARs delivers superior forecasting accuracy. In this context, Table 6 repeats the results of Table 3, but with a BVAR(2) serving as the benchmark with which to compare the NK model. A comparison of the two tables shows that the forecasting accuracy of the BVAR is indeed often superior to what it was without the priors (said otherwise, the relative MSE reported are often higher than in Table 3). Nevertheless, similar observations about the model’s forecasting properties can be made: the NK model can forecast output, interest rates, and money well compared to the benchmark, and as the forecasting horizon increases, several of these differences become statistically significant. However, the results from the Diebold and Mariano (1995) test now reject that the forecasting

³¹Assume the following regression equation:

$$e_t^B = \gamma(e_t^B - e_t^M) + \epsilon_t,$$

where e_t^B and e_t^M represent the forecasting errors from the VAR benchmark model and the NK model, respectively. The null hypothesis is $H_0 : \gamma = 0$. Under the null, the errors made by the VAR benchmark cannot be explained (and thus potentially reduced) by information arising from the NK model.

advantages of the NK model are significant; for interest rates and for long horizons, some of the test still present significant differences.

4.3 Discussion and Econometric Issues

Three caveats need to be discussed when assessing the results of Table 3 to Table 6. First, the two sets of forecasts that enter into the computation of the Diebold and Mariano (1995) test rely on the estimated values of model parameters, rather than population values. As discussed in McCracken and West (2002) and Corradi and Swanson (forthcoming), parameter estimation error might thus affect the limiting distribution of the test statistic and therefore the inference taken from it. In particular, since parameter estimation error introduces an additional variance factor, the test might reject the null hypothesis of equal forecasting accuracy too often. In that context, the instances where we reported that the New Keynesian model had a significantly superior forecasting accuracy relative to that of the VAR benchmarks should be interpreted with caution. Controlling for the potential biases introduced by parameter estimation error would therefore strengthen our econometric analysis.

Second, in Table 3 to Table 6, we have assessed the relative forecasting accuracy of the New Keynesian model for each of the four variables individually. It might then be difficult to interpret instances where the NK model performs better for one variable but worse for another.³² Further, the Diebold and Mariano (1995) test assumes that it is point forecasts that are of interest, whereas it might be important to assess how well the model predicts the whole spectrum of possible values.

Computing density forecasts from both models and then comparing them has the potential to offer such a multivariate, across possible outcomes, comparison between the two models. Corradi and Swanson (2005) contains a thorough discussion of the issues involved in computing and comparing density forecasts and the proposed tests in the literature. As a first step towards a comprehensive assessment of the models' density forecasts, we apply the test developed in Amisano and Giacomini (forthcoming). Let $\log \hat{f}_{NK,t}(\mathbf{Y}_{t+1})$ be the predicted density function for our four variables (in vector \mathbf{Y}) at time $t + 1$, arrived at by estimating the NK model with data up to time t , but evaluated at the data point, \mathbf{Y}_{t+1} , that actually occurred. Intuitively, this logarithmic score rewards a density forecast that assigns a high probability to the events that actually occurred. By defining an equivalent quantity for the benchmark VAR and comparing

³²In addition, this strategy introduces sequential test bias, where applying the same test several times potentially leads to critical values being incorrectly sized.

the two all forecasts, the test can suggest the model with the superior accuracy for density forecasts: if higher score values for the density forecasts arise from the New Keynesian model (compared to those from the VAR benchmark) this naturally suggest superior forecasting power for the model.³³

Table 7 reports the result of the test. The positive and relatively high value of the test statistic, 0.96 suggests that the New Keynesian model’s predictions for the density compare well to those of the VAR benchmark; however, the second column of the table shows that the difference between the log scores are not statistically significant.

Finally, note that the tests we use rely on asymptotic arguments to establish critical values. Bootstrapping methods, although at first more computationally intensive, can provide less fragile assessments of critical values and should be explored in a comprehensive examination of the New Keynesian model’s forecasting accuracy. In such future work, the bootstrap-based evaluation methods in Corradi and Swanson (forthcoming) (for point forecasts) and Corradi and Swanson (2006) (for density forecasts) would likely play the leading role.

Considering these caveats, what can one safely conclude from our results about the forecasting accuracy of the New Keynesian model for Canadian data? Our position is as follows: while no overwhelming evidence has been presented that the model possesses superior forecasting accuracy to popular time-series benchmarks like VARs, our results nevertheless strongly indicate that the model could provide an important contribution to the forecasting of Canada time series.³⁴

5 Conclusion

Since the introduction of real business cycle models, researchers have often identified dimensions along which these structural models seemed at odds with features of observed data. Further, extensions of the simple real business structure to New Keynesian models with nominal rigidities and multiple sources of volatility often had difficulties replicating observed features of the data, like the strong autocorrelation properties of inflation or output, unless several layers of shocks

³³Let $\log \hat{f}_{VAR,t}(Y_{t+1})$ be the similar statistic for the benchmark VAR and $LR_t = \log \hat{f}_{NK,t}(Y_{t+1}) - \log \hat{f}_{VAR,t}(Y_{t+1})$ the difference between the two scores. The statistic is computed as $AG = \frac{\overline{LR}}{\hat{\sigma}_{LR}/\sqrt{n}}$ where \overline{LR} is the sample average of LR_t and $\hat{\sigma}_{LR}$ is a heteroscedastic and autocorrelation(HAC)-consistent estimate of its standard deviation.

³⁴For example, it seems unlikely that the results on forecast encompassing –it pays to combine the NK forecasts with those from the VAR benchmarks– will be overturned by more refined econometric tests.

and frictions are introduced in the models. The emerging evidence, presented in this paper and in other recent contributions³⁵, showing that simple New Keynesian models may have comparable or even better out-of-sample forecasting accuracy relative to simple time series benchmarks may thus seem surprising.

This evidence suggests that restrictions derived from DSGE specifications may help models outperform unrestricted time series benchmarks in out-of-sample exercises. The main trade-off identified by Clements and Hendry (1998) in discussing this conjecture is that of sampling variability (introduced in the unrestricted specification by the estimation of numerous parameters) versus inconsistency (introduced in restricted models by imposing possibly false restrictions).

The econometric tests we report show that, at a minimum, restricting a VAR by appealing to the structure of the New Keynesian model has no significant, negative impact on its forecasting performance for Canadian data. Additionally, in the case of some variables like output, the restricted model may in fact have superior forecasting accuracy, particularly as the forecasting horizon increases. As the results show, the forecasting properties of the model for inflation are not as strong, although there are not significantly worse than those of the benchmark VARs.

The first dimension along which our results can be extended is to develop an open-economy specification of our experiments. This will allow both the New Keynesian model and the benchmarks to capture information related to external (principally American) data and the various channels by which they affect the Canadian economy; such channels could prove particularly useful for forecasting Canadian time series.

Second, introducing shifts to the inflation target of monetary authorities may allow the models to better track the recent observed downward trend in inflation. To introduce such a feature, the regime switching environment in Erceg and Levin (2003), where the inflation target of monetary authorities is periodically modified, could be employed. Such an extension could prove particularly important for forecasting the inflation and nominal interest rate series.

Third, it would be important to study whether a difference stationary process for technology, rather than the trend-stationary we employ, would modify our assessment of the forecasting accuracy of the New Keynesian model for Canada. Used with American data by Such a specification of trend growth³⁶ would imply a common trends environment for variables like output,

³⁵As mentioned in the introduction, Ingram and Whiteman (1994) and DeJong, Ingram and Whiteman (2000) and more recent papers like Korenok and Swanson (2005), Adolfson et al. (2006) and Del Negro et al. (forthcoming) contribute to this evidence

³⁶Such specifications are adapted to American data by Ireland (2001b), Korenok and Swanson (2005), and Del Negro et al. (forthcoming) as well as to Euro-area data by Adolfson et al. (2006).

consumption and investment; the natural benchmark to compare forecasts would therefore be a VAR in differences, possibly with cointegrating vectors.³⁷

The comparison carried out in the present paper, between the New Keynesian model and its time series benchmarks, should be interpreted as a first step in a series of increasingly sophisticated econometric assessments of the out-of-sample forecasting accuracy of that model. The model will need to be confronted to several time-series benchmarks at once, in a manner that is less dependent on the relatively small number of data points, and by assessing the entire range of rather than simply checking point estimates; these tasks can be executed using recent bootstrap-based contributions to forecast evaluation in Corradi and Swanson (2005, 2006, forthcoming). Overall however, our results are encouraging for researchers working with New Keynesian models, so that more formal assessment of the forecasting accuracy of these models will likely remain an important and fruitful area of research.

References

- M. Adolfson, J. Lind, and M. Villani. Forecasting performance of an open economy dynamic stochastic general equilibrium model. Sveriges Riksbank Working Paper No. 190, June 2006.
- G. Amisano and R. Giacomini. Comparing density forecasts via weighted likelihood ratio tests. *Journal of Business and Economic Statistics*, forthcoming.
- P. R. Bergin. Putting the “New Open Economy Macroeconomics” to a test. *Journal of International Economics*, 60:3–34, 2003.
- O. J. Blanchard and C. M. Kahn. The solution of linear difference models under rational expectations. *Econometrica*, 48:1305–11, 1980.
- J. Boivin and M. Giannoni. DSGE models in a data-rich environment. Working Paper, March 2005.
- H. Bouakez, E. Cardia, and F. Ruge-Murcia. Habit formation and the persistence of monetary shocks. *Journal of Monetary Economics*, 52:1073–88, 2005.

³⁷Ireland (2001b) presents formal comparison between estimating trend-stationary or difference-stationary RBC models and reports that the better out-of-sample forecasting accuracy arises from the trend stationary specification.

- G. A. Calvo. Staggered prices in a utility-maximizing framework. *Journal of Monetary Economics*, 12:383–398, 1983.
- Y. Chong and D. Hendry. Econometric evaluation of linear macro-economic models. *Review of Economic Studies*, 53:671–690, 1986.
- L.J. Christiano, M. Eichenbaum, and C. Evans. Nominal rigidities and the dynamic effects of a shock to monetary policy. *Journal of Political Economy*, 113:1–45, 2005.
- M. P. Clements and D. F. Hendry. *Forecasting Economic Time Series*. Cambridge University Press, 1998.
- V. Corradi and N. R. Swanson. Predictive density evaluation. In C. W. J. Granger, G. Elliot, and A. Timmerman, editors, *Handbook of Economic Forecasting*, pages 197–284. Elsevier, Amsterdam, 2005.
- V. Corradi and N. R. Swanson. Predictive density and conditional confidence interval accuracy tests. *Journal of Econometrics*, 135:187–228, 2006.
- V. Corradi and N. R. Swanson. Nonparametric bootstrap procedures for predictive inference based on recursive estimation schemes. *International Economic Review*, forthcoming.
- M. Del Negro and F. Schorfheide. Priors from general equilibrium models for VARs. *International Economic Review*, 45:643–673, 2004.
- M. Del Negro, F. Schorfheide, F. Smets, and R. Wouters. On the fit of New-Keynesian models. *Journal of Business and Economic Statistics*, forthcoming.
- A. Dib. An estimated Canadian DSGE model with nominal and real rigidities. *Canadian Journal of Economics*, 36:949–972, 2003a.
- A. Dib. Monetary policy in estimated models of small open and closed economies. Bank of Canada Working Paper No. 2003-27, September 2003b.
- A. Dib. Nominal rigidities and monetary policy in Canada. *Journal of Macroeconomics*, 28:285–304, 2006.
- A. Dib, M. Gamoudi, and K. Moran. Forecasting canadian time series with the new keynesian model. Bank of Canada Working Paper No. 2006-04, March 2006.

- F. X. Diebold and R. S. Mariano. Comparing predictive accuracy. *Journal of Business and Economic Statistics*, 13:253–263, 1995.
- T. Doan, R. Litterman, and C. Sims. Forecasting and conditional projections using realistic prior distributions. *Econometric Review*, 3:1–100, 1984.
- V. Dolar and K. Moran. Estimated DGE models and forecasting accuracy: A preliminary investigation with Canadian data. Bank of Canada Working Paper No. 2002-18, July 2002.
- C. J. Erceg and A. T. Levin. Imperfect credibility and inflation persistence. *Journal of Monetary Economics*, 50:915–944, 2003.
- C. W. J. Granger and P. Newbold. Some comments on the evaluation of economic forecasts. *Applied Economics*, 5:35–47, 1973.
- J. D. Hamilton. *Time Series Analysis*. Princeton University Press, Princeton, 1994.
- D. Harvey, S. Leybourne, and P. Newbold. Testing the equality of prediction mean squared errors. *International Journal of Forecasting*, 13:281–291, 1997.
- D. Harvey, S. Leybourne, and P. Newbold. Tests for forecast encompassing. *Journal of Business and Economic Statistics*, 15:254–259, 1998.
- B. F. Ingram and C. H. Whiteman. Supplanting the Minnesota prior: Forecasting macroeconomic times series using real business cycle model priors. *Journal of Monetary Economics*, 34:497–510, 1994.
- P. N. Ireland. A small, structural, quarterly model for monetary policy evaluation. *Carnegie-Rochester Conference Series on Public Policy*, 47:83–108, 1997.
- P. N. Ireland. Sticky-price models of the business cycle: Specification and stability. *Journal of Monetary Economics*, 47:3–18, 2001a.
- P. N. Ireland. Technology shocks and the business cycle: An empirical investigation. *Journal of Economic Dynamics and Control*, 25:703–719, 2001b.
- P. N. Ireland. Endogenous money or sticky prices. *Journal of Monetary Economics*, 47:3–18, 2003.

- P. N. Ireland. A method for taking models to the data. *Journal of Economic Dynamics and Control*, 47:3–18, 2004.
- O. Korenok and N. R. Swanson. The incremental predictive information associated with using theoretical New Keynesian models vs. simple linear econometric models. *Oxford Bulletin of Economics and Statistics*, 67:905–930, 2005.
- B. T. McCallum and E. Nelson. Nominal income targeting in an open-economy optimizing model. *Journal of Monetary Economics*, 43:553–578, 1999.
- M. W. McCracken and K. W. West. Inference about predictive ability. In M. P. Clements and D. F. Hendry, editors, *A Companion to Economic Forecasting*, pages 299–321. Blackwell, Oxford, 2002.
- F. Smets and R. Wouters. An estimated stochastic dynamic general equilibrium model of the euro area. *Journal of the European Economic Association*, 1:123–1175, 2003.
- F. Smets and R. Wouters. Comparing shocks and frictions in US and Euro area business cycles: A Bayesian DSGE approach. *Journal of Applied Econometrics*, 20:161–183, 2005.
- J. B. Taylor. Discretion versus policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy*, 39:195–214, 1993.
- M. Woodford. *Interest and Prices; Foundations of a Theory of Monetary Policy*. Princeton University Press, 2003.
- T. Yun. Nominal price rigidity, money supply endogeneity, and business cycles. *Journal of Monetary Economics*, 37:345–30, 1996.

Table 1: Maximum-likelihood estimates and standard errors (1981Q3 to 1996Q4)

Parameter	Estimate	Std. Deviation	<i>t</i> -statistic
β	0.990	0.002	609.2
γ	0.056	0.016	3.48
ϱ_π	0.925	0.192	4.82
ϱ_μ	0.578	0.155	3.74
ϱ_y	0.001	0.037	0.03
ρ_v	0.239	0.084	2.85
σ_v	0.007	0.001	6.51
ϕ	0.639	0.065	9.85
A	3.507	0.167	20.91
ρ_A	0.899	0.061	14.64
σ_A	0.016	0.003	4.75
b	0.492	0.066	7.48
ρ_b	0.996	0.005	194.36
σ_b	0.011	0.001	10.1
ρ_z	0.907	0.035	25.82
σ_z	0.023	0.004	5.41
π	1.010	0.003	398.73
η	1.003	0.001	847.91
LL	918.5924		

Note: LL is the maximum log-likelihood value.

Table 2. The Forecasting Experiment (1997Q1 - 2004Q1)

Estimate	Forecast k periods ahead				
	$k = 1$	$k = 2$	$k = 3$	\dots	$k = 8$
1981Q3 \longrightarrow 1996Q4	1997Q1	1997Q2	1997Q3	\dots	1998Q4
1981Q4 \longrightarrow 1997Q1	1997Q2	1997Q3	1997Q4	\dots	1999Q1
1982Q1 \longrightarrow 1997Q2	1997Q3	1997Q4	1998Q1	\dots	1999Q2
1982Q2 \longrightarrow 1997Q3	1997Q4	1998Q1	1998Q2	\dots	1999Q3
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
1988Q1 \longrightarrow 2003Q2	2003Q3	2003Q4	2004Q1	---	---
1988Q2 \longrightarrow 2003Q3	2003Q4	2004Q1	---	---	---
1988Q3 \longrightarrow 2003Q4	2004Q1	---	---	---	---

Table 3: Testing for Equal Forecasting Accuracy
Model and VAR(2) Benchmark, 1997Q1 - 2004Q1

Variable	Relative MSE ^a	DM stat. ^b (<i>p</i> -value)	HLN stat. ^c (<i>p</i> -value)
<i>Forecasting One Period Ahead</i>			
Output	2.13	-2.14 (0.03)	-2.10 (0.05)
Inflation	0.87	1.21 (0.23)	1.19 (0.25)
Interest Rate	0.62	1.62 (0.11)	1.59 (0.12)
Money	0.87	0.68 (0.50)	0.66 (0.51)
<i>Forecasting Two Periods Ahead</i>			
Output	1.29	-0.59 (0.56)	-0.55 (0.58)
Inflation	0.86	0.95 (0.34)	0.90 (0.38)
Interest Rate	0.53	1.83 (0.07)	1.73 (0.09)
Money	0.74	1.11 (0.27)	1.05 (0.31)
<i>Forecasting Four Periods Ahead</i>			
Output	0.57	0.80 (0.42)	0.69 (0.49)
Inflation	0.88	1.30 (0.19)	1.13 (0.27)
Interest Rate	0.51	1.80 (0.07)	1.55 (0.13)
Money	0.64	0.79 (0.43)	0.68 (0.50)
<i>Forecasting Six Periods Ahead</i>			
Output	0.28	1.68 (0.09)	1.29 (0.21)
Inflation	0.83	1.53 (0.13)	1.18 (0.25)
Interest Rate	0.38	2.42 (0.02)	1.86 (0.08)
Money	0.47	0.74 (0.46)	0.57 (0.57)
<i>Forecasting Eight Periods Ahead</i>			
Output	0.19	2.23 (0.03)	1.47 (0.16)
Inflation	0.95	NA (NA)	NA (NA)
Interest Rate	0.32	22.48 (0.00)	14.81 (0.00)
Money	0.32	0.78 (0.44)	0.51 (0.61)

^aMSE (New Keynesian Model) / MSE (VAR Benchmark); values smaller than 1 suggest superior performance by the New Keynesian model.

^bTest statistic from the Diebold and Mariano (1995) test. The null hypothesis posits equal forecasting accuracy between the two models. The statistic is asymptotically normal.

^cHarvey et al. (1997)'s correction of the Diebold and Mariano (1995) test. The statistic follows a t_{N-1} distribution, with N denoting the number of forecasts.

Table 4: Sign Test of Equal Forecasting Accuracy
Model and VAR(2) Benchmark, 1997Q1 - 2004Q1

Variable	Times Benchmark Wins ^a	Test Statistic. ^b (<i>p</i> -value)
<i>Forecasting One Period Ahead</i>		
Output	19	1.86 (0.06)
Inflation	12	-0.74 (0.48)
Interest Rate	9	-1.86 (0.06)
Money	13	-0.37 (0.71)
<i>Forecasting Two Periods Ahead</i>		
Output	14	0.00 (1.00)
Inflation	13	-0.38 (0.71)
Interest Rate	9	-1.89 (0.06)
Money	12	-0.76 (0.45)
<i>Forecasting Four Periods Ahead</i>		
Output	12	-0.39 (0.70)
Inflation	14	0.39 (0.70)
Interest Rate	6	-2.75 (0.00)
Money	12	-0.39 (0.70)
<i>Forecasting Six Periods Ahead</i>		
Output	7	-2.04 (0.04)
Inflation	13	0.41 (0.68)
Interest Rate	1	-4.49 (0.00)
Money	11	-0.41 (0.68)
<i>Forecasting Eight Periods Ahead</i>		
Output	8	-1.28 (0.20)
Inflation	10	-0.43 (0.67)
Interest Rate	1	-4.26 (0.00)
Money	13	0.85 (0.39)

^aNumber of times the (squared) forecasting error of the VAR benchmark is lower than the corresponding forecasting error of the New Keynesian model.

^bThe null hypothesis is that the median difference in the squared forecasting errors of the two models is 0. The statistic is asymptotically normal.

Table 5: Forecast Encompassing: Does the NK model provide *any* information not contained in the VAR(2) Benchmark?

Variable	DM stat. ^a (<i>p</i> -value)	HLN stat. ^b (<i>p</i> -value)
<i>Forecasting One Period Ahead</i>		
Output	3.18 (0.00)	3.12 (0.00)
Inflation	1.90 (0.03)	1.87 (0.04)
Interest Rate	2.90 (0.00)	2.85 (0.00)
Money	1.99 (0.02)	1.95 (0.03)
<i>Forecasting Two Periods Ahead</i>		
Output	3.46 (0.00)	3.27 (0.00)
Inflation	1.69 (0.05)	1.60 (0.06)
Interest Rate	2.39 (0.00)	2.26 (0.02)
Money	2.21 (0.01)	2.09 (0.02)
<i>Forecasting Four Periods Ahead</i>		
Output	2.22 (0.01)	1.92 (0.03)
Inflation	1.87 (0.03)	1.62 (0.06)
Interest Rate	2.21 (0.01)	1.91 (0.03)
Money	1.45 (0.07)	1.26 (0.11)
<i>Forecasting Six Periods Ahead</i>		
Output	2.56 (0.00)	1.97 (0.03)
Inflation	2.08 (0.02)	1.61 (0.06)
Interest Rate	2.88 (0.00)	2.22 (0.02)
Money	1.12 (0.13)	0.86 (0.20)
<i>Forecasting Eight Periods Ahead</i>		
Output	2.82 (0.00)	1.86 (0.04)
Inflation	3.67 (0.00)	2.42 (0.00)
Interest Rate	NA (NA)	NA (NA)
Money	1.00 (0.16)	0.67 (0.26)

^aTest statistic proposed by Harvey et al. (1998). The null hypothesis is that the forecasts from the NK model provide no information not already contained in those from the VAR benchmark.

^bHarvey et al. (1997)'s correction. The statistic follows a t_{N-1} distribution

Table 6: Testing for Equal Forecasting Accuracy
Model and BVAR(2) Benchmark, 1997Q1 - 2004Q1

Variable	Relative MSE ^a	DM stat. ^b (<i>p</i> -value)	HLN stat. ^c (<i>p</i> -value)
<i>Forecasting One Period Ahead</i>			
Output	2.87	-2.92 (0.00)	-2.88 (0.00)
Inflation	0.85	0.89 (0.36)	0.87 (0.39)
Interest Rate	0.88	0.49 (0.63)	0.48 (0.64)
Money	1.02	-0.08 (0.93)	-0.08 (0.94)
<i>Forecasting Two Periods Ahead</i>			
Output	1.88	-1.23 (0.22)	-1.16 (0.25)
Inflation	0.75	1.53 (0.13)	1.45 (0.16)
Interest Rate	0.84	0.56 (0.57)	0.53 (0.60)
Money	1.03	-0.11 (0.91)	-0.11 (0.92)
<i>Forecasting Four Periods Ahead</i>			
Output	0.76	0.36 (0.73)	0.30 (0.77)
Inflation	0.81	1.64 (0.11)	1.42 (0.17)
Interest Rate	0.69	1.13 (0.26)	0.98 (0.34)
Money	0.88	0.26 (0.79)	0.23 (0.82)
<i>Forecasting Six Periods Ahead</i>			
Output	0.36	1.12 (0.26)	0.87 (0.40)
Inflation	0.69	NA (NA)	NA (NA)
Interest Rate	0.52	2.51 (0.01)	1.94 (0.07)
Money	0.66	0.58 (0.56)	0.45 (0.66)
<i>Forecasting Eight Periods Ahead</i>			
Output	0.23	1.18 (0.24)	0.78 (0.45)
Inflation	0.86	NA (NA)	NA (NA)
Interest Rate	0.42	NA (NA)	NA (NA)
Money	0.45	0.87 (0.39)	0.57 (0.57)

^aMSE (New Keynesian Model) / MSE (VAR Benchmark); values smaller than 1 suggest superior performance by the New Keynesian model.

^bTest statistic from the Diebold and Mariano (1995) test. The null hypothesis posits equal forecasting accuracy between the two models. The statistic is asymptotically normal.

^cHarvey et al. (1997)'s correction of the Diebold and Mariano (1995) test. The statistic follows a t_{N-1} distribution, with N denoting the number of forecasts.

Table 7: Testing for Equal Density Forecasting Accuracy

Model and VAR(2) Benchmark, 1997Q1 - 2004Q1, one-step-ahead density forecasts

Test statistic ^a	Significance
0.96	0.34

^aTest statistic from the Amisano and Giacomini (forthcoming) test. The null hypothesis is that of equal log density scores. The test statistic is asymptotically normal

Figure 1: A Monetary Policy Tightening
(Shock occurs at $t = 5$)

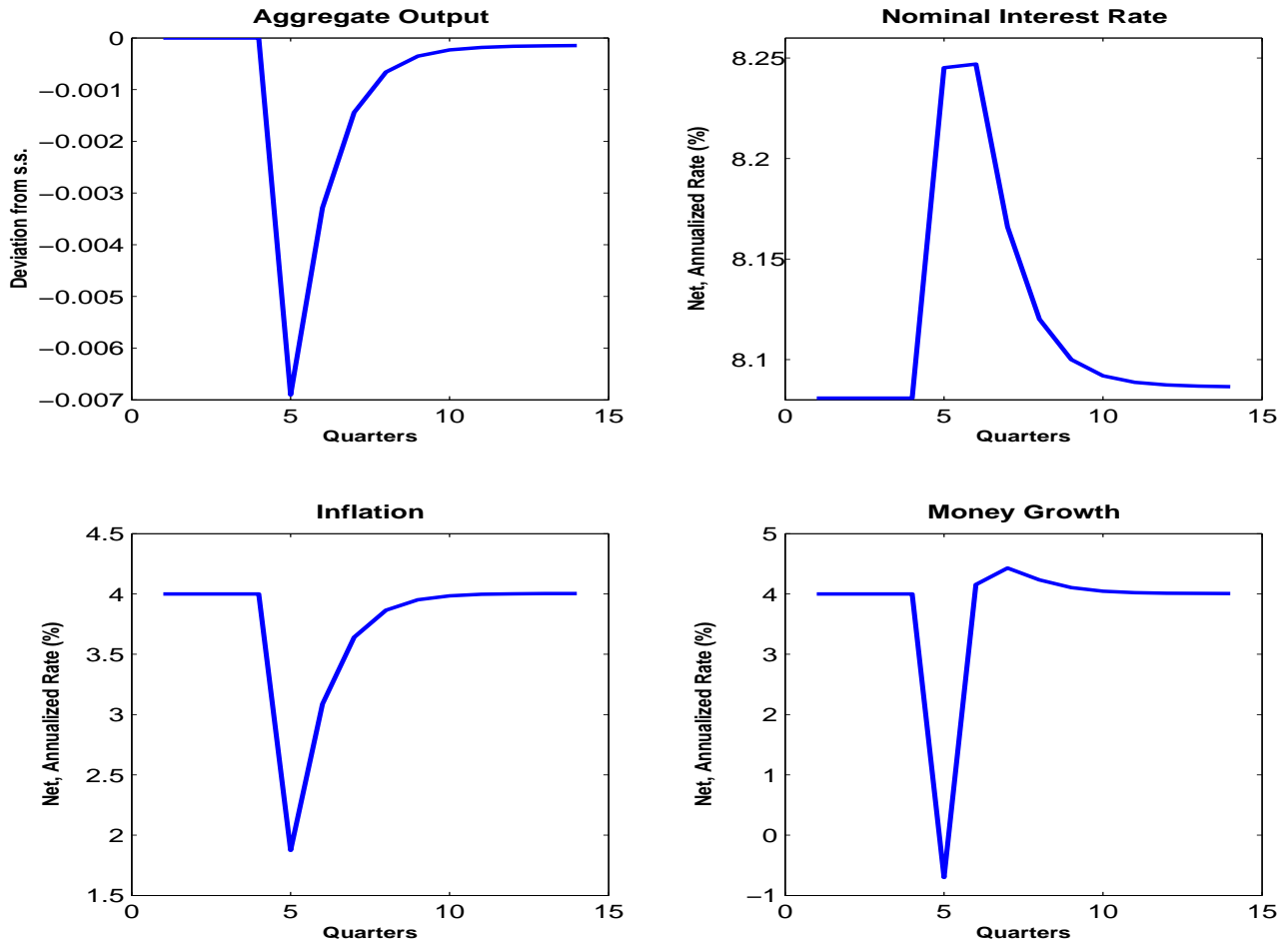


Figure 2: The Economy's Response to a Positive Money-Demand Shock
 (Shock occurs at $t = 5$)

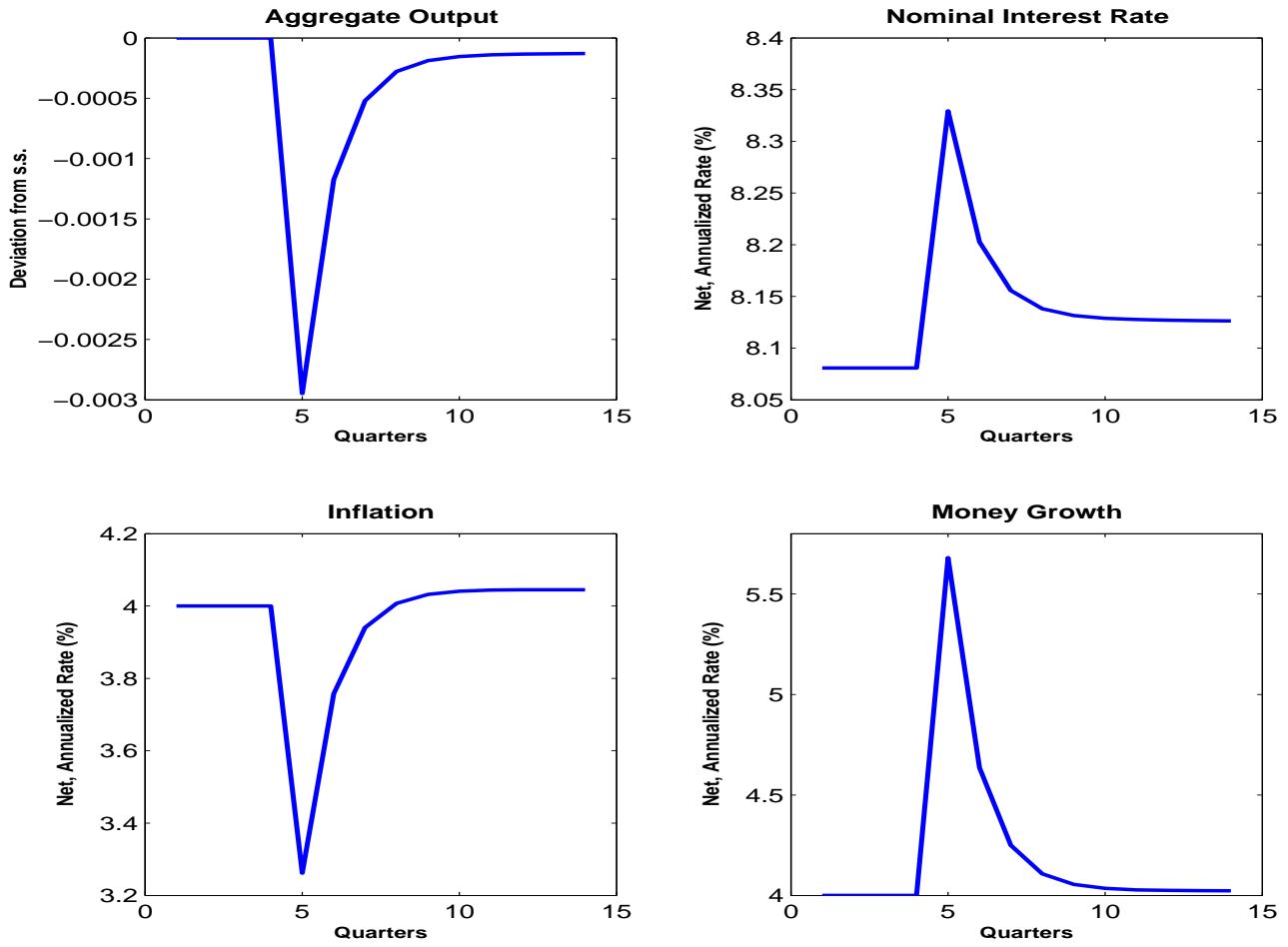


Figure 3: The Economy's Response to a Positive Technology Shock
 (Shock occurs at $t = 5$)

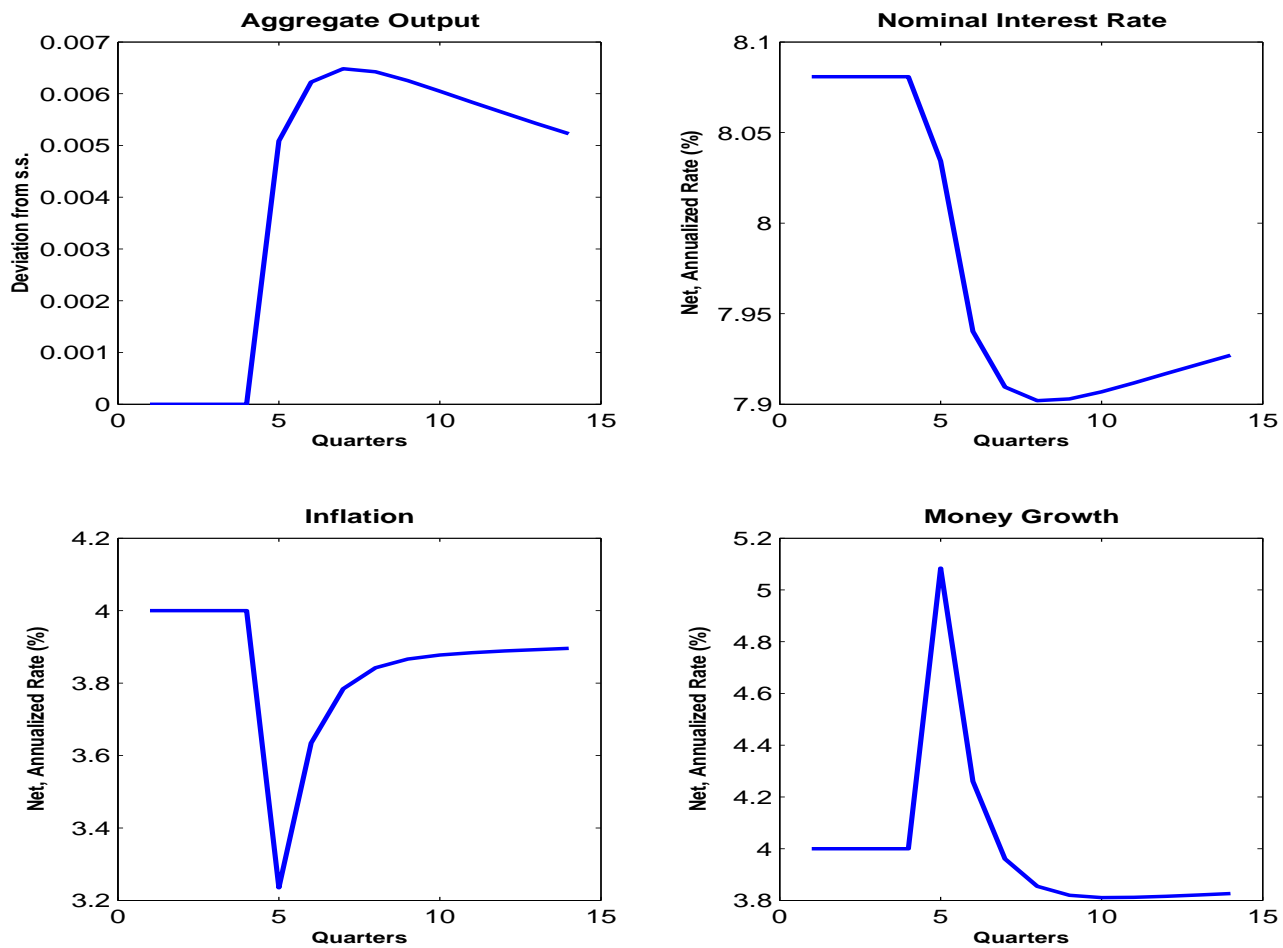


Figure 4: The Economy's Response to a Positive Preference Shock

(Shock occurs at $t = 5$)

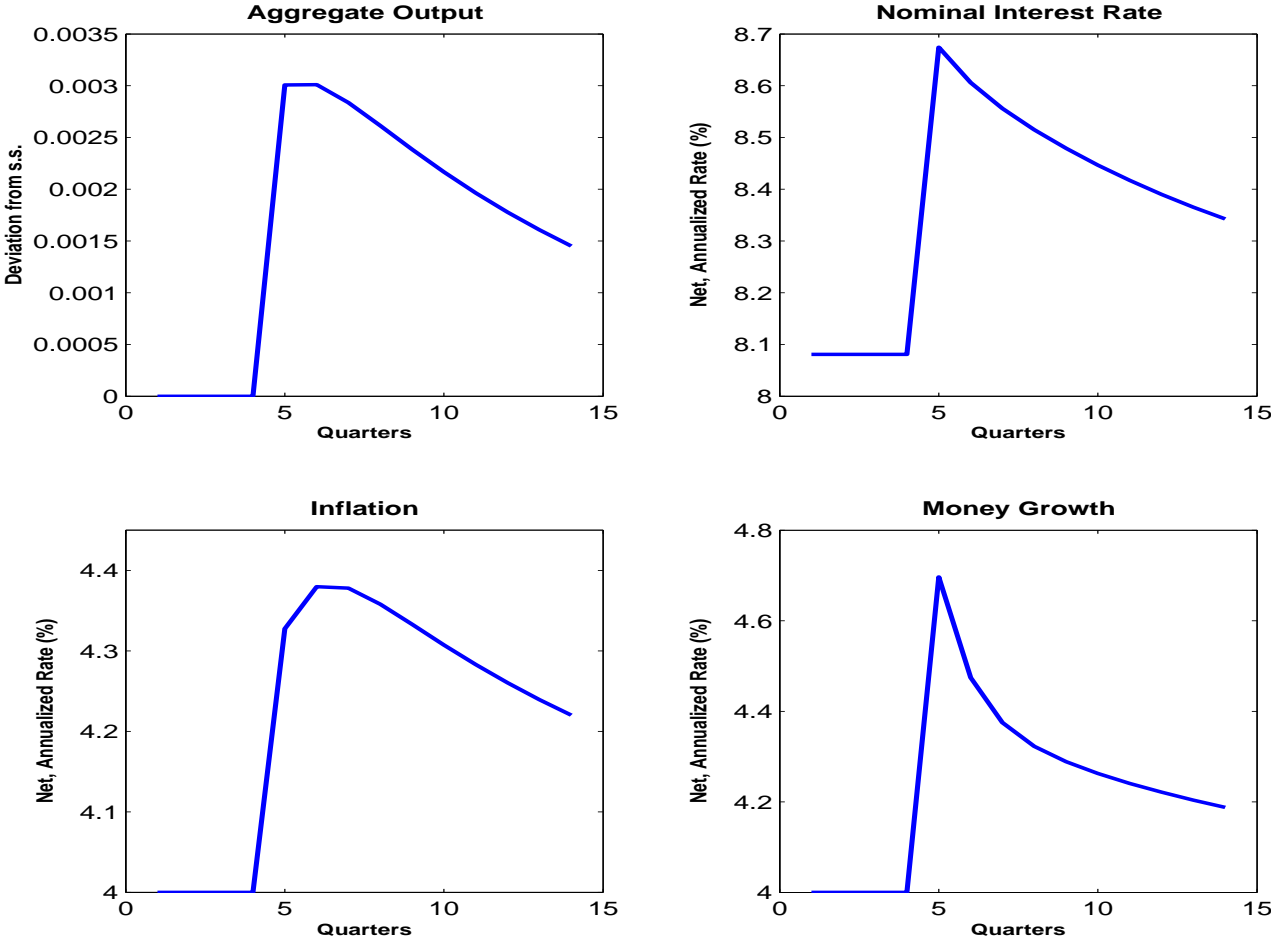


Figure 5. Actual Data and Forecasts from the New Keynesian Model

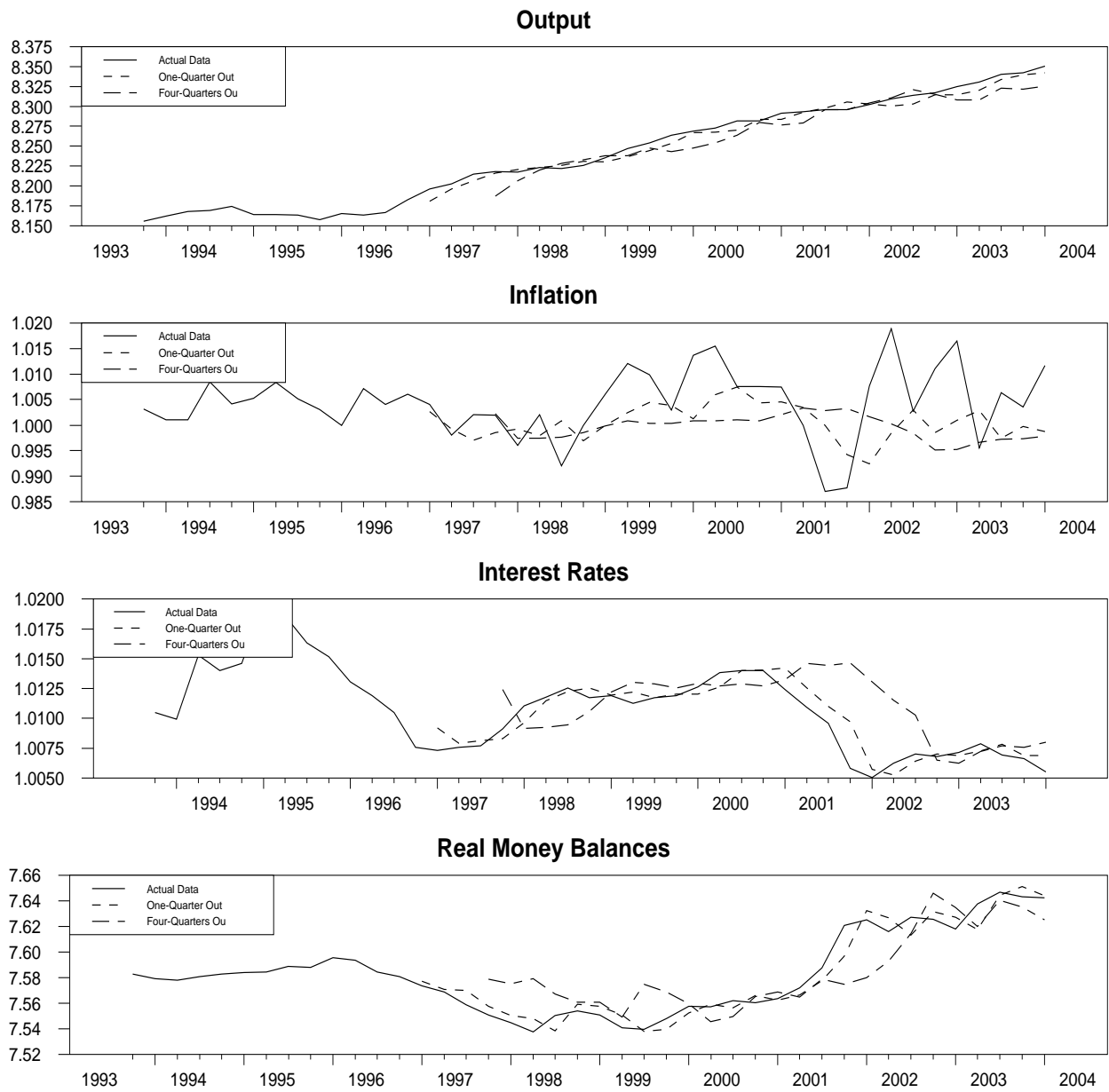


Figure 6. Forecast Errors: One-Quarter Ahead

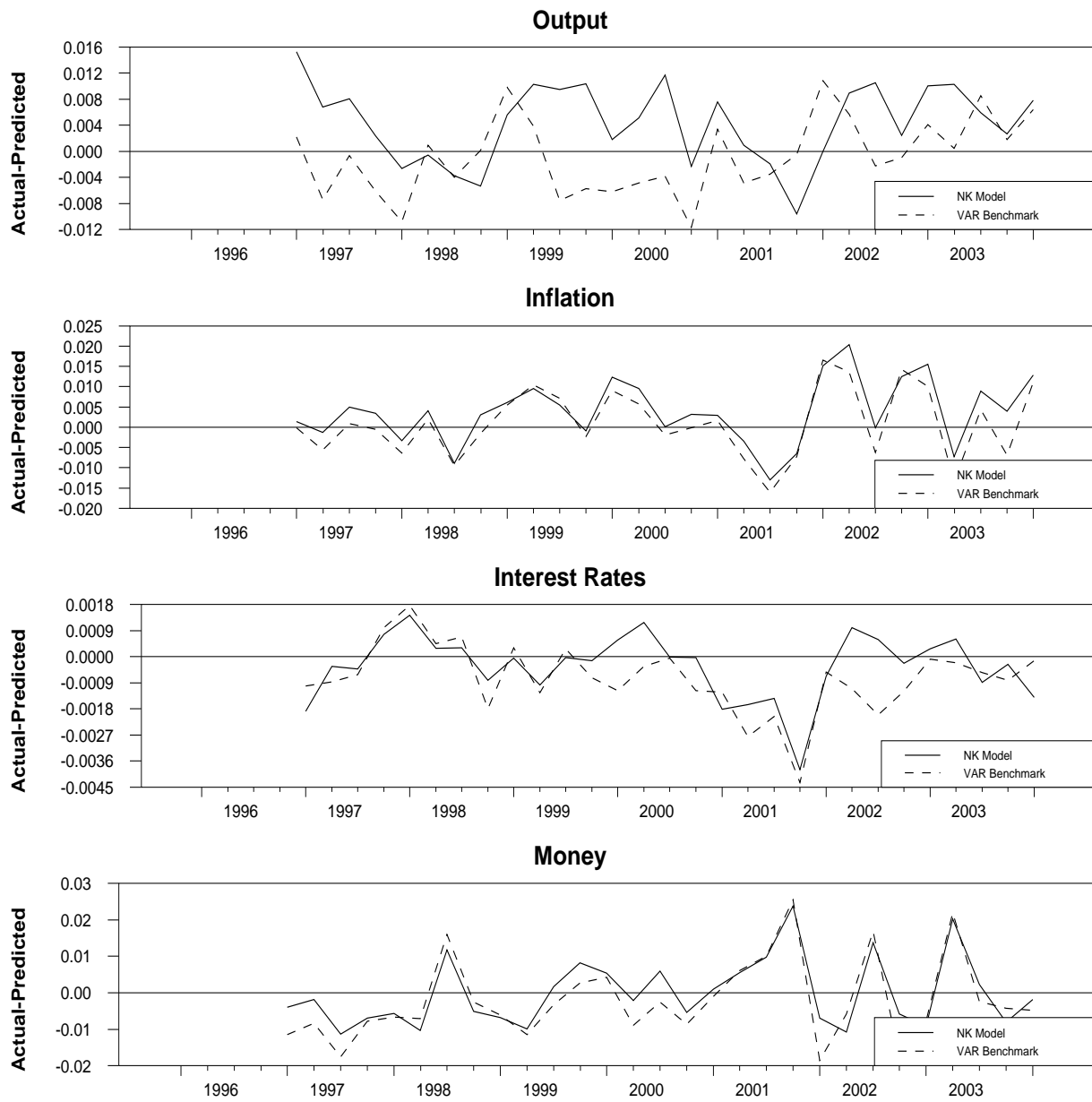


Figure 7. Forecast Errors: Four Quarters Ahead

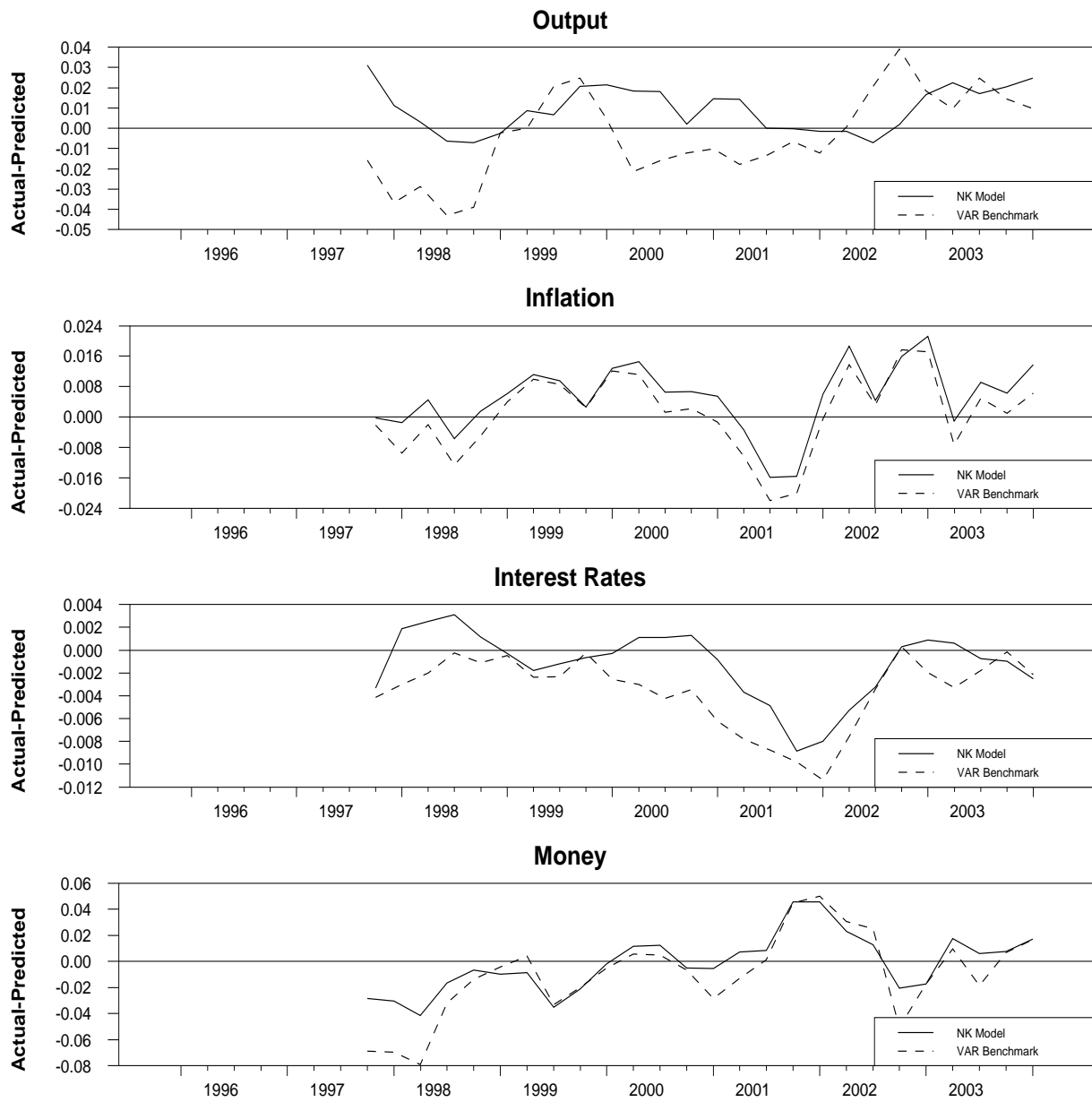
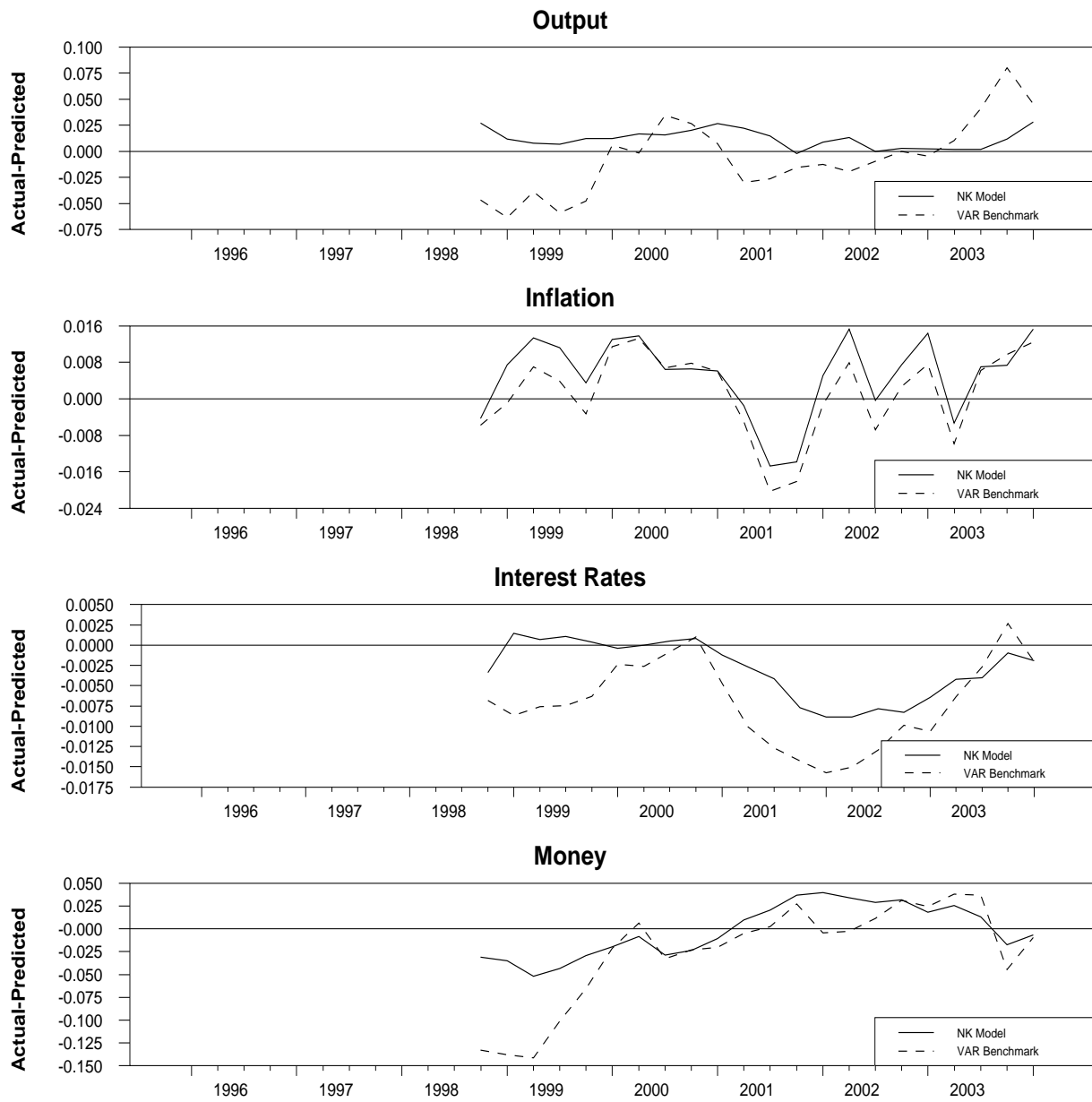


Figure 8. Forecast Errors: Eight Quarters Ahead



A Solving the New Keynesian Model

A.1 The symmetric equilibrium

$$\frac{z_t c_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t; \quad (\text{A.1})$$

$$\frac{z_t b_t^{1/\gamma} m_t^{-\frac{1}{\gamma}}}{c_t^{\frac{\gamma-1}{\gamma}} + b_t^{1/\gamma} m_t^{\frac{\gamma-1}{\gamma}}} = \lambda_t - \beta E_t \left(\frac{\lambda_{t+1}}{\eta \pi_{t+1}} \right); \quad (\text{A.2})$$

$$\frac{\zeta}{1 - h_t} = \lambda_t w_t; \quad (\text{A.3})$$

$$\begin{aligned} \beta E_t \left[\frac{\lambda_{t+1}}{\eta \lambda_t} \left(r_{k_{t+1}} + 1 - \delta + \eta \psi \left(\frac{k_{t+2}}{k_{t+1}} - 1 \right) \frac{k_{t+2}}{k_{t+1}} - (\eta \psi / 2) \left(\frac{k_{t+2}}{k_{t+1}} - 1 \right)^2 \right) \right] \\ = 1 + \eta \psi \left(\frac{k_{t+1}}{k_t} - 1 \right); \end{aligned} \quad (\text{A.4})$$

$$\frac{1}{R_t} = \beta E_t \left[\frac{\lambda_{t+1}}{\eta \lambda_t \pi_{t+1}} \right] \quad (\text{A.5})$$

$$y_t = k_t^\alpha (A_t h_t)^{1-\alpha}; \quad (\text{A.6})$$

$$\frac{\alpha y_t}{k_t} = q_t r_t; \quad (\text{A.7})$$

$$\frac{(1 - \alpha) y_t}{h_t} = q_t w_t; \quad (\text{A.8})$$

$$\tilde{p}_t = \frac{\theta \pi_t}{\theta - 1} \frac{E_t \sum_{k=0}^{\infty} (\beta \phi \pi^{-\theta})^k \lambda_{t+k} y_{t+k} q_{t+k} \left(\prod_{s=1}^k \pi_{t+s}^\theta \right)}{E_t \sum_{k=0}^{\infty} (\beta \phi \pi^{1-\theta})^k \lambda_{t+k} y_{t+k} \left(\prod_{s=1}^k \pi_{t+s}^{\theta-1} \right)}; \quad (\text{A.9})$$

$$\pi_t^{1-\theta} = (1 - \phi) \tilde{p}_t^{1-\theta} + \phi \pi; \quad (\text{A.10})$$

$$y_t = c_t + \eta k_{t+1} - (1 - \delta) k_t - (\psi / 2) \left(\frac{k_{t+1}}{k_t} - 1 \right)^2; \quad (\text{A.11})$$

$$\mu_t = \frac{\eta m_t \pi_t}{m_{t-1}}; \quad (\text{A.12})$$

$$\log(R_t/R) = \varrho_\pi \log(\pi_t/\pi) + \varrho_y \log(y_t/y) + \varrho_\mu \log(\mu_t/\mu) + \log(v_t); \quad (\text{A.13})$$

$$\log(A_t) = (1 - \rho_A) \log(A) + \rho_A \log(A_{t-1}) + \varepsilon_{At}; \quad (\text{A.14})$$

$$\log(b_t) = (1 - \rho_b) \log(b) + \rho_b \log(b_{t-1}) + \varepsilon_{bt}; \quad (\text{A.15})$$

$$\log(z_t) = \rho_z \log(z_{t-1}) + \varepsilon_{zt}; \quad (\text{A.16})$$

$$\log(v_t) = \rho_v \log(v_{t-1}) + \varepsilon_{vt}. \quad (\text{A.17})$$

A.2 Finding The Non-stochastic Steady-State

Setting all shocks to their mean, the economy converges to a steady state in which all variables in (A.1) to (A.17) are constant. Removing the time subscripts to denote the steady-state values of these variables, one is lead to the following system:

$$\mu = \eta\pi; \tag{A.18}$$

$$R = \frac{\pi}{\beta}; \tag{A.19}$$

$$r_k = \frac{\eta}{\beta} - 1 + \delta; \tag{A.20}$$

$$q = \frac{\theta - 1}{\theta}; \tag{A.21}$$

$$\lambda c = \left[1 + b \left(\frac{\mu}{\mu - \beta} \right)^{\gamma-1} \right]^{-1}; \tag{A.22}$$

$$\lambda m = \lambda c b \left(\frac{\mu}{\mu - \beta} \right)^{\gamma}; \tag{A.23}$$

$$\frac{k}{y} = \frac{\alpha q}{r_k}; \tag{A.24}$$

$$\frac{c}{y} = 1 - \delta \left(\frac{k}{y} \right); \tag{A.25}$$

$$wh\lambda = \frac{q(1 - \alpha)(\lambda c)}{(c/y)}; \tag{A.26}$$

$$h = \frac{wh\lambda}{\zeta + wh\lambda}; \tag{A.27}$$

$$y = hA \left(\frac{k}{y} \right)^{\frac{\alpha}{1-\alpha}}. \tag{A.28}$$

A.3 Linearized System

The next step in the solution is to compute a first-order approximation of (A.1) to (A.17) around the steady state. A hatted variable denotes the deviation relative to the steady-state value of the variable. The equations are divided between *Static* and *Dynamic* equations.

A.3.1 Static equations

$$\hat{Y}_t + (\alpha - 1)\hat{h}_t = \alpha\hat{k}_t + (1 - \alpha)\hat{A}_t; \quad (\text{A.29})$$

$$\hat{\mu}_t - \hat{\pi}_t = \hat{m}_t - m_{t-1}; \quad (\text{A.30})$$

$$\hat{Y}_t - \hat{r}_{kt} = \hat{k}_t - \hat{q}_t; \quad (\text{A.31})$$

$$\begin{aligned} (-1 - (\gamma - 1)\lambda c)\hat{C}_t &= \gamma\hat{\lambda}_t + ((\gamma - 1)\lambda((\mu - \beta)/\mu)m)\hat{m}_t \\ &+ (\lambda((\mu - \beta)/\mu)m)\hat{b}_t - \gamma\hat{z}_t; \end{aligned} \quad (\text{A.32})$$

$$\begin{aligned} (-\beta/(\mu - \beta))\hat{R}_t - (\lambda((\gamma - 1)/\gamma)c)\hat{C}_t &= \hat{\lambda}_t + (\lambda((\gamma - 1)/\gamma)(\mu - \beta)/\mu)m + 1/\gamma)\hat{m}_t \\ &+ ((\lambda((\mu - \beta)/\mu)m - 1)/\gamma)\hat{b}_t - \hat{z}_t; \end{aligned} \quad (\text{A.33})$$

$$(h/(1 - h))\hat{h}_t - \hat{w}_t = \hat{\lambda}_t; \quad (\text{A.34})$$

$$\hat{Y}_t - \hat{w}_t - \hat{h}_t = -\hat{q}_t; \quad (\text{A.35})$$

$$\hat{R}_t - \varrho_\mu\hat{\mu}_t - \varrho_\pi\hat{\pi}_t - \varrho_y\hat{y}_t = \hat{v}_t; \quad (\text{A.36})$$

These equations can be rewritten compactly in matrix form as

$$AZ_t = BX_t + CU_t$$

where A, B, and C are 8x8, 8x5, and 8x4 matrices, respectively and we have $Z_t = (\hat{y}_t, \hat{R}_t, \hat{r}_{kt}, \hat{C}_t, \hat{\pi}_t, \hat{w}_t, \hat{h}_t, \hat{\mu}_t)'$ (a vector of endogenous variables), $X_t = (\hat{k}_t, m_{t-1}, \hat{\lambda}_t, \hat{q}_t, \hat{m}_t)'$ (a vector of state and co-state variables) and $U_t = (\hat{A}_t, \hat{b}_t, \hat{v}_t, \hat{z}_t)'$ (a vector of shock variables).

A.3.2 Dynamic equations

The dynamic equations are the following:

$$\beta\hat{\pi}_{t+1} = \hat{\pi}_t - \frac{(1 - \beta\phi)(1 - \phi)}{\phi}\hat{q}_t; \quad (\text{A.37})$$

$$\begin{aligned} (\psi(\beta(1 - \delta) - (1 + \beta)))\hat{k}_{t+1} + (\beta(1 + r_k - \delta))\hat{\lambda}_{t+1} \\ + \beta r_k \hat{r}_{kt+1} + (\beta\psi y/k)\hat{y}_{t+1} - (\beta\psi c/k)\hat{c}_{t+1} = -\psi\hat{k}_t + \hat{\lambda}_t; \end{aligned} \quad (\text{A.38})$$

$$k\hat{k}_{t+1} = (1 - \delta)k\hat{k}_t + y\hat{y}_t - c\hat{C}_t; \quad (\text{A.39})$$

$$\hat{\lambda}_{t+1} - \hat{\pi}_{t+1} = \hat{\lambda}_t - \hat{R}_t; \quad (\text{A.40})$$

$$\hat{m}_t = \hat{m}_t. \quad (\text{A.41})$$

Again, this can be rewritten in matrix form as

$$DX_{t+1} + EZ_{t+1} = FX_t + GZ_t + HU_t$$

where D, E, F, G, and H are 5x5, 5x8, 5x5, 5x8, and 5x4 matrices, respectively.

Using the static and dynamic equations, we can solve the model using the methodology of Blanchard and Kahn (1980), which leads us to the following first-order state-space solution of the system:

$$\begin{aligned}\widehat{s}_{t+1} &= \Phi_1 \widehat{s}_t + \Phi_2 \varepsilon_{t+1}, \\ \widehat{d}_t &= \Phi_3 \widehat{s}_t,\end{aligned}$$

where the matrices Φ_1 , Φ_2 and Φ_3 are functions of the structural parameters of the model.

B Data

The model is estimated using data that spans the period 1981:3 to 2004:1. The data is taken from Statistics Canada's CANSIM database, for which we list the associated mnemonics. Output Y_t is *final domestic demand* [V1992068], of quarterly frequency and in chained 1987 dollars. We convert this series into per-capita terms using the population of age 15 and over.

The interest rate R_t is the three month treasury bill rate ([V122531]), a series of daily frequency, for which we take a quarterly average.

Finally, the money stock M_t is $M2$ [B1630] which is of monthly frequency; we take a quarterly average and convert the resulting series into real, per-capita terms by dividing it with the GDP implicit price deflator ([D100465]) and the population age 15 and over. Output and money data are logged before estimation.

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