

Optimal Retail Price Promotions

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Abstract The paper proposes a dynamic optimization model for a retailer's price promotions of two brands in a product category. There is no general consensus in empirical marketing literature on what are the impacts of a promotion on intertemporal consumer behavior. This paper will focus on three effects: (1) The immediate, and positive, impact of a price deal on the sales of a promoted brand; (2) Brand substitution, where some consumers switch from a nonpromoted brand to a lower priced promoted brand; and (3) Consumers stockpile a promoted brand during a deal period, which affects postpromotional demand. The paper characterizes the magnitude of discounts as well as the timing and durations of promotions. Optimal policies for a myopic and a forward looking retailer are identified. We also provide comparative statics that identify the dependence of discounts and durations upon key model parameters.

Keywords Retailing, Price Promotions, Consumer Stockpiling, Mathematical Programming.

JEL Classification C61, M31, M37.

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1 Introduction

A very popular practice in the marketing of consumer packaged goods are temporary reductions of the retail price. A *retail price promotion*, or *price deal*, is a reduction of a brand's regular consumer price, lasting for a short period of time. Retail price promotions are accompanied by in-store signs and off-shelf displays, and will be featured in newspapers, television, and radio. Price reductions can take various forms: A "shelf-price" reduction applies to any buyer of the brand, whereas price reductions based on coupons apply only to those who redeem a coupon. This paper will be concerned with shelf-price reductions only.

The success of a promotion depends on many things, for instance, the profits made on the promoted item, the cannibalization caused by customers who switch from regular-priced brands to a promoted item, increased sales of products that are complementary to the promoted product, and its ability to increase store traffic and get more sales of items at regular prices (Mulhern and Leone (1991), Blattberg and Neslin (1993)).

When making strategic decisions about promotions, retailers must rely on their expectations of consumers' reactions (Kalwani and Yim (1992)). Here it is helpful to consider the categorization of reactions introduced by Pauwels et al. (2002) who distinguish the *immediate* effects of price promotions (short-term changes in sales) and the *adjustment* (or transient) effects that prevail in the transition period between the immediate response and a long-run equilibrium state. The latter reflects the *permanent* effects of a promotion's impact, being carried forward to affect the long-run state. Anderson and Simester (2004) notes that some confusion here seems to rest in the definition of what "long run" should mean.

In the empirical marketing literature there is mixed evidence about signs and magnitudes of the longer-term effects. Blattberg et al. (1995) write that this is "probably the most debated issue in the promotional literature and one for which the jury is 'still' out" (p. G127). Pauwels et al. (2002) noted that "...the

net impact of promotions on dynamic consumer response remains an empirical puzzle in marketing literature” (p. 424)).

However, as to the *immediate* effect of a price promotion, it is widely agreed that a deal leads to an increase in sales of the promoted brand during the promotional period (Guadagni and Little (1983), Moriarty (1985), Gupta (1988), Blattberg and Neslin (1990), Walters (1991), Mulhern and Leone (1991), Blattberg et al. (1995)). This effect is being caused by consumers who buy the brand earlier and in larger quantities than usually, consumers who switch from competing brands and/or stores, and consumers who only buy the category occasionally. Increased sales of a promoted item can also come from previous nonbuyers who choose to enter the market now (Moriarty (1985)). On the other hand, the brand choice effect imply that a promotion causes a decrease in current sales of competing brands within the category (Guadagni and Little (1983), Kumar and Leone (1988), Walters (1991)).

Pauwels et al. (2002) note that *adjustment* effects can be positive or negative, and their sign and size affect significantly the profitability of the promotion. Adjustment effects include many influences, but this paper will focus upon a single one, *consumers’ stockpiling*. This means that a promoted brand suffers from a reduction of its postpromotion sales due to forward buying. Consumers stockpile the promoted brand, having the rational expectation that the promotion only lasts for a short period of time after which the price returns to its regular level. The empirical evidence of postdeal troughs seems mixed. Neslin et al. (1985), Anderson and Simester (2004) report postdeal troughs; studies by Moriarty (1985), Grover and Srinivasan (1992) find only small or no such effects. On the other hand, economic studies of household inventory behavior suggest that purchase acceleration should exist (Neslin et al. (1985), Gupta (1988)).

The empirical research on the *permanent* effects of promotions is not conclusive and discusses a number of issues, for example, that price deals may result in less loyalty toward the brand. We conclude that the impacts of a promotion are

not universally agreed upon, and cannot possibly be, since variations over product categories and store types can be considerable. There may also be regional and cultural differences in the ways consumers react to deals. The empirical literature has mainly assessed the impacts of promotions on a case-by-case basis, using a variety of statistical methods on data from specific product categories.

The price promotion literature has predominantly been concerned with empirical analyses of the impacts of promotions. There are a few prescriptive studies which are related to the topic of this paper. Rao and Thomas (1973) suggested a dynamic programming model to determine the optimal price-off and the number of times to promote a single brand during a fixed planning horizon. Optimal solutions were derived by numerical methods for a number of parameter sets. Rao (1991) studied retail price promotions in a duopoly and argued that promotional decisions are part of a more general decision problem. Thus, a retailer first chooses a regular price for a brand, and then makes the promotional decisions. The latter have two elements: the depth and the frequency of promotions. Thus, the retailer's pricing problem is solved consecutively. This idea will be exploited in what is to follow. Tellis and Zufryden (1995) proposed an elaborate model for a retailer's optimal timing and depth of discounts, combined with the optimal timing and quantity of the retailer's orders from manufacturers. Trade deals are offered by manufacturers and the model accounts for inventories at both consumer and retail levels. The model incorporates multiple brands and deals and incorporates a consumer response model based on brand choice, category incidence, and quantity events. Due to the complexity of the resulting mathematical programming model, the authors used numerical simulations to characterize optimal solutions and their sensitivity to parameter values.

1.1 Overview of Model and Results

We set up an intertemporal model of a retailer's promotional activities and the aim is to study the problem of optimal duration, timing, and depth of discounts.

The solution of the resulting mathematical programming problem should primarily be seen as a retailer decision support tool, but the issues of promotion frequency and depth of discounts can be interesting in other contexts, too. For example, frequency and depth of discounts may have an impact on consumers' price expectations (Kalwani and Yim (1992)).

Methodologically, the approach is *normative* and *analytical*. Being normative, the paper complements the (sparse) literature on optimal retail price promotions over time. Being analytical, the paper complements the results obtained by numerical methods. As is well known, analytical solutions yield results of greater generality than those obtained by numerical methods. More general results, however, come at the cost that the model needs to be analytically tractable, that is, sufficiently simple. However, we believe that our simplistic model illustrate some (although not all) of the significant issues in the optimal design of retail price promotions.

The model supposes that a retailer has two brands in a specific category, and that one brand only is promoted at a time. (In most of the simulations in Tellis and Zufryden (1995), brands should not be promoted simultaneously. They noted, however, that if one brand carries a very high margin and the other has a very high sensitivity to discounts, it may be profitable to discount both brands). An important reason for promoting brands one at a time is that the retailer incurs an opportunity cost: customers who are loyal to the discounted brand buy the product at the discounted price, but would have bought it at the regular price. When two brands are discounted simultaneously, this effect is reinforced.

We take three effects of a price promotion into account:

1. The direct influence on the sales of a promoted brand during the promotional period
2. Brand substitution: Some consumers switch from the nonpromoted brand to the lower priced, promoted brand

3. Consumers stockpile a promoted brand during a deal period which affects the postdeal demand of the promoted brand.

Our main findings are the following:

- The discount on a brand should be smaller, the more damaging a promotion of the brand will be for the current demand for the other brand
- When the impact of a discount on the demand for brand increases, the depth of a discount should be increased
- The discount on each brand decreases as its regular demand rates increases. For a sufficiently high regular demand, the brand should not be promoted
- The discount of a brand should be smaller, the larger the regular price of the other brand
- If a discount has a low impact on demand, the deal period is short, and the stockpiling effect is large, the discount should be smaller, the higher the brand's regular price (margin)
- The larger the stockpiling effect, the smaller the discount
- The longer the duration of a deal, the higher the discount.
- If the effects of the promotion of brand 1 last *one* period, the optimal discount of brand 2 will be less than the discount that applies when the effects of promoting brand 1 last for two or three periods
- Under a symmetry assumption, brand 1 should have a smaller discount than brand 2 if the stockpiling effect of promoting brand 1 is larger than that of brand 2
- A forward-looking retailer takes into account the impacts of deals on future demand and applies a smaller discount than a myopic retailer
- Promoting the two brands is not equally attractive.

The paper progresses as follows. Section 2 suggests a dynamic model of a retailer's price promotion decisions. The aim is to determine optimally the depth of discounts and the duration and timing of price deals. Section 3 determines an

optimal solution for a myopic and a forward-looking retailer, respectively, and discusses the managerial implications of the model's recommendations. Section 4 concludes and points to some limitations of the model. We also offer some suggestions for future research in the challenging, but complicated area of retail promotions.

2 Dynamic Model of Retail Price Promotions

A retailer has two brands, 1 and 2, in a specific product category. Time t is measured continuously and the retailer's planning period starts at $t = 0$ and ends at $t = T$. The length T of the planning period is fixed. Typically, price promotions are short-run activities, and we can safely omit discounting of future profits.

To characterize the retailer's promotion plan, let time instants θ_1 [θ_2] denote the start [the end] of a promotion of brand 1, and let η_1 [η_2] denote the start [the end] of a promotion of brand 2. Thus, $\theta_2 - \theta_1$ [$\eta_2 - \eta_1$] is the duration of a promotion of brand 1 [2]. If $\theta_2 = \theta_1$ [$\eta_2 = \eta_1$], brand 1 [2] is not promoted. Since we have assumed that deals do not overlap, it must hold that $\theta_2 \leq \eta_1$. There are five subintervals to consider:

- $[0, \theta_1)$: No brand is promoted
- $[\theta_1, \theta_2]$: Brand 1 is promoted
- (θ_2, η_1) : No brand is promoted
- $[\eta_1, \eta_2]$: Brand 2 is promoted
- $(\eta_2, T]$: No brand is promoted.

If $\theta_2 = \eta_1$ the promotion of brand 2 starts at the very moment where the promotion of brand 1 ends. If, for instance, $\theta_1 = \theta_2$, we take it to mean that brand 1 is not promoted.

Our next task is to specify the demand conditions in the five subintervals. As already said, we focus upon three effects of retail price promotions:

- The immediate effect on the demand for a promoted brand
- The effects of brand switching within the category
- Consumers' stockpiling affect negatively postdeal demand.

These modeling choices are crucial since they will be the main drivers of the results that are to follow.

2.1 Demand and Revenue Functions

This section describes demands and revenues during the five subintervals stated above. We adopt the standard assumption that the retailer is concerned with *category revenue*. Let \bar{p}_1 and \bar{p}_2 denote the *regular prices* of brand 1 and 2, respectively, and suppose that these prices have been fixed at time $t = 0$. A regular price is constant over time and is valid during any period in which a brand is not promoted.

A regular price being constant means that the basic demand and cost conditions do not change over time. In particular this means that manufacturers' transfer prices and the retailer's processing costs are constant throughout the retailer's planning period. Defining retail prices p_1 and p_2 as being net of purchase and processing costs, one can view p_1 and p_2 as margins.

It should be noted that most retail promotions are triggered by manufacturers' trade deals (i.e., temporary cuts in transfer prices). A manufacturer's intention is that the retailer should increase her orders and reduce consumer prices. There is, however, a substantial amount of evidence showing that although retailers take advantage of a trade deal, the volume of retailer forward buying and impacts on consumer prices are not clear. For instance, a retailer may stockpile items bought at a discount and sell these goods at regular retail prices later on. A retailer may pocket part (or all) of the price savings offered by manufacturers. In view of the lack of a clear connection between the availability and size of a trade deal and subsequent retail price promotions, our model does not include trade deals.

First time interval, $[0, \theta_1)$, where no brand is promoted

Denote by $\bar{q}_1 > 0, \bar{q}_2 > 0$ the (regular) demand rates of the two brands during the time interval $[0, \theta_1)$. The category revenue rate is

$$\bar{K} = \bar{p}_1 \bar{q}_1 + \bar{p}_2 \bar{q}_2,$$

which we view as the retailer's baseline revenue. The category revenue over the time interval $[0, \theta_1)$ equals $\theta_1 \bar{K}$.

Second time interval, $[\theta_1, \theta_2]$, where brand 1 is promoted

The demand rates of both brands are affected by the depth of the discount of brand 1. Let p_1^* and d_θ denote the *promotion price* and the *discount* (cents-off the regular price), respectively, of brand 1 during its promotion. Thus

$$p_1^* = \bar{p}_1 - d_\theta \iff d_\theta = \bar{p}_1 - p_1^*.$$

In what follows, the discount will be the retailer's decision variable.

Blattberg et al. (1995) state that “*little is known about the shape of the deal effect curve, though it determines the “optimal” dealing amounts*” (p. 127). Here we assume that demand rates are given by the linear functions:

$$q_1(d_\theta) = \bar{q}_1 + \beta_1 d_\theta, \quad q_2(d_\theta) = \bar{q}_2 - \varepsilon_2 d_\theta, \quad (1)$$

in which $\beta_1 > 0$ and $\varepsilon_2 \geq 0$ are constants. In (1), β_1 measures the marginal impact of the promotion of brand 1 on its own demand rate. The higher the discount on brand 1, the larger its demand rate during the promotion period. The parameter ε_2 reflects the effect of brand switching within the category, that is, the impact on the demand for brand 2 of a promotion of brand 1. Thus, an increasing number of brand 2 buyers switch to brand 1 as the discount on the latter increases. For $\varepsilon_2 = 0$, brand 2 customers are completely loyal to their brand.

We assume $\beta_1 > \varepsilon_2$, that is, a promotion has a stronger marginal impact on the demand for the promoted brand 1 than on the demand for brand 2 (e.g., Blattberg and Neslin (1990)). The reason is that brand 1 increases its sales to its regular customers, to occasional buyers, and to consumers who switch from brand 2.

The category revenue rate is

$$K^*(d_\theta) = (\bar{p}_1 - d_\theta)[\bar{q}_1 + \beta_1 d_\theta] + \bar{p}_2 [\bar{q}_2 - \varepsilon_2 d_\theta] = \quad (2)$$

$$\bar{K} + (\bar{p}_1 \beta_1 - \bar{p}_2 \varepsilon_2 - \bar{q}_1) d_\theta - \beta_1 (d_\theta)^2,$$

and the category revenue over the time interval $[\theta_1, \theta_2]$ equals $(\theta_2 - \theta_1)K^*$.

Third time interval, (θ_2, η_1) , where no brand is promoted

In general, postpromotion demand could be affected by the duration of the promotion, the depth of the discount, or both. The duration of the promotion affects postpromotion demands from the simple reason that the longer the duration, the more consumers will have the opportunity to buy and stockpile the promoted brand.

We suppose that postpromotion demand for brand 1 is affected by the depth of discount, such that the steeper the discount, the more stockpiling, and the smaller the postpromotion demand for brand 1. For brand 2, however, *no* such effect is assumed. Our hypothesis thus is that brand 2 buyers may exploit the price differential, by switching to brand 1 during its promotion (cf. (1)), but brand 2 buyers do not stockpile on brand 1.

Demand rates during the time interval (θ_2, η_1) are represented by tildes and are given by

$$\tilde{q}_1(d_\theta) = \bar{q}_1 - \sigma_1 d_\theta, \quad \tilde{q}_2 = \bar{q}_2, \quad (3)$$

in which $\sigma_1 \geq 0$ is a constant. The value of σ_1 reflects the effect of stockpiling of brand 1 buyers.

The category revenue rate is

$$\tilde{K}(d_\theta) = \bar{p}_1[\bar{q}_1 - \sigma_1 d_\theta] + \bar{p}_2 \bar{q}_2 = \bar{K} - \bar{p}_1 \sigma_1 d_\theta,$$

and the category revenue over the time interval (θ_2, η_1) equals $(\eta_1 - \theta_2)\tilde{K}$. Due to our assumptions, we have $\tilde{K} \leq \bar{K}$, which says that postpromotion category revenue will not exceed baseline category revenue.

Fourth time interval, $[\eta_1, \eta_2]$, where brand 2 is promoted

Let $p_2^* = \bar{p}_2 - d_\eta$ and $d_\eta = \bar{p}_2 - p_2^*$ denote the promotion price and the discount, respectively, of brand 2. Demand functions during the promotion of brand 2 are given by

$$\begin{aligned} q_1(d_\theta, d_\eta) &= \tilde{q}_1(d_\theta) - \varepsilon_1 d_\eta = \bar{q}_1 - \sigma_1 d_\theta - \varepsilon_1 d_\eta \\ q_2(d_\theta, d_\eta) &= \tilde{q}_2 + \beta_2 d_\eta = \bar{q}_2 + \beta_2 d_\eta, \end{aligned} \quad (4)$$

in which $\varepsilon_1 \geq 0$ and $\beta_2 > 0$ are constants. In (4), the parameter ε_1 measures substitution from brand 1 to brand 2. The parameter β_2 measures the direct impact of promoting brand 2 on its own demand. We assume $\beta_2 > \varepsilon_1$. (Grover and Srinivasan (1992) note that cross-promotional effects most likely are asymmetric, which means that the ε -parameters in (1) and (4) have different values. The reason is a difference in brand equity).

Notice that the inclusion of the term $\sigma_1 d_\theta$ in (4) means that the stockpiling effect of the promotion of brand 1 is present not only in the immediate postdeal period (θ_2, η_1) , but also in the time interval (η_1, η_2) during which brand 2 is promoted. If the stockpiling effect has vanished before time η_1 , one omits the term $\sigma_1 d_\theta$ in (4).

The category revenue rate is

$$\begin{aligned} K^o(d_\theta, d_\eta) &= \bar{p}_1[\tilde{q}_1(d_\theta) - \varepsilon_1 d_\eta] + (\bar{p}_2 - d_\eta)[\tilde{q}_2 + \beta_2 d_\eta] = \\ &\tilde{K} + (\bar{p}_2 \beta_2 - \bar{p}_1 \varepsilon_1 - \bar{q}_2) d_\eta - \beta_2 (d_\eta)^2, \end{aligned} \quad (5)$$

and the category revenue over the time interval $[\eta_1, \eta_2]$ equals $(\eta_2 - \eta_1)K^o$.

Fifth time interval, $(\eta_2, \mathbf{T}]$, where no brand is promoted

The postdeal demand rate of brand 2, but not of brand 1, is affected by the promotion of brand 2 and we specify the postpromotion demand rates as follows:

$$\hat{q}_1(d_\theta, d_\eta) = \bar{q}_1 - \sigma_1 d_\theta, \quad \hat{q}_2(d_\theta, d_\eta) = \bar{q}_2 - \sigma_2 d_\eta, \quad (6)$$

in which $\sigma_2 \geq 0$ is a parameter that reflects the influence of the promotion of brand 2 on the postdeal demand rate of this brand. If the effect of the promotion of brand 1 has expired before time η_2 , one omits the term $\sigma_1 d_\theta$ in (6).

The category revenue rate is

$$\hat{K}(d_\theta, d_\eta) = \tilde{K} - \bar{p}_2 \sigma_2 d_\eta, \quad (7)$$

and the category revenue over the time interval $(\eta_2, T]$ equals $(T - \eta_2)\hat{K}$. It holds that $\hat{K} \leq \bar{K}$, that is, postpromotion category revenue after two promotions will not exceed the baseline revenue.

2.2 Profit Function

Although displays and feature advertising could influence demand, the possible synergies between these activities and price discounts have only been sporadically researched in the empirical literature (Blattberg et al. (1995)). We suppose that advertising and displaying a brand does not affect the demand rates during a promotion. The implication is that advertising and display enter the model as a cost only, penalizing promotions with long durations and making it less attractive to promote all the time.

For both brands, the costs of feature advertising and display are $C(t) = at$, where a is a positive constant. Thus, costs are independent of which brand is promoted and they will depend only on the length of the time interval during

which a brand is promoted. (For simplicity, advertising and display activities start at the same time where a price deal starts. One may argue that advertising should start before a promotion starts. This situation can easily be handled, but does not add much to the understanding of optimal promotions).

When the brands are promoted over time periods $[\theta_1, \theta_2]$ and $[\eta_1, \eta_2]$, respectively, advertising costs amount to

$$\int_{\theta_1}^{\theta_2} atdt = \frac{a}{2}(\theta_2^2 - \theta_1^2) \triangleq A_\theta, \quad \int_{\eta_1}^{\eta_2} atdt = \frac{a}{2}(\eta_2^2 - \eta_1^2) \triangleq A_\eta. \quad (8)$$

The profit of the retailer is category revenues minus advertising costs over the planning period $[0, T]$:

$$J(\theta_1, d_\theta, \theta_2, \eta_1, d_\eta, \eta_2) = \theta_1 \bar{K} + (\theta_2 - \theta_1)K^* - A_\theta + \\ (\eta_1 - \theta_2)\tilde{K} + (\eta_2 - \eta_1)K^o - A_\eta + (T - \eta_2)\hat{K}.$$

3 Optimal Retail Price Promotions

The retailer's problem is a multi-stage decision problem and an optimal promotion plan is a set of decisions $\{\theta_1, d_\theta, \theta_2, \eta_1, d_\eta, \eta_2\}$, determining the depths of the discounts as well as the durations and timing of the two promotions. We determine an optimal plan under two alternative assumptions concerning the retailer's optimizing behavior:

- The retailer is a dynamic optimizer who takes into account that the current decision affects the state of the system, now and in the future (Rao and Thomas (1978), Tellis and Zufryden (1995))
- The retailer is myopic and believes that a current decision influences the current state of the system only, or does not care about the influence of the current decision upon future states (Vilcassim and Chintagunta (1995)).

3.1 Forward-Looking Retailer

Rao (1991) suggested to view promotional decision making as a three-stage problem. In stage one, the retailer fixes the regular prices of the brands. In stage two, the depths of promotions are determined, and in stage three the frequencies of promotions are decided. Such sequential reasoning may resemble actual promotional decision making behavior where a retailer defines a promotion *menu*, which is a rule that sets the regular prices as well as the depths of possible discounts. When it comes to the decision whether or not to discount, and the answer is affirmative, the menu is invoked. Then it remains to be determined for how long a period the deal should go on. This paper adopts a similar approach. Thus, Section 2.1.1 determines the optimal discounts and this provides the menu since regular prices have been fixed already. (The determination of these prices is a straightforward exercise in optimization). Given the menu, the decision whether or not to discount, and for how long a period, amounts in our setup to the determination of the duration and timing of the two promotions. This problem is dealt with in Section 2.1.3.

3.1.1 Optimal Discounts When Durations are Fixed For the determination of the optimal discounts in the menu, the advertising costs in (8) are irrelevant and can be disregarded. Our first result is the following:

Proposition 1 *Whenever positive, optimal discounts of brands 1 and 2 are given by*

$$d_{\theta}^* = \frac{1}{2\beta_1} \left[\bar{p}_1\beta_1 - \bar{p}_2\varepsilon_2 - \bar{q}_1 - \bar{p}_1\sigma_1 \frac{T - \theta_2}{\theta_2 - \theta_1} \right] \quad (9)$$

$$d_{\eta}^* = \frac{1}{2\beta_2} \left[\bar{p}_2\beta_2 - \bar{p}_1\varepsilon_1 - \bar{q}_2 - \bar{p}_2\sigma_2 \frac{T - \eta_2}{\eta_2 - \eta_1} \right] \quad (10)$$

and the optimal profit over the planning period $[0, T]$ is

$$\begin{aligned} J(d_\theta^*, d_\eta^*) = & -[A_\theta + A_\eta] + T\bar{K} - (T - \eta_2)\bar{p}_2\sigma_2d_\eta^* - (T - \theta_2)\bar{p}_1\sigma_1 + \\ & (\eta_2 - \eta_1)[(\bar{p}_2\beta_2 - \bar{p}_1\varepsilon_1 - \bar{q}_2)d_\eta^* - \beta_2(d_\eta^*)^2] + \\ & (\theta_2 - \theta_1)[(\bar{p}_1\beta_1 - \bar{p}_2\varepsilon_2 - \bar{q}_1)d_\theta^* - \beta_1(d_\theta^*)^2]. \end{aligned} \quad (11)$$

Eq. (11) shows that the optimal profit consists of

- $A_\theta + A_\eta$: Total advertising cost
- $T\bar{K}$: Aggregate category revenue if there were no promotions at all
- $-[(T - \eta_2)\bar{p}_2\sigma_2d_\eta^* + (T - \theta_2)\bar{p}_1\sigma_1d_\theta^*]$: Aggregate loss of postpromotional revenues, caused by the two promotions
- The last two terms represent incremental revenues, generated by the two promotions. In the first term we have

$$(\bar{p}_2\beta_2 - \bar{p}_1\varepsilon_1 - \bar{q}_2)d_\eta^* - \beta_2(d_\eta^*)^2 = K^o - \bar{K}, \quad (12)$$

which is the difference between the category revenue rate during the promotion of brand 2 and the prepromotion category revenue rate. In the second term we have

$$(\bar{p}_1\beta_1 - \bar{p}_2\varepsilon_2 - \bar{q}_1)d_\theta^* - \beta_1(d_\theta^*)^2 = K^* - \bar{K}, \quad (13)$$

which has a similar interpretation. By the optimality of the discounts, the differences on the right-hand sides of (12) and (13) are positive; otherwise there should be no promotion.

3.1.2 Postoptimality Analyses Partial differentiations in (9) and (10) provide the following results.

- *Effect of promoting brand 2 [1] on the current demand for brand 1 [2]*: It holds that $\frac{\partial d_\theta^*}{\partial \varepsilon_2} < 0$, $\frac{\partial d_\eta^*}{\partial \varepsilon_1} < 0$, which is intuitive. The discount on a brand should be

smaller, the more damage a promotion of this brand will do to the current demand for the other brand. Thus, if brand substitution is significant, the retailer should discount moderately or even refrain from promoting. Another interpretation lies in brand loyalty related to consumer responses to price cuts (Guadagni and Little (1983)). If, for example, brand 2 customers are very loyal to their brand ($\varepsilon_2 \simeq 0$), brand 1 can have a deep discount without affecting demand for brand 2 very much. To put this into perspective, Raju et al. (1990) found that brands with a strong brand loyalty are less often promoted than brands with weaker loyalty.

- *Effect of promoting brand i on its current demand*: It holds that $\frac{\partial d_\theta^*}{\partial \beta_1} < 0$, $\frac{\partial d_\eta^*}{\partial \beta_2} > 0$, which is intuitive. As the marginal impact of a discount on the demand for brand increases, the depth of a discount should be increased.
- *Effect of a brand's regular demand on its discount*: It holds that $\frac{\partial d_\theta^*}{\partial \bar{q}_1} < 0$, $\frac{\partial d_\eta^*}{\partial \bar{q}_2} < 0$, which means that the discount on each brand decreases as its regular demand rates increases. For a sufficiently high regular demand, the brand should not be promoted. The intuition is simple: when demand already is high, there is less need for a promotion.
- *Effect of the regular price (margin) of brand 1 [2] on the discount of brand 2 [1]*: It holds that $\frac{\partial d_\theta^*}{\partial \bar{p}_2} < 0$, $\frac{\partial d_\eta^*}{\partial \bar{p}_1} < 0$, saying that the discount of brand 1 [2] should be smaller, the larger the regular price of brand 2 [1]. The reason is brand substitution: When, for example, brand 1 is sold at a discount, brand 2 buyers switch to brand 1. They pay the discounted price, but if they had stayed with brand 2, they would have paid the regular price \bar{p}_2 . The higher that price, the larger the opportunity loss for the retailer.
- *Effect of the regular price (margin) of a brand on its discount*: It holds that $\frac{\partial d_\theta^*}{\partial \bar{p}_1} = \frac{1}{2\beta_1} \left[\beta_1 - \sigma_1 \frac{T-\theta_2}{\theta_2-\theta_1} \right]$, $\frac{\partial d_\eta^*}{\partial \bar{p}_2} = \frac{1}{2\beta_2} \left[\beta_2 - \sigma_2 \frac{T-\eta_2}{\eta_2-\eta_1} \right]$. It suffices to illustrate the first result and we see that $\frac{\partial d_\theta^*}{\partial \bar{p}_1} < 0$ if $\beta_1(\theta_2 - \theta_1) < \sigma_1(T - \theta_2)$. The latter inequality can be fulfilled if a discount has a low impact β_1 on demand, the deal period $\theta_2 - \theta_1$ is short, and the stockpiling effect σ_1 is large. Under

such circumstances, the discount of brand 1 should be smaller, the higher the brand's regular price (margin). The intuition is that a high-margin brand should have only a light (or no) discount if discounting is inefficient in raising demand, the deal is limited to a very short period, and the postpromotional decrease in demand is considerable.

- *Effect of stockpiling on a discount:* It holds that $\frac{\partial d_\theta^*}{\partial \sigma_1} < 0$, $\frac{\partial d_\eta^*}{\partial \sigma_2} < 0$, which is intuitive: The larger the stockpiling effect (decrease in postpromotional demand), the smaller the discount.
- *Effect of the duration of a deal on the discount:* We have $\frac{\partial d_\theta^*}{\partial(\theta_2 - \theta_1)} > 0$, $\frac{\partial d_\eta^*}{\partial(\eta_2 - \eta_1)} > 0$, which states that the longer the duration of a deal, the higher the discount. The intuition lies in the fact that running a deal over a longer period of time means higher costs of advertising and display. To pay for these costs, more extra demand needs to be generated which requires a steeper discount.

Next we examine the impact on optimal discounts of changing the stockpiling effect in the demand specifications. These effects are reflected in the σ 's appearing in the demand functions. Note that the results so far developed have assumed that the postpromotional effects of the first promotion last *three* periods. Note, due to the finite horizon date T , the stockpiling effect of the second promotion can last one period only.

- *Postpromotional effects last for one or two periods:* If the effects of the promotion of brand 1 last *two* periods, the only change is in the postprom . on revenue \widehat{K} , cf. (7), which obviously increases. The interesting case is when the effects of promoting brand 1 last *one* period. Then three things change: the revenue K^o during the promotion of brand 2, the optimal discount d_η^* , and the postpromotion revenue \widehat{K} . Using (10) with $\sigma_2 = 0$ shows that the optimal discount d_η^* of brand 2 will be *less* than the discount that applies when the effects of promoting brand 1 last for two or three periods. To see the intuition of this result, note that when the effects of the first promotion

last one period only, the depth of the first discount does not influence that of the second discount (cf. (10)). Hence, even after a deep discount of brand 1, demand has recovered when it comes to the promotion of brand 2. Hence the discount of this brand can be lighter.

Finally, we wish to compare the two optimal discounts. Clearly, given the number of parameters involved, this task requires a symmetry assumption:

$$\begin{aligned} \bar{p}_1 = \bar{p}_2 = \bar{p}, \quad \bar{q}_1 = \bar{q}_2 = \bar{q}, \quad \beta_1 = \beta_2 = \beta \\ \varepsilon_1 = \varepsilon_2 = \varepsilon, \quad \theta_2 - \theta_1 = \eta_2 - \eta_1, \end{aligned} \quad (14)$$

which means that the two brands have the same regular price and demand rate. Moreover, the impact of a promotion of a brand on the brand's own demand is the same for both brands, as is the impact of a promotion on the demand for the other brand. Finally, the durations of the promotions are equal. Using (9) and (10) yields

$$d_\theta^* \begin{cases} > \\ = \\ < \end{cases} d_\eta^* \iff \sigma_1(T - \theta_2) \begin{cases} < \\ = \\ > \end{cases} \sigma_2(T - \eta_2). \quad (15)$$

in which it holds that $T - \theta_2 > T - \eta_2$. Thus, postpromotion effects will last longer for brand 1 than for brand 2 (which is an artifact of the finite horizon). Using (15) one can conclude that brand 1 should have a smaller discount than brand 2 ($d_\theta^* < d_\eta^*$) if $\sigma_1 > \sigma_2$. The latter means that a promotion of brand 1 causes more damage to postpromotion demand than a promotion of brand 2.

Consider the extreme cases

$$(i): \sigma_1 = 0, \sigma_2 > 0, \quad (ii): \sigma_1 > 0, \sigma_2 = 0, \quad (iii): \sigma_1 = \sigma_2 = 0,$$

and recall that a σ -parameter being zero means that there are no stockpiling effect of a promotion on future demand. It holds in Case (i) that $d_\theta^* > d_\eta^*$ and $d_\theta^* < d_\eta^*$ in Case (ii). The intuition here is simple: a discount can be steeper if

there is no postpromotional decrease in demand. In Case (iii) the brands are completely symmetric and will have the same discount, $d_\theta^* = d_\eta^*$.

3.1.3 Optimal Durations with Fixed Discounts This subsection determines the optimal timing and duration of the promotion periods in the case where the discounts have already been fixed as part of a promotion menu. The problem then is to determine $\theta_1, \theta_2, \eta_1, \eta_2$ so as to maximize

$$J = - \left[\frac{a}{2}(\theta_2^2 - \theta_1^2) + \frac{a}{2}(\eta_2^2 - \eta_1^2) \right] + \\ \theta_1 \bar{K} + (\theta_2 - \theta_1) K^* + (\eta_1 - \theta_2) \tilde{K} + (\eta_2 - \eta_1) K^o + (T - \eta_2) \hat{K},$$

in which

$$\begin{aligned} \bar{K} &= \bar{p}_1 \bar{q}_1 + \bar{p}_2 \bar{q}_2 \\ K^* &= \bar{K} + (\bar{p}_1 \beta_1 - \bar{p}_2 \varepsilon_2 - \bar{q}_1) d_\theta - \beta_1 (d_\theta)^2 \\ \tilde{K} &= \bar{K} - \bar{p}_1 \sigma_1 d_\theta \\ K^o &= \tilde{K} + (\bar{p}_2 \beta_2 - \bar{p}_1 \varepsilon_1 - \bar{q}_2) d_\eta - \beta_2 (d_\eta)^2 \\ \hat{K} &= \tilde{K} - \bar{p}_2 \sigma_2 d_\eta \end{aligned}$$

and the discounts d_θ and d_η are numbers. We need to satisfy the constraints

$$\theta_1 \geq 0, \theta_2 - \theta_1 \geq 0, \eta_1 - \theta_2 \geq 0, \eta_2 - \eta_1 \geq 0, T - \eta_2 \geq 0. \quad (16)$$

This is a quadratic programming problem and the constraints in (16) have the following interpretation. The first and fifth are obvious. The second and fourth state that a promotional period cannot be negative, and the third one reflects the assumption that promotions must not overlap. Our main results for the quadratic programming problem are stated in Proposition 2. The proof of the proposition is straightforward and omitted.

Proposition 2 *If $0 < \theta_1 < \theta_2 < \eta_1 < \eta_2 < T$, the optimal time instants and durations are given by*

$$\begin{aligned}\theta_1 &= \frac{1}{a}[K^* - \bar{K}], & \theta_2 &= \frac{1}{a}[K^* - \tilde{K}] \\ \eta_1 &= \frac{1}{a}[K^o - \tilde{K}], & \eta_2 &= \frac{1}{a}[K^o - \hat{K}]\end{aligned}\quad (17)$$

and

$$\begin{aligned}\theta_2 - \theta_1 &= \frac{1}{a}[\bar{K} - \tilde{K}] = \frac{\bar{p}_1 \sigma_1 d_\theta}{a} \geq 0 \\ \eta_2 - \eta_1 &= \frac{1}{a}[\tilde{K} - \hat{K}] = \frac{\bar{p}_2 \sigma_2 d_\eta}{a} \geq 0,\end{aligned}\quad (18)$$

respectively. The durations of the promotions are related by

$$\eta_2 - \eta_1 = \frac{\bar{p}_1 \sigma_1 d_\theta + \bar{p}_2 \sigma_2 d_\eta}{a} - (\theta_2 - \theta_1). \quad (19)$$

3.1.4 Postoptimality Analyses

- *Relationship between durations.* By (19), the duration of the second promotion is a linearly decreasing function of the duration of the first promotion (and vice versa). This is a consequence of the fact that the problem of determining the duration of promotions essentially is one of allocating a fixed amount of time, $\kappa \triangleq (\bar{p}_1 \sigma_1 d_\theta + \bar{p}_2 \sigma_2 d_\eta) / a > 0$, among two promotions. The quantity κ has the following interpretation. Its numerator equals $\bar{K} - \hat{K}$ and represents the loss in revenue per unit of time, caused by the promotions. The denominator is the advertising cost per unit of time. Hence κ is the decrease in postpromotion revenue per advertising dollar. If the advertising cost parameter a increases, the duration of at least one of the promotions must be reduced. At least one promotion can be extended in time if the loss of revenue diminishes. To

compare the durations of the promotions consider the inequalities

$$\theta_2 - \theta_1 \begin{cases} > \\ = \\ < \end{cases} \eta_2 - \eta_1 \iff \bar{p}_1 \sigma_1 d_\theta \begin{cases} > \\ = \\ < \end{cases} \bar{p}_2 \sigma_2 d_\eta, \quad (20)$$

derived from (19). Suppose that $\sigma_1 = \sigma_2$, that is, the impacts of the promotions on postpromotion demands are the same. The upper inequalities in (20) show that brand 1 is promoted for a longer period of time than brand 2 if $\bar{p}_1 = \bar{p}_2$ and $d_\theta > d_\eta$. Thus, having two brands with the same regular price, the retailer should promote for the longer period of time the brand that gets the largest discount.

- *Relationship between depth of discount and duration.* The results in (18) show that the duration of a promotion increases with the depth of the (fixed) discount. Thus, a promotion with a deep discount should last longer than one involving a more modest discount. This confirms a result in Rao and Thomas (1973) and has already been noted in Section 2.2.2 - although it certainly disagrees with the practice of having short campaigns with deep discounts.
- *Time interval between promotions.* Recalling that K^* and K^o are the category revenues during the promotion of brands 1 and 2, respectively, we get from (17)

$$\eta_1 - \theta_2 = \frac{1}{a}(K^o - K^*),$$

and hence

$$K^o \begin{cases} > \\ = \\ < \end{cases} K^* \implies \eta_1 \begin{cases} > \\ = \\ < \end{cases} \theta_2. \quad (21)$$

The expression in (21) states the following. If $K^o > K^*$, the category revenue earned by promoting brand 2 exceeds that of brand 1 and then there will be

no promotion in the time interval $[\theta_2, \eta_1]$. Otherwise, the category revenue when promoting brand 2, K^o , falls short of that of brand 1, K^* ; Then the promotion of brand 2 starts when the promotion of brand 1 ends. The reason is that it pays to extend as far as possible the more profitable promotion of brand 1 and this is done by setting its terminal date θ_2 equal to the starting date η_1 of the second promotion.

3.2 Myopic Retailer

When the retailer is myopic, the problem of determining optimal discounts is straightforward. We report the results without proofs. Optimal discounts (denoted by δ_θ^* and δ_η^*) are symmetric, and are given by

$$\begin{aligned}\delta_\theta^* &= \frac{1}{2\beta_1} [\bar{p}_1\beta_1 - \bar{p}_2\varepsilon_2 - \bar{q}_1] \\ \delta_\eta^* &= \frac{1}{2\beta_2} [\bar{p}_2\beta_2 - \bar{p}_1\varepsilon_1 - \bar{q}_2].\end{aligned}\tag{22}$$

The optimal duration and timing of promotions are the same as for the forward-looking retailer.

From Proposition 1 we have the optimal discounts of a forward-looking retailer:

$$\begin{aligned}d_\theta^* &= \frac{1}{2\beta_1} [\bar{p}_1\beta_1 - \bar{p}_2\varepsilon_2 - \bar{q}_1] - \frac{\bar{p}_1\sigma_1(T - \theta_2)}{2\beta_1(\theta_2 - \theta_1)} \\ d_\eta^* &= \frac{1}{2\beta_2} [\bar{p}_2\beta_2 - \bar{p}_1\varepsilon_1 - \bar{q}_2] - \frac{\bar{p}_2\sigma_2(T - \eta_2)}{2\beta_2(\eta_2 - \eta_1)}.\end{aligned}\tag{23}$$

Using (22) and (23) yields

$$\begin{aligned}d_\theta^* &= \delta_\theta^* - \frac{\bar{p}_1\sigma_1(T - \theta_2)}{2\beta_1(\theta_2 - \theta_1)} \\ d_\eta^* &= \delta_\eta^* - \frac{\bar{p}_2\sigma_2(T - \eta_2)}{2\beta_2(\eta_2 - \eta_1)}\end{aligned}\tag{24}$$

which shows that $d_\theta^* \leq \delta_\theta^*$ and $d_\eta^* \leq \delta_\eta^*$. Thus, a forward-looking retailer applies a *smaller* discount in both promotions. This illustrates a main tenet of dynamic behavior: being forward-looking is being cautious. To account for the postpromotion impacts of promotions, a forward-looking retailer modifies the myopic decision rule and sets smaller discounts. (An exception is when postpromotional effects are completely absent ($\sigma_1 = \sigma_2 = 0$); then a forward-looking retailer can act myopically).

If in (24) we have (i) $T = \eta_2$ or (ii) $T = \theta_2$, then d_η^* equals δ_η^* . Both results are expected. In Case (i) the second promotion ends at the horizon date. Since any retailer ignores effects that lie beyond the horizon date she can behave myopically. (Clearly, this is an end-game phenomenon). In Case (ii), there is no second promotion and the same remark applies.

4 Conclusions

The paper has suggested a dynamic planning model for a retailer who wishes to determine optimal discounts, timing, and duration of promotions for two brands in a specific category. The model also reports a key performance measure, category profits, in each time period as well as payoffs-to-go from the start of each time period. We calculated optimal discounts on each brand, and their timing and duration. An important part of our work is the postoptimality analysis, where in particular the dependence of optimal promotional decisions on demand characteristics was studied.

In order to have an analytically tractable setup, a number of simplifications were made in the modeling process. Among these are:

- The driving force behind retail price promotions, trade deals, was omitted. Clearly, in practice the availability, the duration, and the magnitude of a transfer price discount influence in some way or another the design of the retailer's price promotions. An interesting modification of the model would

be to superimpose a (parsimonious) trade deal pattern on the five-period planning horizon, to see how the design of such a scheme would impact the retailer's promotional decisions. Such an extension is complicated, also from the reason that the extent of the retailer pass-through of trade deals needs to be modeled. A further modification would involve the manufacturers of the two brands as active decision makers in the game

- Demand functions are linear. Since the shape of the deal effect curve has only little or no empirical support (Blattberg et al. (1995)), it seems worthwhile to (i) extend the model to include more general demand specifications, and (ii) to intensify the empirical work on assessing the shape of demand functions during promotional periods
- Feature advertising and display activities did not influence demand during a promotion. To remedy this simplification, one would need to incorporate the levels of these activities as decision variables that affect demand levels during promotions. An obstacle here is the lack of knowledge of the simultaneous effects of advertising, displays, and discounts on demand. Also in this area there is a clear need for empirical research
- Our setup allows for at most one promotion of each of the brands. A more elaborate framework would involve more periods such that multiple promotions of each brand would be possible.

Acknowledgements The paper has benefited from the comments of seminar participants at GERAD, HEC Montréal and CentER, Tilburg University. This work was partially supported by NSERC-Canada.

5 Appendix

The appendix presents the solution of the dynamic programming problem in Section 2.2.1. Denote by J_i the value function at stage $i \in \{1, 2, \dots, 5\}$. The value

function measures the optimal profit-to-go as of stage i and will be calculated in backward time.

Stage five starts at time $t = \eta_2$ and we have

$$J_5 = (T - \eta_2)\hat{K} = (T - \eta_2)(\bar{K} - \bar{p}_1\sigma_1d_\theta - \bar{p}_2\sigma_2d_\eta).$$

At **stage 4**, which starts at $t = \eta_1$, we find J_4 from

$$\begin{aligned} J_4 = & -A_\eta + \max_{d_\eta} \{(\eta_2 - \eta_1)K^o + J_5\} = -A_\eta + \\ & \max_{d_\eta} \{(\eta_2 - \eta_1)[\bar{p}_1(\bar{q}_1 - \sigma_1d_\theta - \varepsilon_1d_\eta) + \\ & (\bar{p}_2 - d_\eta)(\bar{q}_2 + \beta_2d_\eta)] + J_5\}. \end{aligned}$$

Note that K^o is a strictly concave function of d_η . Performing the maximization yields the optimal discount of brand 2. Whenever positive, it is given by

$$d_\eta^* = \frac{1}{2\beta_2} \left[\bar{p}_2\beta_2 - \bar{p}_1\varepsilon_1 - \bar{q}_2 - \bar{p}_2\sigma_2 \frac{T - \eta_2}{\eta_2 - \eta_1} \right],$$

and using d_η^* one can calculate J_4 :

$$\begin{aligned} J_4 = & -A_\eta - (T - \eta_2)\bar{p}_2\sigma_2d_\eta^* + (T - \eta_1)\bar{K} + \\ & (\eta_2 - \eta_1)[(\bar{p}_2\beta_2 - \bar{p}_1\varepsilon_1 - \bar{q}_2)d_\eta^* - \bar{p}_1\sigma_1d_\theta - \beta_2(d_\eta^*)^2]. \end{aligned}$$

At **stage 3**, which starts at $t = \theta_2$, e determine J_3 :

$$\begin{aligned} J_3 = & (\eta_1 - \theta_2)\tilde{K} + J_4 = \\ & -A_\eta - (T - \eta_2)\bar{p}_2\sigma_2d_\eta^* + (T - \theta_2)[\bar{K} - \bar{p}_1\sigma_1d_\theta] + \\ & (\eta_2 - \eta_1)[(\bar{p}_2\beta_2 - \bar{p}_1\varepsilon_1 - \bar{q}_2)d_\eta^* - \beta_2(d_\eta^*)^2]. \end{aligned}$$

At **stage 2**, starting at $t = \theta_1$, we determine J_2 :

$$J_2 = -A_\theta + \max_{d_\theta} \{(\theta_2 - \theta_1)K^* + J_3\} = -A_\theta + \max_{d_\theta} \{(\theta_2 - \theta_1)[(\bar{p}_1 - d_\theta)(\bar{q}_1 + \beta_1 d_\theta) + \bar{p}_2(\bar{q}_2 - \varepsilon_2 d_\theta)] + J_3\}.$$

Note that K^* is strictly concave in d_θ . Performing the maximization provides the optimal discount for brand 1:

$$d_\theta^* = \frac{1}{2\beta_1} \left[\bar{p}_1\beta_1 - \bar{p}_2\varepsilon_2 - \bar{q}_1 - \bar{p}_1\sigma_1 \frac{T - \theta_2}{\theta_2 - \theta_1} \right].$$

Then J_2 can be calculated:

$$\begin{aligned} J_2 = & -A_\theta - A_\eta - (T - \eta_2)\bar{p}_2\sigma_2 d_\eta^* - (T - \theta_2)\bar{p}_1\sigma_1 d_\theta^* + (T - \theta_1)\bar{K} + \\ & (\eta_2 - \eta_1)[(\bar{p}_2\beta_2 - \bar{p}_1\varepsilon_1 - \bar{q}_2)d_\eta^* - \beta_2(d_\eta^*)^2] + \\ & (\theta_2 - \theta_1)[(\bar{p}_1\beta_1 - \bar{p}_2\varepsilon_2 - \bar{q}_1)d_\theta^* - \beta_1(d_\theta^*)^2]. \end{aligned}$$

At **stage 1**, which starts at time zero, we have the profit for the entire planning period:

$$\begin{aligned} J = & \theta_1\bar{K} + J_2 = \\ & T\bar{K} - [A_\theta + A_\eta] - [(T - \eta_2)\bar{p}_2\sigma_2 d_\eta^* + (T - \theta_2)\bar{p}_1\sigma_1 d_\theta^*] + \\ & (\eta_2 - \eta_1)[(\bar{p}_2\beta_2 - \bar{p}_1\varepsilon_1 - \bar{q}_2)d_\eta^* - \beta_2(d_\eta^*)^2] + \\ & (\theta_2 - \theta_1)[(\bar{p}_1\beta_1 - \bar{p}_2\varepsilon_2 - \bar{q}_1)d_\theta^* - \beta_1(d_\theta^*)^2]. \end{aligned}$$

Finally, inserting the optimal discounts into J yields the optimal value of the retailer's objective function, in terms of the parameters only.

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