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WELFARE IMPLICATIONS OF A SECOND LENDER IN THE INTERNATIONAL MARKETS

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Resumen

El rol de un prestamista adicional en los mercados internacionales – tal como el FMI u otra institución similar - ha sido ampliamente cubierto en discusiones académicas y entre políticos. Sin embargo, no existe una respuesta clara ni un consenso acerca de sus implicaciones en términos de bienestar. Por un lado, se plantea que las consecuencias de riesgo moral de un prestamista adicional podrían ser lo suficientemente importantes como para compensar los potenciales beneficios asociados a una fuente adicional de financiamiento. Por otro lado, también se plantea que las consecuencias de riesgo moral podrían ser más bien menores y, en consecuencia, la presencia de un segundo prestamista podría implicar ganancias de bienestar. El aporte de este artículo es doble. Primero, éste provee una perspectiva numérica sobre los efectos de bienestar de un segundo prestamista. Segundo, permite entender de mejor modo las dinámicas que se generan en el escenario con dos prestamistas. El principal resultado es que el segundo prestamista no aumenta el bienestar para un amplio rango de parámetros. Sin coordinación, las estimaciones implican pérdidas de bienestar que varían entre 1.5% y 6% del PIB, dependiendo de la severidad de los castigos impuestos por el segundo prestamista. Alternativamente, si un mecanismo de coordinación es impuesto, de manera tal que el segundo prestamista actúe como un prestamista de última instancia, entonces, el modelo reproducirá el escenario con solo un prestamista donde el primer prestamista deja de participar en el mercado.

Abstract

The role of an extra lender in the international markets – such the IMF or another similar institution - has been widely covered in academic discussions and among policy makers. However, there is neither a clear answer nor a consensus about its welfare consequences. On the one hand, it is argued that the moral hazard consequences of an extra lender could be strong enough to offset any positive effect of an additional source of funding. On the other hand, it is argued that moral hazard consequences could be negligible and, therefore, the second lender's presence could be welfare improving. The aim of this paper is twofold. First, it provides a numerical perspective about the welfare effect of an active second lender. Second, it sheds light on the debt dynamics in the two-lender case. The main result is that the second lender is not beneficial from a welfare standpoint for a wide range of parameters. Without coordination, the estimates imply welfare losses that range from 1.5% to 6% of GDP, depending on the severity of the second lender's penalties. On the other hand, if a coordination mechanism is imposed, such as the second lender acting as a lender of last resort, then the model will mimic a one-lender model where the first lender is crowded out from the market.

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1. Introduction and Motivation

The sequence of defaults during the nineties opened a strong discussion about the role of the IMF to prevent or reduce the effect of similar episodes.¹ A high degree of consensus has been reached with respect to the elimination of currency and term mismatches, better institutional infrastructure and bail-ins, among other points. However, the direct role of the IMF in the lending does not have such consensus (see Tirole, 2002 and Isard, 2005). Basically, there are two positions. The *dovish* approach considers that moral hazard implications from IMF's lending activities are small² (Allen and Gale, 2000; Eichengreen, 1999; Obstfeld, 2000). Meanwhile, the *hawkish* position claims that extra lending will promote over borrowing with its related consequences (Schwartz, 1999; Meltzer Commission, 2000).

This paper provides a quantitative approach to the role of the IMF and/or other similar international organizations in the international finance architecture. In this sense, the main goal is to evaluate if the moral hazard consequences of an extra lender are strong enough to counteract the potential benefits of an extra lender (*hawkish*) or vice versa (*dovish*). The paper develops a model where potential IMF activities can be envisioned as those of a second lender in the international markets (hereinafter “the second lender”) and the private market lending will be referred as “the first lender”³. The approach follows the classic framework of Eaton and Gersovitz (1981) in which risk sharing is limited to non-contingent bonds, and repayment is enforced by the threat of exclusion from the financial market and the loss of a fraction of output.⁴

¹ This discussion has been highly influenced by the Argentina's default on the international bonds of over \$82 billion in 2001 and the IMF's involvement in the Asian financial crisis of 1997-98, where it provided more than \$36 billion to Indonesia, Korea, and Thailand.

² The dovish and hawk terms are due to Tirole (2002).

³ This paper is devoted to understand the implications of the direct involvement of the IMF in the international markets as an active lender. In this sense, other activities where the IMF is involved are not considered – such as surveillance, and technical assistance - and, consequently, the implications and conclusions of this paper should be restricted to this particular dimension.

⁴ This approach has been extended to study several dimensions of the sovereign debt problem. For example, Yue (2006) studies the implications of renegotiation between lenders and borrowers, while Arellano (2004) explores the interactions between the tradable and no tradable sectors.

The existence of a second lender introduces a richer set of dynamics than the standard one-lender model, the most important of which is that the threat to be excluded from the capital markets is virtually void, because if the country defaults with the first (second) lender, the country can still borrow from the second (first) lender. Additionally, the presence of a second lender will also generate extra incentives to default on one lender (even if the agent is under default with the other lender), because there is a higher probability to be forgiven by at least one of the lenders. This kind of dynamics directly affects the tightness of the endogenous borrowing constraints, which has direct repercussions on the actual access to international financial markets.

To evaluate how important these effects are, the welfare loss in terms of equivalent consumption is estimated for a wide range of parameters. A key aspect in this evaluation is the degree of benevolence of the second lender, where the term benevolence refers to the severity of the penalties with respect to those observed in the private markets (the first lender). This point is implicitly embodied in the hawkish and dovish visions. The hawkish vision should advocate for high penalties to discourage the adverse moral hazard effects (i.e., no benevolence), and the dovish vision will ask for low penalties because the moral hazard effects are negligible. In practice, there is not clear evidence about the degree of IMF's benevolence, but its goals consider simultaneously the stability of the international financial system and the direct welfare of the member countries, as measured by, for instance, poverty reduction.⁵ Therefore, it is plausible that if the second lender evaluates both the financial stability and the welfare of borrowers (member countries), and the private market only considers the stability of the financial system, then, under the prior that low penalties are positively related to countries' welfare, the second lender's penalties could be lower than the first lender's penalties. Another potential argument in this line could be that it is dynamically inconsistent to apply the penalty after the default event⁶.

⁵ The Articles of Agreement of the International Monetary Fund state as one of its purposes "*To facilitate the expansion and balanced growth of international trade, and to contribute thereby to the promotion and maintenance of high levels of employment and real income and to the development of the productive resources of all members as primary objectives of economic policy*" (International Monetary Fund, 2004).

⁶ Kenneth and Wright (2000) propose a long-term debt relationship that is sustainable, although the contract is continually subject to renegotiation.

Theoretically, if markets are complete, the policy recommendation is direct: penalties should be high enough to discourage any potential default. If the markets are not complete, the policy recommendation is not that straightforward. Zame (1993) shows that in a world with uncertainty and incomplete financial markets, default *may* promote efficiency, because the possibility of default improves the efficiency of markets and does so in a way that simply opening new markets does not.⁷ In a similar line of research, Dubey, Genakoplos and Shubik (2001) show cases where it is better to set intermediate default penalties instead of extremely high penalties, because higher penalties may tend to reduce the incentives to trade securities in order to achieve a higher degree of risk sharing.

The previous paragraph points out the existence of important trades-off related to the second lender's presence, which contrasts with the lack of empirical and numerical studies in this area. In fact, most of the evidence about the moral hazard implications is subject to important criticisms (Morris and Shin, 2006). As a consequence of the lack of numerical and empirical research in this area, the discussion about the future financial architecture has been supported to a great extent by conceptual viewpoints or case-by-case arguments, where it is common to observe judgments that are more normative than positive. Additionally, the existence of more than one lender has not been explored in depth in the literature. Some exceptions are Krueger and Uhlig (2005), and Phelan (1995). Krueger and Uhlig study a problem with multiple principals and agents, where the agents can walk away from the contract and borrow from another principal. Their main result is that no, partial and full risk sharing can be supported depending on the agents' discount factor with respect to principal's degree of impatience. In a similar line, Phelan (1995) considers an environment where agents can only leave the contract at the beginning of the period, without knowledge of their period income, with the conclusion that insurance still occurs and that a non-degenerate long-run distribution of consumption exists.

Therefore, numerical evidence about the implications of an active second lender is particularly relevant for discussing the optimal degree of IMF involvement in international markets. In this regard, the main conclusion of this paper is that the second lender does not

⁷ Zame (1990) proves that the default equilibrium is close to Walrasian equilibrium, provided that the default penalty and the number of securities are sufficiently large.

play a beneficial role, on the contrary it is welfare reducing. Second, the dynamics of the model imply that the borrower has strong incentives to always default on one lender in order to obtain funding from the other one. Under good financial status (i.e., no default at all) the market exclusion penalty is virtually void and, therefore, the maximum sustainable level of debt that both lenders are willing to lend decreases substantially, which clearly is not optimal for impatient borrowers. Finally, if the second lender acts as a lender of last resort, then the two-lender model will mimic a one-lender model where the first lender is totally crowded out from the market.

The rest of the paper is organized as follows. The next section describes the model and defines the equilibrium. Section 3 contains the numerical analysis, with special attention to the welfare implications of the second lender. Section 4 contains a description of the main factors behind the welfare losses due to a second lender (i.e., flip-and-back behavior and lower endogenous borrowing). Section 5 modifies the setting, imposing a coordination mechanism, such that the second lender acts as a lender of last resort. Section 6 concludes.

2. The Model

The model consists of a small open economy that receives a stochastic endowment every period. The representative household decides how much to consume and borrow from two international lenders. The international financial market only considers a non-contingent bond with a maturity of one period. Financial contracts are non enforceable and the borrower can default on the debt of both lenders. If the borrower defaults on one lender, the economy will not have access to that market and it will also incur direct output costs, but the economy can still borrow from the other lender. In this sense, the financial autarky scenario requires that the borrower be in default with both lenders at the same time.

Households

Households are identical and have preferences given by:

$$E_0 \sum_{t=0}^{\infty} \beta^t u(c_t) \quad (1)$$

where $0 < \beta < 1$ is the discount factor, c_t is the consumption in period t and $u(\cdot)$ is strictly concave, increasing and twice continuously differentiable. Households receive stochastic endowment streams of y goods. This process is assumed to have a compact support and to be a Markov process drawn from a probability space (y, ζ, π) , where y is the set of endowment realizations, ζ is a σ -algebra of its subsets, and π is a measure defined on ζ .

The budget constraint conditional on non default equates purchases of the consumption good to the endowment less debt repayments, plus the issues of new debt (b_t^f, b_t^s) at price (q_t^f, q_t^s) , where the superscripts “ f ” and “ s ” denotes the first and second lender, respectively. Or, equivalently, the budget constraint can be written as:

$$c_t + q_t^f b_t^f + q_t^s b_t^s = y_t + b_{t-1}^f + b_{t-1}^s \quad (2)$$

where $b^j > 0$ means that the country is a net saver of the $j = f, s$ lender.

In case of default with lender $j = f, s$, the debt with this lender is erased and the household can not borrow or save with lender j during a determined period and, additionally, it faces a direct output cost equal to λ^j .⁸ The output loss penalty can be rationalized by a number of different ways, the most commonly mentioned being the problems derived from disruptions in international trade (Cole and Kehoe, 2000; Bulow and Rogoff, 1989b).⁹ Based on the previous description, the budget constraint in this scenario is the following:

$$c_t + q_t^{-j} b_t^{-j} = (1 - \lambda^j) y_t + b_{t-1}^{-j} \quad (3)$$

⁸ In the absence of penalties, the reputation models of sovereign debt implies zero debt in equilibrium because default will happen with probability one (Bulow and Rogoff, 1989). Some authors have introduced modifications to deal with this issue. Amador (2003) considers a short-sighted government that faces political shocks. Kletzer and Wright (2000) work with lack of commitment from lenders, which allows supporting international borrowing in equilibrium. Krueger and Uhlig (2005) also work in this line.

⁹ In a more quantitative dimension, Rose (2002) finds evidence and of a sizeable – 8% per year – decline in bilateral trade flows following the initiation of debt renegotiation by a country in a sample covering 200 trading partners over the period 1948 to 1997.

where $-j$ denotes the complement of j .

If the borrower defaults on both lenders, then, the budget constraint is reduced to the financial autarky case. It means that the representative household will consume each period its net output:

$$c_t = (1 - \lambda^f - \lambda^s)y_t \quad (4)$$

Lenders

Lenders have access to an international credit market in which they can borrow and lend at a constant rate $r > 0$. The first lender can be envisioned as the international private market composed of a continuum of agents that behave competitively. In other words, the first lender maximizes the following function:

$$\psi_t^f = q_t^f b_t^f - \frac{(1 - d_t^f)}{1 + r} b_t^f \quad (5)$$

where d_t^f is the probability of default on the first lender, the prices are taken as given and, as a consequence of the competitive assumption, the expected profit (ψ_t^f) is equal to zero.

With respect to the second lender, it is also assumed that this lender makes zero expected profit. This assumption serves to make the implications of an extra lender comparable to the one-lender model with competitive private markets. In effect, if the zero expected profit condition is not imposed, then the welfare implications of the two-lender model could be reflecting the transfer of resources from the second lender to households and not necessarily the effect related to its lending activities. Given the previous elements, the second lender maximizes the following function:

$$\psi_t^s = q_t^s b_t^s - \frac{(1 - d_t^s)}{1 + r} b_t^s \quad (6)$$

The direct implication from equations (5) and (6) is:

$$q_t^f = \frac{(1-d_t^f)}{1+r}, \quad q_t^s = \frac{(1-d_t^s)}{1+r} \quad (7)$$

There are some direct implications from condition (7). The first one is that if the borrower is saving with both lenders, then the probability of default is zero ($d_t^f = d_t^s = 0$) and, therefore, the debt prices are the same and equal to the inverse of the gross risk-free interest rate ($q_t^f = q_t^s = 1/(1+r)$). The second implication is derived from the default probabilities. The lender with lower penalties will have a default probability at most equal to the other lender, which in turn will imply that the low-penalty price will have as its upper bound the high-penalty debt price. Finally, the bond prices lie in the closed interval $[0, 1/(1+r)]$, because $0 \leq d^j \leq 1$.

Recursive Problem

This section presents the recursive formulation of the model. The state variables correspond to the debt with the first and second lender (b^f, b^s) , the endowment in each period (y) and, finally, the financial status of the borrower –summarized in a dummy variable D^j that takes on a value 1 if borrower is in default with lender j –. The financial status can take four alternative values: i) good financial record, it means that the household is not in default with both lenders ($D^f = 0, D^s = 0$); ii) default with the second lender ($D^f = 0, D^s = 1$); iii) default with the first lender ($D^f = 1, D^s = 0$); and iv) default with both lenders or, equivalently, financial autarky ($D^f = 1, D^s = 1$). Therefore, given the financial status, the value function of households could be represented as follow:

Household in good financial status

$$v(b^f, b^s, y, D^f = 0, D^s = 0) = \text{Max} \left\{ \begin{array}{l} w^o(b^f, b^s, y, D^f = 0, D^s = 0), \\ w^s(b^f, b^s, y, D^f = 0, D^s = 0), \\ w^f(b^f, b^s, y, D^f = 0, D^s = 0), \\ w^{fs}(b^f, b^s, y, D^f = 0, D^s = 0), \end{array} \right\} \quad (8)$$

Household in default with the second lender:

$$v(b^f, 0, y, D^f = 0, D^s = 1) = \text{Max} \left\{ \begin{array}{l} w^s(b^f, 0, y, D^f = 0, D^s = 1), \\ w^{fs}(b^f, 0, y, D^f = 0, D^s = 1) \end{array} \right\} \quad (9)$$

Household in default with the first lender:

$$v(0, b^s, y, D^f = 1, D^s = 0) = \text{Max} \left\{ \begin{array}{l} w^f(0, b^s, y, D^f = 1, D^s = 0), \\ w^{fs}(0, b^s, y, D^f = 1, D^s = 0) \end{array} \right\} \quad (10)$$

Household in default with both lenders:

$$v(0, 0, y, D^f = 1, D^s = 1) = w^{fs}(0, 0, y, D^f = 1, D^s = 1) \quad (11)$$

where:

v = value function

w^j = expected utility under the financial states denoted by the following superscripts: o = no default, f = default with the first lender, s = default with the second lender, fs = default with both lenders.

The previous equations point out the exponential nature of the model's dynamic when more lenders are added. In this particular case – two lenders -, the households in good financial status have four options with respect to their debt. They can pay both lenders, default on one of them and continue borrowing from the other lender or, finally, they can default on both lenders. By contrast, if the households are under default with one lender, they can only choose between repaying and not repaying their current debt. These options will introduce a rich dynamic in terms of payment behavior and they can allow us to understand the actual effects of active lending by the IMF and/or another similar institution.

Additionally, the existence of two potential lenders expands the set of prices in this economy. The traditional models with one lender just have one price schedule for bonds. However, the two-lender case is characterized by four possible price schedules: i) the first lender's price of bonds under good financial records; ii) the first lender's price of bonds

under default with the second lender; iii) the second lender's price of bonds under good financial records; and iv) the second lender's price of bonds under default with the first lender. Roughly speaking, the four prices are required because the probability of default varies with the financial status of households and, therefore, the required price to obtain zero expected profit should also change.

From the previous elements it should be clear that the financial status of households is the key element to determine the set of consumption and borrowing options. The transition law of this state –i.e., financial status - is the outcome between the default decision taken by the households and the forgiveness of lenders. For instance, if the financial status in this period is good ($D^f = D^s = 0$) and the households choose to default on lender j , then the financial status in the next period could be good again if the lender j forgave the household (with probability θ^j) or, alternatively, the household could be excluded from that market another period (with probability $1-\theta^j$).

As is traditional in this kind of model, the components to evaluate in each state are the value to honor the debt vis à vis the default decision. The value of each one of the options in this problem is summarized in the following specifications.

a) Household in good financial status and honors its debt ($D^f = D^s = 0$)

$$w^o(b^f, b^s, y, D^f = 0, D^s = 0) = \underset{b^{f'}, b^{s'}}{\text{Max}} \quad u(y - q^f b^{f'} - q^s b^{s'} + b^f + b^s) \\ + \beta \sum_s v(b^{f'}, b^{s'}, y', D^{f'} = 0, D^{s'} = 0) \pi(s' | s) \quad (12)$$

b) Household in default with the second lender and honors its debt ($D^f = 0, D^s = 1$)

$$w^s(b^f, 0, y, D^f = 0, D^s = 1) = \underset{b^{f'}}{\text{Max}} \quad u((1 - \lambda^s)y - q^f b^{f'} - b^f) \\ + \beta \sum_s [\theta^s v(b^{f'}, 0, y', D^{f'} = 0, D^{s'} = 0) \\ + (1 - \theta^s) v(b^{f'}, 0, y', D^{f'} = 0, D^{s'} = 1)] \pi(s' | s) \quad (13)$$

c) Household in default with the first lender and honors its debt ($D^f = 1, D^s = 0$)

$$\begin{aligned}
w^f(0, b^s, y, D^f = 1, D^s = 0) &= \text{Max}_{b^{s'}} u((1 - \lambda^f)y - q^s b^{s'} - b^s) \\
&+ \beta \sum_s [\theta^f v(0, b^{s'}, y', D^{f'} = 0, D^{s'} = 0) \\
&+ (1 - \theta^f)v(0, b^{s'}, y', D^{f'} = 1, D^{s'} = 0)] \pi(s' | s)
\end{aligned} \tag{14}$$

d) Household in default with both lenders ($D^f = 1, D^s = 1$)

$$\begin{aligned}
w^{fs}(0, 0, y, D^f = 1, D^s = 1) &= u((1 - \lambda^f - \lambda^s)y) \\
&+ \beta \sum_s [\theta^f \theta^s v(0, 0, y', D^{f'} = 0, D^{s'} = 0) \\
&+ \theta^f (1 - \theta^s)v(0, 0, y', D^{f'} = 1, D^{s'} = 0) \\
&+ \theta^s (1 - \theta^f)v(0, 0, y', D^{f'} = 0, D^{s'} = 1) \\
&+ (1 - \theta^f)(1 - \theta^s)v(0, 0, y', D^{f'} = 1, D^{s'} = 1)] \pi(s' | s)
\end{aligned} \tag{15}$$

e) Other combinations.

$$\begin{aligned}
w^s(b^f, b^s, y, D^f = 0, D^s = 0) &= w^s(b^f, 0, y, D^f = 0, D^s = 1) \\
w^f(b^f, b^s, y, D^f = 0, D^s = 0) &= w^f(0, b^s, y, D^f = 1, D^s = 0) \\
w^{fs}(b^f, b^s, y, D^f = 0, D^s = 0) &= w^{fs}(0, 0, y, D^f = 1, D^s = 1)
\end{aligned}$$

Honoring the debt and not defaulting is a period-by-period decision. In each period, the household must evaluate the expected value to stay in the credit contract versus the expected value of being excluded from a particular market or both. The set of policy decisions can be characterized by default sets contingent on the states. Concretely, let $DS_i^j(b^f, b^s) \subset Y$ be the set of endowment shock y for which default in debt $j = f, s$ is optimal given the household's debt and its financial status $i = o, f, s$ - i.e., no default, .default with the first lender, and default with the second lender, respectively -. That is

a) Good Financial Status ($D^f = D^s = 0$)

$$DS_0^s(b^f, b^s) = \{y \in Y : w^s \text{ and / or } w^{fs} = \text{Max}(w^o, w^f, w^s, w^{fs})\}$$

$$DS_0^f(b^f, b^s) = \{y \in Y : w^f \text{ and / or } w^{fs} = \text{Max}(w^o, w^f, w^s, w^{fs})\}$$

b) Default with the Second Lender ($D^f = 0, D^s = 1$)

$$DS_s^f(b^f, 0) = \{y \in Y : w^{fs} = \text{Max}(w^s, w^{fs})\}$$

c) Default with the First Lender ($D^f = 1, D^s = 0$)

$$DS_f^s(0, b^s) = \{y \in Y : w^{fs} = \text{Max}(w^f, w^{fs})\}$$

Recursive Equilibrium

The recursive equilibrium for this economy is defined as a set of policy functions for:

i) consumption $c(b^f, b^s, y, D^f, D^s)$, ii) asset holdings $b^f'(b^f, b^s, y, D^f, D^s)$ and $b^s'(b^f, b^s, y, D^f, D^s)$, iii) default sets $DS_{i=o,f,s}^{j=f,s}(b^f, b^s)$, and price functions $q^{j=f,s}(b^f, b^s, y, D^f, D^s)$ such that¹⁰:

1. Given the bond price schedules $q^{j=f,s}(b^f, b^s, y, D^f, D^s)$, policy functions for $c(b^f, b^s, y, D^f, D^s)$, $b^f'(b^f, b^s, y, D^f, D^s)$, $b^s'(b^f, b^s, y, D^f, D^s)$, and $DS_{i=o,f,s}^{j=f,s}(b^f, b^s)$ solve the household's problem.
2. Bond prices $q^{j=f,s}(b^f, b^s, y, D^f, D^s)$ are such that they reflect the default probabilities and are consistent with lenders' expected zero profits in order to clear loan markets.

¹⁰ The existence of this kind of equilibrium has been proved for several similar settings (Eaton and Ergovitz, 1981; Chatterjee, Corbae, Nakajima and Rios-Rull, 2002; Yue, 2006, among others). Therefore, the existence of equilibrium is extrapolated to the model with two lenders, which is ratified by the numerical convergence of the model (section 3).

Some General Characteristics

Proposition 1: Default sets are growing in debts. For all $b^{j1} \leq b^{j2}$, if default on the $j = f, s$ lender is optimal for b^{j2} for some states y , then, $DS_i^j(b^{j2}, b^{-j}) \subseteq DS_i^j(b^{j1}, b^{-j})$ - $i = o, f, s$ and $j = f, s$ -.

Proof. See Appendix 1.

Proposition 2: Default sets are shrinking in penalties. a) If the output penalty of the non-defaulted debt λ increases to λ' ($\lambda < \lambda'$), then $DS_i^j(b^f, b^s)' \subseteq DS_i^j(b^f, b^s)$, where the superscript 'denotes λ' . b) If the market exclusion penalty of the no defaulted debt $1 - \theta$ increases to $1 - \theta'$ ($\theta < \theta'$), then $DS_i^j(b^f, b^s)' \subseteq DS_i^j(b^f, b^s)$, where the superscript 'denotes θ' .

Proof. See Appendix 1.

Proposition 3: The household in good financial status tends to default on the lender with lower penalties. For $\lambda^j < \lambda^{-j}$, $\theta^j = \theta^{-j}$, $D^f = D^s = 0$, and $b^f = b^s = 0$, if the household decides to default only on one type of debt, and it is planning to issue new debt equal to \bar{b} , then it will default on the lender j - i.e, the lender with lower penalties-.

Proof. See Appendix 1.

Propositions 1 and 2 are self explanatory. With respect to Proposition 3, it says that if the household will default on just one lender and will issue new debt, then it will prefer to default on the lender with lower penalties. The logic behind this result lies in the following elements: i) the lower output loss penalty ($\lambda^j < \lambda^{-j}$), and ii) the fact that the lender with higher penalties face lower default probabilities and therefore will charge lower interests rate to the household (i.e., higher debt price).

3. Numerical Analysis

Functional Forms and Parameters

The household has constant relative risk aversion (CRRA) preferences over consumption:

$$u(c) = \frac{c^{1-\gamma}}{1-\gamma} \quad (15)$$

The process of endowment considers stochastic cumulative growth, where the log growth rate follows an autoregressive process of first order. The main feature of this specification is that a positive shock to the growth rate implies a permanent higher level of output and, therefore, the output trend is stochastic. This process was originally proposed by Aguiar and Gopinath (2005) to explain differences between emerging and developed economies in a business cycles context¹¹. The functional form of the endowment process is characterized by:

$$y_t = \Gamma_t = g_t \Gamma_{t-1} = \prod_{s=0}^t g_s \quad (16)$$

$$\ln(g_t) = (1 - \rho_g) \ln(\mu_g) + \rho_g \ln(g_{t-1}) + \varepsilon_t^g \quad (17)$$

where g_t = productivity growth rate in period t and ε_t^g = shock to the growth rate in period t . In order to obtain a well defined problem, it is necessary to assume that $\beta \cdot \mu_g^{1-\gamma} < 1$,¹² with a discount rate here $0 < \beta < 1$.

The model is simulated with parameters taken from previous models that study the business cycle and default events for Argentina on a quarterly basis (Arellano, 2005; Yue, 2006; Aguiar and Gopinath, 2005 and 2006). The parameters are reported in Table 1. The coefficient of relative risk aversion is 2, a standard value in the literature. The risk-free interest rate is set at 1% per quarter, which is the average quarterly yield on 3-month US treasury bills.¹³ The discount rate is equal to 0.80. This parameter is relatively low, but this is required to generate a reasonable default rate in equilibrium. This requirement also applies to other models with lack of commitment. In effect, Arellano (2003), Aguiar and Gopinath (2004b), Chatterjee et al. (2002), and Yue (2006) employ a similar degree of impatience to

¹¹ In the particular case of models with lack of commitment, this approach has been employed by Yue (2006) and Aguiar and Gopinath (2006). This kind of specification is particularly useful to model default in emerging economies. Concretely, Aguiar and Gopinath (2004b) show that a specification with stochastic growth can generate default rates ten times higher than the standard endowment specification, which allows to improve the match of some moments with their empirical counterparts in variables such as spreads and default rates, among others.

¹² Aguiar and Gopinath (2004).

¹³ This value has been employed by Arellano (2003), Aguiar and Gopinath (2004), and Yue (2006), among others.

calibrate their models¹⁴. Technological parameters are taken from Aguiar and Gopinath (2005) and they were obtained from a calibration exercise for the Argentinean economy. The long-run mean of growth is calibrated to 0.6% per quarter or, equivalently, $\mu_g = 1.006$. The growth rate's persistence is equal to $\rho_g = 0.17$ and, finally, the long-run volatility of the growth rate corresponds to $\sigma_g = 3\%$.

The penalties for the first lender will be fixed at the values commonly employed in the literature (Table 1). The output loss is $\lambda^f = 2\%$ or, equivalently, when a country defaults, its output declines by 2%. The support for this number is Puhán and Sturzenegger (2005), and it has been set as the benchmark to calibrate virtually all the numerical models with partial commitment. The probability of reentering financial markets after default is set at $\theta^f = 0.1$, which is consistent with Gelos et al. (2004) and it implies that the economy is denied market access for 2.5 years on average. Meanwhile, the penalties related to the second lender will be sensibilized in order to achieve a deeper understanding of the role and implications of the second lender

Table 1
Benchmark Parameters

Parameter	Coefficient
<u>Preference Parameters</u>	
Coef. Risk Aversion ($1 - \gamma$)	2
Quarterly Risk Free Interest Rate (r)	1%
Discount Factor (β)	0.8
<u>Technological Parameters</u>	
Mean Growth (μ_g)	1.006
Volatility Growth (σ_g)	3%
Shocks Persistence (ρ_g)	0.17
<u>The First Lender's Penalties</u>	
Output Loss (λ^f)	2%
Market Exclusion (θ^f)	10%

Note: Parameters obtained from Arellano (2003); Aguiar and Gopinath (2005), Chatterjee et al. (2002); Yue (2006); Puhán and Sturzenegger (2005); Gelos et al. (2004).

¹⁴ In fact, Yue (2006) employs a discount rate equal to 0.74.

Simulation

The model was solved numerically by value function iteration. The computational aspects are covered in Appendix 2. However, it is important to mention that the recursive formulation was redefined in order to facilitate the problem's numerical solution. Basically, the vector (b^f, b^s) will be represented by the total debt $b = b^f + b^s$, and the share of the first lender's debt hold by the household $\delta = b^f/b$. Therefore, the household's portfolio can be summarized by (b, δ) . The main advantage of this representation is that reduces the computational requirements to simulate the model accurately, because it has more degrees of freedom to determine the grid of both variables – for example, the grid for b can be finer than the grid for δ ¹⁵.

Table 2 presents the benchmark scenario with one and two lenders, and also contains the empirical counterpart for some of the moments, where they correspond to the Argentinean economy and were obtained from Yue (2006), and Aguiar and Gopinath (2006). The baseline scenario considers a similar level of penalties for both lenders ($\lambda^f = \lambda^s = 2\%$ and $\theta^f = \theta^s = 10\%$). The statistics correspond to the average of 1000 simulations, where each simulation is 2000 periods long, but only the last 100 periods were considered to obtain the stationary distribution of the model economy. It is necessary to keep in mind that the welfare implications of two lenders are the focus of the analysis; therefore, the computed moments' comparison neither pretends to evaluate the model nor to conclude something about the optimal penalties based on the fit of some particular specification. In this sense, this section only illustrates the main numerical implications of the two-lender model.

In terms of the standard macro variables, the consumption volatility and the current account-output correlation are practically the same in both models, observing a slightly higher current account volatility in the two-lender model. Concretely, the current account volatility is 1.08% and 0.87% times the output volatility in the two-lender and one-lender model, which has an empirical counterpart equal to 1.35%. With respect to the spreads, the

¹⁵ In equilibrium, and given a set of reasonable parameters, households tend to hold only one kind of debt. In fact, when they are in good financial status with both lenders, they hold both debts only during the first period. This point is developed in detail in section 4.1.

statistic does not modify too much. The average spread increases from 0.17% to 0.22% in the two-lender model and its empirical value is 1.02%; meanwhile, the spread volatility increases from 0.07% to 0.12%, which compares with an observed volatility equal to 0.42%. However, the differences between these two models increase substantially when both lenders apply different penalties, where the two lender model allows to close partially the gap between their simulated and empirical counterpart in several variables. One example is the spread's volatility, which increases to 0.56% for an output loss penalty of the second lender equal to $\lambda^s = 0.5\%$.¹⁶

Table 2
One and Two Lenders Models
Numerical Solution

Statistic ¹	Data ¹	Two Lenders ² $\lambda^f=2\%$, $\lambda^s=2\%$ $\theta^f=10\%$, $\theta^s=10\%$	One Lender $\lambda=2\%$, $\theta=10\%$
Std(c)/Std(y)	1.15	1.07	1.06
Std(nx/y)	1.35%	1.08%	0.87%
Corr(nx/y,y)	-0.88	-0.15	-0.20
Std(spread)	0.42%	0.12%	0.07%
Average Spread	1.02%	0.22%	0.17%
Corr(spread,y)	-0.59	-0.02	-0.03
Corr(spread,nx)	0.49	-0.09	0.12
Default First Lender	0.69%	0.09%	0.16%
Default Second Lender	n.a.	0.09%	-
Periods Under Total Default	n.a.	1.00%	1.74%
Hold Freq. (%)			
First Lender's Bond	n.a.	50.34%	-
Second Lender's Bond		49.66%	-

Notes: 1: these figures correspond to Argentina (Aguar and Gopinath, 2005 and Yue, 2006). 2= the second lender's output penalty was set at 1.999% in order to solve the model, because of with two equal bonds the model does not converge to an equilibrium with debt.

Welfare Implications

To evaluate the impact of two lenders on welfare, the household's expected utility is calculated with one and two lenders under different penalties for the second lender and, based on these estimations, it is obtained the change on consumption that makes the

¹⁶ The sensitivity analysis is shown in Appendix 3.

household indifferent between both models. The change in consumption φ that satisfies the previous requirement is defined as:

$$\varphi = \left(\frac{E \left[\sum_{i=0}^n \beta^i c_{1,i}^{1-\sigma} \right]}{E \left[\sum_{i=0}^n \beta^i c_{2,i}^{1-\sigma} \right]} \right)^{\frac{1}{1-\sigma}} - 1 \quad (18)$$

where:

$c_{j,i}$ = consumption in period i with $j=1, 2$ lenders.

If $\varphi > 0$, the household is better off with only one lender. The converse also holds. The simulations are characterized by the following initial states: i) high endowment shock and low debt; and ii) low endowment realization and high debt, where high (low) is denoted by the median plus (minus) 1.5 standard deviation of the grid for each variable. Additionally, both scenarios assume that the household is in good financial status at the beginning ($D^f = D^s = 0$). Notice that this approach does not compute the welfare consequence on the stationary distribution. The advantage of this method is to control by the initial values, making more comparable the estimations of φ under different settings.

The welfare impact was computed for a wide range of penalties: $\lambda^s = 0.5\%$ to 4.0% , and $\theta^s = 0\%$ to 20% (tables 3 and 4). This is a key component in the analysis and is intended to capture the discussion about how much softer (or harder) should the second lender (IMF) be in its lending activities. As was mentioned, one could easily -and mistakenly- interpret that the IMF should be softer than the private market, because its statements consider elements like poverty reduction or unemployment. However, the elements discussed in the first section illustrate that is not clear that the existence of a softer lender will be welfare improving. Therefore, the numerical exercises consider an extensive sensitivity analysis with respect to this issue.

The case where the second lender imposes the same penalties as the first lender implies a welfare cost between 2.9% and 3.4% depending on the simulation's initial states. These estimates are higher than the standard estimations of welfare losses due to incomplete financial markets (2% or lower). The elements behind this higher welfare loss could be the consequences of higher incentives to default on at least one lender having as consequence a higher and more frequently applied output loss penalty, and the maximum level of debt that can support the two-lender model is lower than that of the one-lender model. These issues are discussed in detail in section 4.

The simulations also reveal that the welfare cost is highly sensitive to the severity of the penalties, where higher penalties mean a lower welfare cost. The sensitivity exercise for output loss penalties (λ^s) indicates that the welfare cost can increase from 1.5% to 5.1% in case that the output penalty decreases from $\theta^s = 4\%$ to $\theta^s = 0.5\%$ when the agent starts with the highest income realization. These numbers do not change much if the initial endowment is the lowest realization; the main difference is that the lowest output penalty generates a welfare cost of 6.2%. The sensitivity analysis for several market exclusion penalties is similar to the previous one. In effect, if the market exclusion ranges from $\theta^s = 0\%$ to $\theta^s = 20\%$, the welfare cost increase from near 1.5% to 5.7% in the high-endowment/low-debt scenario. Meanwhile, the welfare cost increases from 1.4% to 4.9% in the low-endowment/high-debt scenario.

These results suggest that the second lender model could be Pareto improving if the penalties were high enough to compensate the incentives to maintain a good financial record with just one lender. However, there are at least two elements to consider. First, it is difficult to think of a set of sanctions that could generate output losses larger than the output penalties shown in tables 3 and 4 - notice that the simulations consider the permanent market exclusion penalty ($\theta^s = 0\%$).¹⁷ Second, beyond the feasibility of imposing such penalties, stronger penalties could be subject to dynamic inconsistency issues. In other

¹⁷ The model is not simulated with higher penalty levels because these imply a broader set of values to simulate the model, since the maximum level of debt increases with the severity of the penalties and, therefore, the grid for the debt increases too, with the potential cost in terms of numerical accuracy.

words, is it credible that the second lender (i.e., IMF) impose extremely high penalties after a default episode? Probably the answer is no.

In summary, the welfare net losses due to a second source of funding are robust to a wide range of penalties (both output loss and market exclusion penalties). Accordingly, the policy recommendation is straightforward: this kind of help does not really help. Similar conclusions have been reached in different contexts. In effect, the contingent credit lines designed by the IMF failed because adherence to this instrument could be interpreted as a bad signal about the fundamentals of “the benefited countries”, with the natural repercussions on access to financial markets. In this case, the potential costs are derived from a signaling problem instead of from the dynamic consequences of the incentives faced by borrowers, but the consequence shows some similarities with the two-lender model, since both cases are characterized by losses due to lower access to international funding.

Table 3
Welfare Impact of the Second Lender
Sensitivity Analysis: Output Loss Penalty

Scenario	$\lambda^s=4\%$, $\lambda^f=2\%$ $\theta^s=10\%$, $\theta^f=10\%$	$\lambda^s=3\%$, $\lambda^f=2\%$ $\theta^s=10\%$, $\theta^f=10\%$	$\lambda^s=2\%$, $\lambda^f=2\%$ $\theta^s=10\%$, $\theta^f=10\%$	$\lambda^s=1\%$, $\lambda^f=2\%$ $\theta^s=10\%$, $\theta^f=10\%$	$\lambda^s=0.5\%$, $\lambda^f=2\%$ $\theta^s=10\%$, $\theta^f=10\%$
<u>Welfare Cost (%)</u>					
High Income and Low Debt	1.5%	1.9%	2.9%	5.1%	5.1%
Low Income and High Debt	1.5%	2.1%	3.4%	5.1%	6.2%

Note: the statistics were computed over 1000 simulations of sequences of 2000 periods.

Table 4
Welfare Impact of the Second Lender
Sensitivity Analysis: Market Exclusion Penalty

Scenario	$\lambda^s=2\%$, $\lambda^f=2\%$ $\theta^s=0\%$, $\theta^f=10\%$	$\lambda^s=2\%$, $\lambda^f=2\%$ $\theta^s=5\%$, $\theta^f=10\%$	$\lambda^s=2\%$, $\lambda^f=2\%$ $\theta^s=10\%$, $\theta^f=10\%$	$\lambda^s=2\%$, $\lambda^f=2\%$ $\theta^s=15\%$, $\theta^f=10\%$	$\lambda^s=2\%$, $\lambda^f=2\%$ $\theta^s=20\%$, $\theta^f=10\%$
<u>Welfare Cost (%)</u>					
High Income and Low Debt	1.4%	1.9%	2.9%	5.5%	5.7%
Low Income and High Debt	1.5%	2.1%	3.4%	4.2%	4.9%

Note: the statistics were computed over 1000 simulations of sequences of 2000 periods.

4. Borrower and Lender Policies: Understanding the Welfare Implications

This section illustrates the model's dynamic with two lenders and its repercussions in terms of welfare losses. The analysis will be focused on debt level and default decisions, because these elements are the main departures with respect to the standard one-lender model. On one hand, if default events are observed more (less) frequently, then the country will face more (less) frequent output penalties. On the other hand, if the maximum level of debt feasible to attain is reduced (increased), then this will impact negatively (positively) the borrower's welfare.

Flip-and-Back Behavior

To illustrate the agents' policies clearly, graphs 1 and 2 show the debt's prices of the two-lender-model when the household is in good financial status ($D^f = D^s = 0$) and when it is on default with some lender ($D^f = 0, D^s = 1$ or $D^f = 1, D^s = 0$). The first lender is characterized by $\lambda^f = 2\%$ and $\theta^f = 10\%$, while the second lender applies lower penalties $\lambda^s = 1\%$ and $\theta^s = 10\%$. In this exercise, the equilibrium prices under good financial status are positive for the first lender's debt (q^f) and zero for the second lender's debt (q^s) (graphs 1 and 2), which implies that in practice the household can only borrow from the first lender (i.e., high penalty lender). However, the feasible debt to attain from the first lender is lower with respect to the case when the household is in default with the second lender - the debt prices are lower - (graphs 1 and 2). To attain higher debt levels, the household will go into debt with the first lender at some positive debt price and, simultaneously go into debt with the second lender at a zero price (i.e., no debt). Therefore, the household will be able to default on the second lender during the following period, which allows him to have access to a higher debt level from the first lender. These dynamics are observed virtually for all penalties employed in tables 1 and 2, which suggests that the good financial record status is not feasible to sustain under the set of current parameters.

The rationality behind this result is the following. The good financial status implies that the borrower can default on one lender without facing the market exclusion cost, because he can continue borrowing from the other lender. Therefore, the endogenous borrowing constraint that emerges from this case is too binding; that is lenders are not willing to lend too many resources. From this perspective, the household could be better off if he did not have access to some specific market, because it could borrow a higher amount from only one lender. The alternative followed by the household is to borrow from both lenders in order to default on the low-penalty lender the next period. In equilibrium, the counterpart of this dynamic is that the low-penalty lender will lend at zero price. In this sense, given the difference in penalties, the low penalty debt market is passively closed.

After that, the model will be characterized by dynamics similar to those of the one-lender model. The only differences are: a) the lender with the lower penalty will impose an output loss penalty to the borrower until he forgives him, which is some kind of sunk cost in this model; b) the effective market exclusion threat is lower than the one observed in the absence of a second lender, because the borrower could be forgiven by the second lender in the meantime;¹⁸ and c) if the household is holding debt from the lender with higher penalties and he is forgiven by the other, then the household could default on that debt and start borrowing from the second lender (see next exercise). The net effect of these elements on the borrowing constraint is ambiguous in principle, because the element (a) will reduce the incentive to default,¹⁹ but (b) and (c) introduce a higher default probability. However, the numerical simulations in the next section show that the net effect is a reduction in borrowing limits.

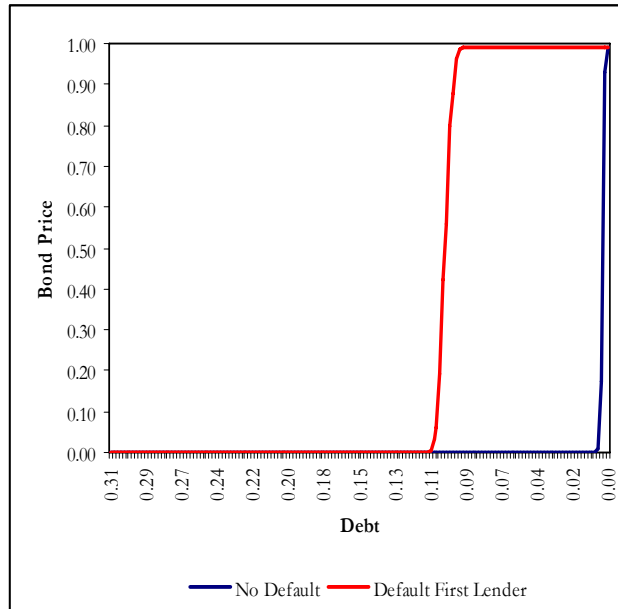
When the household is in good financial status only with the first lender ($D^f = 0, D^s = 1$) and the second lender forgives him, the incentives to default on the first lender are strong enough to forbid an equilibrium where the household does not default on some lender. The elements behind this result are the same as before plus the fact that the

¹⁸ The higher probability to reenter is given by $\theta^s(1-\theta^f) \geq 0$. This probability represents the case where the second lender forgives the borrower, but the first lender does not.

¹⁹ The concavity of the preferences makes more costly to face financial autarky with two output penalties (λ^f, λ^s) .

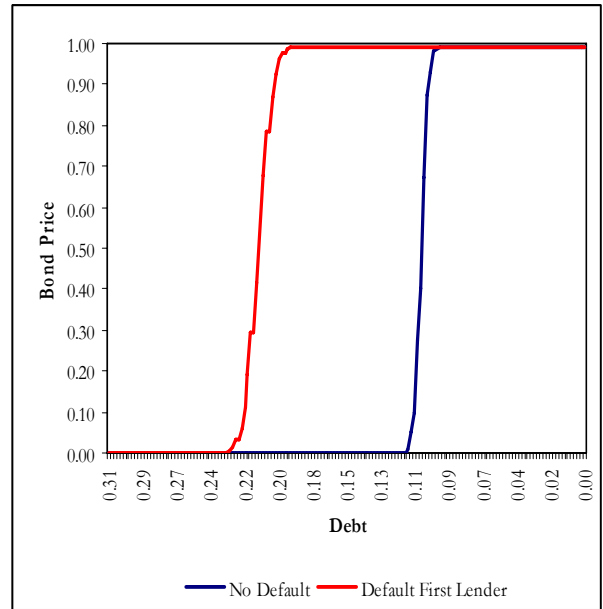
household has the extra benefit of discharge the debt hold with the first lender. The same behavior is observed when the household only holds low-penalty debt and is forgiven by the high-penalty lender.

Graph 1
 Price of the Second Lender Bonds (q^s)
 Under Good Financial Record and Default with the
 First Lender
 $\lambda^f = 2\%, \lambda^s = 1\%, \theta^f = \theta^s = 10\%$



Note: The endowment state is the average shock.

Graph 2
 Price of the First Lender Bonds (q^f)
 Under Good Financial Record and Default with
 the Second Lender
 $\lambda^f = 2\%, \lambda^s = 1\%, \theta^f = \theta^s = 10\%$



Note: The endowment state is the average shock.

The previous result points out dynamics clearly marked by a permanent default status. If the borrower is in good financial status, he will prefer to default on the low-penalty lender in order to be able to borrow more debt from the high-penalty lender. The second default round will be observed when the low-penalty lender forgives the borrower and so on. The consequences of this flip-and-back behavior – or permanent non-loyalty behavior - are that the household will tend to be in good financial status with only one lender and will therefore face a permanent output loss penalty. In other words, if the borrower can have access to both lenders, then it is not sustainable to borrow from both lenders and stay in a good financial status.

The flip and back behavior is illustrated by the relationship between the percentage of periods that the household holds debt with each one of the lenders and the frequency with which lenders forgive the defaulter (i.e., market exclusion penalty θ 's). Basically, if one lender forgives the defaulter more often (higher θ), then the household will default with higher frequency on the other lender and, therefore, it will hold more frequently debt from the lender with higher θ . This relationship is clearly illustrated in Table 5, if the second lender's market exclusion penalty is equal to $\theta^s = 20\%$ implies that household will hold only debt of the second lender a 66.79% of periods, but if the second lender increases the penalty to $\theta^s = 0\%$ (i.e., the lender never forgives), the household will only hold debt of the second lender just 1.09% of the periods. It is worth to mention that the sensitivity analysis for different values of output loss penalties does not generate such dynamic (Table 5). In this case, the frequency with which the household will hold low-penalty debt is modified just slightly for different penalties, which ratifies the persistence of the flip-and-back behavior even for high output loss penalties.

The permanent non-loyalty behavior is a salient feature of the model. In order to check its robustness for different degrees of borrower impatience, the two-lender model was simulated with discount factors equal to 0.85, 0.90, and 0.95. In terms of default decisions, the borrower's policies do not change significantly for discount factors lower than 0.90. A similar exercise was conducted for different output penalties (λ 's).²⁰ Specifically, the output loss was sensibilized for values between $\lambda^s = 2\%$ and 4%, which is equivalent to twice the standard value estimated in the literature. However, even for $\lambda^s = 4\%$ is not feasible to sustain a good financial status.

The flip-and-back robustness imposes a strong point to take into account. On one hand, discount factors higher than 0.90 are virtually incompatible with this kind of model, because at those levels is difficult to replicate default episodes. On the other hand, it is nearly impossible to think in feasible sanctions that can result in output loss of more than 3%. In summary, even though it could be possible to sustain equilibriums characterized by the absence of default, the required parameter configuration is highly restrictive.

²⁰ The sensibilizations with respect to the market exclusion penalties show similar conclusions.

In some sense, the flip-and-back behavior generates a dynamic that could be similar to a one-lender model with stochastic penalties. In the presence of flip-and-back behavior, the household will face a permanent and variable output loss (λ^f , λ^s or $\lambda^f + \lambda^s$), and the market exclusion penalty will depend on θ^f and θ^s according to the household's financial status. Therefore, the two-lender model imposes an extra source of welfare loss due to a new element of uncertainty (let us say "penalty risk").

Table 5
Holding Frequency of Debt With Both Lenders
According to Output and Market Exclusion Penalty

Statistic	Output Loss Penalty Sensibility				
	$\theta^s=10\%$, $\theta^f=10\%$ $\lambda^s=4\%$, $\lambda^f=2\%$	$\theta^s=10\%$, $\theta^f=10\%$ $\lambda^s=3\%$, $\lambda^f=2\%$	$\theta^s=10\%$, $\theta^f=10\%$ $\lambda^s=2\%$, $\lambda^f=2\%$	$\theta^s=10\%$, $\theta^f=10\%$ $\lambda^s=1\%$, $\lambda^f=2\%$	$\theta^s=10\%$, $\theta^f=10\%$ $\lambda^s=0.5\%$, $\lambda^f=2\%$
<u>Holding Debt Frequency (%)</u>					
Second Lender's Bonds	50.45%	50.43%	50.34%	50.40%	50.18%
First Lender's Bonds	49.55%	49.57%	49.66%	49.60%	49.78%
Statistic	Market Exclusion Penalty Sensibility				
	$\theta^s=0\%$, $\theta^f=10\%$ $\lambda^s=2\%$, $\lambda^f=2\%$	$\theta^s=5\%$, $\theta^f=10\%$ $\lambda^s=2\%$, $\lambda^f=2\%$	$\theta^s=10\%$, $\theta^f=10\%$ $\lambda^s=2\%$, $\lambda^f=2\%$	$\theta^s=15\%$, $\theta^f=10\%$ $\lambda^s=2\%$, $\lambda^f=2\%$	$\theta^s=20\%$, $\theta^f=10\%$ $\lambda^s=2\%$, $\lambda^f=2\%$
<u>Holding Debt Frequency (%)</u>					
Second Lender's Bonds	1.09%	34.01%	50.34%	60.19%	66.79%
First Lender's Bonds	98.91%	65.99%	49.66%	39.81%	33.21%

Note: the statistics were computed over 1000 simulations employing the last 1000 observations of sequences of 2000 periods.

Endogenous Borrowing Constraints: Lower Indebtedness

The presence of two lenders not only introduce an extra cost due to the behavior previously discussed (a permanent output loss penalty and penalty risk), but also reduces the sustainable level of debt in equilibrium. As was mentioned, the presence of a second lender implies lower expected penalties from default and, therefore, the optimal response of lenders is to increase the interest rate (i.e., a lower debt price), which renders unfeasible levels of debt that were feasible with a single lender. To illustrate this point, graphs 3 and 4 show the maximum debt levels with two lenders according to the borrower's financial status for

different levels of penalties, employing as benchmark the one-lender model with an output loss penalty equal to $\lambda^f = 2\%$ and $\theta^f = 10\%$ ²¹.

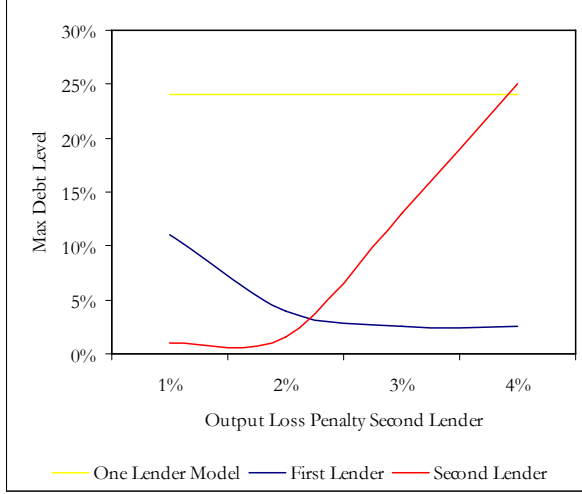
The case for borrowers in good financial status is illustrated in graph 3. The maximum level of debt with two lenders is substantially lower when the household has a good financial status, which explains the default's attractiveness on the lender with lower penalties (see previous section)²². Moreover, the maximum level of debt attainable in a two-lender model is comparable to the one-lender scenario only for cases with output losses greater than 4%. This simulation also highlights that the potential costs of a second lender could be exacerbated if both lenders competed under similar conditions (same penalties). If the penalties are too close ($\lambda^f = \lambda^s = 2\%$), then both markets could be virtually shut down in equilibrium when the borrower starts from a good financial status.

Graph 4 shows debt limits when the household is on the default on one lender. The line for the first (second) lender means the maximum attainable debt level for different output loss penalties conditional on the household being in default on the second (first) lender. These simulations point out again the effect of a second lender in terms of lowering the maximum level of attainable debt. The benchmark case ($\theta^f = \theta^s = 2\%$) implies a maximum debt of around 4% lower than the one-lender case for $\theta = 2\%$. Finally, it is possible to close the borrowing gap between one and two lenders with higher output loss, but this will have as a counterpart a reduction in welfare due to the presence of the flip-and-back behavior.

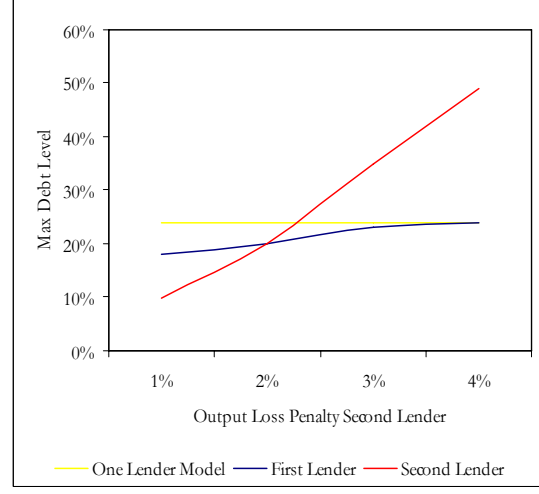
²¹ The maximum level of debt is defined as the maximum level of debt that can be issued at some positive price.

²² Notice that the first lender is the higher penalty lender for the first two observations ($\lambda^s = 1\%, 2\%$), after that the first lender will have the lower penalties.

Graph 3
Maximum Level of Debt
Household in Good Financial Status
 $\lambda^f = 2\%, \theta^f = \theta^s = 10\%$



Graph 4
Maximum Level of Debt
Household in Default with One Lender
 $\lambda^f = 2\%, \theta^f = \theta^s = 10\%$



5. Welfare Impact of Coordination

This section modifies the setting in the direction of imposing some coordination mechanism between lenders. Now, if the household defaults on the second lender, the first lender will also apply the penalties. On the contrary, if the household defaults on the first lender, it can still borrow from the second lender. This mechanism resembles the idea that the second lender is a lender of last resort, where the first lender is the private market, which is willing to participate only if the borrower has not defaulted on the lender of last resort.

In terms of the setting in section 3, the previous assumption implies the following new set of value functions:

$$v(b^f, b^s, y, D^f = 0, D^s = 0) = \text{Max} \left\{ \begin{array}{l} w^o(b^f, b^s, y, D^f = 0, D^s = 0), \\ w^f(b^f, b^s, y, D^f = 0, D^s = 0), \\ w^{fs}(b^f, b^s, y, D^f = 0, D^s = 0), \end{array} \right\} \quad (8')$$

$$v(0, b^s, y, D^f = 1, D^s = 0) = \text{Max} \left\{ \begin{array}{l} w^f(0, b^s, y, D^f = 1, D^s = 0), \\ w^{fs}(0, b^s, y, D^f = 1, D^s = 0) \end{array} \right\} \quad (10')$$

$$v(0, 0, y, D^f = 1, D^s = 1) = w^{fs}(0, 0, y, D^f = 1, D^s = 1) \quad (11')$$

Table 6 presents the welfare cost estimates for different output loss penalties²³. The main result is that if the lender of last resort condition is imposed, the two lenders model implies positive welfare gains with respect to the standard one-lender model. For instance, if the second lender's output loss penalty is 3%, the welfare measure implies that the presence of a lender of last resort is equivalent to an increase between 2.5% and 2.3% in consumption with respect to the one-lender model. Table 7 shows similar results for the sensitivity analysis of market exclusion penalties.

The coordination mechanism increases the welfare due to the combination of two elements. First, the simulations show that around 100% of lending is explained by the lender of last resort, and the first lender provides no funding at all. In other words, the two-lender model will mimic the one-lender model. The logic behind this result is simple. The first lender is not willing to lend under good financial status —because the household will default on its debt, and if the household defaults on the lender of last resort, the first lender cannot lend money to the household. The second element is higher implicit penalties, which in turn implies higher feasible debt levels. In effect, the new setting implies that the implicit market exclusion penalty of the second lender (lender of last resort) is equivalent to financial autarky, and the output loss penalty is equal to $\lambda^f + \lambda^s$. These new penalties allow higher access to international markets, which is clearly illustrated in Table 8.

An interesting result emerges from this section. The lender of last resort mechanism will mimic the one-lender model where the first lender is virtually crowded out from the market and the lending activities are conducted only by the second lender. In this sense, the welfare gains of a lender of last resort are equivalent to the gains derived from higher penalties in a one-lender model. Therefore, this result suggests that the role of a

²³ The benchmark to compute the welfare implications is the one-lender case with penalties equal to $\theta = 2\%$ and $\lambda = 10\%$.

second lender is not beneficial *per se*, because the benefits are derived from the higher penalties.

Table 6
Welfare Impact of a Lender of Last Resort
Sensitivity Analysis: Output Loss Penalty

Scenario	$\lambda^s=3\%$, $\lambda^f=2\%$ $\theta^s=10\%$, $\theta^f=10\%$	$\lambda^s=2\%$, $\lambda^f=2\%$ $\theta^s=10\%$, $\theta^f=10\%$	$\lambda^s=1\%$, $\lambda^f=2\%$ $\theta^s=10\%$, $\theta^f=10\%$
<u>Welfare Cost (%)</u>			
High Income and Low Debt	-2.6%	-0.8%	-0.3%
Low Income and High Debt	-2.3%	-0.3%	-0.1%

Note: the statistics were computed over 1000 simulations of sequences of 2000 periods.

Table 7
Welfare Impact of a Lender of Last Resort
Sensitivity Analysis: Market Exclusion Penalty

Scenario	$\lambda^s=2\%$, $\lambda^f=2\%$ $\theta^s=5\%$, $\theta^f=10\%$	$\lambda^s=2\%$, $\lambda^f=2\%$ $\theta^s=10\%$, $\theta^f=10\%$	$\lambda^s=2\%$, $\lambda^f=2\%$ $\theta^s=20\%$, $\theta^f=10\%$
<u>Welfare Cost (%)</u>			
High Income and Low Debt	-3.6%	-0.8%	-0.8%
Low Income and High Debt	-3.4%	-0.3%	-0.2%

Note: the statistics were computed over 1000 simulations of sequences of 2000 periods.

Table 8
Maximum Debt-Output Ratio

Scenario	Statistic
<u>One-Lender Model</u>	
$\lambda = 2\%$ and $\theta=10\%$	19.02%
<u>Two-Lender Model</u>	
<u>Market Exclusion</u>	
$\lambda^s=2\%$, $\lambda^f=2\%$ and $\theta^s=5\%$, $\theta^f=10\%$	79.06%
$\lambda^s=2\%$, $\lambda^f=2\%$ and $\theta^s=10\%$, $\theta^f=10\%$	56.14%
$\lambda^s=2\%$, $\lambda^f=2\%$ and $\theta^s=20\%$, $\theta^f=10\%$	39.53%
<u>Output Loss</u>	
$\lambda^s=1\%$, $\lambda^f=2\%$ and $\theta^s=10\%$, $\theta^f=10\%$	44.39%
$\lambda^s=2\%$, $\lambda^f=2\%$ and $\theta^s=10\%$, $\theta^f=10\%$	56.14%
$\lambda^s=3\%$, $\lambda^f=2\%$ and $\theta^s=10\%$, $\theta^f=10\%$	70.09%

Note: the statistics were computed over 1000 simulations employing the last 1000 observations of sequences of 2000 periods.

6. Conclusions

Default episodes have had a high impact on emerging economies recently. This situation opens several questions on how to face these events and/or reduce their occurrence, without neither a consensus nor a unique approach to most of those questions. This paper studies sovereign default in a small open economy with two lenders. This extension is done with the purpose of understanding the welfare implications of an active presence of the IMF or other similar institution in the international financial markets. In equilibrium, the presence of this second lender reduces the lending activity of private markets and increases the bond prices to compensate for the higher default risk with respect to the scenario with only one lender. The main implication of this is that the borrower's welfare decreases, with welfare losses ranging between 1.5% and 6.2% of equivalent consumption.

This finding is robust to a wide range of penalties applied by the second lender. The downsides related to a second lender are two. First, there is a kind of sunk cost due to the output loss penalty generated by the flip-and-back or, equivalently, a permanent non-loyalty behavior, because the presence of two lenders introduces strong incentives to default on one lender at each moment. Second, these higher default incentives will also imply that lenders will be less willing to lend money to households, which reduces agents' ability to smooth consumption and, given that borrowers are more impatient than lenders, it has a direct welfare effect on borrowers.

The previous results assume no coordination between lenders. If a setting that replicates a lender of last resort environment is imposed, the model will mimic the one-lender case where the lending is done by the lender of last resort. Therefore, from a welfare point of view, there are no gains with an extra lender. However, from a broader perspective, the virtual shutdown of the first lender is a result that could be contrary to the spirit of institutions like the IMF, where the extra lending is supposed to complement private market lending and not shut down it.

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Appendix 1

Proposition 1: Default sets are growing in debts. For all $b^{j1} \leq b^{j2}$, if default on the $j = f, s$ lender is optimal for b^{j2} for some states y , then, $DS_i^j(b^{j2}, b^{-j}) \subseteq DS_i^j(b^{j1}, b^{-j})$ - $i = o, f, s$ and $j = f, s$ -.

This proof is similar to that found in Eaton and Gersovitz (1981), Arellano (2006) and Yue (2006).

Case $i = s, j = f$. For all $\{y\} \in DS_s^f(b^{f^2}, 0)$ implies that $u((1 - \lambda^f - \lambda^s)y) + \beta E[\theta^f \theta^s v(D^{f'} = 0, D^{s'} = 0) + (1 - \theta^f) \theta^s v(D^{f'} = 1, D^{s'} = 0) + (1 - \theta^f) \theta^s v(D^{f'} = 0, D^{s'} = 1) + (1 - \theta^f)(1 - \theta^s) v(D^{f'} = 1, D^{s'} = 1)] \geq u((1 - \lambda^s)y - q^f b^{f'} + b^f) + \beta E[\theta^s v(D^{f'} = 0, D^{s'} = 0) + (1 - \theta^s) v(D^{f'} = 0, D^{s'} = 1)]$ (a). Since $(1 - \lambda^s)y - q^f b^{f'} + b^f \geq (1 - \lambda^s)y - q^f b^{f'} + b^{f^1}$ for all $b^{f'}$, $u((1 - \lambda^s)y - q^f b^{f'} + b^f) + \beta E[\theta^s v(D^{f'} = 0, D^{s'} = 0) + (1 - \theta^s) v(D^{f'} = 0, D^{s'} = 1)] \geq u((1 - \lambda^s)y - q^f b^{f'} + b^{f^1}) + \beta E[\theta^s v(D^{f'} = 0, D^{s'} = 0) + (1 - \theta^s) v(D^{f'} = 0, D^{s'} = 1)]$ (b). It follows directly from (a) and (b) that $\{y\} \in D_s^f(b^{f^1}, 0)$.

This approach is extended to all $i = o, f, s$ and $j = f, s$ combinations.

Proposition 2: Default sets are shrinking in penalties. a) If the output penalty of the non-defaulted debt λ increases to λ' ($\lambda < \lambda'$), then $DS_i^j(b^f, b^s)' \subseteq DS_i^j(b^f, b^s)$, where the superscript ' denotes λ' . b) If the market exclusion penalty of the no defaulted debt $1 - \theta$ increases to $1 - \theta'$ ($\theta < \theta'$), then $DS_i^j(b^f, b^s)' \subseteq DS_i^j(b^f, b^s)$, where the superscript ' denotes θ' .

The proof follows the same approach as Proposition 1. With respect to the output loss penalty (a), the starting point is $(1 - \lambda)y \geq (1 - \lambda')y$ for all y , then a similar procedure can be applied to conclude that $D_i^j(b^f, b^s) \subseteq D_i^j(b^f, b^s)'$.

The part related to the market exclusion penalty (b) follows a similar approach. For example, the default set on the second lender for the case of good financial status ($D^f = 0, D^s = 0$) is based on $\theta \cdot v(D^{f'} = 0, D^{s'} = 0) + (1 - \theta) \cdot v(D^{f'} = 0, D^{s'} = 1) \geq \theta' \cdot v(D^{f'} = 0, D^{s'} = 0) + (1 - \theta') \cdot v(D^{f'} = 0, D^{s'} = 1)$, because $v(D^{f'} = 0, D^{s'} = 0) \geq v(D^{f'} = 0, D^{s'} = 1)$. Based on the previous inequality, the procedure is straightforward.

Proposition 3: The household in good financial status tends to default on the lender with lower penalties. For $\lambda^j < \lambda^{-j}$, $\theta^j = \theta^{-j}$, $D^f = D^s = 0$, and $b^f = b^s = 0$, if the household decides to default only on one type of debt, and it is planning to issue new debt equal to \bar{b} , then it will default on the lender j - i.e, the lender with lower penalties-

By contradiction, let us assume that, for some y , the household will default on the high-penalty lender - by simplicity assume that the first lender corresponds to the high-penalty lender- then $w^f(0, 0, y, D^f = 1, D^0 = 0) \geq w^s(0, 0, y, D^f = 0, D^0 = 1)$ and, equivalently, $v(0, 0, y, D^f = 0, D^s = 1) \geq v(0, 0, y, D^f = 1, D^s = 0)$. This will imply $u((1 - \lambda^f)y - q^s \bar{b}) + \beta E[\theta^f v(0, \bar{b}, D^{f'} = 0, D^{s'} = 0) + (1 - \theta^f) v(0, \bar{b}, D^{f'} = 1, D^{s'} = 0)] \geq$

$u((1 - \lambda^s)y - q^f \bar{b}) + \beta E[\theta^s v(\bar{b}, 0, D^{f'} = 0, D^{s'} = 0) + (1 - \theta^s)v(\bar{b}, 0, D^{f'} = 0, D^{s'} = 1)]$. The previous expression can be written as $u((1 - \lambda^f)y - q^s \bar{b}) - u((1 - \lambda^s)y - q^f \bar{b}) \geq \beta E[(1 - \theta^l)(v(\bar{b}, 0, D^f = 1, D^s = 0) - v(\bar{b}, 0, D^f = 0, D^s = 1))] \geq 0$. Now, if the preferences are increasing in their argument, we will have $(\lambda^s - \lambda^f)y - (q^s - q^f)\bar{b} \geq 0$. Notice that the first element is negative ($\lambda^s < \lambda^f$) and $y \geq 0$. As for the second element, it is positive. In effect, from Proposition 2, the default set for the low-penalty lender is bigger than set for the high-penalty lender and, therefore, by construction, $q^l \leq q^h$ and, from the statement, $\bar{b} \leq 0$. In summary, the first term is negative and the second term is positive, which means that the inequality is not satisfied.

Appendix 2 Solving Method

The model economy is no stationary, because a realization of the growth shock affects the endowment permanently. To deal with this issue, the model is detrended by the lagged endowment level y_{t-1} . This adjustment modifies neither the nature of the decision problem nor the set of involved states. In effect, let denote the normalized equivalent of any variable x_t by \hat{x}_t (i.e., $\hat{x}_t = x_t/y_{t-1}$), then, if x_t is in the agent's information set at t , so is the detrended variable \hat{x}_t . Additionally, the policy rule for \hat{x}_t is equivalent to x_t , because y_{t-1} is taken as given at the beginning of t .

To solve the model numerically, the discrete state-space method was used. The endowment shocks are discretized into a twelve state Markov chain using the quadrature approach developed by Hussey and Tauchen (1991). The asset space is divided into 500 equally spaced grids, where the limits are set in order to make certain that the upper and lower bound are not bindings in equilibrium. The portfolio composition considered initially 11 points, covering a share of debt with the first lender from 0% to 100%. However, the numerical solution is not modified if it only includes 3 states – 0%, 50% and 100% -. The reason is that, in equilibrium, agents hold only one class of asset ($\delta = 0\%$ *or* 100%) and the intermediate option ($\delta = 50\%$) is only employed to jump from good financial status ($D^f = D^s = 0$) to default on one lender ($D^f = 1$ *or* $D^s = 1$).

Given the previous grids, the procedure to compute the equilibrium of the model economy is the following:

1. Guess an initial price of discounted loans for the first and second lender's debt for each financial status q^0 .
2. Guess an initial value function for each financial status v^0 .

3. Given a price for loans, the household's optimization problem is solved. This procedure includes finding the value function as well as the default decisions. For this purpose, the value function is iterated using the Bellman equation to find the fixed out v^* , such that for a given bond price the value function converges.
4. From (3) the default sets are derived by comparing the value functions of default and no default on each lender.
5. Using the default set derived in step (4) and the zero expected profit condition, the new price of discounted bonds q^1 is computed. If the new prices are sufficiently close to the initial guess, stop iterating; otherwise go back step 1 and set $q^0 = q^1$.

Appendix 3

Sensitivity Analysis

Tables A2.1 and A2.2 show the model's sensitivity to different values of output and market exclusion penalties. In general terms, the two-lender model helps to close the gap in several moments characterized by a lower estimated volatility with respect to the empirical one. The consumption volatility is 1.06 times the output volatility in the one-lender model and its observed value is 1.35. On the other hand the two-lender model generates volatilities in the range of 1.07 to 1.47 depending on the penalty on the output loss (Table A2.1).²⁴ This feature also applies for the current account volatility.

Regarding spreads, the estimated average and volatility are improved with respect to its empirical counterpart. Spread volatility increases from 0.07% to 0.12%-0.56% according to the output penalty assigned to the second lender, which compares with a historical volatility of 0.42%. Nonetheless, the sensitivity of this statistic is relatively low to changes in the market exclusion penalty. For instance, if the market exclusion of the second lender decreases to 20%,²⁵ then the volatility increases only to 0.16% (Table 2.2).

Overall, the numerical implications of two lenders can be summarized in higher volatility and spreads. An element behind the higher volatility is the flip-and-back behavior, because agents can change from one schedule price to another facing different prices and maximum debt level in each case, where the difference in bond prices and debt level are determined by how different the penalties are. In the particular case of consumption, the existence of two potential lenders can either increase or reduce the agents' capacity to smooth consumption. On one hand, if the household faces an extremely low endowment realization, then the agent can default on one lender and still borrow from the other, which

²⁴ The same feature emerges from the market exclusion penalty sensitivity (Table A2.2).

²⁵ It means an expected market reentry in 1.25 years; this figure compares with estimates in the range of 2.5 years.

clearly increases the capacity to smooth consumption across states. On the other hand, the existence of two lenders reduces the maximum level of debt that the market can support.

Table A2.1
Sensitivity Analysis: Output Loss Penalty

Statistic	Data ¹	Two Lenders					One Lender $\theta=10\%$ $\lambda=2\%$
		$\theta^s=10\%$, $\theta^f=10\%$ $\lambda^s=4\%$, $\lambda^f=2\%$	$\theta^s=10\%$, $\theta^f=10\%$ $\lambda^s=3\%$, $\lambda^f=2\%$	$\theta^s=10\%$, $\theta^f=10\%$ $\lambda^s=2\%^2$, $\lambda^f=2\%$	$\theta^s=10\%$, $\theta^f=10\%$ $\lambda^s=1\%$, $\lambda^f=2\%$	$\theta^s=10\%$, $\theta^f=10\%$ $\lambda^s=0.5\%$, $\lambda^f=2\%$	
Std(c)/Std(y)	1.15	1.92	1.33	1.07	1.25	1.47	1.06
Std(nx/y)	1.35%	6.03%	3.28%	1.08%	2.95%	4.35%	0.87%
Corr(nx/y,y)	-0.88	-0.04	-0.05	-0.15	-0.04	-0.02	-0.20
Std(spread)	0.42%	0.15%	0.13%	0.12%	0.18%	0.56%	0.07%
Average Spread	1.02%	0.25%	0.24%	0.22%	0.31%	0.17%	0.17%
Corr(spread,y)	-0.59	-0.08	-0.03	-0.02	-0.14	-0.08	-0.03
Corr(spread,nx)	0.49	-0.02	-0.11	-0.09	0.27	0.24	0.12

Note: 1: these figures correspond to Argentina. 2= the low output penalty was set at 1.999% in order to solve the model, because of with two equal bonds the model does not converge to an equilibrium with debt. Sources: Aguiar and Gopinath (2005), Yue (2006).

Table A2.2
Sensitivity Analysis: Market Exclusion Penalty

Statistic	Data ¹	Two Lenders					One Lender $\theta=10\%$ $\lambda=2\%$
		$\theta^s=0\%$, $\theta^f=10\%$ $\lambda^s=2\%$, $\lambda^f=2\%$	$\theta^s=5\%$, $\theta^f=10\%$ $\lambda^s=2\%$, $\lambda^f=2\%$	$\theta^s=10\%$, $\theta^f=10\%$ $\lambda^s=2\%^2$, $\lambda^f=2\%$	$\theta^s=15\%$, $\theta^f=10\%$ $\lambda^s=2\%$, $\lambda^f=2\%$	$\theta^s=20\%$, $\theta^f=10\%$ $\lambda^s=2\%$, $\lambda^f=2\%$	
Std(c)/Std(y)	1.15	1.06	1.21	1.07	1.11	1.20	1.06
Std(nx/y)	1.35%	0.76%	2.66%	1.08%	1.79%	2.73%	0.87%
Corr(nx/y,y)	-0.88	-0.23	-0.07	-0.15	-0.07	-0.04	-0.20
Std(spread)	0.42%	0.07%	0.13%	0.12%	0.15%	0.16%	0.07%
Average Spread	1.02%	0.13%	0.23%	0.22%	0.29%	0.33%	0.17%
Corr(spread,y)	-0.59	-0.27	-0.05	-0.02	-0.03	-0.15	-0.03
Corr(spread,nx)	0.49	0.71	-0.07	-0.09	0.05	0.08	0.12

Note: 1: these figures correspond to Argentina. 2= the low output penalty was set at 1.999% in order to solve the model, because of with two equal bonds the model does not converge to an equilibrium with debt. Sources: Aguiar and Gopinath (2005), Yue (2006).

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