Translating Lukasiewicz's logics into Classical Logic: a grade of difficulty

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Resumo

Neste trabalho são vistos alguns conceitos sobre as traduções entre lógicas e uma breve revisão sobre as lógicas de Lukasiewicz. Então, como resultado central, mostra que não existe qualquer tradução esquemática das lógicas proposicionais de Lukasiewicz na lógica proposicional clássica. Desta maneira, pode ser avaliado o quão difícil é encontrar uma tal tradução.

Introduction

The use of functions between logics in order to obtain some logical results has been developed since the beginning of the 20th century as we can see in several books and articles on logic such as Feitosa, D'Ottaviano (2000) or D'Ottaviano, Feitosa, (1999b) and others mentioned in the bibliography. But the idea of constructing a theory of such functions is not so old. The theory of translation between logics intends to analyze logical properties having these functions as tools.

In general it is not very difficult to get a translation or better a conservative translation from the classical calculus into some other calculus, but the inverse of this result is always hard. In this work we will consider this problem associated to Lukasiewicz's calculi. In D'Ottaviano, Feitosa, (1999 a) we see a family of conservative translations from the classical propositional calculus (**CPC**) into the n-valued Lukasiewicz's

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calculi ($\mathbf{L}_n$), for each n ∈ . In Feitosa, Baar (200_) it is shown that, using facts about algebraic semantic, there is a conservative translation for each case. But this is a non-constructive result.

So, in Section 3, we will present a result that shows us a high grade of complexity for generating those functions. It can give us some idea about how difficult is to translate conservatively Lukasiewicz's logics into classical logic, even in the propositional case.

1. The translations

Now we are going to review some basic concepts about a theory of translation between logics.

A *consequence operator* over E is a mapping **C**: *P*(E) → ρ(E) such that, for every A, B ⊆ E:

(i) A ⊆ **C**(A)
(ii) A ⊆ B ⇒ **C**(A) ⊆ **C**(B)
(iii) **C**(**C**(A)) ⊆ **C**(A).

Certainly, for every consequence operator **C**, through (i) and (iii), we have the equality **C**(**C**(A)) = **C**(A). Sometimes the consequence operator is named "closure operator".

A consequence operator **C** over E is *finitary* if for every A ⊆ E, **C**(A) = ∪{**C**(A_0) / A_0 is a finite subset of A}.

Let **C** be a consequence operator over E. A set A is *closed* in E if **C**(A) = A, and A is *open* if the complement of A, denoted by CA, is closed. An element x ∈ E is *dense* in E if **C**({x}) = E.

A *logic* or *deductive system* is a pair **L** = (L, **C**), where L is any set (the domain) and **C** is a consequence operator over L.

Well, besides this very general construction about logics without a subjacent formal language, connectives and others specific logical symbols, when we are working with logical calculus we usually have in our minds all these elements. Thus, let us try even in a general way to retrieve some of those elements in our theory.

Considering a formal language L, let **For**(L) be the defined set of formulas of L. A *consequence operator over* L has as domain the set **For**(L).

Given a formal language L, let us consider the free algebra of formulas of L, which is generated by the set of atomic formulas also denoted by **For**(L). A *replacing* is an endomorphism s on **For**(L), that is, s ∈ **Hom**(**For**(L), **For**(L)).

Let L be a formal language, s ∈ **Hom**(**For**(L), **For**(L)) and **C** a consequence operator over L. The consequence operator **C** is *structural* if s**C**(Γ) ⊆ **C**(s(Γ)), for every Γ ⊆ **For**(L). The operator **C** is *standard* if **C** is structural and finitary.

The concept of logical system permits us to characterize particular cases of logics, which we can claim that the operator is finitary, structural or standard.

A *logical system* defined over L is a pair L = (L, **C**), where L is a formal language and **C** is a consequence operator over L.

If L is a logical system, the set **For**(L) is also denoted by **For**(L). Let L = (L, **C**) be a logical system and Δ ⊆ **For**(L), a *theory* Δ of L is a closed set in L.

C(∅) and **For**(L) are, for sure, the smallest and biggest theories respectively associated to the consequence operator **C**. An element of the theory Δ of L is named a *theorem* of Δ. As a theorem of L we mean a theorem of **C**(∅). The set of theorems of L is denoted **Teo**(L) = {α / α ∈ **C**(∅)}.

After these initial concepts about the logical systems we can give a definition of translation between such systems, which have been named Tarski's logical systems.

Let L_1 and L_2 be two logical systems. A *translation* from L_1 to L_2 is a mapping T: $L_1 \rightarrow L_2$ such that for every subset of formulas $\Gamma \cup \{\alpha\} \subseteq$ **For**(L_1):

$$\alpha \in \mathbf{C}_1(\Gamma) \Rightarrow T(\alpha) \in \mathbf{C}_2(T(\Gamma)).$$

For logical systems with correct deductibility, that is, $\alpha \in \mathbf{C}(\Gamma) \Leftrightarrow \Gamma \mid\!- \alpha$, the function T is a translation if and only if:

$\Gamma \mid\!-_1 \alpha \Rightarrow T(\Gamma) \mid\!-_{L_2} T(\alpha)$.

As from set theory we have $T(\varnothing) = \varnothing$, considering $\Gamma = \varnothing$ every translation takes theorems of L_1 into theorems of L_2, that is:

$\mid\!-_{L_1} \alpha \Rightarrow \mid\!-_{L_2} T(\alpha)$.

Proposition 1.1: A mapping T: $L_1 \rightarrow L_2$ is a translation iff $T(\mathbf{C}_1(\Gamma)) \subseteq \mathbf{C}_2(T(\Gamma))$, for every $\Gamma \subseteq \mathbf{For}(L_1)$, with $T(\Gamma) = \{T(\alpha) / \alpha \in \Gamma\}$.

Theorem 1.2: Let T: $L_1 \rightarrow L_2$ be a mapping between logical systems. The following sentences are equivalent:

(i) T is a translation

(ii) the inverse image of a closed set is a closed set

(iii) the inverse image of an open set is an open set.

A mapping is *closed* if the image of every closed set is still a closed set. A *conservative mapping* from the logical system L_1 into the logical system L_2 is a function f: $L_1 \rightarrow L_2$ such that for every formula α of L_1:

$\mid\!-_{L_1} \alpha \Leftrightarrow \mid\!-_{L_2} f(\alpha)$.

A *conservative translation* is a mapping T from L_1 into L_2, such that for every set $\Gamma \cup \{\alpha\} \subseteq \mathbf{For}(L_1)$:

$\alpha \in \mathbf{C}_1(\Gamma) \Leftrightarrow T(\alpha) \in \mathbf{C}_2(T(\Gamma))$. In the logical systems with correct deductibility a function T is a conservative translation iff:

$\Gamma \mid\!-_{L_1} \alpha \Leftrightarrow T(\Gamma) \mid\!-_{L_2} T(\alpha)$.

Two logical systems L_1 and L_2 are *L-homeomorphics* if there is a bijective function T: $L_1 \rightarrow L_2$, such that T and T^{-1} are translations. In that case, the function T is named an *L-homeomorphism*.

Proposition 1.3: Let T: $L_1 \rightarrow L_2$ be a bijection. Then T is an L-homeomorphism iff $T(\mathbf{C}_1(\alpha)) = \mathbf{C}_2(T(\alpha))$, for every $\alpha \subseteq \mathbf{For}(L_1)$.

Proposition 1.4: Let T: $L_1 \rightarrow L_2$ be a function between logical systems. Then T is an L-homeomorphism iff T is a conservative and bijective translation.

Let us consider that L_1 is a language with only unary and binary connectives and such that $\sigma_0, \sigma_1, \sigma_2, \ldots$ denote the atomic formulas of L_1. Now, let L_2 be another language, so a mapping f: $L_1 \rightarrow L_2$ is *schematic* if exists schemes[2] A, $B_{\&}$, $C_{\#}$ of L_2 such that:

(i) $f(\sigma) = A(\sigma)$, for every atomic formula of L_1;

(ii) $f(\&\alpha) = B_{\&}(f(\alpha))$, for each unary connective & of L_1;

(iii) $f(\alpha \neq \beta) = C\neq(f(\alpha), f(\beta))$, for each binary connective of L_1.

A schematic mapping is a homomorphism between languages because it preserves the algebraic structure of the algebra of formulas associated with the respective languages.

A schematic mapping is *literal* if it translates each connective in itself, that is, $f(\&\alpha) = \&f(\alpha)$ and $f(\alpha \neq \beta) = f(\alpha) \neq f(\beta)$. The function T is a *schematic translation* if T is a translation and schematic mapping.

2. About Lukasiewicz's logics

The Lukasiewicz's logics were initially introduced in 1920 and 1922 by semantical matrices as the following.

Let us consider the sets:

$$L_n = \begin{cases} \{0, 1/n\text{-}1, 2/n\text{-}1, 3/n\text{-}1, \ldots, 1\} \text{ if } n \in \text{[illegible]} \text{ and } n \geq 2 \\ \{s/m : 0 \leq s \leq m, \text{ com } s \in \text{[illegible]} \text{ and } m \in \text{[illegible]}^*\} \text{ if } n = \aleph_0 \\ [0, 1] \text{ (real interval) if } n = \aleph_1 \end{cases}$$

So a *Lukasiewicz n-valued matrix* has the form:

$M_n = ((L_n, \neg, \rightarrow, \wedge, \vee, \leftrightarrow), \{1\}),$

[2] Schemes preserve the form or figure of formulas such that for every replacing in the parts the form keeps the same.

where $\neg$ is an unary symbol to the negation and $\rightarrow, \wedge, \vee, \leftrightarrow$ are binary symbols for the conditional, conjunction, disjunction and biconditional, respectively, defined on the set L_n as below:

$$\neg x =_{df} 1-x$$
$$x \rightarrow y =_{df} \min\{1, 1-x+y\}$$
$$x \vee y =_{df} (x \rightarrow y) \rightarrow y = \max\{x, y\}$$
$$x \wedge y =_{df} \neg(\neg x \vee \neg y) = \min\{x, y\}$$
$$x \leftrightarrow y =_{df} (x \rightarrow y) \wedge (y \rightarrow x) = 1-|x-y|.$$

In 1920 Lukasiewicz introduced his 3-valued logic with the following tables:

α	$\neg\alpha$
0	1
½	½
1	0

$\rightarrow$	0	½	1
0	1	1	1
½	½	1	1
1	0	½	1

for negation and conditional extending the classical interpretation for these connectives. The other connectives were defined as above. This way we can see that M_2 coincides with the classical matrix.

Let us indicate the set of tautologies of M_n by Taut(M_n). Then it is possible to demonstrate these results (see Malinowski, 1993):

Proposition 2.1: Taut(M_n) $\subseteq$ Taut(M_2), for every $n \in$.

Proposition 2.2: (Lukasiewicz and Tarsk (1930)) Taut(M_n) $\subseteq$ Taut(M_m) $\Leftrightarrow$ m-1/n-1.

Proposition 2.3: Taut($M_{\aleph 1}$) = Taut($M_{\aleph 0}$).

Proposition 2.4: Taut($M_{\aleph 0}$) = $\cap${Taut(M_n) / $n \geq 2$ and $n \in$ }.

Only in a second moment was a search for axiomatizations to Lukasiewicz's logics started. Like we can see in Malinowski (1993, p. 39):

> "It can be easily verified that the content of any Lukasiewicz matrix is closed under the detachment rule (for the original implication of the matrix in question). A proof that finite matrices are axiomatizable was given in Lukasiewicz and Tarski (1930). Nevertheless, the problem of formulation of a concrete finite axiomatization of $E(M_n)$ [$E(M_n)$ coincides with $Taut(M_n)$] (with the exception of the case $n = 3$) remained open till 1952; see Rosser and Turquette (1952). Wajsberg (1931) showed that the $(\neg, \rightarrow)$-fragment of Lukasiewicz's three-valued propositional calculus may be axiomatized, accepting the rules MP [*Modus ponens*] and SUB [Substitution], in the following way:
>
> W1. $p\rightarrow(q\rightarrow p)$
>
> W2. $(p\rightarrow q)\rightarrow((q\rightarrow r)\rightarrow(p\rightarrow r))$
>
> W3. $(\neg p\rightarrow\neg q)\rightarrow(q\rightarrow p)$
>
> W4 $((p\rightarrow\neg p)\rightarrow p)\rightarrow p$".

Lukasiewicz also proposed an axiomatization to the infinite valued system using five axioms and conjectured that the axiomatization was complete for the semantical matrix.

Cignoli, D'Ottaviano and Mundici (1994) say:

> "Wajsberg 1935, p. 240, mentioned that he had proof the Lukasiewicz's conjecture, however his proof never appeared published (see Tarski 1956, p. 51, or Tarski 1983, p. 51).
>
> The first proof published about the Lukasiewicz's conjecture was that by Rose and Rosser 1958. Chang 1959 presented another proof of algebraic character. An exposition of these works is in Rosser 1960" [Author's free translation from original in Portuguese].

All that tell us that it was a little difficult to clean the bridge between the semantical and syntactical aspects of Lukasiewicz's logics.

3. Limits for a translation from $\mathbf{L}_\lambda$ into **CPC**

In this section we will show that there is not any conservative and schematic translation, or better, any conservative and schematic mapping from $\mathbf{L}_\lambda$ into **CPC**, for $\lambda = n \in$ ‥ or $\lambda = \aleph_0$. We will follow a similar way to that one of Epstein (1990) for $\mathbf{L}_3$.

In the following we will interpret the schemes $A(\varphi)$, $B_\&(t(\alpha))$ and $C_{\neq}(t(\alpha), t(\beta))$ in **CPC**. Let us indicate a tautology by $\mathbf{1} \Leftrightarrow (p \rightarrow p)$ and a contradiction by $\mathbf{0} \Leftrightarrow (p \wedge \neg p)$. Considering a formula in which only one propositional variable p occurs, its translations A(p) has exactly one possibility among the following equivalencies in **CPC**:

$\vdash_{CPC} A(p) \Leftrightarrow \mathbf{0}$

$\vdash_{CPC} A(p) \Leftrightarrow p$

$\vdash_{CPC} A(p) \Leftrightarrow \neg p$

$\vdash_{CPC} A(p) \Leftrightarrow \mathbf{1}$.

Theorem 3.1: There is not any conservative and schematic mapping t from $\mathbf{L}_\lambda$ to **CPC**.

Proof: Let us suppose that there is a conservative and schematic mapping t from $\mathbf{L}_\lambda$ to **CPC**. We will need to consider just the connectives $\neg$ and $\rightarrow$.

- It is not possible to occur $B_\neg(t(\alpha)) \Leftrightarrow \mathbf{0}$ because in $\mathbf{L}_\lambda$, for every formula α, we have $\vdash_{\mathbf{L}_\lambda} \alpha \leftrightarrow \neg\neg\alpha$. Thus, there is a formula α such that $\vdash_{\mathbf{L}_\lambda} \neg\alpha$ and through the conservative mapping t we have $\vdash_{CPC} B_\neg(t(\alpha))$ or $\vdash_{CPC} \mathbf{0}$.
- It is not possible to occur $B_\neg(t(\alpha)) \Leftrightarrow \mathbf{1}$ because there is a formula α in $\mathbf{L}_\lambda$ such that $\vdash_{\mathbf{L}_\lambda} \neg\alpha$. Applying the conservative mapping t we have $\vdash_{CPC} B_\neg(t(\alpha))$ or $\vdash_{CPC} \mathbf{1}$.

• It is not possible to occur $B_{\neg}(t(\alpha)) \Leftrightarrow t(\alpha)$ because there is a formula α in $\mathbf{L}_\lambda$ such that $\vdash_{\mathbf{L}_\lambda} \alpha$ and $\vdash_{\mathbf{L}_\lambda} \neg\alpha$. So, by the conservative mapping t we have $\vdash_{CPC} t(\alpha)$ and $\vdash_{CPC} B_{\neg}(t(\alpha))$ or $\vdash_{CPC} t(\alpha)$ and $\vdash_{CPC} t(\alpha)$.

Thus, $B_{\neg}(t(\alpha)) \Leftrightarrow \neg t(\alpha)$, that is, $t(\neg\alpha) \equiv \neg t(\alpha)$.

Now we need to consider the scheme $C_{\rightarrow}(t(\alpha), t(\beta))$. As in the Lukasiewicz's logics, there is exactly one designed element 1, but there are other true values, for any formula α. Let us denote its valuation by $e(\alpha) = T = 1$ and $e(\alpha) = F \neq 1$.

• It is not possible to occur in $\mathbf{L}_\lambda$ that $e(\alpha) = T$, $e(\beta) = T$ and $v(C_{\rightarrow}(t(\alpha), t(\beta))) = 0$, because in **CPC** we would have $\vdash_{\mathbf{CPC}} C_{\rightarrow}(t(\alpha), t(\beta)) \Leftrightarrow \mathbf{0}$. But considering $\alpha \equiv (p\rightarrow p)$ and $\beta \equiv (p\rightarrow p)\rightarrow(p\rightarrow p)$ we have $\vdash_{\mathbf{L}_\lambda} \alpha$, $\vdash_{\mathbf{L}_\lambda} \beta$ and $\vdash_{\mathbf{L}_\lambda} \alpha\rightarrow\beta$. By the conservative mapping t, $\vdash_{CPC} t(\alpha)$, $\vdash_{CPC} t(\beta)$ and $\vdash_{CPC} C_{\rightarrow}(t(\alpha), t(\beta))$.

• It is not possible to occur that $e(\alpha) = T$, $e(\beta) = F$ and $v(C_{\rightarrow}(t(\alpha), t(\beta))) = 1$. Considering $\alpha \equiv (p\rightarrow p)$ and $\beta \equiv \neg(p\rightarrow p)$ we have $\vdash_{\mathbf{L}_\lambda} \alpha$, $\vdash_{\mathbf{L}_\lambda} \neg\beta$ and $\vdash_{\mathbf{L}_\lambda} \alpha\rightarrow\beta$. By the conservative mapping t, we have $\vdash_{CPC} t(\alpha)$, $\vdash_{CPC} \neg t(\beta)$ and $\vdash_{CPC} C_{\rightarrow}(t(\alpha), t(\beta))$.

• It is not possible to occur that $e(\alpha) = F$, $e(\beta) = T$ and $v(C_{\rightarrow}(t(\alpha), t(\beta))) = 0$. Considering $\alpha \equiv \neg(p\rightarrow p)$ and $\beta \equiv (p\rightarrow p)$ we have $\vdash_{\mathbf{L}_\lambda} \neg\alpha$, $\vdash_{\mathbf{L}_\lambda} \beta$ and $\vdash_{\mathbf{L}_\lambda} \alpha\rightarrow\beta$. By the conservative mapping t, we have $\vdash_{CPC} \neg t(\alpha)$, $\vdash_{CPC} t(\beta)$ and $\vdash_{CPC} C_{\rightarrow}(t(\alpha), t(\beta))$.

• It is not possible to occur that $e(\alpha) = F$, $e(\beta) = F$ and $v(C_{\rightarrow}(t(\alpha), t(\beta))) = 0$. Considering $\alpha \equiv \neg(p\rightarrow p)$ and $\beta \equiv \neg(p\rightarrow p)$, we have $\vdash_{\mathbf{L}_\lambda} \neg\alpha$, $\vdash_{\mathbf{L}_\lambda} \neg\beta$ and $\vdash_{\mathbf{L}_\lambda} \alpha\rightarrow\beta$. By the conservative mapping t we have $\vdash_{CPC} \neg t(\alpha)$, $\vdash_{CPC} \neg t(\beta)$ and $\vdash_{CPC} C_{\rightarrow}(t(\alpha), t(\beta))$.

Thus, $C_{\rightarrow}(t(\alpha), t(\beta)) \Leftrightarrow t(\alpha) \rightarrow t(\beta)$.

We still need to show the impossibility of a function with these characteristics. Well, the formulas of the following types are theorems of $\mathbf{L}\aleph_0$ and therefore of $\mathbf{L}_\lambda$:

Ax_1 $\alpha\rightarrow(\beta\rightarrow\alpha)$

Ax_2 $(\alpha \to \beta) \to (\neg\beta \to \neg\alpha)$.

The schemes Ax_1 and Ax_2 were introduced by Lukasiewicz for $\mathbf{L}\aleph_0$. In this way, the type of formulas above are theorems of $\mathbf{L}\aleph_0$ and any other $\mathbf{L_n}$. But the scheme:

Ax_3 $(\alpha \to (\beta \to \gamma)) \to ((\alpha \to \beta) \to (\alpha \to \gamma))$,

is not a $\mathbf{L}_\lambda$-theorem because in the presence of these three schemes we obtain the classical propositional calculus $\mathbf{L_2} \equiv$ **CPC** as in Hamilton (1978). The schemes Ax_1 and Ax_3 together give us the usual Deduction Theorem that is known to be non-valid in $\mathbf{L}_\lambda$.

Thus, taking $\sigma \equiv [\alpha \to (\beta \to \gamma)] \to [(\alpha \to \beta) \to (\alpha \to \gamma)]$ we get $t(\sigma) \Leftrightarrow [t(\alpha) \to (t(\beta) \to t(\gamma))] \to [(t(\alpha) \to t(\beta)) \to (t(\alpha) \to t(\gamma))]$ and then $|\text{-}\mathbf{L}_\lambda\ \sigma$, but $|\text{-}_{\mathbf{CPC}}\ t(\sigma)$, that is a contradiction to the fact that t is a conservative mapping.

Even though it is not part of the proof we can consider also the scheme A(p).

- Since $|\text{-}\mathbf{L}_\lambda$ p, then it is not possible to occur $|\text{-}_{\mathbf{CPC}}$ $A(p) \Leftrightarrow \mathbf{1}$.
- As $|\text{-}\mathbf{L}_\lambda\ \neg p$ and $t(\neg p) \equiv \neg A(p)$, then it is not possible to occur $|\text{-}_{\mathbf{CPC}}$ $A(p) \Leftrightarrow \mathbf{0}$.

Then we have that $A(p) \equiv p$ or $A(p) \equiv \neg p$.

This result shows us that it is not easy to translate from Lukasiewicz's into the classical logic. Even though it was shown the possibility of getting such a function, maybe the function supposed in (Feitosa, Baar, 200_) be a non recursive function and so it keeps us far from that translation.

Abstract

This work presents some concepts involving translation between logics and a revision of Lukasiewicz's logics. Then, as the central result, it shows there is not a schematic translation from any Lukasiewicz's logic into the classical propositional logic. This can give us some idea of how difficult it is to find such a function.

Bibliography

ACKERMANN, R. (1967) **Introduction to many-valued logics**. New York: Dover.

BOICESCU, V., FILIPOIU, A., GEORGESCU, G., RUDEANU, S. (1991) **Lukasiewicz-Moisil algebras**. Amsterdam: North-Holland. (Annals of Discrete Mathematics, v. 49)

BOLC, L., BOROWIK, P. (1992) **Many-valued logics: 1 Theoretical Foundations**. Berlin: Springer-Verlag.

CIGNOLI, R. (1980) Some Algebraic Aspects of Many-Valued Logics. In: ARRUDA, A. I., DA COSTA, N. C. A., SETTE, A. M. (Eds.) **Proceedings of the Third Brazilian Conference on Mathematical Logic**. São Paulo: Sociedade Brasileira de Lógica, p. 49-69.

CIGNOLI, R. L. O., D'OTTAVIANO, I. M. L., MUNDICI, D. (1994) **Álgebras das lógicas de Lukasiewicz**. Campinas: UNICAMP/CLE. (Coleção CLE, v. 12)

D'OTTAVIANO, I. M. L., FEITOSA, H. A. (1999(a)) Many-valued logics and translations. In: CARNIELLI, W. (Ed.) Multi-valued logics. ***Journal of Applied Non-Classical***, v. 9, n. 1, p. 121-140.

D'OTTAVIANO, I. M. L., FEITOSA, H. A. (1999(b)) Conservative translations and model-theoretic translations. ***Manuscrito***, v. XXII, n. 2, p. 117-132.

D'OTTAVIANO, I. M. L., FEITOSA, H. A. (2000) Paraconsistent logics and translations. In the Festschriftt in honour of N. C. A. da Costa. ***Synthese***, v. 125, n. 1-2, p. 75-97.

EBBINGHAUS, H. D., FLUM, J., THOMAS, W. (1984) **Mathematical logic**. New York: Springer-Verlag.

EPSTEIN, R. L. (1990) **The Semantic Foundations of Logic.** Volume 1: Propositional logics. Dordrecht: Kluwer Academic Publishers.

FEITOSA, H. A. (1997) **Traduções conservativas** (Conservative tranlations). Campinas: UNICAMP. (Tese de Doutorado em Lógica)

FEITOSA, H. A., BAAR T. (200_) *Is it possible to translate Lukasiewicz logics into classical logics?* (To appear)

FEITOSA, H. A., D'OTTAVIANO, I. M. L. (2001) Conservative Translations. ***Annals of Pure and Applied Logics***, Amsterdam, v.108, p.205-227.

HAMILTON, A. G. (1978) **Logic for Mathematicians**. Cambridge: Cambridge University Press.

MALINOWSKI, G. (1993) **Many-valued Logics**. Oxford: Clarendon Press.

MENDELSON, E. (1964) **Introduction to Mathematical Logic**. Princeton: D. Van Nostrand.

RASIOWA, H. (1974) **An Algebraic Approach to Non-classical Logics**. Amsterdam: North-Holland.

WÓJCICKI, R. (1988) **Theory of Logical Calculi: Basic Theory of Consequence Operations**. Dordrecht: Kluwer, 1988. (Synthese Library, v. 199)