Ingeniería y Ciencia, ISSN 1794–9165
Volumen 4, número 7, junio de 2008, páginas 7–22

Algunas integrales que involucran a la función hipergeométrica generalizada

Some Integrals involving generalized hypergeometric function

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Recepción: 06-ago-2007/Modificación: 21-nov-2007/Aceptación: 23-nov-2007
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Resumen

Recientemente Virchenko y colaboradores trataron una generalización de la función gamma $\Gamma\left(\begin{array}{c} a,b;c; \\ u,v \end{array};p,\tau\right) = v^{-a}\int\limits_0^\infty x^{u-1}e^{-px}\,{}_2R_1\left(a,b;c;\tau;-\frac{x}{v}\right)dx$, donde ${}_2R_1\left(a,b;c;\tau;x\right)$ es la función hipergeométrica generalizada presentada por Dotsenko en 1991. El objeto de este artículo es obtener algunos resultados que involucran casos especiales de esta función y obtener formas computables para los mismos.

Palabras claves: función hipergeométrica generalizada, función gamma generalizada.

Abstract

Recently Virchenko et al have treated a generalized gamma function $\Gamma\left(\begin{array}{c} a,b;c; \\ u,v \end{array};p,\tau\right) = v^{-a}\int\limits_0^\infty x^{u-1}e^{-px}\,{}_2R_1\left(a,b;c;\tau;-\frac{x}{v}\right)dx$, where ${}_2R_1(a,b;c;\tau;x)$ is the generalized hypergeometric function presented for Dotsenko in 1991. The object of this paper is to obtain some more results involving especial case of this function and computational forms as obtained.

Key words: generalized hypergeometric function, generalized gamma function.

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1 Introducción

La generalización τ de la función hipergeomética de Gauss notada por ${}_2R_1(a,b;c;\tau;z)$, fue presentada recientemente por Nina Virchenko [1] de la forma

$$ {}_2R_1(a,b;c;\tau;z) = \frac{\Gamma(c)}{\Gamma(a)\Gamma(b)} \sum_{k=0}^{\infty} \frac{\Gamma(a+k)\Gamma(b+\tau k)}{\Gamma(c+\tau k)} \frac{z^k}{k!} \tag{1} $$

donde a, b, c son parámetros complejos, $\tau \in \mathbb{R}$, $\tau > 0$, $c \neq 0, -1, -2, ...$, $|z| < 1$.

Castillo y colaboradores [2] presentaron las siguientes representaciones simples para la función ${}_2R_1(a,b;c;\tau;z)$:

$$ {}_2R_1(a,b;c;\tau;z) = \frac{\Gamma(c)}{\tau\Gamma(b)\Gamma(c-b)} \sum_{k=0}^{\infty} \frac{(1-c+b)_k}{k!} \times $$
$$ \left[\beta\left(\frac{k+b}{\tau} - a, 1\right)(-z)^{-a}\ {}_2F_1\left(a, a - \frac{k+b}{\tau}; 1 + a - \frac{k+b}{\tau}; \frac{1}{z}\right) + \right. $$
$$ \left. \beta\left(\frac{k+b}{\tau}, -\frac{k+b}{\tau} + a\right)(-z)^{-\frac{k+b}{\tau}} \right] \tag{2} $$

$$ \mathrm{Re}\,(c) > \mathrm{Re}\,(b) > 0,\ |\arg\ (-z)| < \pi $$

$(a)_k = \frac{\Gamma(a+k)}{\Gamma(a)}$ es el símbolo de Pocchammer;

$$ {}_2R_1(a,b;c;\tau;z) = \frac{\Gamma(c)}{\tau\Gamma(b)\Gamma(c-b)} \sum_{k=0}^{\infty} \frac{(1-c+b)_k}{k!} \frac{\Gamma\left(\frac{k+b}{\tau}\right)}{\Gamma\left(\frac{k+b}{\tau}+1\right)} \times $$
$$ {}_2F_1(a, \frac{k+b}{\tau}; \frac{k+b}{\tau} + 1; z). \tag{3} $$

$$ \tau > 0,\ \mathrm{Re}\,(c) > \mathrm{Re}\,(b) > 0\,, $$

las cuales facilitan los desarrollos simbólicos para obtener algunos resultados complejos que las involucran.

Recientemente, Castillo [3] presentó un conjunto de resultados de la forma

$$ \int_a^b x^{\alpha-1} f(x) \times\ {}_2R_1(a,b;c;\tau;t(x))dx\,, $$

los cuales constituyen fórmulas computables para cierto tipo de generalización de la función beta introducida por Ben Nakhi y Kalla. En este artículo se establece otro conjunto de resultados de la forma

$$\int_a^b A(x)\, e^{-pw(x)}\ {}_2R_1\left(a, b; c; \tau; \varphi(x)\right) dx\,,$$

los cuales son expresiones de una nueva forma unificada de la función gamma. Inicialmente se presentan algunos resultados existentes, los cuales serán generalizados posteriormente.

1.1 Algunas integrales impropias con límite de integración infinito que involucran $A(x)\, e^{-pw(x)}\ {}_2F_1(a, b; c; \varphi(x))$

A continuación se presentan algunas integrales impropias con límites de integración infinitos que contienen $A(x)\, e^{-pw(x)}\ {}_2F_1(a, b; c; \varphi(x))$ [4, págs. 318-321, Nos (1) − (13)], las cuales han sido calculadas por otros investigadores:

1 $$\int_0^\infty x^{\alpha-1} e^{-px}\ {}_2F_1(a, b; c; -wx)\, dx = w^{-\alpha}\Gamma\left[\begin{array}{l} c, \alpha, a-\alpha, b-\alpha \\ a, b, c-\alpha \end{array}\right]\times$$
$${}_2F_2\left(\alpha, \alpha-c+1; \alpha-a+1, \alpha-b+1; \frac{p}{w}\right) + \frac{p^{a-\alpha}}{w^a}\Gamma\left[\begin{array}{l} c, b-a, \alpha-a \\ b, c-a \end{array}\right]\times$$
$${}_2F_2\left(a, a-c+1; a-\alpha+1, \alpha-b+1; \frac{p}{w}\right) + \frac{p^{b-\alpha}}{w^b}\Gamma\left[\begin{array}{l} c, a-b, \alpha-b \\ a, c-b \end{array}\right]\times$$
$${}_2F_2\left(b, b-c+1; b-a+1, b-\alpha+1; \frac{p}{w}\right)$$
$$\operatorname{Re} p,\ \operatorname{Re} \alpha > 0;\ |\arg w < \pi|\,.$$

2 $$\int_0^\infty x^{c-1} e^{-px}\ {}_2F_1(a, b; c; -wx)\, dx = \frac{\Gamma(c)}{p^c}\left(\frac{p}{w}\right)^{\frac{(a+b-1)}{2}} e^{\frac{p}{2w}} W_{\frac{(1-a-b)}{2},\ \frac{(a-b)}{2}}\left(\frac{p}{w}\right)$$
$$\operatorname{Re} c,\ \operatorname{Re} p > 0;\ |\arg w < \pi|\,.$$

3 $$\int_0^\infty x^{c-1} e^{-px}\ {}_2F_1(a, 1-a; c; -wx)\, dx = \sqrt{\frac{p}{\pi w}}\frac{\Gamma(c)}{(2p)^c} K_{a-\frac{1}{2}}\left(\frac{p}{2w}\right)$$

$$\operatorname{Re} c,\ \operatorname{Re} p > 0;\ |\arg w < \pi|.$$

4 $$\int_0^\infty x^{c-1} e^{-px}\ {}_2F_1\left(\frac{1}{2}, 1; c; -wx\right) dx = \sqrt{\frac{\pi}{w}} \frac{\Gamma(c)}{p^{c-\frac{1}{2}}} e^{\frac{p}{w}} \operatorname{erf} c\left(\sqrt{\frac{p}{w}}\right)$$

$$\operatorname{Re} c,\ \operatorname{Re} p > 0;\ |\arg w < \pi|.$$

5 $$\int_0^\infty x^{\alpha-1} e^{-px}\ {}_2F_1(a, b; c; 1-wx)\, dx = \Gamma\left[\begin{array}{l} c, \alpha, a-\alpha, b-\alpha, c-a-b-\alpha \\ a, b, c-\alpha, c-b \end{array}\right] \times$$
$$w^{-\alpha}{}_2F_2\left(\alpha, c-a-b-\alpha; \alpha-a+1, \alpha-b+1; -\frac{p}{w}\right) + \Gamma\left[\begin{array}{l} c, b-a, \alpha-a \\ b, c-a \end{array}\right] \times$$
$$\frac{p^{a-\alpha}}{w^a}{}_2F_2\left(a, c-b; a-b+1, a-\alpha+1; -\frac{p}{w}\right) + \Gamma\left[\begin{array}{l} c, a-b, \alpha-b \\ a, c-b \end{array}\right] \times$$
$$\frac{p^{b-\alpha}}{w^b}{}_2F_2\left(b, c-a; b-a+1, b-\alpha+1; -\frac{p}{w}\right)$$

$$\operatorname{Re}\alpha,\ \operatorname{Re} p,\ \operatorname{Re}(c-a-b+\alpha) > 0;\ |\arg w < \pi|.$$

6 $$\int_0^\infty x^{\alpha-1} e^{-p/x}\ {}_2F_1(a, b; c; -wx)\, dx w^{-\alpha}\Gamma\left[\begin{array}{l} c, \alpha, a-\alpha, b-\alpha \\ a, b, c-\alpha \end{array}\right] \times$$
$${}_2F_2(a-\alpha, b-\alpha; 1-\alpha, c-\alpha; wp) + p^{\alpha}\Gamma(-\alpha)\,{}_2F_2(a, b; c, \alpha+1; wp)$$

$$\operatorname{Re}(a-\alpha),\ \operatorname{Re}(b-\alpha),\ \operatorname{Re} p > 0;\ |\arg w < \pi|.$$

7 $$\int_0^\infty e^{-p\sqrt{x}}\ {}_2F_1\left(a, b; \frac{3}{2}; -wx\right) dx = \frac{2p^{a+b-2}}{w^{\frac{(a+b)}{2}}} S_{1-a-b, a-b}\left(\frac{p}{\sqrt{w}}\right)$$

$$\operatorname{Re} p > 0;\ |\arg w < \pi|.$$

8 $$\int_0^\infty x^{\alpha-1} e^{-p\sqrt{x}}\ {}_2F_1(a, b; c; -wx)\, dx = \frac{2p^{2a-2\alpha}}{w^a}\Gamma\left[\begin{array}{l} c, b-a, 2\alpha-2a \\ b, c-a \end{array}\right] \times$$
$${}_2F_3\left(\alpha, a-c+1; a-b+1, a-\alpha+1, a-\alpha+\frac{1}{2}; -\frac{p^2}{4w}\right) + \frac{2p^{2b-2\alpha}}{w^b} \times$$
$$\Gamma\left[\begin{array}{l} c, a-b, 2\alpha-2b \\ a, c-b \end{array}\right]{}_2F_3\left(b, b-c+1; b-a+1, b-\alpha+1, b-\alpha+\frac{1}{2}; -\frac{p^2}{4w}\right)$$

 Ingeniería y Ciencia, ISSN 1794–9165

$$+w^{-\alpha}\Gamma\left[\begin{array}{l} c,\alpha,a-\alpha,b-\alpha \\ a,b,c-\alpha \end{array}\right]{}_2F_3\left(\alpha,\alpha-c+1;\alpha-a+1,\alpha-b+1,\frac{1}{2};-\frac{p^2}{4w}\right)$$

$$-\frac{p}{w^{\alpha+1/2}}\Gamma\left[\begin{array}{l} c,\alpha+1/2,a-\alpha-1/2,b-\alpha-1/2 \\ a,b,c-\alpha-1/2 \end{array}\right]\times$$

$${}_2F_3\left(\alpha-c+\frac{3}{2},\alpha+\frac{1}{2};\frac{3}{2},\alpha-a+\frac{3}{2},\alpha-b+\frac{3}{2};-\frac{p^2}{4w}\right)$$

$$\mathrm{Re}\,\alpha,\ \mathrm{Re}\,p>0;\ |\arg w<\pi|\,.$$

9 $$\int_0^\infty x^{-\frac{1}{2}}e^{-p\sqrt{x}}\ {}_2F_1\left(a,b;\frac{1}{2};-wx\right)dx=2w^{\frac{-a-b}{2}}p^{a+b-1}S_{1-a-b,a-b}\left(\frac{p}{\sqrt{w}}\right)$$)

$$\mathrm{Re}\,p>0;\ |\arg w<\pi|\,.$$

10 $$\int_0^\infty x^{\alpha-1}e^{-p\sqrt{x}}\ {}_2F_1\left(a,b;c;1-wx\right)dx=\frac{2p^{2a-2\alpha}}{w^a}\Gamma\left[\begin{array}{l} c,b-a,2\alpha-2a \\ b,c-a \end{array}\right]\times$$

$${}_2F_3\left(\alpha,c-b;a-b+1,a-\alpha+1,a-\alpha+\frac{1}{2};\frac{p^2}{4w}\right)+\frac{2p^{2b-2\alpha}}{w^b}\times$$

$$\Gamma\left[\begin{array}{l} c,a-b,2\alpha-2b \\ a,c-b \end{array}\right]{}_2F_3\left(b,c-a;b-a+1,b-\alpha+1,b-\alpha+\frac{1}{2};\frac{p^2}{4w}\right)+$$

$$w^{-\alpha}\Gamma\left[\begin{array}{c} c,\alpha,a-\alpha,b-\alpha,c-a-b+\alpha \\ a,b,c-\alpha,c-b \end{array}\right]{}_2F_3\left(\alpha,c-a-b+\alpha;\frac{1}{2},\alpha-a+1,\right.$$

$$\left.\alpha-b+1;\frac{p^2}{4w}\right)-\Gamma\left[\begin{array}{c} c,\alpha+\frac{1}{2},a-\alpha-\frac{1}{2},b-\alpha-\frac{1}{2},c-a-b+\alpha+\frac{1}{2} \\ a,b,c-\alpha,c-b \end{array}\right]\times$$

$$\frac{p}{w^{\alpha+\frac{1}{2}}}{}_2F_3\left(\alpha+\frac{1}{2},c-a-b+\alpha+\frac{1}{2};\frac{3}{2},\alpha-a+\frac{3}{2},\alpha-b+\frac{3}{2};\frac{p^2}{4w}\right)$$

$$\mathrm{Re}\,\alpha,\ \mathrm{Re}\,p,\ \mathrm{Re}\,(c-a-b+\alpha)>0;\ |\arg w<\pi|\,.$$

11 $$\int_0^\infty x^{\alpha-1}e^{-p/\sqrt{x}}\ {}_2F_1\left(a,b;c;-wx\right)dx=w^{-\alpha}\Gamma\left[\begin{array}{l} c,\alpha,a-\alpha,b-\alpha \\ a,b,c-\alpha \end{array}\right]\times$$

$${}_2F_3\left(a-\alpha,b-\alpha;\frac{1}{2},1-\alpha,c-\alpha;-\frac{p^2w}{4}\right)+2p^{2\alpha}\Gamma\left(-2\alpha\right){}_2F_3\left(a,b;c,\alpha+1,\right.$$

$$\left.\alpha+\frac{1}{2};-\frac{p^2w}{4}\right)-w^{1/2-\alpha}p\Gamma\left[\begin{array}{l} c,\alpha-1/2,a-\alpha+1/2,b-\alpha+1/2 \\ a,b,c-\alpha+1/2 \end{array}\right]\times$$

$$ {}_2F_3\left(\alpha-\alpha+1/2,b-\alpha+1/2;3/2,3/2-\alpha,c-\alpha+1/2;-\frac{p^2w}{4}\right) $$

$$ \mathrm{Re}\,p,\ \mathrm{Re}\,(a-\alpha),\ \mathrm{Re}\,(b-\alpha)>0;\ |\arg w<\pi|\,. $$

12 $$ \int\limits_0^y x^{\alpha-1}(y-x)^{c-1}e^{-px}\ {}_2F_1\left(a,b;c;1-\frac{x}{y}\right)dx=\Gamma\left[\begin{array}{c} c,\alpha,c-a-b+\alpha \\ c-a+\alpha,c-b+\alpha \end{array}\right]\times $$
$$ y^{c+\alpha-1}\ {}_2F_2\left(\alpha,c-a-b+\alpha;c-a+\alpha,c-b+\alpha;-py\right) $$

$$ y,\ \mathrm{Re}\,c,\ \mathrm{Re}\,\alpha,\ \mathrm{Re}\,(c-a-b+\alpha)>0\,. $$

13 $$ \int\limits_0^y x^{\alpha-1}(y-x)^{c-1}e^{-p\sqrt{x}}\ {}_2F_1\left(a,b;c;1-\frac{x}{y}\right)dx=y^{c+\alpha-1}\times $$
$$ \Gamma\left[\begin{array}{c} c,\alpha,c-a-b+\alpha \\ c-a+\alpha,c-b+\alpha \end{array}\right]{}_2F_3\left(\alpha,c-a-b+\alpha;c-a+\alpha,c-b+\alpha,\frac{1}{2};\right. $$
$$ \left.\frac{p^2y}{4}\right)-py^{c+\alpha-1/2}\Gamma\left[\begin{array}{c} c,\alpha+1/2,c-a-b+\alpha+1/2 \\ c-a+\alpha+1/2,c-b+\alpha+1/2 \end{array}\right]{}_2F_3\left(\alpha+1/2,\right. $$
$$ \left.c-a-b+\alpha+1/2;c-a+\alpha+1/2,c-b+\alpha+1/2,\frac{3}{2};\frac{p^2y}{4}\right) $$

$$ y,\ \mathrm{Re}\,c,\ \mathrm{Re}\,\alpha,\ \mathrm{Re}\,(c-a-b+\alpha)>0\,. $$

2 Resultados

En esta sección se presentan algunas integrales impropias con límite de integración infinito que involucran en el integrando a la función $A(x)\,e^{-pw(x)}\ {}_2R_1\left(a,b;c;\varphi(x)\right)$. Aquí se presenta el desarrollo simbólico para uno de los resultados, señalando el procedimiento seguido. Para obtener los otros resultados de la sección 2.1 el procedimiento es similar; también es importante destacar el hecho de que cada uno de los resultados que aquí se obtienen, corresponden a casos especiales de la función gamma generalizada, por lo tanto generalizan a los presentados en la sección 1.1.

2.1 Algunas integrales impropias con límite de integración infinito que involucran $A(x)\, e^{-pw(x)}\ {}_2R_1(a,b;c;\varphi(x))$

A continuación se presentan los resultados obtenidos, junto con sus condiciones de validez, los cuales generalizan a los presentados en la sección 1.1.

1 Calcular

$$I=\int_0^\infty x^{\alpha-1}e^{-px}\ {}_2R_1(a,b;c;\tau;-wx)\,dx\,.$$

Note que I es un caso especial de la función gamma generalizada.

Al usar el resultado (3) en I se tiene

$$\begin{aligned} I &= \frac{\Gamma(c)}{\tau\Gamma(b)\Gamma(c-b)}\int_0^\infty x^{\alpha-1}e^{-px}\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\frac{\Gamma\left(\frac{k+b}{\tau}\right)}{\Gamma\left(\frac{k+b}{\tau}+1\right)}\times \\ &\quad {}_2F_1(a,\frac{k+b}{\tau};\frac{k+b}{\tau}+1;-wx)\,. \end{aligned}$$

Aquí es posible el intercambio de la suma con la integral puesto que la serie converge uniformemente

$$\begin{aligned} I &= \frac{\Gamma(c)}{\tau\Gamma(b)\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\frac{\Gamma\left(\frac{k+b}{\tau}\right)}{\Gamma\left(\frac{k+b}{\tau}+1\right)}\times \\ &\quad \int_0^\infty x^{\alpha-1}e^{-px}\ {}_2F_1(a,\frac{k+b}{\tau};\frac{k+b}{\tau}+1;-wx)\,. \end{aligned}$$

La integral es similar al resultado 1 de la sección 2, entonces

$$\begin{aligned} I &= \frac{\Gamma(c)}{\tau\Gamma(b)\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\frac{\Gamma\left(\frac{k+b}{\tau}\right)}{\Gamma\left(\frac{k+b}{\tau}+1\right)}\times \\ &\quad \left[w^{-\alpha}\Gamma\left[\begin{array}{c}\frac{k+b}{\tau}+1,\alpha,a-\alpha,\frac{k+b}{\tau}-\alpha\\ a,\frac{k+b}{\tau},\frac{k+b}{\tau}+1-\alpha\end{array}\right]\times\right. \\ &\quad {}_2F_2\left(\alpha,\alpha-\frac{k+b}{\tau};\alpha-a+1,\alpha-\frac{k+b}{\tau}+1;\frac{p}{w}\right)+ \\ &\quad \frac{p^{a-\alpha}}{w^a}\Gamma\left[\begin{array}{c}\frac{k+b}{\tau}+1,\frac{k+b}{\tau}-a,\alpha-a\\ \frac{k+b}{\tau},\frac{k+b}{\tau}+1-a\end{array}\right]\times \\ &\quad {}_2F_2\left(a,a-\frac{k+b}{\tau};a-\alpha+1,\alpha-\frac{k+b}{\tau}+1;\frac{p}{w}\right)+ \end{aligned}$$

$$\frac{p^{\frac{k+b}{\tau}-\alpha}}{w^{\frac{k+b}{\tau}}}\Gamma\left[\begin{array}{c}\frac{k+b}{\tau}+1, a-\frac{k+b}{\tau}, \alpha-\frac{k+b}{\tau}\\ a, 1\end{array}\right]\times$$
$$\left.{}_2F_2\left(\frac{k+b}{\tau}, 0; \frac{k+b}{\tau}+1-a, \frac{k+b}{\tau}+1-\alpha; \frac{p}{w}\right)\right],$$

pero ${}_2F_2\left(\frac{k+b}{\tau}, 0; \frac{k+b}{\tau}+1-a, \frac{k+b}{\tau}+1-\alpha; \frac{p}{w}\right)=1$, luego

$$\begin{aligned}
I \;=\; & \frac{\Gamma(c)w^{-\alpha}}{\tau\Gamma(b)\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\Gamma\left[\begin{array}{c}\alpha, a-\alpha, \frac{k+b}{\tau}-\alpha\\ a, \frac{k+b}{\tau}+1-\alpha\end{array}\right]\times\\
& {}_2F_2\left(\alpha, \alpha-\frac{k+b}{\tau}; \alpha-a+1, \alpha-\frac{k+b}{\tau}+1; \frac{p}{w}\right)+\\
& \frac{\Gamma(c)p^{a-\alpha}}{\tau\Gamma(b)\Gamma(c-b)w^{a}}\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\Gamma\left[\begin{array}{c}\frac{k+b}{\tau}-a, \alpha-a\\ \frac{k+b}{\tau}+1-a\end{array}\right]\times\\
& {}_2F_2\left(a, a-\frac{k+b}{\tau}; a-\alpha+1, \alpha-\frac{k+b}{\tau}+1; \frac{p}{w}\right)+\frac{\Gamma(c)p^{-\alpha}}{\tau\Gamma(b)\Gamma(c-b)}\times\\
& \sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\frac{p^{\frac{k+b}{\tau}}}{w^{\frac{k+b}{\tau}}}\Gamma\left[\begin{array}{c}a-\frac{k+b}{\tau}, \alpha-\frac{k+b}{\tau}, \frac{k+b}{\tau}\\ a\end{array}\right]
\end{aligned}\tag{4}$$

$$\tau,\ \operatorname{Re} p,\ \operatorname{Re}\alpha>0;\ |\arg w<\pi|.$$

Este resultado generaliza a 1, para $\tau=1$ en (4)

Prueba. En efecto, sea $\tau=1$, considérese el primer término de I como

$$\begin{aligned}
A \;=\; & \frac{\Gamma(c)w^{-\alpha}}{\Gamma(b)\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\Gamma\left[\begin{array}{c}\alpha, a-\alpha, k+b-\alpha\\ a, k+b+1-\alpha\end{array}\right]\times\\
& {}_2F_2\left(\alpha, \alpha-k-b; \alpha-a+1, \alpha-k-b+1; \frac{p}{w}\right).
\end{aligned}$$

Usando la expansión en serie de ${}_2F_2\left(\frac{p}{w}\right)$ se tiene

$$\begin{aligned}
A \;=\; & \frac{\Gamma(c)\Gamma(\alpha)\Gamma(a-\alpha)w^{-\alpha}}{\Gamma(a)\Gamma(b)\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{(1-c+b)_k\Gamma(b-\alpha+k)}{k!\Gamma(1+b-\alpha+k)}\times\\
& \sum_{n=0}^{\infty}\frac{(\alpha)_n(\alpha-b-k)_n}{(\alpha-a+1)_n(\alpha-b+1-k)_n}\frac{\left(\frac{p}{w}\right)^n}{n!};
\end{aligned}$$

pero $(a)_n=\frac{\Gamma(a+n)}{\Gamma(a)}$ y $\Gamma(a-n)=(-1)^n\frac{\Gamma(a)}{(1-a)_n}$, [4, pág. 758] entonces

$$A \;=\; \frac{\Gamma(c)\Gamma(\alpha)\Gamma(a-\alpha)\Gamma(1+\alpha-b)w^{-\alpha}}{\Gamma(a)\Gamma(b)\Gamma(c-b)\Gamma(\alpha-b)}\sum_{k=0}^{\infty}\frac{(1-c+b)_k\Gamma(b-\alpha+k)}{k!\Gamma(1+b-\alpha+k)}\times$$

$$\sum_{n=0}^{\infty} \frac{(\alpha)_n \Gamma(\alpha-b+n)(b-\alpha+1)_k (b-\alpha-n)_k}{(b-\alpha+1-n)_k \Gamma(1+\alpha-b+n)(\alpha-a+1)_n (b-\alpha)_k} \frac{\left(\frac{p}{w}\right)^n}{n!}.$$

De donde

$$\begin{aligned} A &= \frac{\Gamma(c)\Gamma(\alpha)\Gamma(a-\alpha)\Gamma(b-\alpha)\Gamma(1+\alpha-b)w^{-\alpha}}{\Gamma(a)\Gamma(b)\Gamma(c-b)\Gamma(\alpha-b)\Gamma(b-\alpha+1)} \times \\ & \sum_{n=0}^{\infty} \frac{(\alpha)_n \Gamma(\alpha-b+n)}{\Gamma(1+\alpha-b+n)(\alpha-a+1)_n} \frac{\left(\frac{p}{w}\right)^n}{n!} \times \\ & \sum_{k=0}^{\infty} \frac{(1-c+b)_k (b-\alpha-n)_k}{k!(b-\alpha+1-n)_k}, \end{aligned}$$

se sigue que

$$\begin{aligned} A &= \frac{\Gamma(c)\Gamma(\alpha)\Gamma(a-\alpha)\Gamma(b-\alpha)\Gamma(1+\alpha-b)w^{-\alpha}}{\Gamma(a)\Gamma(b)\Gamma(c-b)\Gamma(\alpha-b)\Gamma(b-\alpha+1)} \times \\ & \sum_{n=0}^{\infty} \frac{(\alpha)_n \Gamma(\alpha-b+n)}{\Gamma(1+\alpha-b+n)(\alpha-a+1)_n} \frac{\left(\frac{p}{w}\right)^n}{n!} \times \\ & {}_2F_1(1-c+b, b-\alpha-n; b-\alpha+1-n; 1), \end{aligned}$$

y teniendo en cuenta que ${}_2F_1(a,b;c;1) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)}$ se obtiene

$$\begin{aligned} A &= \frac{\Gamma(c)\Gamma(\alpha)\Gamma(a-\alpha)\Gamma(b-\alpha)\Gamma(1+\alpha-b)w^{-\alpha}}{\Gamma(a)\Gamma(b)\Gamma(\alpha-b)\Gamma(c-\alpha)} \times \\ & \sum_{n=0}^{\infty} \frac{(\alpha)_n (\alpha-c+1)_n \Gamma(\alpha-b+n)}{\Gamma(1+\alpha-b+n)(\alpha-a+1)_n (\alpha-b)_n} \frac{\left(\frac{p}{w}\right)^n}{n!}, \end{aligned}$$

esto es,

$$A = w^{\alpha}\Gamma\left[\begin{array}{c} \alpha, c, a-\alpha, b-\alpha \\ a, b, c-\alpha \end{array}\right] {}_2F_2\left(\alpha, \alpha-c+1; \alpha-a+1, \alpha-b+1; \frac{p}{w}\right).$$

Similarmente,

$$B = \frac{p^{a-\alpha}}{w^a}\Gamma\left[\begin{array}{c} c, b-a, \alpha-a \\ b, c-a \end{array}\right] {}_2F_2\left(a, a-c+1; a-\alpha+1, \alpha-b+1; \frac{p}{w}\right)$$

$$C = \frac{p^{b-\alpha}}{w^b}\Gamma\left[\begin{array}{c} c, a-b, \alpha-b \\ a, c-b \end{array}\right] {}_2F_2\left(b, b-c+1; b-a+1, b-\alpha+1; \frac{p}{w}\right).$$

Combinando estos resultados se obtiene 1 de la sección 1.1.

Usando procedimientos similares al anterior se obtuvieron los siguientes resultados:

2

$$\int_0^\infty x^{c-1}e^{-px}\ {}_2R_1\left(a,b;c;\tau;-wx\right)dx=\frac{\Gamma(c)}{\tau\Gamma(b)\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\times$$

$$\beta\left(\frac{k+b}{\tau}-a,1\right)w^{-a}p^{a-c}\ {}_2F_2\left(a,a-\frac{k+b}{\tau};1+a-\frac{k+b}{\tau},1+a-c;\frac{p}{w}\right)+$$

$$\frac{\left(\frac{p}{w}\right)^{\frac{b}{\tau}}p^{-c}}{\Gamma\left(a\right)}\sum_{k=0}^{\infty}(1-c+b)_k\,\Gamma\left(\frac{k+b}{\tau}\right)\Gamma\left(a-\frac{k+b}{\tau}\right)\Gamma\left(c-\frac{k+b}{\tau}\right)\frac{\left(\frac{p}{w}\right)^{\frac{k}{\tau}}}{k!}$$

$$\tau,\ \text{Re}\ c,\ \text{Re}\ p>0;\ \arg\ |\ w\ |<\pi\,.$$

3

$$\int_0^\infty x^{c-1}e^{-px}\ {}_2R_1\left(a,1-a;c;\tau;-wx\right)dx=\frac{\Gamma(c)\Gamma\left(c-a\right)w^{-a}p^{a-c}}{\tau\Gamma(1-a)\Gamma(c+a-1)}\sum_{k=0}^{\infty}\frac{(2-c-a)_k}{k!}\times$$

$$\beta\left(\frac{k+1-a}{\tau}-a,1\right)\ {}_2F_2\left(a,a-\frac{k+1-a}{\tau};1+a-\frac{k+1-a}{\tau},1+a-c;\frac{p}{w}\right)$$

$$+\frac{\Gamma(c)\left(\frac{p}{w}\right)^{\frac{1--a}{\tau}}p^{-c}}{\tau\Gamma(1-a)\Gamma(c+a-1)}\sum_{k=0}^{\infty}(2-c-a)_k\,\beta\left(\frac{k+1-a}{\tau},a-\frac{k+1-a}{\tau}\right)\times$$

$$\Gamma\left(c-\frac{k+1-a}{\tau}\right)\frac{\left(\frac{p}{w}\right)^{\frac{k}{\tau}}}{k!}$$

$$\tau,\ \text{Re}\ c,\ \text{Re}\ p>0;\ \arg\ |\ w\ |<\pi\,.$$

4

$$\int_0^\infty x^{c-1}e^{-px}\ {}_2R_1\left(\frac{1}{2},1;c;\tau;-wx\right)dx=\frac{\Gamma(c)\Gamma\left(c-\frac{1}{2}\right)\left(\frac{p}{w}\right)^{\frac{1}{2}}}{\tau\Gamma(c-1)p^c}\sum_{k=0}^{\infty}\frac{(2-c)_k}{k!}\times$$

$$\beta\left(\frac{k+1}{\tau}-\frac{1}{2},1\right){}_2F_2\left(\frac{1}{2},\frac{1}{2}-\frac{k+1}{\tau};\frac{3}{2}-\frac{k+1}{\tau},\frac{3}{2}-c;\frac{p}{w}\right)+\frac{\Gamma(c)\left(\frac{p}{w}\right)^{\frac{1}{\tau}}}{\tau\Gamma(c-1)p^c}\times$$

$$\sum_{k=0}^{\infty}(2-c)_k\,\beta\left(\frac{k+1}{\tau},\frac{1}{2}-\frac{k+1}{\tau}\right)\Gamma\left(c-\frac{k+1}{\tau}\right)\frac{\left(\frac{p}{w}\right)^{\frac{k}{\tau}}}{k!}$$

$$\tau,\ \text{Re}\ c,\ \text{Re}\ p>0;\ \arg\ |\ w\ |<\pi\,.$$

5 $$\int_0^\infty x^{\alpha-1}e^{-px}\ {}_2R_1\left(a,b;c;\tau;1-wx\right)dx =$$

$$\frac{w^{-\alpha}}{\tau}\Gamma\left[\begin{array}{c} c,\alpha,a-\alpha,1-a-\alpha \\ a,b,c-b \end{array}\right]\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\frac{\Gamma\left(\frac{k+b}{\tau}-\alpha\right)}{\Gamma\left(\frac{k+b}{\tau}+1-\alpha\right)}\times$$

$${}_2F_2\left(\alpha,1-a-\alpha;\alpha-a+1,\alpha-\frac{k+b}{\tau}+1;-\frac{p}{w}\right)+\frac{p^{a-\alpha}}{w^a}\Gamma\left[\begin{array}{c} c,\alpha-a \\ b,c-b \end{array}\right]\times$$

$$\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\frac{\Gamma\left(\frac{k+b}{\tau}-a\right)}{\Gamma\left(\frac{k+b}{\tau}+1-a\right)}{}_2F_2\left(a,1;a-\frac{k+b}{\tau}+1,a-\alpha+1;-\frac{p}{w}\right)+$$

$$\frac{p^{\frac{b}{\tau}-\alpha}}{\tau w^{\frac{b}{\tau}}}\Gamma\left[\begin{array}{c} c \\ a,b,c-b \end{array}\right]\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\Gamma\left(\frac{k+b}{\tau}\right)\Gamma\left(a-\frac{k+b}{\tau}\right)\times$$

$$\Gamma\left(\alpha-\frac{k+b}{\tau}\right)\left(\frac{p}{w}\right)^{\frac{k}{\tau}}\ {}_1F_1\left(\frac{k+b}{\tau};\frac{k+b}{\tau}-\alpha+1;-\frac{p}{w}\right)\Bigg]$$

$$\tau>0,\ \mathrm{Re}\,\alpha,\ \mathrm{Re}\,p,\ \mathrm{Re}\,(1-a+\alpha)>0;\ |\arg w<\pi|.$$

6 $$\int_0^\infty x^{\alpha-1}e^{-p/x}\ {}_2R_1\left(a,b;c;\tau;-wx\right)dx=\frac{\Gamma(c)\Gamma\left(\alpha\right)\Gamma\left(a-\alpha\right)w^{-\alpha}}{\tau\Gamma\left(a\right)\Gamma(b)\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}$$

$$\frac{\Gamma\left(\frac{k+b}{\tau}-\alpha\right)}{\Gamma\left(\frac{k+b}{\tau}+1-\alpha\right)}\ {}_2F_2\left(a-\alpha,\frac{k+b}{\tau}-\alpha;1-\alpha,\frac{k+b}{\tau}+1-\alpha;wp\right)+\frac{\Gamma(c)}{\tau\Gamma(b)}\times$$

$$\frac{\Gamma\left(-\alpha\right)p^{\alpha}}{\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\frac{\Gamma\left(\frac{k+b}{\tau}\right)}{\Gamma\left(\frac{k+b}{\tau}+1\right)}{}_2F_2\left(a,\frac{k+b}{\tau};\frac{k+b}{\tau}+1,\alpha+1;wp\right)$$

$$\tau>0,\ \mathrm{Re}\,(a-\alpha),\ \mathrm{Re}\,p,\ \mathrm{Re}\left(\frac{k+b}{\tau}-\alpha\right)>0;\ |\arg w<\pi|.$$

7 $$\int_0^\infty e^{-p\sqrt{x}}\ {}_2R_1\left(a,b;\frac{3}{2};\tau;-wx\right)dx=\frac{\Gamma(\frac{3}{2})}{\tau\Gamma(b)\Gamma(\frac{3}{2}-b)}\sum_{k=0}^{\infty}\frac{\left(b-\frac{1}{2}\right)_k}{k!}\times$$

$$\frac{\Gamma\left(\frac{k+b}{\tau}\right)}{\Gamma\left(\frac{k+b}{\tau}+1\right)}\frac{2p^{a+\frac{k+b}{\tau}-2}}{w^{(a+\frac{k+b}{\tau})/2}}S_{1-a-\frac{k+b}{\tau},a-\frac{k+b}{\tau}}\left(\frac{p}{\sqrt{w}}\right)$$

$$\tau\ \mathrm{Re}\,p>0;\ |\arg w<\pi|.$$

8 $$\int_0^\infty x^{\alpha-1}e^{-p\sqrt{x}}\ {}_2R_1\left(a,b;c;\tau;-wx\right)dx=\frac{\Gamma(c)}{\tau\Gamma(b)\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\times$$

$$\frac{\Gamma\left(\frac{k+b}{\tau}\right)}{\Gamma\left(\frac{k+b}{\tau}+1\right)}\left[\frac{2p^{2a-2\alpha}}{w^{a}}\Gamma\left[\begin{array}{l}\frac{k+b}{\tau}+1,\frac{k+b}{\tau}-a,2\alpha-2a\\ \frac{k+b}{\tau},\frac{k+b}{\tau}+1-a\end{array}\right]\times\right.$$
$${}_2F_3\left(a,a-\frac{k+b}{\tau};a-\frac{k+b}{\tau}+1,a-\alpha+1,a-\alpha+\frac{1}{2};-\frac{p^2}{4w}\right)+$$
$$\frac{2p^{2\frac{k+b}{\tau}-2\alpha}}{w^{\frac{k+b}{\tau}}}\Gamma\left[\begin{array}{l}\frac{k+b}{\tau}+1,a-\frac{k+b}{\tau},2\alpha-2\frac{k+b}{\tau}\\ a\end{array}\right]+w^{-\alpha}\Gamma\left[\begin{array}{l}\frac{k+b}{\tau}+1,\alpha,\\ a,\ \frac{k+b}{\tau},\end{array}\right.$$
$$\left.\begin{array}{l}a-\alpha,\frac{k+b}{\tau}-\alpha\\ \frac{k+b}{\tau}+1-\alpha\end{array}\right]{}_2F_3\left(\alpha,\alpha-\frac{k+b}{\tau};\alpha-a+1,\alpha-\frac{k+b}{\tau}+1,\frac{1}{2};-\frac{p^2}{4w}\right)$$
$$-\frac{p}{w^{\alpha+\frac{1}{2}}}\Gamma\left[\begin{array}{l}\frac{k+b}{\tau}+1,\alpha+\frac{1}{2},a-\alpha-\frac{1}{2},\frac{k+b}{\tau}-\alpha-\frac{1}{2}\\ a,\ \frac{k+b}{\tau},\frac{k+b}{\tau}-\alpha+\frac{1}{2}\end{array}\right]{}_2F_3\left(\alpha-\frac{k+b}{\tau}+\frac{1}{2},\right.$$
$$\left.\left.\alpha+\frac{1}{2};\frac{3}{2},\alpha-a+\frac{3}{2},\alpha-\frac{k+b}{\tau}+\frac{3}{2};-\frac{p^2}{4w}\right)\right]$$

$$\mathrm{Re}\,c>\mathrm{Re}\,b>0,\ \tau,\ \mathrm{Re}\,p,\ \mathrm{Re}\,\alpha>0;\ |\arg w|<\pi\,.$$

9 $$\int_0^\infty x^{-\frac{1}{2}}e^{-p\sqrt{x}}\ {}_2R_1\left(a,b;\frac{1}{2};\tau;-wx\right)dx=\frac{2\sqrt{\pi}}{\tau\Gamma(b)\Gamma(1/2-b)}\sum_{k=0}^{\infty}\frac{(1/2+b)_k}{k!}\times$$
$$\frac{\Gamma\left(\frac{k+b}{\tau}\right)}{\Gamma\left(\frac{k+b}{\tau}+1\right)}\ w^{-\frac{\left(a+\frac{k+b}{\tau}\right)}{2}}p^{a+\frac{k+b}{\tau}-1}S_{1-a-\frac{k+b}{\tau},a-\frac{k+b}{\tau}}\left(\frac{p}{\sqrt{w}}\right)$$

$$\tau>0,\ 1/2>\mathrm{Re}\,b>0,\ \mathrm{Re}\,p>0;|\arg w|<\pi\,.$$

10 $$\int_0^\infty x^{\alpha-1}e^{-p\sqrt{x}}\ {}_2R_1\left(a,b;c;\tau;1-wx\right)dx=\frac{\Gamma(c)}{\tau\Gamma(b)\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\times$$
$$\frac{\Gamma\left(\frac{k+b}{\tau}\right)}{\Gamma\left(\frac{k+b}{\tau}+1\right)}\left[\frac{2p^{2a-2\alpha}}{w^{a}}\Gamma\left[\begin{array}{l}\frac{k+b}{\tau}+1,\frac{k+b}{\tau}-a,2\alpha-2a\\ \frac{k+b}{\tau},\frac{k+b}{\tau}+1-a\end{array}\right]\times\right.$$
$${}_2F_3\left(a,1;a-\frac{k+b}{\tau}+1,a-\alpha+1,a-\alpha+\frac{1}{2};\frac{p^2}{4w}\right)+$$
$$\frac{2p^{2\frac{k+b}{\tau}-2\alpha}}{w^{\frac{k+b}{\tau}}}\Gamma\left[\begin{array}{l}\frac{k+b}{\tau}+1,a-\frac{k+b}{\tau},2\alpha-2\frac{k+b}{\tau}\\ a\end{array}\right]{}_2F_3\left(\frac{k+b}{\tau},\frac{k+b}{\tau}+1-a;\right.$$
$$\left.\frac{k+b}{\tau}+1-\alpha,\frac{k+b}{\tau}-\alpha+1,\frac{k+b}{\tau}-\alpha+\frac{1}{2};\frac{p^2}{4w}\right)+$$
$$+w^{-\alpha}\Gamma\left[\begin{array}{l}\frac{k+b}{\tau}+1,\alpha,a-\alpha,\frac{k+b}{\tau}-\alpha,1-a+\alpha\\ a,\ \frac{k+b}{\tau},\frac{k+b}{\tau}+1-\alpha\end{array}\right]{}_1F_2\left(\alpha;\frac{1}{2},\alpha-\frac{k+b}{\tau}+1;\right.$$

$$\left.\frac{p^2}{4w}\right) - \frac{p}{w^{\alpha+\frac{1}{2}}}\Gamma\left[\begin{array}{c} \frac{k+b}{\tau}+1, \alpha+\frac{1}{2}, a-\alpha-\frac{1}{2}, \frac{k+b}{\tau}-\alpha-\frac{1}{2}, 1-a+\alpha+\frac{1}{2} \\ a, \frac{k+b}{\tau}, \frac{k+b}{\tau}+1-a \end{array}\right]$$

$$\left.{}_2F_3\left(\alpha+\frac{1}{2}; 1-a+\alpha+\frac{1}{2}; \frac{3}{2}, \alpha-a+\frac{3}{2}, \alpha-\frac{k+b}{\tau}+\frac{3}{2}; \frac{p^2}{4w}\right)\right]$$

$$\text{Re } c > \text{Re } b > 0,\ \tau,\ \text{Re } p,\ \text{Re } \alpha,\ \text{Re } (1-a-\alpha) > 0;\ |\arg w| < \pi\,.$$

11 $$\int_0^\infty x^{\alpha-1} e^{-\frac{p}{\sqrt{x}}}\ {}_2R_1\left(a, b; c; \tau; -wx\right) dx = \frac{\Gamma(c)}{\tau\Gamma(b)\Gamma(c-b)} \sum_{k=0}^\infty \frac{(1-c+b)_k}{k!} \times$$

$$\frac{\Gamma\left(\frac{k+b}{\tau}\right)}{\Gamma\left(\frac{k+b}{\tau}+1\right)}\left[w^{-\alpha}\Gamma\left[\begin{array}{c} \frac{k+b}{\tau}+1, \alpha, a-\alpha, \frac{k+b}{\tau}-\alpha \\ a, \frac{k+b}{\tau}, \frac{k+b}{\tau}+1-\alpha \end{array}\right]\times\right.$$

$${}_2F_3\left(a-\alpha, \frac{k+b}{\tau}-\alpha; \frac{1}{2}, 1-\alpha, \frac{k+b}{\tau}+1-\alpha; -\frac{p^2 w}{4}\right) + 2p^{2\alpha}\Gamma\left(-2\alpha\right) \times$$

$${}_2F_3\left(a, \frac{k+b}{\tau}; \frac{k+b}{\tau}+1, \alpha+1, \alpha+\frac{1}{2}; -\frac{p^2 w}{4}\right) -$$

$$w^{\frac{1}{2}-\alpha} p\Gamma\left[\begin{array}{c} \frac{k+b}{\tau}+1, \alpha-\frac{1}{2}, a-\alpha+\frac{1}{2}, b-\alpha+\frac{1}{2} \\ a, \quad \frac{k+b}{\tau}-\alpha+\frac{3}{2} \quad \frac{k+b}{\tau}, \end{array}\right]$$

$${}_2F_3\left(a-\alpha+\frac{1}{2}, \frac{k+b}{\tau}-\alpha+\frac{1}{2}; \frac{3}{2}, \frac{3}{2}-\alpha, \frac{k+b}{\tau}-\alpha+\frac{3}{2}; -\frac{p^2 w}{4}\right)$$

$$\tau,\ \text{Re}\,(p)\,,\ \text{Re}\,(a-\alpha)\,,\ \text{Re}\ \left(\frac{k+b}{\tau}-\alpha\right) > 0; |\arg\left(w\right)| < \pi\,.$$

12 $$\int_0^y x^{\alpha-1}\left(y-x\right)^{c-1} e^{-px}\ {}_2R_1\left(a, b; c; \tau; \left(1-\frac{x}{y}\right)^\tau\right) dx =$$

$$y^{\alpha+c-1}\frac{\Gamma\left(\alpha\right)\Gamma\left(c\right)}{\Gamma\left(b\right)} \sum_{n=0}^\infty \frac{\left(a\right)_n \Gamma\left(b+\tau n\right)}{\Gamma\left(\alpha+c+\tau n\right) n!}\ {}_1F_1\left(\alpha, \alpha+c+\tau n; -py\right)$$

$$\tau, y,\ \text{Re } c,\ \text{Re } \alpha,\ \text{Re}\,(c-a-b+\alpha) > 0\,.$$

13 $$\int_0^y x^{\alpha-1}\left(y-x\right)^{c-1} e^{-p\sqrt{x}}\ {}_2R_1\left(a, b; c; \tau; \left(1-\frac{x}{y}\right)^\tau\right) dx =$$

$$y^{\alpha+c-1}\frac{\Gamma\left(c\right)\Gamma\left(\alpha\right)}{\Gamma\left(b\right)} \sum_{n=0}^\infty \frac{\left(a\right)_n \Gamma\left(b+\tau n\right)}{\Gamma\left(\alpha+c+\tau n\right) n!}\ {}_1F_2\left(\alpha; \frac{1}{2}, \alpha+c+\tau n; \frac{p^2 y}{4}\right) +$$

$$-py^{\alpha+c-\frac{1}{2}}\frac{\Gamma(c)\Gamma\left(\alpha+\frac{1}{2}\right)}{\Gamma(b)}\sum_{n=0}^{\infty}\frac{(a)_n\Gamma(b+\tau n)}{\Gamma\left(\alpha+c+\frac{1}{2}+\tau n\right)n!}\times$$
$$_1F_2\left(\alpha+\frac{1}{2};\alpha+c+\tau n+\frac{1}{2},\frac{3}{2};\frac{p^2y}{4}\right).$$

$$\tau,y,\ \mathrm{Re}(c),\ \mathrm{Re}(\alpha),\ \mathrm{Re}(c-a-b+\alpha)>0\,.$$

3 Aplicación

En el 2001 Nina Virchenko y colaboradores [5] establecieron una generalización para la función gamma de la siguiente forma

$$\Gamma\left(\begin{array}{c}a,b;c;\\u,v\end{array};p,\tau\right)=v^{-a}\int_0^{\infty}x^{u-1}e^{-px}\ {}_2R_1\left(a,b;c;\tau;-\frac{x}{v}\right)dx \tag{5}$$

$$\tau,\mathrm{Re}\ p,\mathrm{Re}\ u>0;\ |\arg v|<\pi\ \text{y}\ c\neq 0,-1,-2,...$$

De acuerdo con los resultados obtenidos en el presente trabajo, es posible obtener una fórmula computable para (5) de la siguiente forma:
Al hacer $w=v^{-1}$ en (5), y usar el resultado 1 de la sección 2.1 se tiene

$$\Gamma\left(\begin{array}{c}a,b;c;\\u,v\end{array};p,\tau\right)=\frac{\Gamma(c)}{\tau\Gamma(b)\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\Gamma\left[\begin{array}{c}u,a-u,\frac{k+b}{\tau}-\alpha\\a,\frac{k+b}{\tau}+1-u\end{array}\right]\times$$
$$_2F_2\left(u,u-\frac{k+b}{\tau};u-a+1,u-\frac{k+b}{\tau}+1;vp\right)+$$
$$\frac{\Gamma(c)p^{a-u}}{\tau\Gamma(b)\Gamma(c-b)}\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\Gamma\left[\begin{array}{c}\frac{k+b}{\tau}-a,u-a\\\frac{k+b}{\tau}+1-a\end{array}\right]\times$$
$$_2F_2\left(a,a-\frac{k+b}{\tau};a-u+1,u-\frac{k+b}{\tau}+1;vp\right)+\frac{\Gamma(c)p^{-u}}{\tau\Gamma(b)\Gamma(c-b)}\times$$
$$\sum_{k=0}^{\infty}\frac{(1-c+b)_k}{k!}\frac{p^{\frac{k+b}{\tau}}}{v^{a-\frac{k+b}{\tau}}}\Gamma\left[\begin{array}{c}a-\frac{k+b}{\tau},u-\frac{k+b}{\tau},\frac{k+b}{\tau}\\a\end{array}\right]$$

$$\tau,\ \mathrm{Re}\ p,\ \mathrm{Re}\ u>0;\ |\arg w<\pi|\,.$$

4 Conclusiones

De acuerdo con el desarrollo de este trabajo se nota lo siguiente:

- Cada uno de los resultados de la sección 1.1, son formas computables para cada integral propuesta, además, éstas facilitan los cálculos numéricos de las mismas dado que muchos de estos resultados aparecen tabulados en el libro de Prudnikov (véase [4]).
- Cada integral es un caso especial de la función Gamma generalizada

$$\Gamma\left(\begin{matrix} a,b;c; \\ u,\nu \end{matrix}\;;p\right) = v^{-a}\int_0^{\infty} x^{u-1}e^{-px}\;{}_2F_1(a,b;c;-\frac{x}{v})dx\,.$$

  lo que indica que se tienen diversas formas computables para la función Gamma.

- Una aplicación interesante de estas integrales es el hecho que permiten obtener nuevas funciones de densidad de probabilidad, en estos últimos años el doctor Kalla y su grupo de colaboradores han encontrado nuevas funciones de densidad de probabilidad y también han generalizado algunas que ya existen.
- Cada resultado de este trabajo es un caso especial de la función Gamma generalizada

$$\Gamma\left(\begin{matrix} a,b;c; \\ u,v \end{matrix}\;;p,\tau\right) = v^{-a}\int_0^{\infty} x^{u-1}e^{-px}\;{}_2R_1\left(a,b;c;\tau;-\frac{x}{v}\right)dx\,.$$

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