Low-Bid Auction Versus High-Bid Auction for Siting Noxious Facilities in a Two-City Region: An Exact Approach

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Abstract Two auctions have been proposed in the literature for siting noxious facilities: the high-bid and the low-bid auctions. In this paper, we pursue the analysis of these auctions made by O'Sullivan (1993), were he concludes that the high-bid auction has the edge over the low-bid auction. We point out that O'Sullivan has made an approximation for the expected value of the compensation obtained with the high-bid auction, and we show how to obtain the exact value. We discuss a paradox linked with O'Sullivan's result, which mitigates his conclusions, and we show that with exact compensation, the high-bid auction mechanism is indeed far superior to the low-bid auction.

Keywords Facilities siting, noxious, NIMBY, auction scheme, Nash equilibrium, low-bid auction, high-bid auction.

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1 Introduction

"Some things are always in the wrong place" (O'Hare (1977)). Prisons, airports, trash disposal plants, landfills and waste incinerators have this characteristic. They require large amounts of land and generate local environmental costs. The term NIMBY often crops up during discussions on the construction of new facilities. It stands for 'Not in my backyard', an attitude where everyone knows the facility is necessary but no one is willing to host it.

Two auctions mechanism have been proposed to overcome this syndrome. Kunreuther *et al* (1986,1987) suggested a mechanism called the low-bid auction. They assume that when each jurisdiction[1] knows its own preferences, but has no information on others, then a maxi-min bidding strategy is a prudent one to follow and is consistent with the elimination of dominated strategies. The auction mechanism that O'Sullivan (1993) proposed, called the high-bid auction, replaces the maxi-min strategy with the Nash-equilibrium bidding functions and uses a new information set.

O'Sullivan analyzes, in his information set, the efficiency of low-bid and high-bid auctions. In a specific situation where only two jurisdictions are participating in the auction, this low-bid auction is outclassed by the high-bid scheme. However, O'Sullivan has supposed that "*it is reasonable to assume that the expected compensation (the expected higher bid) equals the conjectured bid associated with the average local environmental cost above it's own cost*". By doing this, he has willingly introduced an approximation for the expected value of the compensation in order to determine the equilibrium bidding function. Hence, the main goal of this paper is to qualify O'Sullivan's results and to incorporate the exact value for the expected compensation in the cost formula in order to lead a new comparative analysis. We (i) point out two problems in O'Sullivan's high-bid

[1] By jurisdictions, we mean communities composed of individual having the right to make decisiones on their own behalf. That is State, region, district or city.

auction mechanism with approximation (in that case the low-bid auction is not outclassed by the high-bid scheme) and (ii) show that with exact compensation, the high-bid auction mechanism is indeed far superior to the low-bid auction.

This paper is organized as follows. In section 2, we describe the auction game, the basic assumptions, the desirable properties of these mechanisms. In section 3, we present the different values for the compensation and compute the equilibrium bidding functions in the different auctions. In section 4, we present and criticize the previous results. In Section 5, using the exact compensation, we compare the efficiency properties of the high-bid and low-bid auctions. Finally, Section 6, presents some conclusions and discusses opportunities for future research.

2 The bases of auction mechanisms

2.1 Presentation of auctions

We consider the same economic situation as O'Sullivan (1993): two cities want to build incinerators for the treatment of the garbage. It may be unwise, to site an incinerator in every city that produces household garbage because a few large centralized facilities generally are considered safer, more environmentally sound and more efficient that many small facilities. These two cities (also called "agents" in the present paper) could use a sealed-bid auction to pick a site for the regional facility. We shall study two such bids: the high-bid and the low-bid auctions. For both, the first stages are identical. At first, nature chooses the local environmental cost generated by the noxious facility, if it were sited in each city. Let e_i represent the local environmental cost for city i. We suppose that environmental costs are independent random draws from a common distribution $F(e)$ with support $[\underline{e}, \bar{e}]$. Therefore, the cities differ in the local environmental costs generated by the noxious facility.

Each agent submits a bid, b_i, for hosting the noxious facility. The auction organizer collects these bids. The city with the lowest bid "wins" the auction, hosts the facility and is compensated with: the bid that it submits in a *low-bid auction* and the bid of the other city in a *high-bid auction*. The non-host city pays a tax equals to its bid to have the facility located in the host city.

The problem is then the following. Assume that each city knows its own environmental cost e_i. What bid b_i should it choose?

2.2 Assumptions

The model is based on five principal assumptions. Firstly, we suppose that cities are risk-neutral. Secondly, information is asymmetric and incomplete: the information on local environmental costs is private. Thirdly, the default option for cities is to have a facility installed in their backyards for their own waste. Fourthly, we admit that the payment by the non-host city is funded by a tax that does not distort locational choices. This rules out the cases where e_i would endogenously depend on the result of the auction. Finally, we assume that all cities have both to process the same level of waste.

2.3 Definitions and basic concepts

In order to simplify the paper, we present here three common concepts of major importance: the *conjectured bidding function*, the *expected cost function* and the *certain cost* associated with operating its own facility.

Let i, denote a city which is a potential site. The *conjectured bidding function* is $B : [\underline{e}, \overline{e}] \to \Re$ and is strictly increasing, for reasons to be explained in Section 2.4. City i assumes that city j bids $B(e_j)$ and city j assumes that city i bids $B(e_i)$. These conjectures will be validated in a Nash equilibrium. Each city chooses a bid to minimize its expected loss, taking into account this conjectures behavior of its opponents. The main issue under discussion in this paper is to find find the equilibrium bidding function of the two auctions analyzed. Indeed, with these functions, we can analyze the efficiency properties of the auctions.

To determine *expected cost function* we introduce the probability that i has the lowest bid and thus hosts the facility, given that its bid is b_i. It is:

$$\text{Prob}\{B(e_j) > b_i\} = 1 - F(B^{-1}(b_i)),$$

where B^{-1} is the inverse of the conjectured bidding function, which is well defined since B is strictly increasing. Thus the expected cost for the city i, is:

$$C^{\text{E}}(e_i) = F(B^{-1}(b_i))b_i + [1 - F(B^{-1}(b_i))](e_i - \varGamma_i),$$

where the exponent E indicate that this is the expected cost and $\varGamma_i$ is the level of compensation (certain or expected, see Section 3).

Finally, we assume following again O'Sullivan (1993) that the *certain cost* associated with operating its own facility, since both cities produce the same amount of waste, is:

$$C^{\text{C}}(e_i) = \gamma + \frac{e_i}{2},$$

where the exponent C indicates that this is the certain cost of a city facility, and γ is a measure of scale economies for the noxious facility.

2.4 Desirable properties

Following Richardson and Kunreuther (1993), we retain three properties based on the Nash equilibrium solution concept. Firstly, the auction mechanism is *Revenue-Neutral* (RN) if the amount of compensation is equal to the amount of tax collected. Secondly, the auction mechanism is *Nash-Efficient* (NE) if every Nash equilibrium is Pareto efficient. In that case, the auction mechanism is *Individually Rational* (IR) because every equilibrium outcome Pareto-dominates the default option. Formally, the auction mechanism is NE and thus IR if, for all city i: $C^{\text{E}}(e_i) < C^{\text{C}}(e_i)$. Thirdly, the auction scheme is *Incentive-Compatible* (IC) if announcing the truth is a Nash equilibrium strategy. We also call an auction *weakly incentive-compatible* (WIC) when, in the Nash equilibrium, the city with the lower environmental costs submits the lowest bid and then hosts the facility. Thus, if the auction mechanism is IC or WIC the facility is located in the low-cost city. Formally, the auction mechanism is IC if $B(e_i) = e_i$ and WIC if B is a strictly increasing function.

3 Compensation and bidding functions

3.1 Different values for the compensation

The level of compensation in the cost formula varies from one auction to the other. In the *low-bid auction*, the host city receives its own bid in compensation. So, $\Gamma_i = B(e_i)$ which is a certain value. In the *high-bid auction*, the host city receives the bid of the other city in compensation. In that case, the cities have to anticipate the level of compensation. O'Sullivan (1993) assumes that the expected compensation equals the conjectured bid associated with the average local environmental cost above e_i: $\Gamma_i = B(\frac{e_i+\overline{e}}{2})$.

In this paper we introduce the exact value for the expected compensation. The question is to evaluate $E(\Gamma_i)$ *knowing* that plater i's bid is less than $B(e_j)$, or equivalently that $B^{-1}(b) < e_j$. Let us denote $\widetilde{e} = \widetilde{e}(b) = B^{-1}(b)$ for simplicity. We have the following result: conditioned on the fact that $\widetilde{e} < e_j$, the random variable e_j is distributed as:

$$\text{Prob}(e_j \leq x \mid \widetilde{e} < e_j) = \frac{F(x) - F(\widetilde{e})}{1 - F(\widetilde{e})}\ , \quad x \geq \widetilde{e},$$

and 0 otherwise. Therefore, we have for $x \geq \widetilde{e}$:

$$E(\Gamma_i \mid \widetilde{e} < e_j) = E(B(e_j) \mid \widetilde{e} < e_j) = \int_{\widetilde{e}}^{\overline{e}} B(u) \frac{F'(u)}{1 - F(\widetilde{e})}\ du.$$

3.2 Optimal bidding functions

Equilibrium bidding functions ca be derived by differentiating the expected cost function with respect to b_i. For the optimum bid, the derivative is equal to zero. O'Sullivan (1993) obtains the equilibrium for the low-bid auction (l) and the high-bid auction (h). We present here his results and compute the new equilibrium function for the high-bid auction with the exact compensation (he). We assume that local environmental costs are uniformly distributed over the unit interval, so $F(x) = x$ and $F'(x) = 1$. Among the possible probability distributions, this choice amounts to providing cities with a minimal information. According to the Nash equilibrium principle, the optimal choice should coincide with the value of the bidding function at e_i. Substituting $b = B(e_i)$, that is $\widetilde{e} = e_i$, we obtain the following bidding functions.

For the *low-bid auction*:

$$\frac{dC_l^{\mathrm{E}}}{db_i} = 2B(e_i) - e_i - B'(e_i)(1 - 2e_i) = 0 \quad \Rightarrow \quad B_l(e_i) = \frac{1}{4}e_i + \frac{1}{8} .$$

This function B_l is the unique solution of the differential equation which is well defined over the interval [0,1]. For the *high-bid auction* with $\Gamma_i = B(\frac{e_i+1}{2})$, we have:

$$\frac{dC_h^{\mathrm{E}}}{db_i} = B(e_i) - e_i + B'(e_i)e_i + B\left(\frac{e_i + 1}{2}\right) = 0 . \tag{1}$$

We provide in the Appendix the general solution to this functional equation. Among these solutions, the only one which makes sense in the present context has been found by O'Sullivan:

$$B_h(e_i) = \frac{2}{5}e_i - \frac{1}{10} .$$

Finally, for the *high-bid auction* with exact compensation, we have:

$$E(\Gamma_i \mid \widetilde{e} < e_j) = \int_{\widetilde{e}}^{\overline{e}} B(u) \frac{F'(u)}{1 - F(\widetilde{e})} \, du , \tag{2}$$

so that necessarily:

$$\frac{dC_{he}^{\mathrm{E}}}{db_i} = B'(e_i)e_i + 2B(e_i) - e_i = 0 \; .$$

The solution of this differential equation is of a general form:

$$B_{he}(e_i) = \frac{e_i}{3} + \frac{c}{e_i^2} \; ,$$

and the only solution which is regular at $e_i = 0$ is:

$$B_{he}(e_i) = \frac{e_i}{3} \; .$$

All of these bidding functions are strictly increasing in local environmental costs. The corresponding functions are:

$$C_l^{\mathrm{E}}(e_i) = e_i B_l(e_i) + (1-e_i)[e_i - B_l(e_i)] = -\frac{1}{2}e_i^2 + e_i - \frac{1}{8},$$
$$C_h^{\mathrm{E}}(e_i) = e_i B_h(e_i) + (1-e_i)\left[e_i - B_h\left(\frac{e_i+1}{2}\right)\right] = -\frac{2}{5}e_i^2 + \frac{4}{5}e_i - \frac{1}{10},$$
$$C_{he}^{\mathrm{E}}(e_i) = e_i B_{he}(e_i) + (1-e_i)\left[e_i - \frac{1}{1-e_i}\int_{e_i}^{1} B_{he}(t)dt\right] = -\frac{13}{30}e_i^2 + \frac{9}{10}e_i - \frac{1}{6} \; .$$

4 O'Sullivan's results: presentation and comments

4.1 Previous results

Having analyzed efficiency properties of the low-bid auction and the high-bid auction with $\Gamma_i = B\left(\frac{e_i+\bar{e}}{2}\right)$, O'Sullivan obtains the following results:

- *(i)* In the low-bid auction, in most cases the budget is not balanced since the compensation that the host receives is very likely not equal to the total payments from the other bidder. Conversely, the high-bid auction is revenue-neutral.
- *(ii)* These two schemes are Nash-efficient but for the auction with low-bid compensation, the city with the average local environmental cost is indifferent

between participating in the auction and operating its own facility. In contrast, under an auction with high-bid compensation, the average city prefers the auction to operating its own facility (Figure[1,left]).

- *(iii)* The low-bid and high-bid auction mechanisms are not incentive-compatible but they are weakly incentive-compatible:the equilibrium bidding function is strictly increasing with local environmental costs (Figure [1,right]).

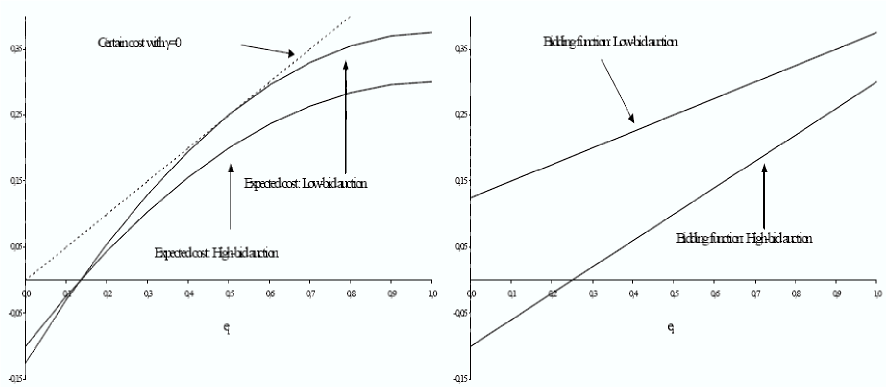

Figure 1: O'Sullivan's results.

O'Sullivan concludes that the high-bid auction mechanism is far superior to the low-bid auction. Concerning this conclusion, we shall however discuss in the next paragraphs that problems can occur with O'Sullivan bidding function $B_h(e_i)$.

4.2 Comments on the participation constraint

Concerning result *(ii)*, it is necessary to point out that the expected cost in a low-bid auction is not always superior to the expected cost in a high-bid auction:

$$C_l^E(e_i) < C_h^E(e_i), \quad \forall e_i \in \left[0; 1 - \frac{1}{2}\sqrt{3}\right].$$

So, if the cities choose the auction in which they want to participate, it is very likely that all do not prefer the same:

$$Pr\left[C_l^E(e_i) < C_h^E(e_i)\right] = 1 - \frac{1}{2}\sqrt{3} \approx 0.133\ .$$

In order to overcome the Nimby syndrome, it might be more appropriate to create a mechanism that permits to achieve unanimity on the choice of the auction (Kunreuther *et al.* (1993)). In that sense, the high-bid auction es not sufficient.

4.3 O'Sullivan's bidding function paradox

Using the function $B_h(e_i)$, a city with local environmental cost lower than 0.25 is willing to pay for the right to host the facility and collect the expected compensation. This might be realistic, but in some situations, this leads to an unexpected outcome.

As an example, suppose that two cities (i and j) agree to share a facility. The local environmental costs for cities are $e_i = 0$ and $e_j = 0.2$. In the Nash equilibrium, each bidding agent submits a bid equal to $B_h(e_i) = B_h(0) = -0.1$ and $B_h(e_j) = B_h(0.2) = -0.02$. So, if each city follows the Nash equilibrium bidding function, the city i has the lowest bid (the facility is located there) and it receives the bid of the non-host city j as compensation: -0.02. Note that a negative compensation can be analyzed as a tax. City j pays its bid to have the facility located in the other city: -0.02. But a negative tax can be analyzed as a compensation. Therefore, city j, which had to pay a tax, receives a compensation of 0.02 and the host-city pays a tax of 0.1. This paradoxical situation arises when bids are such as:

$$B_h(e_j) + B_h(e_i) < 0,$$

i.e. when $e_i + e_j < 0.5$. Is this situation due to the fact that cities anticipate a large compensation? Does the potential manna encourages cities to under-bid to the negative value? In order to investigate these questions, it is interesting to see how O'Sullivan's proposal compares with the exact compensation we have proposed.

Starting with (2), we can write

$$E(\mathit{\Gamma}_i \mid \widetilde{e} < e_j) = \int_{\widetilde{e}}^{\overline{e}} B(u) \frac{F'(u)}{1 - F(\widetilde{e})}\, du \sim \int_{e_i}^{\overline{e}} B(u) \frac{F'(u)}{1 - F(e_i)}\, du \sim B\left(\frac{e_i + \overline{e}}{2}\right),$$

where the last approximation is an equality when $B(e_i)$ is linear and when F is the uniform distribution (that is the case for O'Sullivan (1993)). Accordingly, we can interpret O'Sullivan's proposal as the combination of a) the assumption that player i believes that the environmental cost of the other player is distributed over the interval $[e_i, \overline{e}]$ instead of the interval $\left[B^{-1}(b), \overline{e}\right]$, and b) the approximation of the resulting integral using Euler's scheme, which is actually exact here. O'Sullivan's bidding function therefore that of a city which assumes that its compensation is not function of its bid, but only of its environmental cost. In fact the expected compensation is strictly increasing in b_i: any deviations from the equilibrium bidding function has an influence on the expected compensation. Introducing the dependency on b_i modifies the assumed behavior of agents and generates a different optimal bidding function.

Observe that:

$$E(\mathit{\Gamma}_i \mid \widetilde{e} < e_j) > E(\mathit{\Gamma}_i \mid e_i < e_j)\ ,$$

and that $B_{he}(e_i) > 0\ \forall e_i$. So, the paradoxical situation described above is due to the approximation made by O'Sullivan, which underestimates the actual expected compensations (and costs) of cities.

The comments presented in this Section allow us to claim that *the high-bid auction (such as O'Sullivan formalized it) has not the edge over low-bid auction* because it often generates a paradoxical situation and a conflict on the choice of the auction. This situation is probably due to the approximation.

5 New comparative analysis

We compare here the efficiency properties of the low-bid auction and high-bid auction with exact compensation. The first and the third results of O'Sullivan still hold true with the exact compensation. Some modifications arise for result *(ii)*:

- *(ii)* Low-bid auction and high-bid auction with exact compensation are Nash-efficient. However, the expected cost in a low-bid auction is now always superior to the expected cost in a high-bid auction:

  $$C_l^E(e_i) > C_{he}^E(e_i), \quad \forall e_i\,.$$

  Consequently, if a vote is set up in order to determine which auction will be used, then all the cities will prefer the high-bid auction (Figure [2,left]).

  Furthermore, O'Sullivan's paradox disappears because offers are always positive (Figure [2,right]).

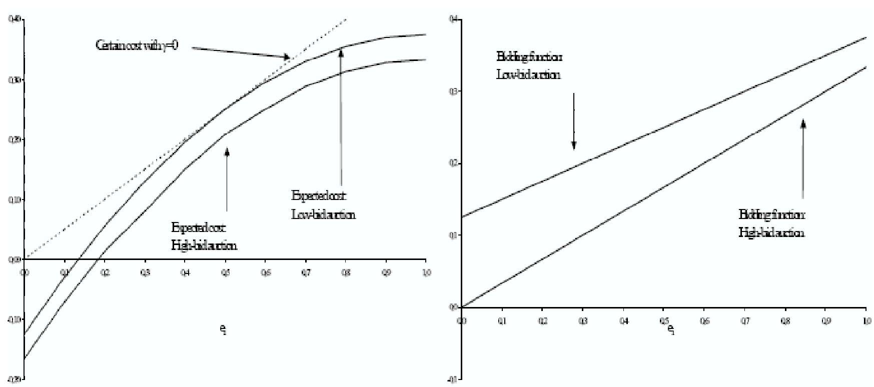

Figure 2: Exact expected cost and bidding functions.

To sum up, with exact compensation, we conclude that:*the high-bid auction mechanism is far superior to the low-bid auction.* Problems identified in

O'Sullivan's results disappear: high-bid auction is always preferred to the low-bid auction and we never have paradoxical situation where the host city pays a tax.

6 Conclusions and suggestions for future research

In this paper we qualify O'Sullivan's results and incorporate the exact value for the expected compensation in the cost formula in order to lead a new comparative analysis between high-bid and low-bid auctions.

The comments presented in Section 4 allow us to claim that *the high-bid auction (such as O'Sullivan formalized it) has not the edge over low-bid auction* because it often generates a paradoxical situation and a conflict on the choice of the auction. However, with the exact value for the expected compensation in the cost formula, all these problems vanish and finally *the high-bid auction has the edge over low-bid auction*.

Future research may continue in several directions. Fist of all, it is possible to conduct an experimental study with the double goal of testing the behavior of players during an auction (Nash *vs* maxi-min), and the theoretical results obtained in this paper. Finally, it would be interesting to generalize existing mechanisms to more than two cities.

Appendix

This appendix provides the technical details on the solution of the functional equation (1). We propose a method based on series expansions. The analysis suggests the change of variable $u = 1-e_i$, and of function $h(u) = B_h(1-u)(1-u)$. Equation (1) is then rewritten as:

$$0 = h'(u) - \frac{1}{1-u/2}\, h\left(\frac{u}{2}\right) + 1 - u\,. \tag{3}$$

We look for solutions that are such $h(u)$ is defined when $u \in [0,1]$ and $h(1) = 0$. We construct a solution of (3) in the form of an analytical series in the vicinity of $u = 0$. Accordingly, set $h(u) = \sum_{k=0}^{\infty} h_k u^k$. Replacing this series in (3) (multiplied by the term $1 - u/2$) and rearranging terms according to powers of u, we get:

$$0 = -\sum_{k=0}^{\infty} (k+1) h_{k+1} u^k + \sum_{k=0}^{\infty} h_k u^k \left(\frac{k}{2} + 2^{-k}\right) - 1 + \frac{3}{2}u - \frac{1}{2}u^2 \,. \tag{4}$$

Identifying the coefficients of u^p to 0 in Equation (4), we find that for every $p \geq 0$[2]:

$$(p+1) h_{p+1} = \frac{1}{2} h_p \left(p + 2^{1-p}\right) - 1_{\{p=0\}} + \frac{3}{2} 1_{\{p=1\}} - \frac{1}{2} 1_{\{p=2\}} \,. \tag{5}$$

Starting from recurrence (5), we obtain for $k \geq 2$:

$$h_k = h_3 \frac{3}{k} 2^{3-k} \prod_{m=3}^{k-1} \left(1 + \frac{2^{1-m}}{m}\right) , \tag{6}$$

and the initial values:

$$\begin{aligned} h_1 &= h_0 - 1, \\ h_2 &= \frac{1}{2} h_1 + \frac{3}{4} = \frac{1}{2} h_0 + \frac{1}{4}, \\ h_3 &= \frac{5}{12} h_2 - \frac{1}{6} = \frac{1}{12} \left(\frac{5}{2} h_0 - \frac{3}{4}\right) . \end{aligned}$$

The general solution to Equation (3) has therefore the following form:

$$\begin{aligned} h(u) = h_0 + (h_0 - 1)u + \left(\frac{1}{2} h_0 + \frac{1}{4}\right) u^2 \\ + \frac{1}{4} \left(\frac{5}{2} h_0 - \frac{3}{4}\right) \sum_{k=3}^{\infty} u^k \frac{2^{3-k}}{k} \prod_{m=3}^{k-1} \left(1 + \frac{2^{1-m}}{m}\right). \end{aligned} \tag{7}$$

[2] The notation $\mathbf{1}_{\{A\}}$ stands for 1 if A is true, 0 otherwise.

The constant h_0 remains to be determined. The particular solution such that $h(1) = 0$ is obtained when:

$$\left(\frac{5}{2}h_0 - \frac{3}{4}\right)\left(1 + \frac{1}{4}S\right) = 0,$$

where S is the value of the series in (7) when $u = 1$. Since $S > 0$, then $h_0 = 3/10$. The resulting solution, after reverting the changes of functions and variables, is $B_h(e_i) = 2e_i/5 - 1/10$.

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