Water Evaporation: Apparent Anomaly and its Resolution

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Abstract

The kinetic theory expression for the rate of evaporation of water gives a rather large numerical value in comparison to what is observed in the laboratory or globally. This paradox is resolved by recalling that evaporation and condensation processes must be considered simultaneously.

Keywords: Kinetic theory, evaporation and condensation water processes, physics education.

Resumen

La expresión de la teoría cinética para la tasa de evaporación del agua da un gran valor numérico para lo que se observa en el laboratorio o en el mundo. Esta paradoja se resuelve recordando que los procesos de evaporación y condensación deben ser considerados al mismo tiempo.

Palabras clave: Teoría cinética, procesos de evaporación del agua, educación en física.

While teaching thermodynamics to undergraduates the following interesting observation was made by us. First consider a beaker with closed lid containing a liquid in dynamic equilibrium with its saturated vapor. The net rate of change of mass of the liquid

$$R \equiv (J_e - J_c) A = 0, \qquad (1)$$

where A is the surface area, J_e the gross mass evaporation rate and J_c the gross mass condensation rate expressed in g/cm^2/s. Application of Maxwellian kinetic theory to the vapour phase then leads to the following well known formula [1,2] for the gross evaporation rate

$$J_e = \left(\frac{m}{2\pi \ kT}\right)^{\frac{1}{2}} p, \qquad (2)$$

where m is the mass of a molecule, T the absolute temperature, p the equilibrium vapor pressure and k the Boltzmann constant. At the face value numerical application of Eq. (2) leads to anomalously large evaporation rate in the case of water and its logical resolution is also called for. This fact is illustrated below by two explicit examples.

Example 1

Take water in an open beaker kept in a hot dry room so that humidity has little apparent role to play. Numerically, using [3]:

$$m = 3.0 \times 10^{-23} \text{ g}, \ T = 300 \text{ K}, \ p = 3.67 \times 10^4 \text{ dyne/cm}^2,$$
$$k = 1.38 \times 10^{-16} \text{ erg/K}, \qquad (3)$$

we find from Eq. (2) that

$$J_e = 0.4 \text{ g/cm}^2\text{/s} \qquad (4)$$

The above theoretical value is obviously couple of orders of magnitude larger than what is observed in practice. This paradox is explained by remembering that a thin boundary layer of air above the liquid surface would very quickly become populated by water molecules. These water molecules would perform a random walk, *i.e.*, would travel in zigzag fashion encountering random collisions with air molecules and, at times, returning back to the liquid and thereby condensing. The thickness of the said boundary layer is a few mean free paths and the condensation process is assisted by the fact that the temperatures as well as mass of the water molecules are comparable to those of the air molecules. Hence, the net rate of evaporation is governed by the non-zero difference $J_e - J_c$ rather than by J_e itself. The fact that the air in the room is on the average

dry is irrelevant; what is relevant is the immediate environment of the evaporating water.

Example 2

Consider geophysical data on the net volume of water [4] evaporated by the sun over the globe viz. 95000 cubic miles per year. Multiplying it by water's density, dividing by the surface area of the earth and time duration we find the geophysical rate

$$J_e^{geo} \sim 2.6 \times 10^{-6} \ \mathrm{g/cm^2/s}, \qquad (5)$$

which is several orders of magnitude smaller than Eq. (4). This paradox is resolved by remembering that the terrestrial sources of water viz. oceans, lakes, rivers, etc. have above them a zone of air saturated to some extent with water vapor so that evaporation and condensation processes go on simultaneously.

It is hoped that the above numerical values of gross/net evaporation rates together with the physical explanations offered will be of great interest to the students as well as teachers of thermodynamics. Before ending the readers may be reminded that in the case of gas-filled tungsten filament lamps the net evaporation of tungsten atoms from the glowing filament is considerably retarded [5] by the presence of a gas sheath called Langmuir sheath.

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