NEW RATIO ESTIMATORS OF THE MEAN USING SIMPLE RANDOM SAMPLING AND RANKED SET SAMPLING METHODS

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ABSTRACT
In this paper, modified ratio estimators of the population mean of the variable of interest are suggested involving the first or third quartiles of an auxiliary variable that is correlated with the variable of interest. The newly suggested estimators are investigated under simple random sampling (SRS) and ranked set sampling (RSS) methods. It is found that these estimators are approximately unbiased and the RSS estimators are more efficient than those based on SRS method for the same quartile and sample size. Also, it is found that, when the two quartiles are compared, estimators based on the third quartile are more efficient than the first quartile.

RESUMEN
En este trabajo se sugiere modificar el estimador de razón de la media poblacional de la variable de interés involucrando el primeo y tercer cuartil de la variable auxiliar que está correlacionada con la variable de interés. Los nuevos estimadores sugeridos son investigados bajo los métodos del muestreo simple aleatorio y muestreo por rangos ordenados (RSS). Ss halló que estos estimadores son aproximadamente insesgados y que los estimadores de RSS son más eficientes que los basados en el método SRS para el mismo cuartil, tamaño de la muestra. También se halló que cuando los dos cuartiles son comparados, los estimadores basados en el tercer cuartil son más eficientes que los del primer cuartil.

KEY WORDS: simple random sampling, ranked set sampling, quartile, auxiliary variable, study variable.

MSC: 62D05

1. INTRODUCTION

The usual SRS ratio estimator for the population mean μ_Y of the variable of interest *Y* is defined as

$$\hat{\mu}_{YSRS} = \mu_X \left(\frac{\bar{Y}_{SRS}}{\bar{X}_{SRS}} \right), \quad (1)$$

where $\bar{X}_{SRS} = \frac{1}{m}\sum_{i=1}^{m} X_i$ and $\bar{Y}_{SRS} = \frac{1}{m}\sum_{i=1}^{m} Y_i$ are the sample means of the auxiliary variable *X* and the variable of interest *Y*, respectively, provided that the mean of *X* is known. The variances of $\bar{X}_{SRS}$ and $\bar{Y}_{SRS}$ respectively, are

$$\text{Var}\left(\bar{X}_{SRS}\right) = \frac{\sigma_X^2}{m} \text{ and } \text{Var}\left(\bar{Y}_{SRS}\right) = \frac{\sigma_Y^2}{m}. \quad (2)$$

The mean square error (MSE) of $\hat{\mu}_{YSRS}$ is given by

$$\text{MSE}\left(\hat{\mu}_{YSRS}\right) \cong \frac{1-f}{m}\left(\sigma_Y^2 + R^2\sigma_X^2\left(1-2D\right)\right) \tag{3}$$

where $f = \dfrac{m}{M}$, M is the population size and m is the sample size, σ_Y^2 and σ_X^2 are the population variances of the variables X and Y, respectively, $R = \dfrac{\mu_Y}{\mu_X}$, $D = \rho\dfrac{C_Y}{C_X}$, $\rho = \dfrac{\sigma_{XY}}{\sigma_X\sigma_Y}$, $C_X = \dfrac{\sigma_X}{\mu_X}$, $C_Y = \dfrac{\sigma_Y}{\mu_Y}$ and $\sigma_{XY} = Cov\left(X,Y\right)$, see Cochran (1977). In sampling situations one can use some parameters of the auxiliary variable such as the mean, median, variance and the coefficient of variation to increase the efficiency of the estimators. Srivastava and Jhajj (1981) suggested a class of estimators of the population mean providing that the mean and variance are known. Singh and Tailor (2003) considered a modified ratio estimator by exploiting the known value of correlation coefficient. Upadhyaya and Singh (1999) suggested two ratio-type estimators when the coefficient of variation and the coefficient of kurtosis of the auxiliary variable are valuable. Many authors suggested several modifications of the usual ratio estimator for the population mean. For more details about these modifications, see for example, Sisodia and Dwivedi (1981) and Kadilar and Cingi (2004).

RSS was introduced by McIntyre (1952) for estimating mean pasture and forage yields as a more efficient and cost effective method than the commonly used simple random sampling in the situations where visual ordering of the sample units can be done easily, but the exact measurement of the units is difficult and expensive. Takahasi and Wakimoto (1968) provided the necessary mathematical theory of RSS. Samawi and Muttlak (1996) studied the used of RSS to estimate the population ratio and suggested the RSS estimator of the population ratio as

$$\hat{R}_{RSS} = \frac{\bar{Y}_{RSS}}{\bar{X}_{RSS}}, \tag{4}$$

where $\bar{X}_{RSS} = \dfrac{1}{m}\sum_{i=1}^{m} X_{i(i)}$ and $\bar{Y}_{RSS} = \dfrac{1}{m}\sum_{i=1}^{m} Y_{i[i]}$ with variance given by

$$\text{Var}\left(\hat{R}_{RSS}\right) \cong \frac{R^2}{m}\left(\frac{\sigma_x^2}{\mu_x^2} + \frac{\sigma_y^2}{\mu_y^2} - 2\rho\frac{\sigma_x\sigma_y}{\mu_x\mu_y} - \left(\frac{\sum_{i=1}^{m} T_{x(i)}^2}{m\mu_x^2} + \frac{\sum_{i=1}^{m} T_{y[i]}^2}{m\mu_y^2} - 2\frac{\sum_{i=1}^{m} T_{xy(i)}^2}{m\mu_x\mu_y}\right)\right), \tag{5}$$

where $T_{x(i)} = \mu_{x(i)} - \mu_x$, $T_{y[i]} = \mu_{y[i]} - \mu_y$ and $T_{xy(i)} = \left(\mu_{x(i)} - \mu_x\right)\left(\mu_{y[i]} - \mu_y\right)$.

Jemain and Al-Omari (2006) proposed multistage median ranked set sampling for estimating the population mean. Jemain and Al-Omari (2006) suggested double quartile ranked set sampling for estimating the population mean. For more details about RSS see Balakrishnan and Li (2006), Tien-suwan et al., (2007), Al-Omari and Jaber (2008), Bouza (2002), Bouza (2001), Al-Saleh and Al-Ananbeh (2007), and Al-Saleh and Al-Omari (2002).

In this paper, modified ratio estimators for the population mean μ_Y are suggested by exploiting the first or third quartiles of the auxiliary variable X. The suggested estimators are studied under simple random sampling and ranked set sampling methods.

2. SUGGESTED ESTIMATORS

Let $\left(X_1,Y_1\right)$, $\left(X_2,Y_2\right)$, ..., $\left(X_m,Y_m\right)$ be a bivariate random sample with probability density function (pdf) $f(x,y)$, cumulative distribution function (cdf) $F(x,y)$, means μ_X,μ_Y, variances σ_X^2,σ_Y^2 and correlation coefficient ρ. Assume that the ranking is performed on the variable *X* to estimate the mean of the variable *Y*. Let $\left(X_{11},Y_{11}\right)$, $\left(X_{12},Y_{12}\right)$, ..., $\left(X_{1m},Y_{1m}\right)$, $\left(X_{21},Y_{21}\right)$, $\left(X_{22},Y_{22}\right)$, ..., $\left(X_{2m},Y_{2m}\right)$, ..., $\left(X_{m1},Y_{m1}\right)$, $\left(X_{m2},Y_{m2}\right)$, ..., $\left(X_{mm},Y_{mm}\right)$ be m independent bivariate random samples each of size *m*.

2.1 UNDER SRS

The suggested simple random sampling (SRS) ratio estimators of the population mean μ_Y of the variable of interest *Y* are defined as

$$\hat{\mu}_{YSRS1}=\bar{Y}_{SRS}\left(\frac{\mu_X+q_1}{\bar{X}_{SRS}+q_1}\right) \text{ and } \hat{\mu}_{YSRS3}=\bar{Y}_{SRS}\left(\frac{\mu_X+q_3}{\bar{X}_{SRS}+q_3}\right), \tag{6}$$

where $\bar{X}_{SRS}$ and $\bar{Y}_{SRS}$ are the sample means of the variables *X* and *Y*, q_1 and q_3 are the first and third quartiles of *X*, respectively. By using Taylor series method, $\hat{\mu}_{YSRSh}$ can be approximated as:

$$\hat{\mu}_{YSRSh}\cong\bar{Y}_{SRS}-K_h\left(\bar{X}_{SRS}-\mu_X\right)+K_hG_h\left(\bar{X}_{SRS}-\mu_X\right)^2-G_h\left(\bar{X}_{SRS}-\mu_X\right)\left(\bar{Y}_{SRS}-\mu_Y\right), \tag{7}$$

where $K_h=\dfrac{\mu_Y}{\mu_X+q_h}$ and $G_h=\dfrac{1}{\mu_X+q_h}$ for $h=1,3$. Using the first degree of approximation, the estimator in (7) is given by

$$\hat{\mu}_{YSRSh}\cong\bar{Y}_{SRS}-K_h\left(\bar{X}_{SRS}-\mu_X\right), \tag{8}$$

with bias and MSE, respectively, are

$$\text{Bias}\left(\hat{\mu}_{YSRS}\right)\cong 0, \tag{9}$$

and

$$\text{MSE}\left(\hat{\mu}_{YSRSh}\right)\cong\text{Var}\left(\bar{Y}_{SRS}\right)+K_h^2\text{Var}\left(\bar{X}_{SRS}\right)-2K_h\text{Cov}\left(\bar{X}_{SRS},\bar{Y}_{SRS}\right), \tag{10}$$

where $\text{Cov}\left(\bar{X}_{SRS},\bar{Y}_{SRS}\right)=E\left(\left(\bar{X}_{SRS}-\mu_X\right)\left(\bar{Y}_{SRS}-\mu_Y\right)\right)$. It can be noted that for the first order of approximation the two estimators are unbiased.

Using

$$\text{Cov}\left(\bar{X}_{SRS},\bar{Y}_{SRS}\right)=\beta\text{Var}\left(\bar{X}_{SRS}\right), \tag{11}$$

and

$$\operatorname{Var}\left(\bar{Y}_{SRS}\right) \cong \beta^2 \operatorname{Var}\left(\bar{X}_{SRS}\right) + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right), \tag{12}$$

and Equation (2), where $\beta = \rho \dfrac{\sigma_Y}{\sigma_X}$, Equation (10) can be written as

$$\begin{aligned}\operatorname{MSE}\left(\hat{\mu}_{YSRSh}\right) &\cong \left(K_h - \beta\right)^2 \operatorname{Var}\left(\bar{X}_{SRS}\right) + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right) \\ &= \frac{1}{m}\left(K_h^2\sigma_X^2 + \sigma_Y^2 - 2K_h\sigma_X\sigma_Y\rho\right). \end{aligned} \tag{13}$$

2.2 UNDER RSS

In this study we assume that the ranking is performed on the variable *X* for estimating the population mean of the variable *Y*. The RSS method can be described as follows: Select *m* random samples each of size *m* bivariate units from the target population. From the first set of *m* units, the smallest ranked unit *X* is selected together with the associated *Y*, and from the second set of *m* units the second smallest ranked unit *X* is selected together with the associated *Y*. The procedure is continued until from the *m*th set of *m* units the largest ranked unit *X* is selected with the associated *Y*. The procedure can be repeated *n* times to increase the sample size to *nm* RSS bivariate units.

Let $\left(X_{i(1)}, Y_{i[1]}\right)$, $\left(X_{i(2)}, Y_{i[2]}\right)$, …, $\left(X_{i(m)}, Y_{i[m]}\right)$ be the order statistics of $X_{i1}, X_{i2}, ..., X_{im}$ and the judgment order of $Y_{i1}, Y_{i2}, \ldots, Y_{im}$ for $i = 1, 2, ..., m$. Then the RSS units are $\left(X_{1(1)}, Y_{1[1]}\right)$, $\left(X_{2(2)}, Y_{2[2]}\right)$, …, $\left(X_{m(m)}, Y_{m[m]}\right)$. The proposed RSS estimators of the population mean μ_Y involving the first and third quartiles of *X* are defined as:

$$\hat{\mu}_{YRSS1} = \bar{Y}_{RSS}\left(\frac{\mu_X + q_1}{\bar{X}_{RSS} + q_1}\right) \text{ and } \hat{\mu}_{YRSS3} = \bar{Y}_{RSS}\left(\frac{\mu_X + q_3}{\bar{X}_{RSS} + q_3}\right), \tag{14}$$

respectively, where $\bar{X}_{RSS} = \dfrac{1}{m}\sum_{i=1}^{m} X_{i(i)}$ with variance $\operatorname{Var}\left(\bar{X}_{RSS}\right) = \dfrac{\sigma_X^2}{m} - \dfrac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{X(i)} - \mu_X\right)^2$, and $\bar{Y}_{RSS} = \dfrac{1}{m}\sum_{i=1}^{m} Y_{i[i]}$ with variance $\operatorname{Var}\left(\bar{Y}_{RSS}\right) = \dfrac{\sigma_Y^2}{m} - \dfrac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{Y[i]} - \mu_Y\right)^2$. Using Taylor series, $\hat{\mu}_{YRSSh}$ $\left(h = 1, 3\right)$ can be approximated as:

$$\hat{\mu}_{YRSSh} \cong \bar{Y}_{RSS} - K_h\left(\bar{X}_{RSS} - \mu_X\right) + K_h G_h\left(\bar{X}_{RSS} - \mu_X\right)^2 - G_h\left(\bar{X}_{RSS} - \mu_X\right)\left(\bar{Y}_{RSS} - \mu_Y\right). \tag{15}$$

For the first order of approximation the estimator in (15) can be approximated as

$$\hat{\mu}_{YRSSh} \cong \bar{Y}_{RSS} - K\left(\bar{X}_{RSS} - \mu_X\right), \tag{16}$$

with bias and MSE are given by

$$\text{Bias}\left(\hat{\mu}_{YRSSh}\right) \cong 0\,, \tag{17}$$

and

$$\text{MSE}\left(\hat{\mu}_{YRSSh}\right) \cong \text{Var}\left(\overline{Y}_{RSS}\right) + K_h^2 \text{Var}\left(\overline{X}_{RSS}\right) - 2K_h \text{Cov}\left(\overline{X}_{RSS}, \overline{Y}_{RSS}\right), \tag{18}$$

respectively, where $\text{Cov}\left(\overline{X}_{RSS}, \overline{Y}_{RSS}\right) = E\left(\left(\overline{X}_{RSS} - \mu_X\right)\left(\overline{Y}_{RSS} - \mu_Y\right)\right)$. As in the case of SRS, it is clear that to the first order of approximation the RSS estimators are unbiased. Using

$$\text{Cov}\left(\overline{X}_{RSS}, \overline{Y}_{RSS}\right) = \beta \text{Var}\left(\overline{X}_{RSS}\right), \tag{19}$$

and

$$\text{Var}\left(\overline{Y}_{RSS}\right) \cong \beta^2 \text{Var}\left(\overline{X}_{RSS}\right) + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right), \tag{20}$$

Equation (18) can be rewritten as

$$\begin{aligned} \text{MSE}\left(\hat{\mu}_{YRSSh}\right) &\cong \left(K_h - \beta\right)^2 \text{Var}\left(\overline{X}_{RSS}\right) + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right) \\ &= \left(K_h - \beta\right)^2 \left(\frac{\sigma_X^2}{m} - \frac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{X(i)} - \mu_X\right)^2\right) + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right). \end{aligned} \tag{21}$$

4. EFFICIENCY OF THE ESTIMATORS

The efficiency of $\hat{\mu}_{YRSSh}$ $(h = 1, 3)$ with respect to $\hat{\mu}_{YSRSh}$ for estimating the population mean μ_Y is defined as:

$$\begin{aligned} \mathit{eff}\left(\hat{\mu}_{YRSSh}, \hat{\mu}_{YSRSh}\right) &= \frac{\text{MSE}\left(\hat{\mu}_{YSRSh}\right)}{\text{MSE}\left(\hat{\mu}_{YRSSh}\right)} \\ &\cong \frac{\left(K_h - \beta\right)^2 \text{Var}\left(\overline{X}_{SRS}\right) + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right)}{\left(K_h - \beta\right)^2 \text{Var}\left(\overline{X}_{RSS}\right) + \frac{1}{m}\sigma_Y^2\left(1-\rho^2\right)} \\ &= \frac{\left(K_h - \beta\right)^2 \frac{\sigma_X^2}{m} + \frac{\sigma_Y^2}{m}\left(1-\rho^2\right)}{\left(K_h - \beta\right)^2 \left(\frac{\sigma_X^2}{m} - \frac{1}{m^2}\sum_{i=1}^{m}\left(\mu_{X(i)} - \mu_X\right)^2\right) + \frac{\sigma_Y^2}{m}\left(1-\rho^2\right)} \\ &= \frac{1}{1-A}. \end{aligned} \tag{22}$$

where $A = \frac{\left(\frac{\sigma_Y}{\sigma_X}K_h - \rho\right)^2 \sum_{i=1}^{m}\left(F_X^{-1}\left(\frac{i}{m+1}\right)\right)}{m\left(\frac{\sigma_Y}{\sigma_X}K_h - \rho\right)^2 + m\left(1-\rho^2\right)}$, and $\mu_{X(i)} \cong \mu_X + \sigma_X F_X^{-1}\left(\frac{i}{m+1}\right)$ for symmetric distributions.

It is clear that, $eff\left(\hat{\mu}_{YRSSh}, \hat{\mu}_{YSRSh}\right) > 1$. This implies that $\hat{\mu}_{YRSSh}$ $(h=1,3)$ is more efficient than $\hat{\mu}_{YSRSh}$ based on the same number of measured units.

5. SIMULATION STUDY

Table 1: The efficiency and bias values of $\hat{\mu}_{YRSSh}$ $(h=1,3)$ with respect to $\hat{\mu}_{YSRSh}$ for $m = 2,3,4,5,6$ with positive values of ρ using $q_1 = 1.32551$.

ρ		$m=2$	$m=3$	$m=4$	$m=5$	$m=6$
0.99	Eff	1.582	1.729	1.909	2.029	2.142
	Bias of RSS	0.023	0.012	0.008	0.005	0.003
	Bias of SRS	0.039	0.024	0.017	0.013	0.012
0.90	Eff	1.294	1.307	1.346	1.379	1.381
	Bias of RSS	0.034	0.016	0.009	0.008	0.004
	Bias of SRS	0.054	0.033	0.025	0.020	0.014
0.80	Eff	1.289	1.291	1.342	1.365	1.367
	Bias of RSS	0.047	0.023	0.013	0.009	0.007
	Bias of SRS	0.076	0.043	0.033	0.024	0.023
0.70	Eff	1.319	1.337	1.371	1.396	1.398
	Bias of RSS	0.056	0.028	0.016	0.013	0.008
	Bias of SRS	0.088	0.056	0.042	0.034	0.026
0.50	Eff	1.323	1.385	1.433	1.466	1.489
	Bias of RSS	0.082	0.041	0.023	0.015	0.009
	Bias of SRS	0.124	0.081	0.054	0.040	0.039
0.40	Eff	1.366	1.403	1.484	1.524	1.544
	Bias of RSS	0.092	0.046	0.023	0.156	0.014
	Bias of SRS	0.144	0.092	0.062	0.050	0.045
0.30	Eff	1.409	1.484	1.507	1.578	1.612
	Bias of RSS	0.106	0.050	0.023	0.023	0.015
	Bias of SRS	0.162	0.963	0.076	0.061	0.048
0.20	Eff	1.438	1.491	1.614	1.677	1.682
	Bias of RSS	0.118	0.057	0.031	0.023	0.019
	Bias of SRS	0.181	0.111	0.083	0.065	0.049
0.10	Eff	1.470	1.550	1.636	1.733	1.759
	Bias of RSS	0.126	0.056	0.041	0.023	0.016
	Bias of SRS	0.198	0.117	0.091	0.073	0.057
0.01	Eff	1.481	1.583	1.726	1.803	1.883
	Bias of RSS	0.137	0.063	0.035	0.026	0.020
	Bias of SRS	0.220	0.132	0.098	0.079	0.065

A simulation study was conducted to investigate the performance of the suggested estimators based on SRS and RSS methods. The samples were generated from bivariate normal distribution $BN\left(2,4,1,1,\rho\right)$ where $\rho = \pm 0.99,\ \pm 0.90,\ \pm 0.80,\ \pm 0.70, \pm 0.50, \pm 0.40,\ \pm 0.30,\ \pm 0.20,\ \pm 0.10,\ \pm 0.01$. Based on 60,000 replications, the results for $m = 2, 3, 4, 5, 6$ using the first quartile $(q_1 = 1.32551)$ for positive and negative values of ρ are obtained and presented in Tables 1 and 2, respectively. Similarly, the results for the third quartile $(q_3 = 2.67449)$ are presented in Tables 3 and 4, respectively. We summarized the results in the tables using graphs, as shown in Figures 1 and 2. The efficiency of $\hat{\mu}_{YRSSh}$ $(h = 1, 3)$ with respect to $\hat{\mu}_{YRSSh}$ $(h = 1, 3)$ is obtained using (22).

Table 2: The efficiency and bias values of $\hat{\mu}_{YRSSh}$ $(h = 1, 3)$ with respect to $\hat{\mu}_{YSRSh}$ for $m = 2, 3, 4, 5, 6$ with negative values of ρ using $q_1 = 1.32551$.

ρ		$m=2$	$m=3$	$m=4$	$m=5$	$m=6$
- 0.99	Eff	1.868	2.200	2.760	3.246	3.670
	Bias of RSS	0.245	0.128	0.068	0.045	0.033
	Bias of SRS	0.398	0.236	0.176	0.136	0.116
- 0.90	Eff	1.758	2.164	2.660	3.048	3.381
	Bias of RSS	0.232	0.118	0.070	0.047	0.034
	Bias of SRS	0.377	0.227	0.179	0.132	0.108
- 0.80	Eff	1.748	2.066	2.512	2.891	3.133
	Bias of RSS	0.221	0.111	0.066	0.044	0.030
	Bias of SRS	0.359	0.227	0.157	0.136	0.109
- 0.70	Eff	1.688	2.061	2.359	2.636	2.869
	Bias of RSS	0.225	0.107	0.063	0.037	0.029
	Bias of SRS	0.341	0.217	0.151	0.122	0.113
- 0.50	Eff	1.680	1.922	2.148	2.369	2.477
	Bias of RSS	0.189	0.089	0.049	0.035	0.027
	Bias of SRS	0.313	0.194	0.141	0.104	0.090
- 0.40	Eff	1.591	1.811	2.029	2.243	2.331
	Bias of RSS	0.179	0.088	0.050	0.032	0.022
	Bias of SRS	0.276	0.179	0.128	0.102	0.085
- 0.30	Eff	1.558	1.745	1.937	2.095	2.204
	Bias of RSS	0.179	0.077	0.043	0.031	0.024
	Bias of SRS	0.266	0.017	0.120	0.095	0.083
- 0.20	Eff	1.512	1.740	1.895	2.031	2.056
	Bias of RSS	0.163	0.083	0.042	0.028	0.021
	Bias of SRS	0.239	0.162	0.111	0.094	0.073
- 0.10	Eff	1.500	1.646	1.815	1.902	1.933
	Bias of RSS	0.148	0.070	0.039	0.028	0.019
	Bias of SRS	0.225	0.143	0.102	0.085	0.068
- 0.01	Eff	1.435	1.604	1.735	1.808	1.891
	Bias of RSS	0.145	0.067	0.040	0.020	0.017
	Bias of SRS	0.209	0.132	0.099	0.074	0.067

From the simulation results given in Tables 1-4, as well as Figures 1 and 2, we can conclude that:

1. By exploiting the knowledge of the first or third quartiles of the auxiliary variable *X*, a gain in efficiency is obtained using RSS with respect to SRS for estimating the population mean of the variable of interest *Y*.

2. For negative values of ρ, the efficiency is increasing as the magnitude of the correlation coefficient increase. For example for $q_3 = 2.67449$, $m = 5$ and $\rho = -0.50, -0.70, -0.80, -0.90,$ and -0.99, the efficiency values are 1.981, 2.320, 2.548, 2.790, and 3.013 respectively.

Table 3: The efficiency and bias values of $\hat{\mu}_{YRSSh}$ $(h=1,3)$ with respect to $\hat{\mu}_{YSRSh}$ for $m=2,3,4,5,6$ with positive values of ρ using $q_3 = 2.67449$.

ρ		$m=2$	$m=3$	$m=4$	$m=5$	$m=6$
0.99	Eff	1.247	1.390	1.447	1.518	1.575
	Bias of RSS	0.010	0.005	0.003	0.002	0.001
	Bias of SRS	0.016	0.010	0.008	0.007	0.005
0.90	Eff	1.036	1.038	1.039	1.041	1.050
	Bias of RSS	0.003	0.001	0.000	0.000	0.000
	Bias of SRS	0.008	0.003	0.002	0.001	0.001
0.80	Eff	1.032	1.035	1.037	1.039	1.041
	Bias of RSS	0.004	0.002	0.000	0.001	0.000
	Bias of SRS	0.005	0.005	0.004	0.002	0.001
0.70	Eff	1.041	1.050	1.053	1.054	1.056
	Bias of RSS	0.014	0.004	0.002	0.004	0.001
	Bias of SRS	0.019	0.015	0.007	0.006	0.005
0.50	Eff	1.101	1.112	1.127	1.136	1.138
	Bias of RSS	0.031	0.012	0.009	0.005	0.003
	Bias of SRS	0.039	0.026	0.019	0.016	0.013
0.40	Eff	1.108	1.170	1.168	1.184	1.190
	Bias of RSS	0.034	0.016	0.010	0.007	0.005
	Bias of SRS	0.049	0.034	0.027	0.018	0.016
0.30	Eff	1.161	1.185	1.189	1.233	1.249
	Bias of RSS	0.043	0.020	0.108	0.008	0.008
	Bias of SRS	0.067	0.046	0.028	0.022	0.020
0.20	Eff	1.170	1.212	1.274	1.284	1.303
	Bias of RSS	0.047	0.022	0.018	0.007	0.011
	Bias of SRS	0.078	0.048	0.033	0.030	0.027
0.10	Eff	1.208	1.260	1.339	1.350	1.379
	Bias of RSS	0.055	0.028	0.018	0.008	0.009
	Bias of SRS	0.092	0.055	0.042	0.037	0.025
0.01	Eff	1.212	1.304	1.378	1.434	1.465
	Bias of RSS	0.061	0.032	0.019	0.012	0.009
	Bias of SRS	0.100	0.063	0.047	0.039	0.028

3. It is found that the RSS estimators are more efficient both for extremely large or small correlation coefficient. For positive extreme values of the correlation coefficient as $\rho = 0.99$ and 0.01 the efficiency is greater than for other values as $\rho = 0.80$, and 0.70, ect. As an example, for q_1, $m=3$ and $\rho = 0.99$, 0.90, 0.80, 0.70, 0.50, 0.40, 0.30, 0.20, 0.10 and 0.01 the efficiency values are 1.729, 1.307, 1.291, 1.337, 1.385, 1.403, 1.484, 1.491, 1.550 and 1.583, respectively.

4. Based on the given value of either q_1 or q_3 for several particular values of the sample size or correlation coefficient, the bias obtained using RSS is less than the bias obtained using SRS method.

5. The efficiency of $\hat{\mu}_{YRSSh}$ is increasing with the sample size for the fixed value of ρ and quartile. As an example, using $q_1 = 1.32551$, and $\rho = -0.80$ for $m = 2,3,4,5,6$, the efficiency values of RSS estimators are 1.748, 2.066, 2.512, 2.891 and 3.133, respectively.

Table 4: The efficiency and bias values of $\hat{\mu}_{YRSSh}$ $(h=1,3)$ with respect to $\hat{\mu}_{YSRSh}$ for $m=2,3,4,5,6$ with negative values of ρ using $q_3=2.67449$.

ρ		$m=2$	$m=3$	$m=4$	$m=5$	$m=6$
- 0.99	Eff	1.573	2.073	2.562	3.013	3.481
	Bias of RSS	0.135	0.071	0.043	0.028	0.021
	Bias of SRS	0.214	0.144	0.100	0.080	0.072
- 0.90	Eff	1.533	1.970	2.435	2.790	3.069
	Bias of RSS	0.139	0.070	0.035	0.027	0.021
	Bias of SRS	0.197	0.133	0.100	0.076	0.065
- 0.80	Eff	1.479	1.847	2.191	2.548	2.742
	Bias of RSS	0.129	0.059	0.037	0.026	0.017
	Bias of SRS	0.191	0.121	0.089	0.073	0.061
- 0.70	Eff	1.477	1.758	2.073	2.320	2.483
	Bias of RSS	0.117	0.058	0.035	0.023	0.019
	Bias of SRS	0.183	0.118	0.087	0.069	0.054
- 0.50	Eff	1.396	1.617	1.843	1.981	2.081
	Bias of RSS	0.103	0.051	0.032	0.022	0.014
	Bias of SRS	0.154	0.101	0.076	0.064	0.005
- 0.40	Eff	1.343	1.526	1.715	1.817	1.936
	Bias of RSS	0.098	0.046	0.031	0.018	0.012
	Bias of SRS	0.140	0.089	0.068	0.525	0.044
- 0.30	Eff	1.312	1.456	1.653	1.720	1.783
	Bias of RSS	0.086	0.047	0.029	0.017	0.013
	Bias of SRS	0.135	0.085	0.063	0.051	0.041
- 0.20	Eff	1.273	1.420	1.535	0.618	1.671
	Bias of RSS	0.078	0.043	0.022	0.017	0.012
	Bias of SRS	0.122	0.077	0.059	0.045	0.041
- 0.10	Eff	1.251	1.354	1.481	1.522	1.561
	Bias of RSS	0.067	0.041	0.023	0.016	0.007
	Bias of SRS	0.114	0.071	0.051	0.041	0.038
- 0.01	Eff	1.215	1.321	1.412	1.447	1.471
	Bias of RSS	0.067	0.031	0.022	0.014	0.008
	Bias of SRS	0.102	0.064	0.054	0.037	0.031

6. For the estimators considered, the negative values of the correlation coefficient ρ give higher values of the efficiency than the positive values based on the same number of measured units. For example, given that $m=6$ and $q_1=1.32551$, the efficiency values are 1.367 and 3.133 for $\rho=0.80$ and -0.80 respectively. This is may be because for negative values of the correlation coefficient the variance and bias of SRS estimators are greater than their counterparts for positive values of the correlation coefficient.

7. The results obtained using the first quartile $q_1=1.32551$ are more efficient than that of used the third quartile $q_3=2.67449$.

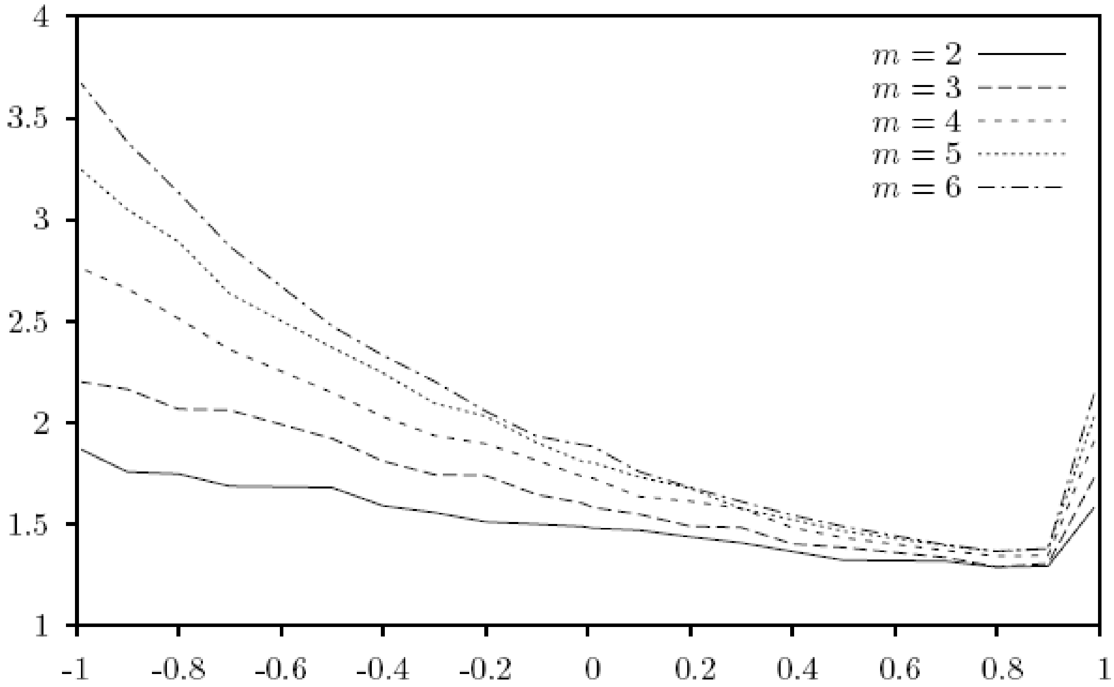

Figure 1: The efficiency of RSS estimators using the first quartile

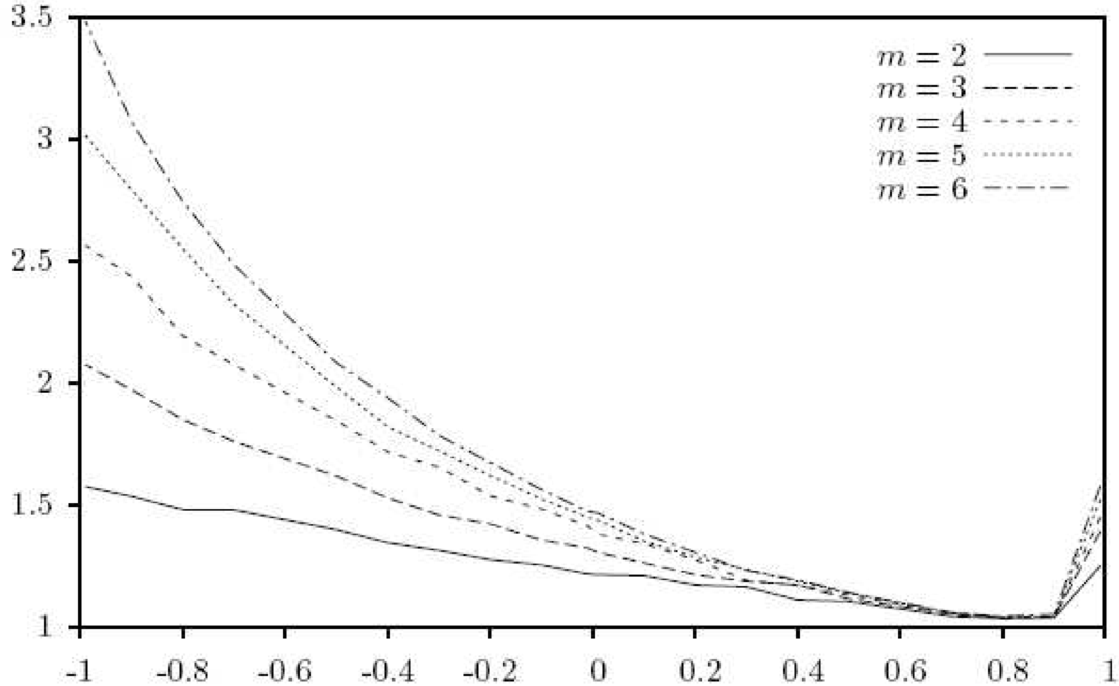

Figure 2: The efficiency of RSS estimators using the third quartile

6. CONCLUSIONS

In this paper, we suggested modified ratio estimators of the population mean using SRS and RSS assuming that the first or third quartiles of the auxiliary variable are available. It is found that these estimators are approximately unbiased of the population mean of the variable of interest for the first order approximation. Also the RSS ratio estimators are more efficient than the SRS based on the same sample size, correlation coefficient and the quartiles.

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