ON THOMPSON TYPE ESTIMATORS FOR THE MEAN OF NORMAL DISTRIBUTION

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ABSTRACT

Let X be a normally distributed with unknown mean μ and variance σ^2. Assume that a prior estimate μ_0 of μ is available. Two Thompson type shrinkage estimators of estimating μ that incorporates prior estimate μ_0 are proposed. These estimators are shown to have a smaller mean squared error in a region around μ_0 in comparison to existing estimators. The expressions for the bias and mean squared error of the proposed t-estimators are obtained. Numerical results are provided when the proposed estimators are t-estimators of level of significance α. Comparisons with the earlier known results show the usefulness of the testimators.

KEY WORDS: normal distribution; shrinkage; preliminary test region; bias ratio; relative efficiency

MSC: 62F10; 62F99

RESUMEN

Sea X una variable distribuida normalmente con media μ y varianza σ^2 desconocidas. Asuma que un estimado a priori μ_0 de μ está disponible. Se proponen dos tipos de estimadores del tipo Thompson de encogimiento (shrinkage) para estimar μ que incorporan el estimado a priori. Se demuestra que estos estimadores poseen menos error cuadrático medio en la región alrededor de μ_0 en comparación con estimadores existentes. Las expresiones para el sesgo y el error cuadrático medio de los t-estimadores son obtenidos. Resultados numéricos son presentados cuando los estimadores propuestos son t-estimadores de nivel de significación. Comparaciones con previos resultados conocidos demuestran la utilidad de los t-estimadores.

1 INTRODUCTION

1.1 The model

The normal distribution is the most widely used distribution in statistics and many other sciences. To be specific, in modeling the normal curve is an excellent approximation to the frequency distributions of observations taken on a variety of variables and as a limiting form of various other distributions (see Davison, 2003). Examples of random variables that have been modeled successfully by the normal distributions are the height and weight of people, diameters of bolts produced by a machine, the IQ of people, the life of electronic products, and so on.

Let $x_1, x_2, \ldots, x_n$ be a random sample of size n from the following normal distribution

$$f(x \mid \mu, \sigma^2) = (1/\sigma\sqrt{2\pi})\exp\{-(x-\mu)^2/2\sigma^2\}; \quad -\infty < x < \infty, -\infty < \mu < \infty, \sigma^2 > 0, \tag{1}$$

where μ being the mean (unknown) and σ^2 is the variance.

1.2 Estimating the mean incorporating a guess

In many problems, the experimenter has some prior guess value regarding the value of μ either due to past experiences or to his familiarity with the behavior of the population under study. However, in certain situations the

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prior information is available only in the form of an initial guess value (natural origin) μ_0 of μ. According to Thompson (1968 a) such guess value may arise for any one of a number of reasons, e.g., we are estimating μ and:

i) we believe μ_0 is close to the true value of μ; or

ii) we fear that μ_0 may be near the true value of μ, i.e., something bad happens if $\mu_0 = \mu$, and we do not know about it.

For such cases, this guess value may be utilized to improve the estimation procedure. For the philosophy and history of shrinkage estimator and its importance see Casella (2002) and Lemmer (2006).

1.3 Background

A standard problem in estimation of the unknown parameter μ when some guess is available in the form of μ_0, is to get an estimator with minimum mean square error or maximum relative efficiency. The commonly used approaches in statistical inference which utilize the prior guess value are the shrunken methods. The problem of estimating the mean μ when μ_0 is available was first discussed by Thompson (1968 a) where he developed single stage shrunken estimators for parameters of normal, binomial, Poisson and gamma distributions. Following this several authors have tried to develop new single stage shrunken estimators by using Thompson type shrinkage weight factors and also proposing new shrinkage weight factors. A general single stage shrunken estimator for the mean μ is defined as follows: (i) Compute the sample mean $\overline{X}$ based on n observations; (ii) Construct a preliminary test region (R) in the space of μ, based on μ_0 and an appropriate criterion. If $\overline{X} \in R$, shrink $\overline{X}$ towards μ_0 by shrinkage factor $0 \le k \le 1$ and use the estimator $k(\overline{X} - \mu_0) + \mu_0$ for μ. But if $\overline{X} \notin R$, take $\overline{X}$ as an estimator of μ.

Thus a single stage shrinkage estimator of μ using the prior estimate μ_0 is given by:

$$\overline{\mu} = \{[\, k(\overline{X} - \mu_0) + \mu_0]\; I_R + [\, \overline{X}\,]\, I_{\overline{R}} \}, \tag{2}$$

where I_R and $I_{\overline{R}}$ are respectively the indicator functions of the acceptance region R and the rejection region $\overline{R}$. The single stage shrunken estimator $\overline{\mu}$ is completely specified if the shrinkage weight factor k and the region R are specified. Consequently, the success of $\overline{\mu}$ depends upon the proper choice of k and R. Several authors have studied estimators of the form $\overline{\mu}$ by choosing different k and R [see Thompson (1968 a, b), Davis and Arnold (1970), Mehta and Srinivasan (1971), Hirano (1977), Kambo, Handa and Al-Hemyari (1992)].

2. THE PROPOSED ESTIMATORS

In this paper we proposed two single stage shrunken estimators for the mean μ when σ^2 is known or unknown denoted by $\tilde{\mu}_i,\ i = 1, 2$, which are a modifications of $\overline{\mu}$ defined in (2). The proposed estimator takes the general form:

$$\tilde{\mu} \;= \{[\, k(\overline{X} - \mu_0) + \mu_0]\, I_R + \left[[(1-k)(\overline{X} - \mu_0) + \mu_0]\right] I_{\overline{R}} \} \tag{3}$$

The main distinguishing feature of this type of single stage estimator from conventional two stage shrinkage testimators is that, the pretest region rejects the prior estimate μ_0 only partially and even if $\overline{X} \notin R$, μ_0 is given some weightage though small in estimation of second stage. The expressions for the bias, mean squared error, and relative efficiency of $\tilde{\mu}$ for the both cases when σ^2 known or unknown are derived and studied theoretically and numerically. Some properties of $\tilde{\mu}$ are studied. Conclusions regarding the constants involved in the proposed estimators are presented. Comparisons with the earlier known results are made. It may be noted here that Kambo, Handa and Al-Hemyari (1990) studied estimator (2) for exponential distribution.

2.1 Estimator with known σ^2

In this section first we define the estimator when σ^2 is known. The bias, mean squared error, and relative efficiency expressions of the proposed testimator are derived. A suitable choice of k is obtained, and finally some properties are also discussed.

Let X be normally distributed with unknown μ and known variance σ^2, assume that a prior estimate μ_0 about μ is available from the past. The first proposed testimator is:

$$\tilde{\mu}_1 = \left\{ \left[k_1(\bar{X} - \mu_0) + \mu_0 \right] I_{R_1} + \left[(1-k_1)(\bar{X} - \mu_0) + \mu_0 \right] I_{\bar{R}_1} \right\}. \tag{4}$$

R_1 is taking as the pretest region of size α for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$, where

$$R_1 = [\mu_0 - z_{\alpha/2} \frac{\sigma}{\sqrt{n}},\ \mu_0 + z_{\alpha/2} \frac{\sigma}{\sqrt{n}}], \tag{5}$$

$z_{\alpha/2}$ is the upper $100(\alpha/2)$ th percentile point of the standard normal distribution.

The bias of $\tilde{\mu}_1$ is given by:

$$\begin{aligned} B(\tilde{\mu}_1 | \mu, R_1) &= E(\tilde{\mu}_1) - \mu \\ &= \int_{\bar{X} \in R_1} \left[k_1(\bar{X} - \mu_0) + \mu_0 \right] f(\bar{X} | \mu, \sigma^2)\, d\bar{X} \ + \int_{\bar{X} \in \bar{R}_1} \left[(1-k_1)(\bar{X} - \mu_0) \right] f(\bar{X} | \mu, \sigma^2)\, d\bar{X} - \mu, \end{aligned} \tag{6}$$

where $f\left(\bar{X} | \mu, \sigma^2\right)$ is the pdf of $\bar{X}$ and $\bar{R}_1$ is the complement of R_1.

Define

$$J_i(a,b) = \frac{1}{\sqrt{2\pi}} \int_a^b Z^i \exp(-Z^2/2)\, dZ, \quad i = 0,1,2,\ldots, \tag{7}$$

where $Z = \sqrt{n_1}\,(\bar{X}_1 - \mu)/\sigma$.

The expression of $B(\tilde{\mu}_1 | \mu, R_1)$ after simplifications yields

$$B(\tilde{\mu}_1 | \mu, R_1) = (\sigma/\sqrt{n}) \left\{ (2k-1)[J_1(\bar{a}_1, \bar{b}_1) - \lambda_1 J_0(\bar{a}_1, \bar{b}_1)] + k\lambda_1 \right\}, \tag{8}$$

where $\lambda_1 = \sqrt{n}\,(\mu_0 - \mu)/\sigma$, $\bar{a}_1 = \lambda_1 - z_{\alpha/2}$ and $\bar{b}_1 = \lambda_1 + z_{\alpha/2}$.

The mean squared error of $\tilde{\mu}_1$ is given by

$$MSE(\tilde{\mu}_1 | \mu, R_1) = E(\tilde{\mu}_1 - \mu)^2 = (\sigma^2/n) \left\{ (2k-1)[J_2(\bar{a}_1, \bar{b}_1) - \lambda_1^2 J_0(\bar{a}_1, \bar{b}_1)] + k^2 \lambda_1^2 + (1-k)^2 \right\}. \tag{9}$$

For numerical computations we may use the relations

$$J_1(a,b) = \left(\exp(-a^2/2) - \exp(-b^2/2) \right) / \sqrt{2\pi}\,, \tag{10}$$

and

$$J_2(a,b) = J_0(a,b) + \left(\exp(-a^2/2) - \exp(-b^2/2) \right) / \sqrt{2\pi}\,. \tag{11}$$

If $\mu = \mu_0$, then expression of bias and MSE of $\tilde{\mu}_1$ respectively simplify to

$$B(\tilde{\mu}_1 | \mu_0, R_1) = 0\,, \tag{12}$$

and

$$MSE(\tilde{\mu}_1|\mu_0, R_1) = (\sigma^2/n)\{(2k-1)J_2(a_1^*, b_1^*) + (1-k)^2\}, \tag{13}$$

where $a_1^* = -z_{\alpha/2}$ and $b_1^* = +z_{\alpha/2}$.

Remark 1: Local minima choice of *k*

When the region R does not depend on *k* a choice (local minima) for *k* is given below:
Select *k* that minimizes $MSE(\tilde{\mu}_1|\mu_0, R)$. From (13) we have

$$\frac{\partial}{\partial k} MSE(\tilde{\mu}_1|\mu_0, R) = (\sigma^2/n)\{(2J_2(a_1^*, b_1^*) - 2(1-k)\}.$$

Now $\frac{\partial}{\partial k} MSE(\tilde{\mu}_1|\mu_0, R) = 0$ gives critical value of *k* as

$$k_1 = 1 - J_2(a_1^*, b_1^*) = 1 - \frac{\int_a^b (\overline{X} - \mu_0)^2 f(\overline{X}|\mu, \sigma^2)\, d\overline{X}}{MSE(\overline{X})}. \tag{14}$$

Clearly $\frac{\partial^2}{\partial k^2} MSE(\tilde{\mu}_1|\mu_0, R) > 0,\ k_1$ is not a function of unknown parameter μ and $0 \le k_1 \le 1$.
It is easy to show that $B(\tilde{\mu}_1|\mu, R_1)$ is an odd function of λ and $MSE(\tilde{\mu}_1|\mu, R_1)$ is an even function of λ. Moreover $\underset{n\to\infty}{Lim}\, B(\tilde{\mu}_1|\mu, R) = 0$ and $\underset{n\to\infty}{Lim}\, MSE(\tilde{\mu}_1|\mu, R) = 0$. This shows that $\tilde{\mu}_1$ is also a consistent estimator.

2.2 Estimator with unknown σ^2

When σ^2 is unknown, it is estimated by $s^2 = \sum_{i=1}^{n_1} (X_i - \overline{X})^2 / (n-1)$. Again taking region R_2 as the pretest region of size α for testing $H_0 : \mu = \mu_0$ against $H_1 : \mu \ne \mu_0$ in the testimator $\tilde{\mu}_1$ defined in equation (4) and denoting the resulting estimator as $\tilde{\mu}_2$. The estimator $\tilde{\mu}_2$ is given by:

$$\tilde{\mu}_2 = \{[k_2(\overline{X} - \mu_0) + \mu_0]I_{R_2} + [(1-k_2)(\overline{X} - \mu_0) + \mu_0]I_{\overline{R}_2}\}. \tag{15}$$

The testimator $\tilde{\mu}_2$ employs the interval R_2 given by:

$$R_2 = \left[\mu_0 - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}, \mu_0 + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}\right], \tag{16}$$

and $t_{\alpha/2,n-1}$ is the upper $100(\alpha/2)\text{th}$ percentile point of the t-distribution with $(n-1)$ degrees of freedom.

The expression for the bias of $\tilde{\mu}_2$ can be written as

$$B(\tilde{\mu}_2|\mu,R_2)=\int_0^{\infty}\int_{\bar{X}\in R_2}\left[k(\bar{X}-\mu_0)+\mu_0\right]f(\bar{X}|\mu,s^2)\;f(s^2|\sigma^2)\,ds^2\,d\bar{X}$$
$$+\int_{\bar{X}\in \bar{R}_2}\left[(1-k)(\bar{X}-\mu_0)\right]f(\bar{X}|\mu,s^2)\;f(s^2|\sigma^2)\,ds^2\,d\bar{X}-\mu, \tag{17}$$

where $f(s^2|\sigma^2)$ is the pdf of s^2. The simplification of the expression of the bias leads to

$$B(\tilde{\mu}_2|\mu,R_2)=(\sigma/\sqrt{n})\left\{(2k-1)\int_0^{\infty}[J_1(a_2,b_2)-\lambda_2 J_0(a_2,b_2)]f(y)\,dy+k\lambda_2\right\}. \tag{18}$$

Using the above simplifications, the expression for the mean squared error is given by

$$MSE(\tilde{\mu}_2|\mu,R_2)=(\sigma^2/n)\left\{(2k-1)\int_0^{\infty}[J_2(a_2,b_2)-\lambda_2^2 J_0(a_2,b_2)]f(y)\,dy+k^2\lambda_2^2+(1-k)^2\right\}, \tag{19}$$

where $\lambda_2=\sqrt{n}\,(\mu_0-\mu)/s$, $a_2=\lambda_2-t_{\alpha/2,n-1}\sqrt{y/(n-1)}$, $b_2=\lambda_2+t_{\alpha/2,n-1}\sqrt{y/(n-1)}$ and y is a random variable with a chi-square disribution with $(n-1)$ degrees of freedom.
In particular when $\mu=\mu_0$ we have after some simple simplifications,

$$B(\tilde{\mu}_2|\mu_0,R)=0, \tag{20}$$

and

$$MSE(\tilde{\mu}_2|\mu_0,R_2)=$$
$$(\sigma^2/n)\left\{(1-k)^2-(2k-1)(2t_{\alpha/2,n-1})\left[\left(\frac{n}{2}\Big/\sqrt{\pi(n-1)}\,\sqrt{(n-1)/2}\right)\right]\left[1+t_{\alpha/2,n-1}^2\big/(n-1)\right]^{n-2}-(1-\alpha)\right\} \tag{21}$$

Remark 2:

Using the same method of Remark 1 the critical value of *k* (local minima) when σ^2 is unknown is given by

$$k_2=1-\frac{\int_0^{\infty}\int_{-t_{\alpha/2(s/\sigma)}}^{t_{\alpha/2(s/\sigma)}}(\bar{X}-\mu_0)^2\,f(\bar{X}/\mu_0,s^2)\;f(s^2/\sigma^2)\,d\bar{X}_1 ds^2}{MSE(\bar{X})}$$
$$=\alpha+\frac{2t_{\alpha/2,n-1}\,\Gamma(n/2)}{\sqrt{(n-1)\pi}\,\Gamma((n-1)/2)\left[1+t_{\alpha/2,(n-1)}^2/n-1\right]^{\frac{n}{2}}} \tag{22}$$

Also k_2 is free from the function of unknown parameters μ and σ^2 and also $0\le k_2\le 1$.

4. NUMERICAL RESULTS AND CONCLUSIONS

The bias ratio $\left(B(\tilde{\mu}_i|\mu_i,R_i)/(\sigma/\sqrt{n})\right)$, mean squared error and relative efficiency of the proposed estimators $\tilde{\mu}_1$ and $\tilde{\mu}_2$ i.e., $RE(\tilde{\mu}_1|\bar{X},R_1)=MSE(\bar{X})/MSE(\tilde{\mu}_1|\mu,R_1)$ and $RE(\tilde{\mu}_2|\bar{X},R_2)=MSE(\bar{X})/MSE(\tilde{\mu}_2|\mu,R_2)$

were computed for different values of constants involved in these estimators. The following conclusions are based on these computations:

Table 1: Relative efficiency $RE(\tilde{\mu}_1 | \bar{X}, R_1)$ and $B(\tilde{\mu}_1|\mu, R_1)/(\sigma/\sqrt{n})$ of proposed estimator $\tilde{\mu}_1$ for different values of α and $|\lambda_1|$

	$\lvert\lambda_1\rvert$						
α	0.0	0.1	0.2	0.3	0.4	0.5	0.6
0.002	44.843 (0.0)	30.797 (0.095)	16.203 0.183)	9.382 (0.261)	6.161 (0.325)	4.467 (0.375)	3.489 (0.410)
0.01	13.042 (0.0)	11.587 (0.084)	8.787 (0.162)	6.342 (0.230)	4.849 (0.287)	3.934 (0.330)	3.176 (0.363)
0.05	4.971 (0.0)	4.831 (0.059)	4.474 (0.116)	4.030 (0.169)	3.603 (0.216)	3.238 (0.260)	2.942 (0.301)
0.1	4.060 (0.0)	4.009 (0.051)	3.810 (0.101)	3.673 (0.150)	3.451 (0.200)	3.224 (0.249)	3.001 (0.299)

	$\lvert\lambda_1\rvert$					
α	0.7	0.8	0.9	1.0	1.5	2.0
0.002	2.881 (0.434)	2.483 (0.448)	2.210 (0.455)	2.018 (0.459)	1.597 (0.499)	1.113 (0.733)
0.01	2.739 (0.387)	2.440 (0.406)	2.228 (0.422)	2.069 (0.440)	1.502 (0.615)	0.905 (0.966)
0.05	2.701 (0.342)	2.498 (0.384)	2.315 (0.429)	2.141 (0.479)	1.306 (0.772)	0.861 (0.989)
0.1	2.781 (0.351)	2.561 (0.405)	2.341 (0.461)	2.123 (0.518)	1.240 (0.811	0.840 (0.857)

(i) For the estimator $\tilde{\mu}_1$, numerical computations were performed by taking the pretest region $R = \left[\mu_0 - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}, \mu_0 + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right]$ which is the acceptance region of size α for testing the hypothesis $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$, $\alpha = 0.02, 0.01, 0.05, 0.1, 0.15$ and $|\lambda| = 0.0(0.1)2.0.$ In Table 1 some sample values of the efficiency of $\tilde{\mu}_1$ relative to $\bar{X}$, and that of $(\sqrt{n}/\sigma)\, B(\tilde{\mu}_1|\mu, R)$ shown in brackets are given for some selected values of α and $|\lambda|$. It is evident from the computations that generally $Eff\,(\tilde{\mu}_1|\mu, R)$ increases as α decreases. Also as $|\lambda|$ increases, the efficiency decreases. It may be remarked that Hirano (1977) recommended using $\alpha = 0.15$. The results of Table 1 compare favorably with the classical estimator. The bias ratio is minimum when $\mu = \mu_0$ and increases with increases in λ.

(ii) For the estimator $\tilde{\mu}_2$, numerical computations were performed by taking the pretest region $R_2 = \left[\mu_0 - t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}, \mu_0 + t_{\alpha/2,n-1}\frac{s}{\sqrt{n}}\right]$ which is the acceptance region of size α for testing the hypothesis $H_0 : \mu = \mu_0$ against $H_1 : \mu \neq \mu_0$ when σ^2 is unknown, $\alpha = 0.02, 0.01, 0.05, 0.1, 0.15$, $|\lambda| = 0.0(0.1)2.0$ and $n = 6, 8, 10, 12$. Some of the computations are given in Table 2.

(iii) The relative efficiency of $\tilde{\mu}_2$ is a decreasing function of n for $0 \le |\lambda| \le 2.0$. The general behavioral pattern of estimator $\tilde{\mu}_2$ is similar to that of $\tilde{\mu}_1$ as the bias ratio and relative efficiency are concerned.

(iv) It is observed that our estimators $\tilde{\mu}_1$ and $\tilde{\mu}_2$ are better in terms of higher relative efficiency when $|\lambda_i| = 0$ and $|\lambda_i| \neq 0$ than the classical and existing shrinkage estimators.

Table 2: Relative efficiency $RE(\tilde{\mu}_2 | \overline{X}, R_2)$ and $B(\tilde{\mu}_2 | \mu, R) / (\sigma/\sqrt{n})$ of proposed estimator $\tilde{\mu}_2$ for different values of $|\lambda_2|$, n and α

		$\|\lambda_2\|$						
α	n	0.0	0.1	0.5	1.0	1.5	1.75	2.0
0.002	6	92.571 (0.0)	47.289 (0.024)	4.798 (0.123)	2.007 (0.371)	1.462 (0.452)	1.335 (0.512)	1.201 (0.652)
	8	77.503 (0.0)	42.529 (0.026)	4.740 (0.145)	2.007 (0.395)	1.453 (0.487)	1.330 (0.552)	1.191 (0.667)
	10	69.498 (0.0)	36.503 (0.028)	4.4668 (0.147)	2.003 (0.411)	1.444 (0.531)	1.382 (0.566)	1.183 (0.676)
0.01	6	21.113 (0.0)	17.466 (0.024)	4.297 (0.144)	2.007 (0.377)	1.449 (0.466)	1.252 (0.531)	1.005 (0.662)
	8	18.547 (0.0)	15.795 (0.027)	4.190 (0.158)	2.007 (0.399)	1.452 (0.493)	1.245 (0.576)	1.025 (0.672)
	10	17.196 (0.0)	14.679 (0.032)	4.116 (0.155)	2.002 (0.451)	1.441 (0.552)	1.238 (0.583)	1.000 (0.699)
0.1	6	4.355 (0.0)	4.277 (0.025)	3.207 (0.154)	2.005 (0.382)	1.265 (0.472)	0.990 (0.552)	0.808 (0.677)
	8	4.251 (0.0)	4.179 (0.031)	3.201 (0.156)	2.005 (0.413)	1.258 (0.478)	0.986 (0.579)	0.807 (0.692)
	10	4.198 (0.0)	4.124 (0.041)	3.201 (0.158)	2.002 (0.466)	1.254 (0.482)	0.985 (0.611)	0.802 (0.715)

ACKNOWLEDGEMENTS:
We are thankful to the referee and to the Editor, whose suggestions improved the paper considerably.

RECEIVED JULY 2008
REVISED JANUARY 2009

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