

Optimal election of qualities in the presence of externalities

Dolores Garcia* Maria Tugores†

October 2002

Abstract

In this paper we obtain the quality and the level of production that a certain sector would have if completely unregulated. We use a vertical differentiation model with variable costs. The market outcome has been compared to the social planner solution, when this accounts for the external costs that the production activity provokes on resident households and for the possibility that an important share of the consumer surplus goes to non-residents. From this perspective, it is found that an optimal solution allows for the coexistence of the high and low qualities, and that the optimal solution does not necessarily imply that the high quality segment should be expanded.

JEL classification: D62, L13, L15.

Keywords: quality, externalities, oligopoly models.

*Department of Economics, Universitat de les Illes Balears

†Reference author: Maria Tugores Department of Economics, Universitat de les Illes Balears, Ctra. Valldemossa Km.7.5, 07071 Palma de Mallorca, Balears, Spain. Tel:+34-971-173050; Fax:+34-971-173426; E-mail: mtugores@uib.es.

1 Introduction

The pattern of specialization is not irrelevant for the capacity of an economy to generate income and welfare. Besides the fact that firms, or the economy as a whole, specialize in certain sectors, the question of the quality level of the good it is sold is likewise important. From the political arena, at least, a great deal of attention is paid to these types of issues.

For the sake of illustration, let us look at a particular instance. In the Spanish context, the role of the specialization pattern may be important. In an economy specialized in tourism, the kind of market segments in which the economy ends up specialized may determine its basic competitiveness factors. Thus, a mass-tourism development strategy, usually means low quality services and therefore, the need of maintaining low prices and costs, with the consequent dependence on a large number of visitors that in aggregate generate congestion problems and externalities as large environmental, cultural and social costs. On the other hand, a high quality tourism development strategy implies competition based on differentiation and innovation, a higher price and a lower number of visitors and therefore, a larger potential for sustainable growth. High quality demand is more inelastic and related to high income consumers.

Given this, in several mature tourism destinations, highly dependent on mass-tourism, has grown concern about the need to change the pattern of specialization, shifting resources from low quality to high quality tourism services. In this context a social debate has arisen about the possible strategies to reduce the environmental and social costs of tourism development and guarantee its sustainability. In this debate, a structural change in favor of high quality tourism has drawn special attention given that more quality in the tourism services would presumably allow the same or higher income with a smaller number of tourists.

In order to clarify this question, we want to compare the quality and the level of production that a certain sector should have in different contexts of decision. We use a vertical differentiation model in which two firms compete in both quality and prices. We solve the market oligopoly equilibrium solution as well as the social planner problem. In order to capture the negative effect that this activity has on total welfare, we include an externality ef-

fect as well as the proportion of consumer's surplus that corresponds to the resident population. Comparing in terms of welfare, we derive the way in which a market should perform in order to gain in efficiency.

After having motivated the importance of modelling how the quality and the output should be chosen in a certain sector, in section 2 we develop an industrial organization model that we will use in order to provide answers to the pointed questions. In section 3 we solve the market equilibrium, in which two firms have to decide either prices or the level of production after having decided the quality to serve. In section 4, we solve the social planner problem, that takes into account not only the producers' surplus but also the consumer surplus of residents, as well as the externality effects that such an activity has on welfare. Finally, we highlight the main conclusions of the paper in section 5.

2 The Model

In this section we present the basic model we use. We consider the simple case of two firms producing a tourism product after having decided the quality of the good they are to produce. We use a vertical product differentiation model both under Cournot and Bertrand competition in which high quality is indexed as u_1 , and low quality as u_2 , with $u_1 > u_2$.

There is a continuum of consumers in the market. They differ in their tastes, described by the parameter $\theta \in [0, \bar{\theta}]$, θ being uniformly distributed with unit density. We define $\bar{\theta}$ as the consumer endowed with the higher taste for quality in the economy. Consumers have the same (indirect) utility function $U = \theta u - p$, if they buy one unit of the good of quality u at a price p , and zero utility if they do not buy the differentiated good. The higher the quality u of the good, the higher the utility U reached by the consumers for any given price p . However, consumers with a higher θ will be willing to pay more for a higher quality good. In accordance with the literature on product differentiation, we assume that consumers can buy at most one unit of the good.

Note that θ can be interpreted as the marginal rate of substitution between income and quality, so that a higher θ corresponds to a lower marginal utility of income and therefore a higher income (Tirole (1988)). Under this

interpretation, the model proposed here is the analog of the models where consumers differ by their incomes rather than by their tastes (Gabszewicz and Thisse (1979, 1980), Shaked and Sutton (1982, 1983), Bonanno (1986), Ireland (1987)).

We assume that there are only two firms in the industry, and that they compete with two strategic variables, their level of production and also the quality of the services they provide. It is likewise considered that there exists a lower bound to the quality level, so that $u_i > 0$. This can be interpreted as a minimum standard legal requirement or as being inherent to the production process. We further assume that current endowments and resources of the economy allow for the use of a maximum quality level, denoted by \bar{u} .

Each firm incurs a cost of the form $C_i(u_i, q_i) = \frac{u_i^2}{2}q_i$, that is, variable costs of quality improvement. This happens when the main burden of quality improvement falls, for instance, on more skilled labor or more expensive raw materials and inputs. We think that it is the case in the tourism sector. This type of costs function have been firstly analyzed by Mussa and Rosen (1978), Gal-Or (1983), and Champsaur and Rochet (1989).

In order to derive the demand expressions for the high and low quality good, we define the taste parameter of the consumer indifferent between buying the high and the low quality good, that is good 1 or good 2. His taste parameter $\theta_{1,2}$ is such that,

$$\theta_{1,2} = \frac{p_1 - p_2}{u_1 - u_2}. \quad (1)$$

The consumer indifferent between buying the low quality good, that is good 2, or not buying at all has the taste parameter $\theta_{0,2} = p_2/u_2$. For such consumer, the purchase of the good of quality u_2 will imply a zero utility level.

The demand functions can be easily built, noting that all the consumers for whom $\bar{\theta} > \theta \geq \theta_{1,2}$ will buy quality u_1 , all those described by $\theta_{1,2} > \theta \geq \theta_{0,2}$ will buy quality u_2 , and those described by $\theta_{0,2} > \theta$ will not buy at all. Notice that we allow for the possibility that the market is not covered, that is, that some consumers may not buy any of the goods, that is $0 < \theta_{0,2}$.

Then, the demand functions for the high and low quality firms are respectively given by:

$$\begin{aligned}
D_1(p_1, p_2) &= \bar{\theta} - \theta_{1,2} = \bar{\theta} - \frac{p_1 - p_2}{u_1 - u_2} \\
D_2(p_1, p_2) &= \theta_{1,2} - \theta_{0,2} = \frac{p_1 - p_2}{u_1 - u_2} - \frac{p_2}{u_2}.
\end{aligned} \tag{2}$$

3 The market equilibrium

In this section we consider the case in which the two firms (hotels) compete in the market as described in the following two-stage game. At the first stage, firms choose the quality of the good they want to produce. At the second stage, a competitive process occurs where firms choose quantities or prices¹. In section 3.1 we analyse the case of price competition at the last stage of the game, while in section 3.2 the case of quantity competition is dealt with. In equilibrium, it will be the case that firms always choose to offer distinct qualities. To solve the problem of firms, we look for the subgame perfect Nash equilibrium of the game. As usual, this will be obtained by backward induction.

3.1 Bertrand competition at the last stage

As we have shown in the previous section, quantities demanded to the high and low quality firm are given respectively by:

$$\begin{aligned}
q_1 &= \bar{\theta} - \frac{p_1 - p_2}{u_1 - u_2} \\
q_2 &= \frac{p_1 - p_2}{u_1 - u_2} - \frac{p_2}{u_2}.
\end{aligned} \tag{3}$$

Firms choose prices in order to maximise their profits $\Pi_i = p_i q_i - C_i$, for any given quality pair (u_1, u_2) . Computing first derivatives with respect to prices and solving the corresponding system of equations, results in the following:

$$\begin{aligned}
p_1 &= \frac{u_1(4u_1\bar{\theta} + 2u_1^2 - 4u_2\bar{\theta} + u_2^2)}{8u_1 - 2u_2} \\
p_2 &= \frac{u_2(2u_1\bar{\theta} + u_1^2 - 2u_2\bar{\theta} + 2u_1u_2)}{8u_1 - 2u_2}.
\end{aligned} \tag{4}$$

¹For further details see Motta (1993).

Profits at the first stage of the game can then be showed to be:

$$\begin{aligned}\Pi_1 &= \frac{u_1^2(u_1 - u_2)(2u_1 + u_2 - 4\bar{\theta})^2}{4(4u_1 - u_2)^2} \\ \Pi_2 &= \frac{u_1^2(u_1 - u_2)(u_1 - u_2 + 2\bar{\theta})^2}{4(4u_1 - u_2)^2}.\end{aligned}\tag{5}$$

The system of first-order conditions to be solved is then the following:

$$\begin{aligned}\frac{\partial \Pi_1}{\partial u_1} &= \frac{u_1(2u_1 + u_2 - 4\bar{\theta})(24u_1^3 - 22u_1^2u_2 + 5u_1u_2^2 - 2u_2^3 - 16u_1^2\bar{\theta} + 12u_1u_2\bar{\theta} - 8u_2^2\bar{\theta})}{4(4u_1 - u_2)^3} = 0 \\ \frac{\partial \Pi_2}{\partial u_2} &= \frac{u_1(u_1 - 2u_2 + 2\bar{\theta})(4u_1^3 - 19u_1^2u_2 + 17u_1u_2^2 - 2u_2^3 + 8u_1^2\bar{\theta} - 14u_1u_2\bar{\theta} - 8u_2^2\bar{\theta})}{4(4u_1 - u_2)^3} = 0.\end{aligned}\tag{6}$$

To find an analytical solution to this system does not seem to be easy. However, we have been able to find a solution once we specify the value of $\bar{\theta}$. For $\bar{\theta} = 1$ it is possible to show that the unique pair of values which solves the system above is given by²:

$$u_1^* = 0,81 \quad u_2^* = 0,39\tag{7}$$

The second derivatives of profits computed at this point are negative; further these values represent a Nash equilibrium of the game since they respect the following conditions:

$$\begin{aligned}\Pi_2(u_2^*, u_1^*) &\geq \Pi_2(u_2, u_1^*) \text{ for } u_2 \leq u_1^* \text{ and} \\ \Pi_2(u_2^*, u_1^*) &\geq \Pi_2(u_2, u_1^*) \text{ for } u_2 \geq u_1^*\end{aligned}\tag{8}$$

$$\begin{aligned}\Pi_1(u_1^*, u_2^*) &\geq \Pi_1(u_1, u_2^*) \text{ for } u_1 \geq u_2^* \text{ and} \\ \Pi_1(u_1^*, u_2^*) &\geq \Pi_1(u_1, u_2^*) \text{ for } u_1 \leq u_2^*\end{aligned}\tag{9}$$

A proof of this result can be found in the appendix below.

We can therefore summarize this result through the following proposition:

²Numerical computations have been performed using the Mathematica program. The exact solutions have more than 10 decimals.

Proposition 1 *Under the assumption of variable costs of quality improvement, the equilibrium of the game in which the duopolist firms first choose qualities and then prices, is such that a firm will select a quality $u_1^* = 0,81$, and the other the quality $u_2^* = 0,39$ (for $\bar{\theta} = 1$).*

3.2 Cournot competition at the last stage

In order to choose quantities at the last stage of the game we have to invert the system of demand functions shown in the previous section. This gives:

$$\begin{aligned} p_1 &= \bar{\theta}u_1 - q_2u_2 - q_1u_1 \\ p_2 &= (\bar{\theta} - q_1 - q_2)u_2. \end{aligned} \quad (10)$$

Then, the profit function for each firm can be written as:

$$\begin{aligned} \Pi_1 &= p_1q_1 - C_1 = (\bar{\theta}u_1 - q_2u_2 - q_1u_1)q_1 - \frac{u_1^2q_1}{2} \\ \Pi_2 &= p_2q_2 - C_2 = (\bar{\theta} - q_1 - q_2)u_2q_2 - \frac{u_2^2q_2}{2}. \end{aligned} \quad (11)$$

Firms choose quantities to maximize their profits, for any given quality pair (u_1, u_2) . First-order conditions are:

$$\begin{aligned} \frac{\partial \Pi_1}{\partial q_1} &= \bar{\theta}u_1 - q_2u_2 - 2q_1u_1 - \frac{u_1^2}{2} = 0 \\ \frac{\partial \Pi_2}{\partial q_2} &= (\bar{\theta} - q_1 - 2q_2)u_2 - \frac{u_2^2}{2} = 0. \end{aligned} \quad (12)$$

Hence, the optimal quantities produced by the high and the low quality firm result in

$$\begin{aligned} q_1 &= \frac{-2u_1^2 + u_2^2 + 4u_1\bar{\theta} - 2u_2\bar{\theta}}{8u_1 - 2u_2} \\ q_2 &= \frac{u_1^2 - 2u_1u_2 + 2u_1\bar{\theta}}{8u_1 - 2u_2}. \end{aligned} \quad (13)$$

At the first stage, firms choose qualities in order to maximise their profits (recall that unit production costs are a quadratic function of quality), given by:

$$\begin{aligned}\Pi_1 &= \frac{u_1(2u_1^2 - u_2^2 - 4u_1\bar{\theta} + 2u_2\bar{\theta})^2}{4(4u_1 - u_2)^2} \\ \Pi_2 &= \frac{u_1^2 u_2 (u_1 - 2u_2 + 2\bar{\theta})^2}{4(4u_1 - u_2)^2}.\end{aligned}\tag{14}$$

The first-order conditions are given by the following expressions:

$$\begin{aligned}\frac{\partial \Pi_1}{\partial u_1} &= \frac{(2u_1^2 - u_2^2 - 4u_1\bar{\theta} + 2u_2\bar{\theta})(24u_1^3 - 10u_1^2 u_2 + 4u_1 u_2^2 + u_2^3 - 16u_1^2 \bar{\theta} + 4u_1 u_2 \bar{\theta} - 2u_2^2 \bar{\theta})}{4(4u_1 - u_2)^3} = 0 \\ \frac{\partial \Pi_2}{\partial u_2} &= \frac{u_1^2 (u_1 - 2u_2 + 2\bar{\theta})(4u_1^2 - 23u_1 u_2 + 2u_2^2 + 8u_1 \bar{\theta} + 2u_2 \bar{\theta})}{4(4u_1 - u_2)^3} = 0.\end{aligned}\tag{15}$$

The symmetric solution to this system is given by $u_1 = u_2 = 2\bar{\theta}$. However, we can derive the second derivatives and check that $\frac{\partial^2 \Pi_1}{\partial u_1^2} = \frac{\partial^2 \Pi_2}{\partial u_2^2} = 4\bar{\theta}/9$, when computed in correspondence of the candidate maximum. In other words, choosing this same quality would give the firms a minimum profit.

We now turn to the question of whether a Nash equilibrium for this game exists at all. It is possible to show that the analytical expressions of u_1 and u_2 which simultaneously satisfy the two conditions mentioned above is not simple. As an example, consider $\bar{\theta} = 1$. In this case we can check that the following pair solves the system of equations: $u_1 = 1,73611$ and $u_2 = 0,710648$. But again, the second derivatives computed to this candidate are positive, leading us to a minimum.

Finally, the following pair also solves the system of equations written above:

$$u_1^* = 0,778638 \quad u_2^* = 0,587366\tag{16}$$

Further, the second derivatives computed at this candidate maximum are negative, being $\frac{\partial^2 \Pi_1}{\partial u_1^2} = -0,1850$ and $\frac{\partial^2 \Pi_2}{\partial u_2^2} = -0,2042$. So, this pair is a local maximum.

However, this is not enough to ensure we have found a Nash equilibrium. We also have to check that firm 2 has no incentive to leapfrog the rival firm and itself produce the highest quality. Likewise, we have to prove, that firm 1 has no incentive to deviate and produce a quality lower than the produced by firm 2.

The appendix contains the proof that the candidate solution above is indeed a Nash equilibrium, that is, satisfies the conditions mentioned above. We can therefore summarize this result through the following proposition:

Proposition 2 *Under the assumption of variable costs of quality improvement, the equilibrium of the game in which the duopolist firms first choose qualities and then quantities, is such that a firm will select a quality $u_1^* = 0,778638$, and the other the quality $u_2^* = 0,587366$ (for $\bar{\theta} = 1$).*

In Table 1 we show the equilibrium values under Bertrand and Cournot competition between this two firms:

Table 1: Market equilibrium values (for $\bar{\theta} = 1$).

Price competition	Quantity competition
$u_1^* = 0.81$	$u_1^* = 0.77$
$u_2^* = 0.39$	$u_2^* = 0.58$
$p_1^* = 0.45$	$p_1^* = 0.46$
$p_2^* = 0.15$	$p_2^* = 0.31$
$q_1^* = 0.27$	$q_1^* = 0.21$
$q_2^* = 0.34$	$q_2^* = 0.24$
$\Pi_1^* = 0.0328$	$\Pi_1^* = 0.0350$
$\Pi_2^* = 0.0242$	$\Pi_2^* = 0.0358$

As to the comparison with the results arising under the Bertrand hypothesis, we note that Cournot competition will give rise to less product differentiation at equilibrium. Note that in the Bertrand case a higher number of consumers is served, since the bottom quality firm tends to cover more of the bottom segment of the market. Further, under Bertrand behaviour the top quality on offer is higher. These features make competition on prices more beneficial to consumers than competition on qualities. However, profits are higher under quantity than under price competition, which is clearly driven by the tougher competition of the Bertrand case.

4 The Social Planner Outcome

Throughout this section we derive the optimal values of qualities and prices that a social planner would choose in order to maximize a certain objective function, under different scenarios with and without externalities. Then we compare this optimal solution to the market outcome, and comment on some policy making implications.

4.1 The objective function of the social planner

As it is common in the industrial organization literature, it has been assumed that the social planner cares about the overall welfare of the relevant population. This total surplus or welfare function typically consists of the sum of the producer and the consumer surplus. Although following this general framework, our welfare specification incorporates some specific features.

Our analysis includes several particularities, basically, we focus on the consequences on residents that may or may not be consumers. First, we consider that only a proportion of the consumers are local, while the remaining ones are foreigners (the good is to be exported). It has been assumed that the local social planner does not care about the surplus of non-resident consumers, and thus this will be ignored. Second, we assume that the consumption –or either, the production– of the good generates negative external effects that affect local residents, independently of whether they participate or not in the market³. With these two particularities in mind, the total surplus function can be written down as:

$$W = PS + \alpha CS - EXT, \quad (17)$$

where PS denotes the aggregate producer surplus; CS the consumer surplus, with α representing the share of consumer surplus that goes to resident consumers, $0 \leq \alpha \leq 1$; and EXT are the externalities caused by the consumption of the good. In our model, the aggregate producer surplus simply corresponds to the sum of profits of firms 1 and 2, that is

³Likewise, it could be considered that the consumption of the good causes external benefits, such as a net positive impact on other sectors of the economy. These possible positive indirect effects are not considered.

$$PS = \Pi_1 + \Pi_2, \quad (18)$$

which have already been used in the previous section. Regarding the consumer surplus, it corresponds to the sum of the differences between the willingness to pay of each consumer and the market price, for each market. The willingness to pay is affected by the quality level of the good as well by the taste parameter θ . Then, the consumer surplus can be expressed as

$$CS = CS_1 + CS_2 = \int_{\theta_{1,2}}^{\bar{\theta}} (\theta u_1 - p_1) d\theta + \int_{\theta_{0,2}}^{\theta_{1,2}} (\theta u_2 - p_2) d\theta, \quad (19)$$

which can more conveniently be expressed as

$$CS = \frac{1}{2}[q_1^2 u_1 + 2q_1 q_2 u_2 + q_2^2 u_2]. \quad (20)$$

As what regards the externality component of the total surplus function, different possibilities may be considered. For instance, it can be assumed that each consumed unit generates a certain external cost, and that such cost may or not be related to quality. In general terms,

$$EXT = EXT_1 + EXT_2, \text{ with } EXT_i = C_i^{EXT}(u_i, q_i), \quad (21)$$

where C^{EXT} stands for the externality function, and i denotes the market segment. We will consider three different scenarios, in which these external costs are decreasing, constant or increasing with the chosen quality level. In our view, the way in which quality affects the negative externality constitutes an empirical matter, which is beyond the scope of this paper. Three specifications of the externality component have been used. When the external costs are decreasing in the quality level, the assumption is that one unit consumed generates a smaller external cost if the quality is higher. If we think for instance of carbon emissions per mile, these are higher when vehicles are of a lower quality. We denote this situation with the superscript D . The total externality function is:

$$C_i^{EXT} = c(2 - u_i)q_i.$$

When the external costs are independent of quality (C), this argument does not enter the C_i^{EXT} function, which is

$$C_i^{EXT} = cq_i.$$

Finally, when C_i^{EXT} is increasing in the quality level (scenario denoted by the superscript I), we have assumed the following specification:

$$C_1^{EXT} = cu_i q_i.$$

This positive relationship between externalities and quality goes as follows. It implies that providing a unit output of relatively high quality causes higher marginal external costs. We believe that this is a reasonable assumption. For instance, it can be justified if it is considered that a high quality unit of output requires a higher consumption of natural resources, and this can be translated into external costs. The justification can also be linked to the way in which consumption changes with income. For normal goods, higher income levels imply higher demands, and as a by-product, higher external costs. Also, other things equal, higher incomes can be related to high quality demands. In our model, however, we impose that all consumers buy one unit of the good only. The fact that the externality increases with quality could be thought of as a way of capturing this latter effect.

This welfare function ultimately depends upon the prices and quality variables, that is

$$W = W(\phi; p_1, p_2, u_1, u_2),$$

where ϕ represents the set of parameters used. The problem of the social planner consists then in choosing the prices as well as the quality levels that should be implemented so that the welfare is maximized. Then,

$$\max_{p_1, p_2, u_1, u_2} W(\phi; p_1, p_2, u_1, u_2). \quad (22)$$

It is worth-mentioning that while the market outcome is sensitive to whether firms compete with quantities or with prices, this is not the case when undertaking the social planner problem. Thus, it can be shown that prices and quantities are interchangeable instruments for the government. In other words, if the social planner were choosing qualities and quantities,

the optimal levels of the decision variables do not vary compared to the scenario in which it chooses qualities and prices –the one here considered.

To solve this maximization problem, we proceed as follows. We derive the first order conditions of the problem, and find the optimal values of the variables involved. The same sequence of decisions used in the resolution of the market problem is here followed; that is, it is considered that the social planner first chooses the quality levels, and then determines prices⁴. Analytically, we find first optimal prices, and then solve for optimal qualities using backwards induction. In the following subsections, we develop the social planner problem in two cases, depending on whether negative externalities are or not present.

4.2 Optimal qualities and prices in the absence of externalities

Let us first find the optimal values of prices. The problem of the social planner in the last stage is

$$\max_{p_1, p_2} W(\phi; p_1, p_2, u_1, u_2) \quad (23)$$

By rearranging the first order conditions, and denoting with the superscript *SP* the choices of the social planner, it is found that:

$$p_1^{SP} = \frac{u_1(2(1-\alpha)\bar{\theta} + u_1)}{2(2-\alpha)} \quad (24)$$

$$p_2^{SP} = \frac{u_2(2(1-\alpha)\bar{\theta} + u_2)}{2(2-\alpha)} \quad (25)$$

Substituting these values into the social welfare function, this can be exclusively expressed in terms of the qualities u_1 and u_2 . It results

$$W(\phi; u_1, u_2) = \frac{u_1(4\bar{\theta}^2 - 4\bar{\theta}u_1 + u_1^2 + u_1u_2 - u_2^2)}{8(2-\alpha)}. \quad (26)$$

The problem in the first stage consists then in choosing the appropriated levels of quality. From the maximization of the function above with respect

⁴Other possibilities might be considered, for instance, that all variables are simultaneously chosen.

to qualities u_1 and u_2 it is found that a maximum exists⁵ when

$$\begin{aligned} u_1^{PS} &= \frac{4\bar{\theta}}{5} \text{ and} \\ u_2^{PS} &= \frac{2\bar{\theta}}{5}. \end{aligned}$$

Substituting back in the expressions for prices, they result in:

$$p_1^{SP} = \frac{4(7 - 5\alpha)\bar{\theta}^2}{25(2 - \alpha)} \quad (27)$$

$$p_2^{SP} = \frac{2(6 - 5\alpha)\bar{\theta}^2}{25(2 - \alpha)}. \quad (28)$$

Notice that there the high quality level doubles that of the low quality segment, and that qualities are independent of the specific portion of the consumer surplus that go to residents. This result generalizes when externalities are present, and always optimal qualities are the same irrespective of the α .parameter. With respect to prices, $p_1^{SP} > p_2^{SP}$ in all instances.

For the sake of comparison, we compare the market and the social planner scenarios for particular values of the parameters. In all instances, the taste parameter has been normalized to 1, that is $\bar{\theta} = 1$. With respect to α , we consider only the extreme cases in which the consumer surplus totally goes to foreigners, or totally to residents. Table 2 summarizes the computation results. The first column includes information that corresponds to the market equilibrium scenario developed in section 3, where the total surplus value has been computed. When there are no resident consumers, total surplus simply coincides with aggregate profits of firms, while when $\alpha = 1$, the whole consumer surplus is summed up.

Optimal qualities are not very different from their market equilibrium counterparts. Thus, the high quality chosen by the social planner is slightly lower compared to the market scenario, while the low quality results slightly higher. As a result, the quality gap decreases.

While optimal quality values do not depend on the α share of consumer surplus that go to resident consumers, quantities do. When there are no resident consumers quantities result smaller compared to the market outcome.

⁵The second order conditions are verified for all vectors of prices and qualities that hereafter constitute maximums, for specific values of the parameters. Thus, the second derivatives and the determinant of the hessian matrix have the required signs.

Table 2: Market and social planner outcomes (Computed for $\bar{\theta} = 1$, $c = 0$)

<i>Market Equilibrium</i>	<i>SP equilibrium</i> (for $\alpha = 0$)	<i>SP equilibrium</i> (for $\alpha = 1$)
$u_1^* = 0.81$	$u_1^{SP} = 0.80$	$u_1^{SP} = 0.80$
$u_2^* = 0.39$	$u_2^{SP} = 0.40$	$u_2^{SP} = 0.40$
$q_1^* = 0.27$	$q_1^{SP} = 0.20$	$q_1^{SP} = 0.40$
$q_2^* = 0.34$	$q_2^{SP} = 0.20$	$q_2^{SP} = 0.40$
$p_1^* = 0.45$	$p_1^{SP} = 0.56$	$p_1^{SP} = 0.32$
$p_2^* = 0.15$	$p_2^{SP} = 0.24$	$p_2^{SP} = 0.08$
$\Pi_1^* = 0.03281$	$\Pi_1^{SP} = 0.048$	$\Pi_1^{SP} = 0$
$\Pi_2^* = 0.02429$	$\Pi_2^{SP} = 0.032$	$\Pi_2^{SP} = 0$
$W^*(\alpha = 0) = 0.0571$	$W^{SP} = 0.08$	–
$W^*(\alpha = 1) = 0.151$	–	$W^{SP} = 0.16$

However, when all consumers are resident, quantities double compared to the case in which they are all non-resident. In the first scenario, the choice of the social planner coincides with that of a monopolist seeking to maximize total profits. This is because welfare and aggregate profits coincide, and it is a well-known result that the equilibrium choices when firms compete strategically are suboptimal. Instead, it can be shown that for $\alpha = 1$, the social planner set prices that coincide with the respective marginal costs of firms, and profits equal zero. For other positive values of α inferior to 1, intermediate results should be expected.

With respect to the aggregate surplus, and as expected, the social planner solutions yield higher welfare values than those derived from the market scenario.

4.3 Optimal qualities in the presence of externalities

Again, we compare some of the results, now focusing on the social planner outcomes in the presence of externalities. We analyze how the optimal choices of the social planner change in varying the way in which the externality costs are affected by the quality of the product. As mentioned before, in the case of increasing marginal external cost, total welfare is negatively affected both by the level of production and the level of quality; we have constant marginal externalities when total welfare is negatively affected by the

level of output, but the quality level does not matter; finally, it is considered the case in which per unit externalities decrease with quality.

Table 3 shows the computed values of the optimal variables for the case in which there are no resident consumers⁶.

Table 3: Different social planner outcomes (Computed for $\bar{\theta} = 1$)

<i>Market solution</i>	<i>Increasing (I)</i> $c = 0.3, \alpha = 0$	<i>Constant (C)</i> $c = 0.3, \alpha = 0$	<i>Decreasing (D)</i> $c = 0.3, \alpha = 0$
$u_1^{SP} = 0.81$	$u_1^I = 0.56$	$u_1^C = 0.94$	$u_1^D = 1.14$
$u_2^{SP} = 0.39$	$u_2^I = 0.28$	$u_2^C = 0.83$	$u_2^D = 1.10$
$q_1^{SP} = 0.27$	$q_1^I = 0.14$	$q_1^C = 0.05$	$q_1^D = 0.08$
$q_2^{SP} = 0.34$	$q_2^I = 0.14$	$q_2^C = 0.05$	$q_2^D = 0.01$
$p_1^{SP} = 0.45$	$p_1^I = 0.44$	$p_1^C = 0.84$	$p_1^D = 1.03$
$p_2^{SP} = 0.15$	$p_2^I = 0.20$	$p_2^C = 0.73$	$p_2^D = 0.99$
$\Pi_1^{SP} = 0.032$	$\Pi_1^I = 0.039$	$\Pi_1^C = 0.022$	$\Pi_1^D = 0.032$
$\Pi_2^{SP} = 0.024$	$\Pi_2^I = 0.022$	$\Pi_2^C = 0.022$	$\Pi_2^D = 0.005$
–	$W^I = 0.027$	$W^C = 0.010$	$W^D = 0.011$

It can be observed that the optimal qualities are higher when the per unit externality is independent or decreasing with quality. Only when accounting for the positive effect of quality on the marginal external cost (I) both, the high and the low quality level decrease with respect to the market outcome.

Quantities decrease in all sectors, even in scenarios C and D , when qualities are to be increased. This fact suggests that there is not a substitution of low by high quality production, but rather, that the production significantly diminishes in all sectors. In cases C and D , prices are higher, since outputs have diminished and qualities have raised. In this two cases, profits diminish for both firms. The intuition is that the diminishment of total external costs exceeds the reduction in the producer surplus.

Under the assumption that higher qualities provoke larger per unit externalities (Case C), quantities diminish as well, but to a lesser extent. The effect on prices and profits, is likewise less important.

⁶The procedure to find the optimal values of the choice variables (qualities and prices) is analogous to the one described in section 4.2, with no externalities. We omit the development of the resolution of the model for the case with externalities because an analytical solution could only be found when the per unit external cost is increasing in quality. For the remaining two cases, we compute the results numerically.

Table 4 shows the computed values of the variables when residents not only suffer the externality, but also consume the output.

Table 4: Different social planner outcomes (Computed for $\bar{\theta} = 1$)

<i>Market solution</i>	<i>Increasing (I)</i> $c = 0.3, \alpha = 1$	<i>Constant (C)</i> $c = 0.3, \alpha = 1$	<i>Decreasing (D)</i> $c = 0.3, \alpha = 1$
$u_1^{SP} = 0.81$	$u_1^I = 0.56$	$u_1^C = 0.94$	$u_1^D = 1.14$
$u_2^{SP} = 0.39$	$u_2^I = 0.28$	$u_2^C = 0.83$	$u_2^D = 1.10$
$q_1^{SP} = 0.27$	$q_1^I = 0.28$	$q_1^C = 0.11$	$q_1^D = 0.17$
$q_2^{SP} = 0.34$	$q_2^I = 0.28$	$q_2^C = 0.11$	$q_2^D = 0.03$
$p_1^{SP} = 0.45$	$p_1^I = 0.32$	$p_1^C = 0.74$	$p_1^D = 0.91$
$p_2^{SP} = 0.15$	$p_2^I = 0.12$	$p_2^C = 0.64$	$p_2^D = 0.87$
$\Pi_1^{SP} = 0.032$	$\Pi_1^I = 0.047$	$\Pi_1^C = 0.033$	$\Pi_1^D = 0.044$
$\Pi_2^{SP} = 0.024$	$\Pi_2^I = 0.023$	$\Pi_2^C = 0.033$	$\Pi_2^D = 0.008$
-	$W^I = 0.054$	$W^C = 0.021$	$W^D = 0.023$

As in the previous subsection, qualities will not vary when changing the α proportion. Logically, optimal quantities increase compared to the $\alpha = 0$ assumption. Because prices are lower, consumers end up buying more units, and quantities increase –other things equal. In aggregate, profits increase for both, the high and the low quality firm.

Again, the optimal qualities are higher than in the market solution when the per unit externality is independent or decreasing with quality and lower when accounting for a positive effect of quality on the marginal external cost. Total quantities still decrease in all the cases.

What conclusions can be obtained from the numerical results presented above? Firstly, results should be carefully interpreted until they prove robust to different specifications of the externality function and additional numerical simulations are performed. This having been said, from our results it seems to arise the idea that qualities should be higher than those the market provides with imperfect competition *when*, the marginal external cost decreases or is constant in u_i .

Instead, if the assumption of per unit external costs increasing with quality is plausible, then recommendations that suggest that firms should devote their efforts to invest in improving the quality of their products do not seem to be supported by theory. This is indeed true when externalities

are absent, since qualities practically do not change compared to the market outcome.

We summarize the previous results in the two following statements:

1. In markets with vertical differentiation and in which externalities are present, a contraction of both the high and low quality segments generally results welfare-improving.
2. In markets with vertical differentiation in which externalities are present, the market equilibrium qualities are suboptimally low if the per unit external cost decreases or is independent of quality. Instead, they result suboptimally high when the per unit externality is increasing in quality.

Our results suggest that little general claims can be made in terms of policy recommendations with respect to the distribution of qualities in the market. Policies directed to increasing qualities in both segments, compared to the market scenario, seem to be justified exclusively when the per unit externality decreases or does not vary with quality. Even in these instances, the social planner solution allows for the co-existence of both segments of the market: the high and the low quality ones. If instead the per unit external cost is increasing in quality, it is lower qualities that should be encouraged through policy.

More interestingly, even when qualities should be made higher, the activity in the market is also diminished. Thus, the recommendation is not to devote more resources to the high quality segment and less resources to the low one; rather, the optimal policy should discourage activity in both segments.

5 Summary and Conclusions

In this paper we investigate the effects that the consideration of the residents' consumer surplus and of negative externalities caused by the production activity have on the optimal decision on the quantity and quality of a certain good. It has been considered a theoretical framework with a vertical differentiation model in which two firms compete in quality and prices. In the market outcome, it turns out that two different qualities exist in equilibrium.

We solve the social planner problem first considering that there are no externalities affecting the welfare of residents. The result in this case is that the high quality chosen by the social planner is slightly lower compared to the market scenario, while the low quality is slightly higher. Adding consumers to the welfare function logically provokes an increase in the optimal output levels. While in the market solution the low quality firm provides more production than the high quality firm, in the social planner solution quantities to be provided by each firm are equal. Of course, total welfare is suboptimal in the market scenario.

Introducing externalities into the welfare function changes results. Total welfare falls down. Qualities are higher than those the market provides when the external costs per unit decreases or is independent of quality. However, if the assumption of per unit external costs increasing with quality is plausible, then this result is reversed.

In all the scenarios we have studied, the social planner solution allows for the co-existence of both segments of the market: the high and the low quality ones. In particular, even when qualities should be made higher, the activity in the market is also diminished. Thus, the policy recommendation is not to devote more resources to the high quality segment and less resources to the low one; rather, the optimal policy should discourage activity in both segments.

We plan to dedicate our further research precisely to study some of political instruments that could be used in order to achieve the social planner outcome. As we have shown, this type of intervention should take into account the way in which quality provokes external costs. Alternative functional forms and assumptions can also be considered to describe the external costs derived from the production.

References

- [1] Bonanno, G., (1986), Vertical Differentiation with Cournot Competition, *Economic Notes*, 15, p.68-91.
- [2] Champsaur, P. and J-C. Rochet, (1989), Multiproduct Duopolists, *Econometrica*, 57, p.533-57.

- [3] Gabszewick, J. and J-F. Thisse, (1979), Price Competition, quality, and income disparities, *Journal of Economic Theory*, 20, p.340-59.
- [4] Gabszewick, J. and J-F. Thisse, (1980), Entry (and exit) in a differentiated industry, *Journal of Economic Theory*, 22, p.327-38.
- [5] Gal-Or, E., (1983), Quality and quantity competition, *Bell Journal of Economics*, 14, p.590-600.
- [6] Ireland, N., (1987), *Product Differentiation and non-price competition*, Basil Blackwell.
- [7] Motta, M., (1993), Endogenous quality choice: price vs. quantity competition, *The Journal of Industrial Economics*, XLI, p.113-31.
- [8] Shaked, A. and J. Sutton, (1982), Relaxing price competition through product differentiation, *Review of Economic Studies*, 49, p.3-14.
- [9] Shaked, A. and J. Sutton, (1983), Natural Oligopolies, *Econometrica*, 51, p.1469-83.
- [10] Tirole, J., (1988), *The theory of industrial organization*, The M.I.T. Press.

Appendix

Proof of Proposition 1

We first prove (a) that the low firm has no incentive to leapfrog the high quality, and then (b) that the high quality firm has no incentive to select a quality lower than its rival.

(a) If the low quality firm decided to provide a quality higher than u_1^* and leapfrog its rival, it would obtain the following profits:

$$\Pi_D = \frac{u_D^2(u_D - u_1^*)(2u_D + 2u_1^* - 4\bar{\theta})^2}{4(4u_D - u_1^*)^2}$$

Then, is easy to see that:

$$\Pi_D(u_1 = u_1^*, u_2 = u_D = 0, 82) = 0, 00003 < \Pi_2(u_1 = u_1^*, u_2 = u_2^*) = 0, 0242$$

(b) If the top quality firm decided to deviate from the proposed equilibrium to produce a quality lower than its rival, it would earn:

$$\Pi_D = \frac{u_2^* u_D (u_2^* - u_D) (2u_2^* - u_D + 2\bar{\theta})^2}{4(4u_2^* - u_D)^2}$$

In this case,

$$\Pi_D(u_1 = u_D = 0,38, u_2 = u_2^*) = 0,0019 < \Pi_1(u_1 = u_1^*, u_2 = u_2^*) = 0,0328$$

There are no incentives to deviate. Proposition 1 is then proved.

Proof of Proposition 2

As we have done above, we first prove (a) that the low firm has no incentive to leapfrog the high quality, and then (b) that the high quality firm has no incentive to select a quality lower than its rival.

(a) If the low quality firm decided to provide a quality higher than u_1^* and leapfrog its rival, it would obtain the following profits:

$$\Pi_D = \frac{u_D (2u_D - u_1^* - 4u_D \bar{\theta} + 2u_1^* \bar{\theta})^2}{4(4u_D - u_1^*)^2}$$

Then, it is possible to see that:

$$\Pi_D(u_1 = u_1^*, u_2 = u_D = 0,78) = 0,0322 < \Pi_2(u_1 = u_1^*, u_2 = u_2^*) = 0,0358$$

(b) If the top quality firm decided to deviate from the proposed equilibrium to produce a quality lower than its rival, it would earn:

$$\Pi_D = \frac{u_2^* u_D (u_2^* - 2u_D + 2\bar{\theta})^2}{4(4u_2^* - u_D)^2}$$

In this case,

$$\Pi_D(u_1 = u_D = 0,57, u_2 = u_2^*) = 0,0325 < \Pi_1(u_1 = u_1^*, u_2 = u_2^*) = 0,0350$$

There are then no incentive for both the top and the low quality firm to deviate. Proposition 2 is then proved.