Local provision of education with opting-out: The role of housing markets and neighbourhood effects

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Abstract

This paper analyses the consequences of introducing private alternatives in a multi-community system with local provision of education. The objective is to reassess two important questions: first, which households opt-out of public education in decentralised education systems with opting-out? second, where do households using private schooling choose to live? With this objective, the model in Bearse, Glomm and Ravikumar (2001) is first extended to an economy with housing markets, property taxation and J communities. This extension proves to be relevant modifying results in the previous literature in two important respects: first, households from more than one income interval may choose the private alternative; second, the richest fraction of households in the economy may not be among households who prefer such option. The model is then extended to encompass neighbourhood effects. Neighbourhood effects are shown to play a key role into the analysis, strongly reducing incentives households in the private sector have to mix with poor households in the public system. Rich (poor) households with children in private schools tend to live with rich (poor) households with children in public schools. It is shown that neighbourhood effects can lead to equilibria showing perfect income stratification either across communities or across educational sectors.

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‡ The author is deeply indebted to Gianni De Fraja for helpful comments, discussions and guidance. This paper would have not been possible either without the encouragement and guidance of my research directors Manuel Martín Rodríguez and Rogelio Velasco Pérez. An earlier version of this paper was presented at the conference of the Public Economic Theory Association (PET'02) in Paris in july 2002. Comments by Charles De Bartolome, Fabien Moizeau and assistants to this conference are also gratefully acknowledged.

1 Introduction

In response to a sizeable decline in students' achievement, recent years have witnessed the emergence of a hot debate about school finance reform in many countries (US, UK and Spain are three examples). During the last decade, the role economists have played in this debate has been continuously increasing. Theoretical and empirical research on the economics of education has contributed to shed light on important issues such as the consequences of peer group effects on sorting across schools and on schools performance, the impact of decentralisation and other forms of school choice on public schools productivity or the likely effects of vouchers programs. The introduction of private schools vouchers programs, aimed at increasing school choice and competition among schools, is one of the most important proposals for primary and secondary school finance reform. For this reason, a voluminous and growing literature focuses on the consequences of introducing private alternatives and different types of vouchers under alternative education finance regimes.

When private alternatives to public education are made available in single jurisdiction economies, two questions emerge: first, which households exert choice and opt-out of the public system? Second, what are the consequences over funding levels and over the distribution of educational benefits across households? Public provision of education with opting-out in a single-jurisdiction framework has been investigated, among others, by Stiglitz (1974), Glomm and Ravikumar (1998) and Epple and Romano (1996). Epple and Romano show that when private alternatives are available, households from the richest income interval in the economy usually opt-out of the public system in order to acquire private education of higher quality. Moreover, the existence of private alternatives reduces political support for and effective public expenditures in education. Their computational model suggests that the impact of vouchers on public expenditures per student is not important, though they significantly increase private school attendance.

The main feature of multi-jurisdictional economies is that school choice can be exerted not only among public and private producers but also between communities within the public sector. Obvously, households that use a private school also have to choose a community in which to reside. Thus, another important question arises in the multi-community case: how do households sort into jurisdictions?

Nechyba (1999, 2002a, 2002b) and Bearse et al. (2001) have investigated the workings of decentralised systems of public education with opting-out. Their answer to the first question posed above coincides with that obtained in the single jurisdiction case: wealthy households are the ones choosing private schooling. They opt-out in order to receive an education of higher quality than that offered in the public sector. For these households, there is no link between residential location and the quality of education their youths receive. Consequently, they show a tendency to move to low housing price communities and to mix there with poor households.

Nechyba (1999, 2002a, 2002b) develops a rich computational model with housing markets and peer group effects operating at the school level and shows that, under certain circumstances, introducing private schools and encouraging attendance to these schools through vouchers can be equity enhancing. The reason is the arrival of wealthy households that use a private school to poor communities. This can increase local expenditures in education because it rises the community fiscal base and reduces attendance to the local public school. This is so even though it reduces political support for public expenditures. In these papers, Nechyba also finds that vouchers systems are likely to rise school quality.

Bearse et al. (2001), in a paper which is central to the present one, aim to evaluate relative performance of centralised and decentralised education finance systems when private alternatives are present in a dynamic framework. Their model sheds light on the role of private alternatives when comparing centralised to decentralised regimes. This role turns out to be important because in the centralised case a larger percentage of households opt-out from the public sector, leading to lower public and total expenditures in static and dynamic economies. Moreover, although households below the 10th percentile lose with decentralisation, overall inequality is lower under this regime for the same reason as in Nechyba's contributions. These results are in stark contrast to those obtained in models without private alternatives (Fernández and Rogerson, 1998 and Bénabou, 1996a, 1996b), in which centralised education finance has been shown to be superior on equity and even efficiency grounds.

The current paper adopts a theoretical perspective to investigate the role of housing markets and neighbourhood effects over households distribution to communities and schools in a multi-community system with local provision of education and opting-out. In particular, we aim to reassess two of the questions that must be answered to understand the impact of private alternatives and vouchers programs in multi-jurisdictional economies: first, which households opt-out of public education in decentralised education systems with opting-out? in other words, do the rich (and only they) always choose private education in such settings? Second, why households that choose private schooling do not typically live in poor communities among poor households?

With these objectives, the model in Bearse et al. (2001) is extended to an economy with housing markets, property taxation, neighbourhood effects and communities. The analysis demonstrates that both housing markets and neighbourhood effects play a key role in the allocation of households to communities and schools. On the one hand, introducing housing markets in a model without neighbourhood effects shows: first, that households from more than one income interval may choose the private alternative and second, that the richest fraction of households in the economy may not be among these households. On the other hand, neighbourhood effects provide a more essential explanation of the fact that households with children in private schools typically do not live in poor communities. Sociological externalities operating at the community level starkly reduce the mixing of rich and poor households and can drive the economy back to perfectly stratified equilibria either across communities, as in classical multi-community models without opting-out (Epple, Filimon and Romer, 1984; 1993), or by schools, as in single-jurisdiction models with private alternatives (Epple and Romano, 1996).

The paper is organised as follows. The next section introduces the basic multi-community model with local provision of education, opting-out and housing markets. This model is analysed in section 3 to answer the question of who opts-out of public education. In section 4, neighbourhood effects are introduced into the basic model to investigate the question of why households in the private sector do not mix with poor ones. Finally, section 5 contains some concluding remarks and discussion.

2 The basic model

2.1 Communities and housing markets

There exists a finite and exogenous number of communities, J, with fixed boundaries, which may differ in the amount of land contained within their limits. Every community has a competitive local housing market. We adopt a simple specification of housing markets borrowed from Epple and Romano

(2002). Houses are homogenous and each household consumes one (and only one) unit of housing at p_h. Every community i has a backward-L housing supply, horizontal at c (where c is the common construction cost) until community land capacity is reached and vertical at that quantity. Each house requires one lot of land. It is assumed that the economy housing capacity is just enough to house the population.

2.2 Education system and taxation

Education is treated as a private good. Educational services are produced from the numeraire, following a technology of production which exhibits constant returns to scale with respect to the number of students and the quantity produced. The cost function $c(x, n) = x \cdot n$ captures this technology. For simplicity sake, it assumes away the influence of peer effects and other inputs such as student ability and effort in the production of education. Moreover, because efficiency differences between public and private schools are not of interest to the model, the technology of production is assumed to be common for public and private producers of education.

Each jurisdiction may impose a proportional property tax, with tax rate t, on the value of housing and use the proceeds to provide public educational services (E). Each community i chooses the pair (E_i, t_i) through a political process, simplified to majority voting. Besides the decentralised public system, there exists a private competitive market for education. Every household can opt-out of the public system and acquire any amount of education in the private market at competitive price $p_x = 1$. As usual in models of education, we consider public and private alternatives as mutually exclusive. A relevant difference between public and private education is that, while a household can acquire as many units of private education as it desires, regardless its community of residence, it is only allowed to send the children to a community's local public school if it resides and pays taxes in it.

2.3 Households and preferences

The system of jurisdictions is inhabited by a continuum of households, each composed by one adult, the decision-maker, and one school-aged children. The population mass is normalised to one. Households are perfectly mobile between jurisdictions and only differ in their exogenously determined endowment of the numeraire, y (income). Income is thus independent of residential location choices[1]. The income distribution is characterized by a continuous density function, $f(y)$.

There are three commodities: education (x), a private composite good (b), used as the numeraire, and housing (h). Because all houses in the system are homogenous and each family consumes one unit of housing, this good can be ignored in the utility function which represents households' preferences over different bundles in the economy.

Axiom 1 *Households share the same preferences represented by a utility function separable in* x *and* b *and increasing in all its arguments:* $U(x, b) = u(x) + z(b)$. $u(x)$*and* $z(b)$*are both twice continuously differentiable in* (x, b) *over all* $(x, b) >> 0$. $u(x)$ *is strictly quasi-concave, while* $z(b)$ *is strictly concave.*

Preferences are, therefore, rational, continuous, strictly convex over and strictly monotone. An important consequence of axiom 1 is that educational services and the numeraire are normal goods.

Axiom 2 $\lim_{x \to 0} u(x) = -\infty$ *and* $\lim_{b \to 0} z(b) = -\infty$

Axiom 2 is for technical convenience. It ensures that any strictly positive combination of is strictly preferred to any bundle with one of the goods equal to zero.

2.4 Households' decision problem and timing

Each household's adult must adopt the following decisions: (i) choose the community in which to reside; (ii) decide to send her child to the local public school or to a private school somewhere; (iii) vote on the pair (E, t) in the

[1] This assumption is typical in multi-community models and makes them most accurate for explaining the workings of urban economies with multiple jurisdictions. In such settings the place of work imposes a weak restriction on the choice of location.

community in which the household resides; and, if her child attends a private school, (iv) allocate income between private education and numeraire consumption. Households are atomistic and, consequently, adults behave as price-takers.

These decisions are made in two stages within a single period. In the first stage, households simultaneously choose communities and schools, taking into account their (correct) expectations over the equilibrium vector of public policies and housing prices $e^* = (E_1, t_1, p_h^1, ..., E_J, t_J, p_h^J)$. In this stage, since the supply of housing is fixed, local housing markets clear. In the second one, once residence and schools decisions are committed, households vote on their community education policy. This sequence of decisions, also found in Nechyba (1999) and Epple and Romano (2002), is essential for solving the non single-peakedness problem that arises in models of public provision of education with opting-out (Stiglitz, 1974). We will come back to this question later on in the paper.

2.5 Definition of equilibrium

In this model, an equilibrium is a partition of households across communities and schools, an allocation (x, b) across households and a vector of community policies and housing prices $e^* = (E_1, t_1, p_h^1, ..., E_J, t_J, p_h^J)$ which satisfy:

1. *Rational choices*: for each household j, the pair (x_j, b_j) associated to the chosen community and school maximises utility among the household feasible set. This implies that no household wants to move to another community or to shift school.

2. *Housing market equilibrium*: housing demand equals housing (fixed) supply in every community.

3. *Majority voting equilibrium*: for all i=1,2,...,J, the pair (E_i, t_i) is majority-preferred by voters in community i, given the partition of households across communities and schools and the price of housing in the community. A pair (E_i, t_i) is majority preferred in community i if the associated pair $(E, p_h^i(1 + t_i))$ is on the community Government Possibility Frontier (GPF) as defined in section 3.3 (i.e. if it satisfies the government budget constraint), and if it is preferred by 50 percent or

more of community i voters in a binary comparison against any other bundle on the GPF.

3 Who opts-out of public education?

In this section, we analyse the basic model and show that in multi-community economies with local provision of education and opting-out a large array of stratification patterns can arise in equilibrium. The analysis leads to some counter-intuitive results. For example it reveals the existence of equilibria in which households from intermediate income intervals choose private schooling, while households from the richest tail of the income distribution send their youths to a (high quality) local public school.

3.1 Induced preferences

In order to clarify how households distribute themselves across communities and schools, we first obtain indirect utility functions corresponding to households choosing public and private education, respectively. From a household point of view, communities are characterised by the pair $(E, p_h(1+t))$, i.e. by the combination of expenditures per student (which ascertains the quality of the local public school) and the gross-of-tax price of housing (which determines the maximum feasible level of private consumption in the community. For this reason, such indirect utility functions are used to depict the indifference map in that space.

On the one hand, a household's decision-maker that sends her children to the local public school does not acquire any private education and, from strict monotonicity, devotes $y - p_h(1+t)$ to consumption of the private composite commodity. The corresponding indirect utility function is therefore:

$$v(E, y-p) = u(E) + z(y - p_h(1+t)) \tag{1}$$

Let p be equal to $p_h(1+t)$ and $M(E, y-p)$ be the slope of indifference curves in this space. This slope is given by:

$$\frac{dp}{dE}|_{v=\overline{v}} = M(E, y{-}p) = -\frac{v_E(E, y-p)}{v_p(E, y-p)} = \frac{u'(E)}{z'(y-p)} > 0 \tag{2}$$

It is equal to the marginal benefit of education in terms of the numeraire. This implies that, in response to a marginal increase in E, a household is willing to accept an increase in the gross-of-tax price of housing equal to the marginal benefit it obtains from education[2].

On the other hand, if the household chooses private education, a demand function for private education, $x(y-p)$, must be obtained. The indirect utility function can then be written as:

$$w(y-p) = u(x(y-p)) + z(y-p-x(y-p)) \qquad (3)$$

In this case, because the child does not attend the local public school, marginal benefit of public educational services is zero and, therefore, indifference curves in (E,p) space are flat at each level of p.

For a utility-maximizing household choosing between public and private schooling in a given community, the induced utility function is,

$$V(E,y-p) = \max\left[v(E,y-p), w(y-p)\right] \qquad (4)$$

The complete indifference map in (E,p) space is in figure 1. $\hat{E}(y-p)$ is the locus of points at which the household is exactly indifferent between public and private schooling. For each pair $(y-p)$, there is only one level of E at which this is satisfied[3]. The indifference map sketched in figure 1 is analogue to that in Epple and Romano (1996)[4]. It shows that, given the gross-of-tax price of housing in the community, p, a household with income y prefers private education for low enough levels of public provision ($E < \hat{E}(\cdot)$), is exactly indifferent between the local public school and private schools for $E = \hat{E}(\cdot)$, and prefers public education for large enough amounts of public educational services ($E > \hat{E}(\cdot)$). The flat part of indifference curves corresponds to the range in which the household prefers the private sector

[2]Furthermore, indifference curves of $v(E,y-p)$ in (E,p) space are strictly concave:
$$\frac{d^2p}{dE^2}|_{v=\overline{v}} = \frac{u''(E)z'(y-p)^2 + u'(E)^2 z''(y-p)}{z'(y-p)^3} < 0$$
because strict quasiconcavity ensures that,
$u''(E)z'(y-p)^2 + u'(E)^2 z''(y-p) < 0$.

[3]First of all, note that continuity of $U(\cdot)$ implies continuity of $v(\cdot)$, $w(\cdot)$ and $V(\cdot)$. Assumption 2 implies that, for any p, when $E = 0$ every household in the community consumes some amount of private education. Moreover, for each pair (y,p), there is a level of E above which the household prefers public education (this is clear because at $E = x(y-p) > 0$, strict monotonicity ensures that $v(\cdot) > w(\cdot)$. Thus, because $U(x,b)$ is continuously increasing in E, there is a unique level of public provision of education in the community at which the household is indifferent between the public and a private alternative. $\hat{E}(y-p)$ is therefore implicitly defined by the equality $v(\hat{E}(y-p), y-p) = w(y-p)$.

[4]The only difference is in the vertical axis. In both cases this axis represents a cost variable: in Epple and Romano it is the income tax rate, while in this paper it is the gross-of-tax price of housing.

and, thus, is indifferent with respect to E. The increasing part corresponds to the range in which the household uses a public school. In such range, an increase in p must be compensated with an increase in E in order to maintain indifference. For any indifference curve, the upper contour set is below it and the lower contour set is above it. Lemma 3 analyses the behaviour of $\hat{E}(y{-}p)$.

Lemma 3 *$\hat{E}(y-p)$ is everywhere increasing in $y-p$.*

Proof. Differentiate with respect to $y-p$:
$d(y{-}p)\left(u'(\hat{E}(y-p))\hat{E}'(y-p)+z'(y-p)\right)=d(y{-}p)\left(z'(y-p-x(y-p)\right).$
Solve for $\hat{E}'(y-p)$ to obtain:

$$\hat{E}'(y{-}p)=\frac{z'(y-p-x(y-p)-z'(y-p)}{u'(\hat{E}(\cdot))}>0 \qquad (5)$$

Given that axiom 2 assures a strictly positive demand for private education when private schooling is chosen, the latter inequality is guaranteed by concavity of $z(\cdot)$. ■

From lemma 3 it is immediate to establish:

Corollary 4 *Within-communities perfect income stratification across schools. If for any (E_j,p_j) a household with income y residing in community j weakly prefers private to public education, then all households with income $y\prime>y(y"<y)$ residing in that community strictly prefer the private (public) option.*

Proof. Let y be such that, given (E_j,p_j), households with income equal to y in community j are indifferent between private and public education. This entails that $E=\hat{E}(y-p)$. Lemma 3 proves that $E=\hat{E}(\cdot)$ rises monotonically with income. Hence, all households with income $y'>y$, satisfy $E<\hat{E}(y'-p)$, and they strictly prefer the private alternative. Similarly, for all households with income $y"<y$, $E>\hat{E}(y''-p)$, and, therefore, they strictly prefer the public option. ■

Corollary 4 shows that, in equilibrium, mixed communities are characterised by perfect income stratification across schools. The poorest households send their children to the local public school and the richest ones opt-out of the public system and acquire private education. This result is a standard prediction in the literature (e.g. Epple and Romano, 1996 and Bearse et al., 2001). Lemma 3 has another important implication: for any level of income, the higher the gross-of-tax price of housing the smaller the amount of public educational services above which the household prefers the local public school. That is to say, other things equal, increases in housing prices or in tax rates have a direct negative impact on private school attendance in a given community.

3.2 Rational residential choices

We now turn to the analysis of optimal residential choices of households. In doing so, we take as given the choice among public and private education. For households choosing private education, it is shown that the rational residential location is the community with the lowest gross-of-tax price of housing. For households sending their children to one of the local public schools, we demonstrate some properties that characterise rational residential choices. For expositional convenience and to avoid uninteresting cases, we adopt the following realistic axiom.

Axiom 5 *All communities provide a positive amount of public educational services.*

Axiom 5 restricts attention to empirically relevant equilibria in which all communities are inhabited by (some) households sending their youths to the local public school.

The analysis proceeds as follows. Lemma 6 establishes an ordering of communities which must be satisfied by any vector of public policies and housing prices to be a candidate for equilibrium. Communities are then numbered according to this ordering. Using such ranking and taking school choices as given, propositions 7 and 8 establish conditions that must be satisfied by residential choices of households with children in private and public schools, respectively, to be rational.

Lemma 6 *In equilibrium, for every pair of communities i and j:* $E_j > E_i \Leftrightarrow p_j > p_i$ *and* $E_j = E_i \Leftrightarrow p_j = p_i$.

Proof. An allocation with $E_j > E_i$ and $p_j \leq p_i$ and cannot be an equilibrium because in that case $u(E_j) > u(E_i)$, $z(y-p_j) \geq z(y-p_i)$ and, therefore, $v(E_j, y-p_j) > v(E_i, y-p_i)$ for all y. Consequently, all households choosing public education in community i would want to move to community j, which is incompatible with our definition of equilibrium. A similar argument serves to prove the second part of the lemma. ■

Housing prices, therefore, serve as screening mechanisms and differences in E are capitalised to some extent into housing prices. Those communities with higher provision levels also have higher gross-of-tax housing prices and those with identical level of provision have equal gross-of-tax housing prices. Henceforth, for expositional convenience, we shall assume that all communities have different gross-of-tax housing prices. All results below extend to the case in which some communities have the same price just by considering them a community group which is treated as a single community. Let communities be numbered such that $(E_i, p_i) << (E_{i+1}, p_{i+1})$ for all $i = 1, 2, ..., J-1$.

Proposition 7 *In equilibrium, all households using private schools reside in the community with the lowest gross-of-tax price of housing (community 1).*

Proof. Strong monotonicity of preferences makes $w(y-p)$ to be everywhere increasing in $y-p$. Because $p_1 < p_j$ for all $j = 2, ..., J$, $y - p_1 > y - p_j$ and $w(y-p_1) > w(y-p_j)$ for all $j = 2, ..., J$. ■

Proposition 7 states that, in equilibrium, households who opt-out of public education reside in the community with the lowest gross-of-tax housing price. This result is easily deduced from the indifference map in figure 1 and occurs because these households only care about the level of private consumption in each community.

Proposition 8 *In equilibrium:*

(i) Perfect income stratification across public schools. Households using public schools are perfectly stratified by income across communities.

(ii) Ascending bundles. Let $\hat{y}_i^u$ be the income of the richest household in community i consuming public education. All communities satisfy the following ascending bundles condition. If $\hat{y}_j^u > \hat{y}_i^u \Rightarrow (E_j, p_j) >> (E_i, p_i)$.

Proof. (i) Slope of indifference curves corresponding to $v(E, y-p)$ in (E,p) space, $M(E, y-p)$, increases monotonically with y[5]:

$$\frac{\partial M(E, y-p)}{\partial y} = \frac{-u'(E)z''(y-p)}{z'(y-p)^2} > 0 \qquad (6)$$

An important consequence of this slope rising in income (SRI) condition is the following single-crossing property: the indifference curve of a household with income y crosses the indifference curve of any other household with different income at most once, and the indifference curve of the wealthier of any two households always cuts that of the poorer from below. The single-crossing property, in turn, implies the following preference ordering, proved in Epple et al. (1993), lemma 1: given $(E_i, p_i) << (E_j, p_j)$,

(a) $v(E_i, y-p_i) \geq v(E_j, y-p_j) \Rightarrow v(E_i, y'-p_i) > v(E_j, y'-p_j);\ \forall y' < y$
(7a)

(b) $v(E_i, y-p_i) \leq v(E_j, y-p_j) \Rightarrow v(E_i, y'-p_i) < v(E_j, y'-p_j);\ \forall y' > y$
(7b)

(7a) and (7b)entail that, in equilibrium, a community cannot be inhabited by households in the public sector from different income intervals.

(ii) By contradiction. Suppose that in equilibrium $(E_j, p_j) >> (E_i, p_i)$ and $\hat{y}_j^u < \hat{y}_i^u$. In that case, the following conditions must hold (a) $v(E_i, \hat{y}_i^u - p_i) \geq v(E_j, \hat{y}_i^u - p_j)$, and (b) $v(E_j, \hat{y}_j^u - p_j) \geq v(E_i, \hat{y}_j^u - p_i)$. From (7a), however, we know that,

$v(E_i, \hat{y}_i^u - p_i) \geq v(E_j, \hat{y}_i^u - p_j) \Rightarrow v(E_i, y'-p_i) > v(E_j, y'-p_j);\ \forall y' < \hat{y}_i^u$.

And, therefore, (b) cannot hold if $\hat{y}_j^u < \hat{y}_i^u$. ■

Corollary 9 *Communities 2 to J show perfect income stratification and are composed by households from single income intervals.*

Proof. Proposition 7 proves that no household in communities 2 to J acquires private educational services. Proposition 8 demonstrates that households in the public sector are perfectly stratified across communities. Hence, communities 2 to J show perfect income stratification and are inhabited by households from single income intervals. ■

[5] This slope rising in income condition is usual in multi-community models (e.g. Westhoff, 1977; Epple et al., 1984). In our model, it is automatically satisfied due to housing homogeneity. If housing were not restricted to be homogenous, this condition would require income elasticity of marginal benefit of education to be larger than the income elasticity of housing demand (see Ross and Yinger, 1999).

3.3 The voting problem

Some features of the voting problem in the model deserve further explanation. At the voting stage, the government budget constraint of any community i is given by:

$$E_i(t_i) = t_i p_h^i \frac{N_i}{n_i}; \forall t_i \geq 0 \quad (8)$$

where n_i is the mass of households sending their child to the local public school in community i and N_i is the community population capacity. Note that voting takes place once community and schooling choices are committed. For this reason, voting outomes do not change the community composition, the price of housing and private school enrolment. We can thus define the Government Possibility Frontier (GPF) of community i in $(E_i, p_h^i(1+t_i))$ space as:

$$p_h^i(1+t_i) = E_i \frac{n_i}{N_i} + p_h^i \quad (9)$$

The GPF of community i shows the maximum level of expenditures per student in the local public school, given the community housing price and the proportion of households using that school in the community (see figure 2). Voters are assumed to know the government budget constraint, the identity among net and gross of tax price of housing $p = p_h(1+t)$.They also know that voting occurs at the final stage, once the choice of community and school has been made. Under these assumptions voters know which alternatives are on the GPF when voting.

Moreover, under the sequence of decisions in our model, preferences over tax rates are single-peaked at the voting stage (as in Nechyba, 1999). The reason is that residential and schooling choices are already committed when voting takes place. The sequence of decisions in the model, thus, allows us to avoid the non single-peakedness problem which arises in previous models of public provision of education when private alternatives are available (Epple and Romano, 1996; Bearse et al., 2001).

Proposition 10 *Given a partition of households across communities and schools and a vector of housing prices $(p_h^1, ..., p_h^J)$, there exists an unique majority voting equilibrium and the median voter is decisive in every community. Moreover, (i) In community 1, the income of the decisive voter, $\widetilde{y}_1$, satisfies:*

$$F(\hat{y}_1^u) - F(\widetilde{y}_1) = \frac{N_i}{2} \quad \textit{(10)}$$

Where $\hat{y}_1^u$ is the income of the richest household choosing the local public school in community 1. (ii) In communities 2 to J the community median income voter is decisive.

Proof. At the voting stage, preferences over tax rates are single-peaked because the choices of community and school are already committed. For all households who have chosen a private school the peak is at $t = 0$. This is because a zero tax rate maximises their level of private consumption and because they do not benefit from expenditures on public education. Households with income y using the local public school in community i reach their peak at the unique t that solves:

$\max_t \; u(E_i) + z(y - p_h^i(1 + t_i))$

s.t. $E_i = t_i p_h^i \dfrac{N_i}{n_i}$

Consequently, by the median voter theorem (Black, 1958), a unique majority voting equilibrium exists in every community and the median voter is pivotal. We now prove (i) and (ii) in the proposition. The proofs follow Epple and Romer (1991) and Epple and Romano (1996).

(i) Consider the case of community 1 and the voter with income $\widetilde{y}_1$. By axiom 5, this voter sends her children to the local public school and thus, by axiom 2, votes for a strictly positive pair $(E, t) >> 0$. Suppose that her most preferred pair on the GPF is given by $(E^*, p_h(1 + t^*))$ in figure 2. This point is determined by the tangency between the community GPF and an indifference curve corresponding to $v(E, \widetilde{y}_1 - p_h(1 + t))$ in the figure. Proposition 10 states that this is the unique majority voting equilibrium in community 1. To prove it, we first consider any point on the GPF to the left of $(E^*, p_h(1 + t^*))$. From the SRI property of indifference curves shown in the proof to proposition 8, all households with income $y > \widetilde{y}_1$ and with their offspring attending the local public school (e.g. households with income y' in figure 2) prefer $(E^*, p_h(1+t^*))$ to any of these points. Given the definition of $\widetilde{y}_1$, these households are half the population in community 1. Therefore, at least half the electorate prefers $(E^*, p_h(1 + t^*))$ to any larger bundle on the GPF and it defeats all of these alternatives. Consider now any point on the GPF to the right of $(E^*, p_h(1 + t^*))$. From the SRI property, all households with income $y < \widetilde{y}_1$ prefer $(E^*, p_h(1 + t^*))$ to any of these points. Moreover, from proposition 1, households sending their youths to a private school have the same preference. Since all these households are half the electorate in community 1, $(E^*, p_h(1 + t^*))$ also defeats any bundle on the GPF to the right of it.

(ii) In any community $i = 2, ..., J$ all households use the local public school. This makes the median income voter most preferred bundle on the GPF to be the unique majority voting equilibrium. To see why note, first, that at least half the population (households with income $y > \widetilde{y}_i$) prefer it to any alternative on the GPF located to the left of it; and, second, that at least half the electorate (voters from households with income $y < \widetilde{y}_i$) prefer it to any alternative on the GPF located to the right of it. Consequently, this bundle defeats all alternatives on the community GPF. ■

3.4 Equilibrium stratification patterns: who opts-out of public education?

We now turn to the question of which households use public schools and which decide to opt-out of the public system and acquire private education. In their model, Bearse et al. (2001) obtain that, in equilibria with no empty communities, rich and poor households mix in the community with the lowest income tax rate and level of provision. This community shows perfect income stratification across schools, with the rich acquiring education in the private market and the poor using the local public school. Middle income households choose the other community and send their youths to that community local public school. This result does not change the basic intuition in Epple and Romano (1996): the richest fraction of households in the economy send their children to private schools because they do not find a public school of high enough quality.

An implication of this result is that, in such equilibria, if households with income y are indifferent between "community 1-private education" and "community 2-public education", then households with income $y' > y$ strictly prefer the former alternative, while households with income $y" < y$ strictly prefer the latter. We begin the analysis proving in Lemma 11 and corollary 12 that this property does not hold in general.

For expositional convenience, and without loss of generality, we set $J = 2$ for all the analysis below. Define $\hat{E}_2(y, p_1, p_2)$ as community 2 level of provision at which households with income y are just indifferent between private education at community 1 and public education at community 2. $\hat{E}_2(y, p_1, p_2)$ is a function, i.e. for each (y, p_1, p_2) there exists a unique level of for which $v(\hat{E}_2(y, p_1, p_2), y - p_2) = w(y - p_1)$. In (E, p) space it coincides with the indifference curve corresponding to $V(E, y-p)$ and to an utility level equal to $w(y-p)$ (see figure 2). Given the actual level of provision in community 2, E_2, households with income y such that $\hat{E}_2(y, p_1, p_2) > E_2$ strictly prefer private education at community 1, while households with income y such that $\hat{E}_2(y, p_1, p_2) < E_2$ strictly prefer living in community 2 and sending their children to the local public school there.

Lemma 11 *For all $p_2 > p_1$, $\hat{E}_2(y, p_1, p_2)$ decreases with income for y satisfying $p_2 - p_1 > x(y-p_1)$, reaches a minimum at y such that $p_2 - p_1 = x(y-p_1)$, and then increases with income for y such that $p_2 - p_1 < x(y-p_1)$.*

Proof. $\hat{E}_2(y, p_1, p_2)$ is implicitly defined by $v(\hat{E}_2(y, p_1, p_2), y - p_2) = w(y-p_1)$. Differentiate this expression with respect to y:

$$dy\left(u'(\hat{E}_2(\cdot))\frac{\partial \hat{E}_2(\cdot)}{\partial y} + z'(y-p_2)\right) = dy\left(z'(y - p_1 - x(y-p_1))\right)$$

Again, for differentiating the right hand side we use axiom 2, which guarantees an interior solution for the utility maximisation problem of households using private schools. Solve for $\frac{\partial \hat{E}_2(\cdot)}{\partial y}$ to obtain:

$$\frac{\partial \hat{E}_2(\cdot)}{\partial y} = \frac{z'(y - p_1 - x(y-p_1) - z'(y-p_2)}{u'(\hat{E}_2(\cdot))} \tag{11}$$

Now, given strict concavity of $z(\cdot)$, this derivative will be positive (negative) if $y - p_1 - x(y-p_1) < y - p_2$ ($y - p_1 - x(y-p_1) > y - p_2$), i.e. if $p_2 - p_1 < x(y-p_1)$ ($p_2 - p_1 > x(y-p_1)$). For completing the proof note that $\lim_{y \to 0} x(y-p) = 0$ and that educational services are normal. ■

Corollary 12 *Let ε be an arbitrarily small positive number. If for given (E_2, p_1, p_2) households with income y are indifferent between "private education-community 1" and "public education-community 2", then:*

(i) households with income $y + \varepsilon$ ($y - \varepsilon$) strictly prefer "public education-community 2" ("private education-community 1") if $p_2 - p_1 > x(y-p_1)$.

(ii) households with income $y + \varepsilon$ ($y - \varepsilon$) strictly prefer "private education-community 1" ("public education-community 2") if $p_2 - p_1 < x(y-p_1)$..

Proof. y satisfies $\hat{E}_2(y, p_1, p_2) = E_2$. Lemma 3 proves that when $p_2 - p_1 > x(y-p_1)$, $\hat{E}_2$ is decreasing in income. This implies $\hat{E}_2(y+\varepsilon, p_1, p_2) < E_2$ and

$\hat{E}_2(y-\varepsilon, p_1, p_2) > E_2$, implying statement (i). To demonstrate statement (ii) just change the direction of all inequalities. ■

Lemma 11 stems from the fact that, for any alternative, marginal utility of income is larger the lower is the consumption of the hicksian commodity. Corollary 12 shows that for such reason, if households indifferent between "private education-community 1" and "public education-community 2" have a demand for private education in community 1 smaller than the difference in gross-of-tax housing prices between community 2 and community 1 (i.e., if for these households consumption of the hicksian commodity is larger in the former alternative), then richer (poorer) households strictly prefer the latter (former) of both alternatives. Note that if this occurs, it seems possible to find equilibria in which households from intermediate income intervals opt-out of the public system, while poorer and richer households remain using public schools.

The result in Lemma 11 is not new. It is implicit in the single jurisdiction case (Epple and Romano, 1996) and an equivalent condition can be obtained for the two community model in Bearse et al. (2001). In the model with central provision, nevertheless, consumption of the hicksian commodity is always larger in the public alternative, precluding situations of the type just described to be an equilibrium. In the model with income taxation and without housing markets in Bearse et al., in turn, whenever marginal utility of income is larger for the alternative "public education-community 2" than for "private education-community 1", the former option provides a strictly lower utility than the latter, for all $y > 0$.

As will be shown below, what it is new in our model is that it is indeed possible to find equilibria in which intermediate income households choose a private school. Moreover, given how $\hat{E}_2(y, p_1, p_2)$ varies as income rises, it seems also possible to find equilibria in which $\hat{E}_2(y, p_1, p_2) > E_2$ holds for two income intervals and, therefore, in which households from those income intervals prefer a private alternative. Proposition 13 reveals through a set of examples that the introduction of housing markets and property taxation into the picture generates existence of this type of equilibria.

Consider an economy corresponding to the model in section 2. This economy has a population mass N normalised to 1. It is composed by two communities, 1 and 2, with capacity equal to N_1 and N_2, respectively. Households preferences are captured by the following utility function, borrowed from Bearse et al. (2001):

$$U(x,b) = \frac{1}{1-\sigma}\left[b^{1-\sigma} + \delta x^{1-\sigma}\right];\ \sigma,\delta > 0 \qquad (12)$$

This utility function is separable in (b,x) and strictly concave for $\sigma,\delta > 0$. It violates axiom 2 but this is inconsequential for the examples in proposition 13. Parameters of the utility function are set at levels in Bearse et al. (2001): $\sigma = 2.23$ and $\delta = 0.0032$. The income cumulative distribution function is given by $F(y)$ with lower and upper bounds $\underline{y}$ and $\overline{y}$. Finally, the cost of producing a unit of housing is $c$and equals the net-of-tax price of housing in community 1, p_h^1.

Proposition 13 *Examples 14, 15 and 16 below are equilibria of the above described economy.*

Proof. See appendix A2. ∎

Example 14 *Communities capacity:* $N_1 = 0.75$*;* $N_2 = 0.25$*.*

Income cumulative distribution function (figure 4):

$F(y) = -0.2232 + 3.6261 \cdot 10^{-2}y - 1.2669 \cdot 10^{-3}y^2 + 3.0696 \cdot 10^{-5}y^3 - 3.8289 \cdot 10^{-7}y^4$

$+2.2992 \cdot 10^{-9}y^5 - 5.2834 \cdot 10^{-12}y^6$*;* $\underline{y} = 8$*;* $\overline{y} = 125$*.*

Vector of public policies and housing prices:

$e^* = (E_1, t_1, p_h^1, E_2, t_2, p_h^2) = (2, 0.2712, 5.9, 6.2, 0.8857, 7)$

Median voters: $(\widetilde{y}_1, \widetilde{y}_2) = (23.3679, 94.7001)$

Partition of households across communities and schools: households with income $y \in [8, 59.88)$ *(with a mass equal to 0.6) choose public education-community 1; households with income* $y \in [59.88, 71.96)$ *(with a mass equal to 0.1) choose private education-community 1; households with income* $y \in [71.96, 117.47]$ *(with a mass equal to 0.25) choose public education-community 2; households with income* $y \in [117.47, 125)$ *(with a mass equal to 0.05) choose private education-community 1.*

Example 15 *Communities capacity:* $N_1 = 0.8$*;* $N_2 = 0.2$*.*

Income cumulative distribution function (figure 5):

$F(y) = -.2133 + 1.8200 \cdot 10^{-2}y + 4.4177 \cdot 10^{-4}y^2 - 1.4110 \cdot 10^{-5}y^3 + 1.2776 \cdot 10^{-7}y^4 - 3.7943 \cdot 10^{-10}y^5$*;* $\underline{y} = 10$*;* $\overline{y} = 120$

Vector of public policies and housing prices:

$e^* = (E_1, t_1, p_h^1, E_2, t_2, p_h^2) = (1.5, 0.1753, 7.4875, 6.6, 0.8684, 7.6)$
Median voters: $(\widetilde{y}_1, \widetilde{y}_2) = (22.9718, 100.9582)$

Partition of households across communities and schools: households with income $y \in [10, 48.09)$ *(with a mass equal to 0.7) choose public education-community 1; households with income* $y \in [48.09, 64.57)$ *(with a mass equal to 0.1) choose private education-community 1; households with income* $y \in [64.57, 120]$ *(with a mass equal to 0.2) choose public education-community 2.*

Example 16 *Communities capacity:* $N_1 = 0.8$*;* $N_2 = 0.2$*.*
Income cumulative distribution function (figure 6):
$F(y) = -3.6344 \cdot 10^{-2} - 2.1132 \cdot 10^{-3} y + 9.8265 \cdot 10^{-4} y^2 - 2.0059 \cdot 10^{-5} y^3 + 1.5755 \cdot 10^{-7} y^4 - 4.3713 \cdot 10^{-10} y^5$*;* $\underline{y} = 8$*;* $\overline{y} = 125$
Vector of public policies and housing prices:
$e^* = (E_1, t_1, p_h^1, E_2, t_2, p_h^2) = (2, 0.2892, 6.05, 4.5, 0.6716, 6.7)$
Median voters: $(\widetilde{y}_1, \widetilde{y}_2) = (26.6958, 70.3533)$

Partition of households across communities and schools: households with income $y \in [8, 54.52)$ *(with a mass equal to 0.7) choose public education-community 1; households with income* $y \in [54.52, 99.18)$ *(with a mass equal to 0.2) choose public education-community 2; households with income* $y \in [99.18, 125]$ *(with a mass equal to 0.1) choose private education- community 1.*

Figures 7, 8 and 9 correspond to examples 14, 15 and 16 respectively. These figures show the level of education received by households from each level of income. In examples 14 and 15 households from an intermediate income interval are not satisfied by the mix of public education and housing prices offered by communities 1 and 2. Therefore, they opt-out of the public system and send their youths to private schools. These schools are of higher quality than the public school in community 1 but of lower quality than the public school in community 2 as can be checked in figures 7 and 8. Notice the sharp increase in the level of public education quality from community 1 to community 2. This is the reason why middle income households opt-out. We do not mean that they would not be happy sending their children to public school in community 2. However, for doing so, they should buy or rent a house in community 2 where the price of housing is high. As a consequence, the mix of numeraire and education consumption they can obtain in community 2 is not the utility maximising alternative in their feasible set. In other words, for these households the level of public provision in community 1 is too low and the price of housing in community 2 is too high.

On the other hand, in the first of these examples, households from the richest interval of income also choose private education, sending their children to private schools of higher quality than the public one in community 2. In the second one, in turn, they reside in community 2 and use the local public school there. In this example, the rich are enough to dominate the political process and establish a local public school of high enough quality there. In such case, rich households are willing to outbid middle income ones from that community in order to use the local public school there. It is remarkable that in equilibria of this type, public support for a high quality public service will not fall as in the single jurisdiction case, at least in the rich community. On the contrary, the model predicts that it is possible to find very high-quality public education in that community. Another implication is that differences in expenditure levels among communities are sharp.

Example 16 serves to prove that equilibria of the type in Bearse et al. (2001) are, of course, possible in our model. In this case, the difference among communities 1 and 2 in public education quality is smaller and middle income households do not opt-out. Rich households, in turn, are not satisfied by public education in this equilibrium and choose private schooling (see figure 9).

In general, households opting-out of public education and choosing a private alternative are those not satisfied by the mix of housing prices, property tax rates and expenditures per student offered by the existing communities. For these households, thus, it pays to spend some extra money for acquiring private educational services. As we have just shown, it turns out that in multi-community settings, the rich are not necessarily those exiting the public education system as in single-jurisdiction economies.

An important question for further research is under which circumstances each type of equilibrium is more likely to arise. Ongoing research by the author suggests that the more polarised the income distribution and the stronger the preferences for education the more likely are equilibria with intermediate income households using a private school or with rich households sending their youths to a local public school. Intuitively, this occurs because in these cases differences in public education quality and, consequently, in housing prices among communities are usually larger.

4 Do households in the private sector mix with the poor?

As in Bearse et al. (2001), the basic model analysed above predicts that households sending their children to private schools move to the lowest housing price community and mix with poor households there. In real world, however, all households choosing private schooling in a metropolitan area do not move to the community with the lowest gross-of-tax housing price. Instead, they reside in different, poor and rich, jurisdictions (see, for example, the evidence in Luengo-Prado and Volij, 2003). There are several reasons that can explain this stylised fact: rigidities of housing markets, endogenous amenities such as commercial activity, crime or social capital and exogenous amenities such as distance to center.

Nechyba (1999, 2002a, 2002b) has shown that the existence of heterogeneity into the housing market and, particularly, of rigidities which prevent all housing types to be available in all communities offers a first solid explanation. His model, however, requires to impose an exogenous distribution of housing types across communities and neighbourhoods and, consequently, is not suited to explain the origin of such heterogeneity or the evolution of community structures. From our point of view, although housing markets rigidities undoubtedly have a role in the allocation of households to communities and schools, a more essential explanation is related to endogenous local amenities.

In this section, we consider one important type of endogenous amenities, the so-called neighbourhood effects. Neighbourhood effects are sociological externalities (peer effects that operate at the school and community level, role models, norms of behaviour, crime, etc.) whereby the utility of a household is affected by the identity of its neighbours. When they are present, location decisions of households using either public or private schooling are affected by each community socio-economic quality. Neighbourhood effects, thus, inhibit incentives households in the private sector have to mix with poor households in the low housing price communities.

The model in this section is an extension of that presented in section 2 to the presence of neighbourhood effects. We hypothesise that neighbourhood effects depend positively on the community mean income. The local externality a household is exposed to in a particular community depends on the level of local social capital and social capital (or at least households' perceptions about it) is supposed to be an increasing function of mean income $L(\overline{y})$, with $L'(\overline{y}) > 0$. To keep the analysis simple, the utility function which captures households (identical) preferences across alternatives is assumed to be separable in $(x, b, L(\overline{y}))$. Hence, demand for education and for the numeraire does not depend on the level of the local externality:

$$U(x, b, L(\overline{y})) = u(x) + z(b) + s(\overline{y}) \qquad (13)$$

This utility function is increasing in all its arguments, strictly quasi-concave in x and strictly concave in b. Indirect utility functions for public and private alternatives in each community are given by:

$$v(E, y-p, \overline{y}) = u(E) + z(y-p) + s(\overline{y}) \qquad (14)$$

$$w(y-p, \overline{y}) = u(x(y-p)) + z(y-p-x(y-p)) + s(\overline{y}) \qquad (15)$$

To establish the way in which households distribute across communities and schools in equilibrium, we must determine (i) the distribution across communities of households using a public school; (ii) the distribution across communities of households who exit the public system; (iii) within each community, the distribution of residents between the local public school and the private sector; and (iv) which households remain in the public system and which opt-out and acquire educational services in the private market. Propositions 17, 18 and 19 analyse the first three questions. They exploit properties of $v(E, y-p, \overline{y})$ and $w(y-p, \overline{y})$ to establish restrictions that must be satisfied by an allocation to be a candidate for equilibrium (given the definition in section 2). The answer to the last question is more difficult. In fact, there are different distributions of households to communities and schools satisfying restrictions in propositions 17 to 19 which are compatible with equilibrium. Proposition 20 analyses this final question for an economy with two communities.

Proposition 17 *In equilibrium,*

1. If $p_i > p_j \Rightarrow u(E_i) + s(\overline{y}_i) > u(E_j) + s(\overline{y}_j)$.

2. Households in the public system are perfectly stratified by income across communities, with richer households living in larger $(p, u(E) + s(\overline{y}))$ *bundles communities.*

Proof. 1. If $p_i > p_j$, private consumption for households using public schooling is larger in community j. In this situation $u(E_i) + s(\overline{y}_i) \leq u(E_j) + s(\overline{y}_j)$ cannot hold in equilibrium because in that case $v(E_i, y-p_i, \overline{y}_i) < v(E_j, y-p_j, \overline{y}_j)$ and all households using the local public school in community i would want moving to community j.

2. Note that given concavity of $z(\cdot)$ and $p_i > p_j$, the following is satisfied for all $y > 0$:

$$\frac{\partial v(E_i, y - p_i, \overline{y}_i)}{\partial y} = z'(y - p_i) > z'(y - p_j) = \frac{\partial v(E_j, y - p_j, \overline{y}_j)}{\partial y} \quad \blacksquare$$

Remarkably, part 1 in proposition 17 does not necessarily require either $E_i > E_j$ or $\overline{y}_i > \overline{y}_j$, although at least one of these inequalities must be satisfied.

Proposition 18 *In equilibrium, for any two communities i and j inhabited by households with children in private schools,*

1. If $p_i > p_j \Rightarrow \overline{y}_i > \overline{y}_j$.

2. Households with children in private schools are perfectly stratified by income across (not necessarily all) communities, with poorer households residing in larger bundles communities.

Proof. 1. If community i and community j are both inhabited by households with children in private schools, given $p_i > p_j$, $\overline{y}_i \leq \overline{y}_j$ cannot hold in equilibrium because, in that case, every household purchasing private education in community i would strictly prefer to reside in community j. The reason is that strict monotonicity makes $u(x(y-p)) + z(y - p - x(y-p))$ to be decreasing in p, consequently, if $p_i > p_j$ households in the private sector living in community i must be compensated with a larger level of social capital.

2. We simply prove that indifference curves corresponding to $w(y - p, \overline{y})$ satisfy a slope rising in income property analogue to that in the proof to proposition 2. This property, in turn, implies the result. The slope of these indifference curves in $(\overline{y}, p)$ space is given by:

$$S(E, y - p, \overline{y}) = -\frac{\partial v(E, y - p, \overline{y})/\partial \overline{y}}{\partial v(E, y - p, \overline{y})/\partial p} = \frac{s'(\overline{y})}{z'(y - p)} > 0$$

The derivative of this slope with respect to income is also positive:

$$\frac{\partial S(E, y - p, \overline{y})}{\partial y} = \frac{-s'(\overline{y}) z''(y - p)}{z'(y - p)^2} > 0. \quad \blacksquare$$

Propositions 17 and 18 imply that the mixing across communities of households who acquire private education with those in the public system is such that poorer (richer) households in the former tend to live with poorer (richer) households in the latter. Proposition 18 also serves to demonstrate that the richest households in the economy will not usually mix with the poorest as predicted by models without neighbourhood effects. The intuition is straightforward. If neighbourhood effects are important, when a household decides to acquire private education and simultaneously chooses the community where to reside, it cares not only about housing prices but also about each community social capital. Because $s'(\overline{y}) > 0$, these households will choose a community with a higher price of housing if the community social capital is large enough as to compensate smaller education and numeraire consumption. That is to say, if these households have to choose between two communities i and j, with $p_i > p_j$ and $\overline{y}_i > \overline{y}_j$, they face a trade-off between housing prices and socio-economic quality. As casual evidence suggests, this trade-off probably makes many of them to prefer communities with higher social quality and housing prices. From our point of view, this result offers a more fundamental and robust answer than housing heterogeneity to the question of interest in this section.

Proposition 19 *Every community inhabited by households with children in private and public schools, shows prefect income stratification across schools, with poorer households sending their children to the local public school.*

Proof. This proposition requires that, within such communities, there is an income level above which all households in the community strictly prefer consuming private education and below which all households strictly prefer the local public school. This will be satisfied is the derivative of $v(E, y-p, \overline{y})$ with respect to income is everywhere smaller than the derivative of $w(y-p, \overline{y})$ with respect to income. Strict concavity of $z(\cdot)$ and axiom 2 guarantee that

$$\frac{\partial v(E_i, y-p_i, \overline{y}_i)}{\partial y} = z'(y-p_i) < z'(y-p_i-x(y-p_i)) = \frac{\partial w(y-p_i, \overline{y}_i)}{\partial y} \quad \blacksquare$$

Corollary 20 *In equilibria in which all communities have households with children in public and private schools, households from the poorest income interval in the economy, i.e. those with income* $y < \widehat{y}_1^U$*, say, (and only they) choose the option "community 1-public education", while households from the richest income interval, i.e. those with income* $y > \widehat{y}_J^P$*, say, (and only they) choose the alternative "community J-private education".*

Proposition 21 studies which households choose public education and which opt-out of public schooling and acquire private educational services. It focuses on the empirically relevant case in which all communities are populated by households in both educational sectors. For expositional convenience, we set $J = 2$. Results remain essentially unchanged once J communities are considered. In a two-community setting, in equilibria in which both communities are inhabited by households in both sectors, corollary 20 guarantees that the poorest fraction in the economy, those with income below $\widehat{y}_1^U$, choose the option "community 1-public education", while households from the richest income interval, those with income above $\widehat{y}_2^P$, prefer the alternative "community 2-private education". Therefore, it just remains to establish how households with income between $\widehat{y}_1^U$ and $\widehat{y}_2^P$choose between the alternatives "community 1-private education" and "community 2-public education".

Let $\hat{E}_{ij}(y, \overline{y}_i, \overline{y}_j, p_i, p_j)$ be community j provision level at which, given $\overline{y}_i < \overline{y}_j$ and $p_i < p_j$, a household of income y is just indifferent between public education in community j and private education in community i. If $\hat{E}_{ij}(y, \overline{y}_i, \overline{y}_j, p_i, p_j) > (<) E_j$, households with income y strictly prefer private education in community i (public education in community j) among these alternatives. In the two-community case,

$$\frac{\partial \hat{E}_{12}(y, \overline{y}_i, \overline{y}_j, p_i, p_j)}{\partial y} = \frac{z'(y - p_1 - x(y - p_1) - z'(y - p_2)}{u'(\hat{E}_{ij}(y, \overline{y}_i, \overline{y}_j, p_i, p_j))} \tag{16}$$

Given strict concavity of $z(\cdot)$, this derivative is positive (negative) if $p_2 - p_1 < (>) x(y - p_1)$. Note that because educational services are a normal good $\hat{E}_{12}$ is first decreasing in y, reaches a minimum at y such that $p_2 - p_1 = x(y - p_1)$ and then increases with y.

Proposition 21 *In a two-community system, in equilibria in which both communities are inhabited by households choosing public and private schooling, with $p_1 < p_2$, three types of distributions of households to communities and schools are compatible with equilibrium:*

1. Perfect income stratification across educational sectors. There exist three border incomes $\widehat{y}_1^U$, $\widehat{y}$, and $\widehat{y}_2^P$, with $\widehat{y}_1^U < \widehat{y} < \widehat{y}_2^P$. Households with income equal to $\widehat{y}$ are just indifferent between the local public school in community 2 and private education in community 1. Households with income y such that $\widehat{y}_2^P > y > \widehat{y}$ choose "private education-community 1", while households with income y satisfying $\widehat{y}_1^U < y < \widehat{y}$ choose "public education-community 2".

2. Perfect income stratification across communities. There exist three border incomes $\widehat{y}_1^U$, $\widehat{y}$, and $\widehat{y}_2^P$, with $\widehat{y}_1^U < \widehat{y} < \widehat{y}_2^P$. Households with income $\widehat{y}$ are just indifferent between "private education-community 1" and "public education-community 2". Households with income y such that $\widehat{y}_2^P > y > \widehat{y}$ choose "public education-community 2", while households with income y satisfying $\widehat{y}_1^U < y < \widehat{y}$ choose "private education-community 1".

3. Non-prefect income stratification. There exist four border incomes $\widehat{y}_1^U$, $\widehat{y}_1$, $\widehat{y}_2$, and $\widehat{y}_2^P$, with $\widehat{y}_1^U < \widehat{y}_1 < \widehat{y}_2 < \widehat{y}_2^P$. Households with income $\widehat{y}_1$ and $\widehat{y}_2$, are just indifferent between "community 1-private education" and "community 2-public education". Households with income $y \in [\widehat{y}_1, \widehat{y}_2]$, choose community 2-public education, while households with income $y \in [\widehat{y}_1^U, \widehat{y}_1]$ and $y \in [\widehat{y}_2, \widehat{y}_2^P]$ prefer the option "community 2-public education".

Proof. From corollary 4, households with income below $\widehat{y}_1^U$ choose the option "community 1-public education", while households with income above $\widehat{y}_2^P$ prefer the alternative "community 2-private education". Proposition 8 establishes how households with income $y \in [\widehat{y}_1^U, \widehat{y}_2^P]$ choose between "community 1-private education" and "community 2-public education". Moreover, because we restrict attention to equilibria in which both communities are inhabited by households in both educational sectors, $\hat{E}_{12}(y, \overline{y}_i, \overline{y}_j, p_i, p_j)$ crosses E_2 in (y, E) space at least once within the relevant income interval. Given behaviour of $\hat{E}_{12}(\cdot)$ in (y, E) space, there exist three possibilities, each corresponding to one type of equilibrium: one crossing from below, one crossing from above and two crossings, the first from above and the second from below:

1. Perfect stratification by income across educational sectors will arise if $\hat{E}_{12}(\cdot)$ crosses E_2 in the (y, E) plane within the relevant income interval once from below. This requires that at the crossing point $p_2 - p_1 < x(\widehat{y} - p_1)$ (figure 10).

2. Perfect stratification by income across communities will characterise equilibrium if $\hat{E}_{12}(\cdot)$ crosses E_2 in (y, E) space within the relevant income interval once from above. This requires that at the crossing point $p_2 - p_1 > x(\widehat{y} - p_1)$ (figure 11).

3. Non prefect income stratification will hold in equilibrium if $\hat{E}_{12}(\cdot)$ crosses E_2 in the (y, E) plane within the relevant income interval twice, the first from above and the second from below. This requires that at the first crossing point $p_2 - p_1 > x(\widehat{y}_1 - p_1)$ and that at the second crossing point $p_2 - p_1 < x(\widehat{y}_2 - p_1)$ (figure 12). ■

Proposition 21 identifies two types of distributions to communities and schools compatible with equilibrium that showing perfect income stratification: one across communities and the other across schools. Moreover, it highlights conditions under which one or the other type of equilibrium prevails. Perfect income stratification across communities will arise if $p_2 - p_1 > x(y - p_1)$ holds in the relevant range $[\widehat{y}_1^U, \widehat{y}_2^P]$ or if this inequality holds in most of it thereby avoiding a second crossing between $\hat{E}_{12}(\cdot)$ and E_2 from below within this interval. That is to say, perfect income stratification across communities is more likely to arise if differences in equilibrium housing prices are large enough to make private consumption in the alternative "community 1-private education" larger than in the alternative "community 2-public education" for households with income $y \in [\widehat{y}_1^U, \widehat{y}_2^P]$ or for most of them. In this case, rich households are able to concentrate in a single community, in which the relative level of social capital and education expenditures are high. Rich households in both public and private schools outbid poor and middle income ones from this community. Households in the private sector in community 1 are middle income households. Given $\overline{y}_1$and $\overline{y}_2$, they are not willing to outbid richer households from the rich community and prefer to increase their consumption of education by opting-out within the poor community.

This kind of equilibrium is more likely to arise in economies with polarised income distributions and high income elasticity of demand for neighbourhood effects. These economies will show greater differences across communities in $\overline{y}$ and E and consequently, in p.

Proposition 21 identifies a third possible equilibrium configuration which constitutes an intermediate case. This equilibrium is a reminiscence from the equilibrium in example 14 of proposition 10.

Interestingly, and not surprisingly, in all types of equilibrium the mixing of rich households with poor ones falls dramatically with respect to an economy without neighbourhood effects.

Before finishing, we should stress that the analysis in this section has not proved existence of equilibrium. It has only identified the distributions of households to communities and schools that are compatible with equilibrium. The analysis of the voting problem in section 3, nevertheless, extends readily to the model with neighbourhood effects and guarantees existence of a unique majority voting equilibrium in every community. Therefore, though we do not provide either a proof of existence or examples of equilibria, there is no reason to think that there may be existence problems once neighbouhood effects are introduced into the picture.

5 Discussion and concluding remarks

The objective of this paper has been to answer two questions that emerge in the analysis of local provision of education with opting-out: first, who opts-out of public education in decentralised systems with opting-out? or, in other words, do the rich (and only they) always choose private schooling? Second, why households choosing private schooling do not always mix with the poor? With this objective, we have developed and analysed a multi-community model with local provision of education and opting-out. The model generalises that in Bearse et al. (2001) to any preference configuration separable in (x, b) and extends it to an economy with housing markets, property taxation, neighbourhood effects and $J \geq 2$ communities. These extensions turn out to be relevant, modifying theoretical results in several important respects.

The analysis in Bearse et al. (2001) shows that in equilibria in which both communities are inhabited, there is a level of income above which all households choose private education and live in the lowest tax rate-public expenditure level community among poor households. Consequently, the richest and the poorest households in the economy mix together in that community while middle income households send their children to the local public school in the other community. This result is similar to that in Epple and Romano (1996) for the single jurisdiction case in that the rich are who opt-out of public education and acquire private educational services. Moreover, it coincides with the view of other authors like Luengo-Prado and Volij (2003).

The model in this paper, however, reveals that a richer array of population distributions to communities and schools is compatible with equilibrium. The basic model without neighbouhood effects served to answer the first question: households choosing a private alternative instead of public schooling are those which do not find a community with a mix of gross-of-tax housing prices and per-student expenditures close enough to their most desired levels of education and numeraire consumption. For these households, it pays to spend some extra money for acquiring private educational services.

In the single jurisdiction case, these households are always the rich ones whose demand for education is much larger than the common provision level. In a multi-community setting, however, this result does not necessarily holds. In such framework, households from different income intervals and, thus, households from one or more intermediate income interval(s) may prefer private schooling for their child. In the model without neighbouhood effects, these households move to the community with the lowest gross-of-tax price of housing (community 1). They choose private schooling and to live community 1 because the level of public provision in that community is too low and the price of housing in the other one is too high.

This model also demonstrates that the richest households in the economy may or may not be among households choosing the private alternative. The reason is that they may be able to establish a local public school of high enough quality in the community with the highest (gross of tax) housing price. In such case, rich households are willing to outbid middle income ones from that community and to use the local public school. In equilibria of this type, public support for a high quality public service will not fall as in the single jurisdiction case, at least in the rich community. On the contrary, the model predicts that it is possible to find very high-quality public education in that community. On the other hand, differences between expenditure levels among communities increase in this kind of equilibria.

Of course, equilibria of the type in Bearse et al. (2001) are also possible in the model with housing. An important question for further research is thus under which circumstances is each type of equilibrium more likely to arise. Ongoing research by the author suggests that the more polarised the income distribution and the stronger the preferences for education the more likely are equilibria with intermediate income households using a private school or with rich households sending their youths to a local public school.

The model with neighbourhood effects was aimed at investigating the second question, i.e. at explaining why households in the private sector do not always mix with the poor in real world. The analysis demonstrates that social externalities operting at the community level provide an answer to this question. From our point of view, this is indeed a more essential reason than housing markets rigidities alluded by Nechyba (1999, 2002a, 2002b). The main implication of the model with neighbourhood effects is that even in multi-community settings with opting-out, there are socio-economic factors that tend to stratify population by income. Poorer (richer) households in the public system tend to live with poorer (richer) households in the private sector. When neighbourhood efects are present, there are two basic equilibrium configurations: one in which households are perfectly stratified by income across communities and other in which perfect income stratification occurs, instead, across schools. Under certain circumstances, thus, these sociological spillovers can drive the economy back to perfect income stratification across communities as in classical multi-community models without opting-out (Epple et al. 1984; 1993).

These results throw doubts about the potential equity enhancing effects of private alternatives and vouchers systems within multi-community mixed education regimes pointed-out in Bearse et al (2001) or in Nechyba (1999). These predictions, however, would be softened if local externalities did not operate at the community but at the neighbourhood level, i.e. if rich neighbourhoods in poor communities were able to establish exclusion mechanisms which prevented externalities generated by poor households to affect its neighbours.

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Appendix A1. Figures

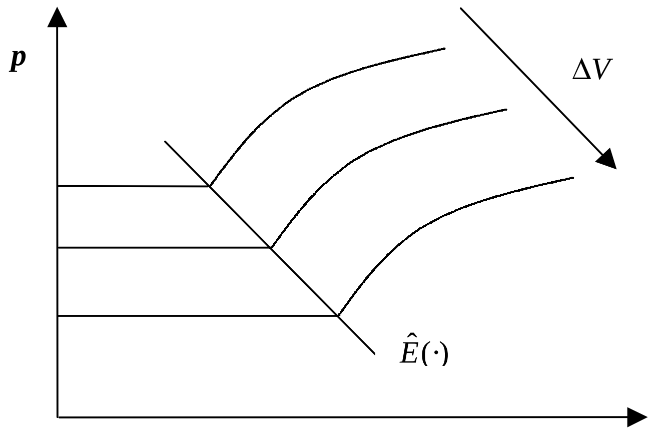

Figure 1

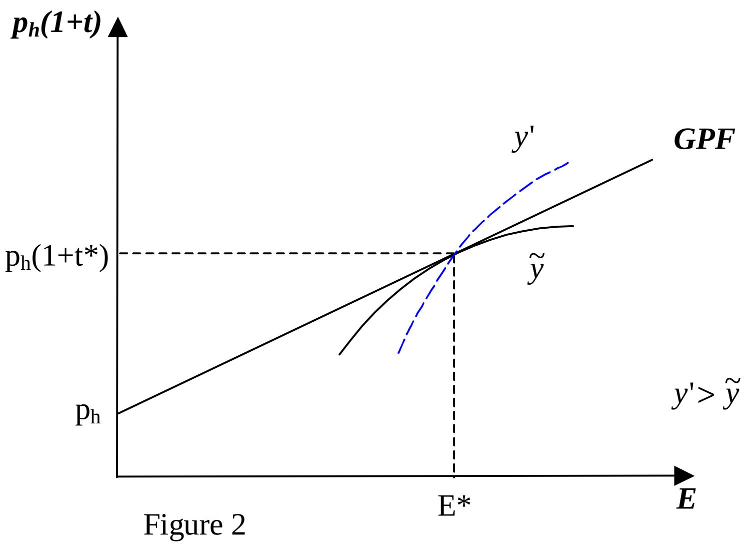

Figure 2

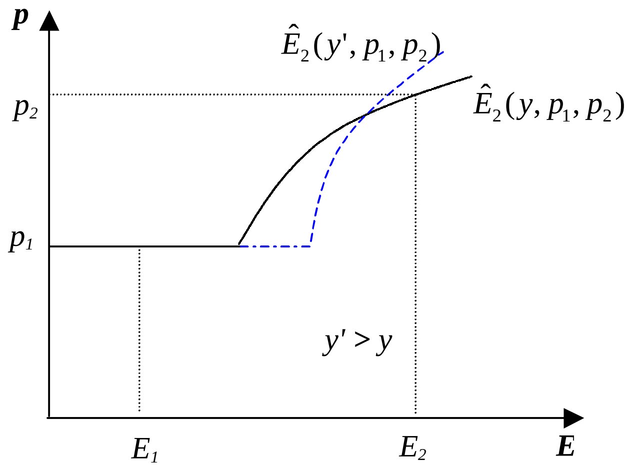

Figure 3

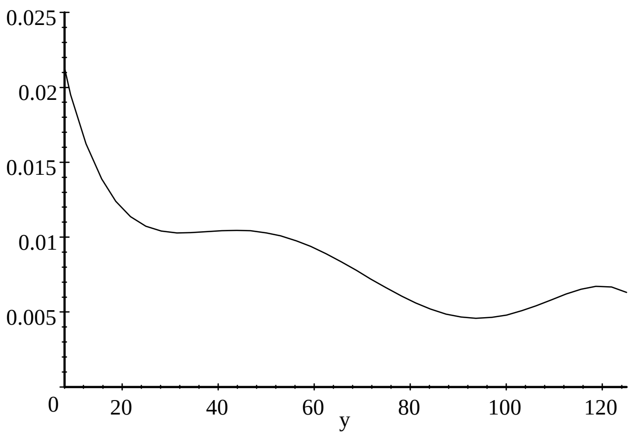

Figure 4. Income distribution density function (example 1)

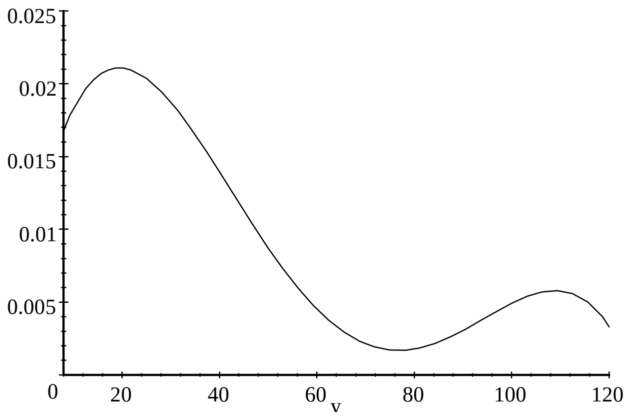

Figure 5. Income distribution density function (example 2)

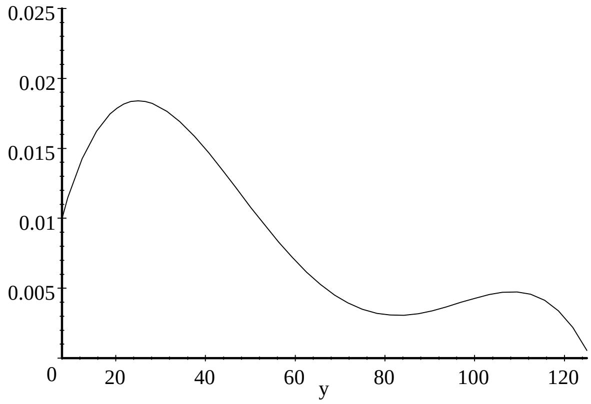

Figure 6. Income distribution density function (example 3)

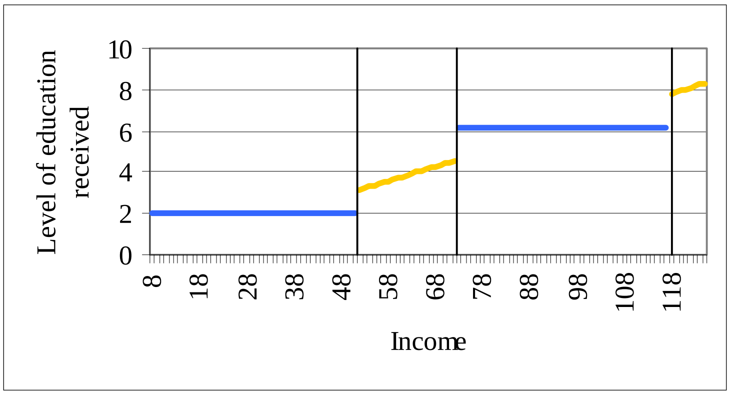

Figure 7. (Example 1)

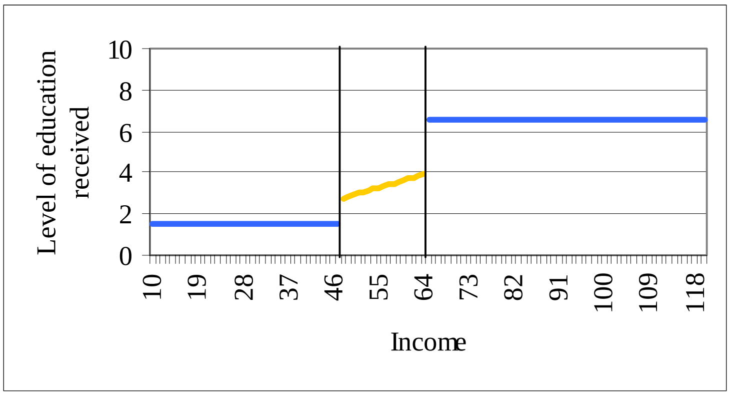

Figure 8. (Example 2)

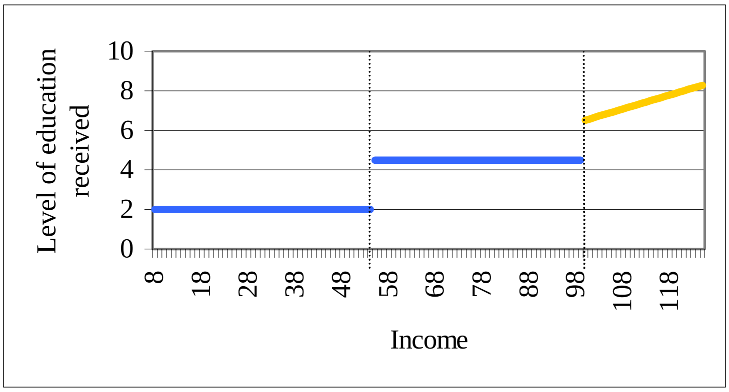

Figure 9. (Example 3)

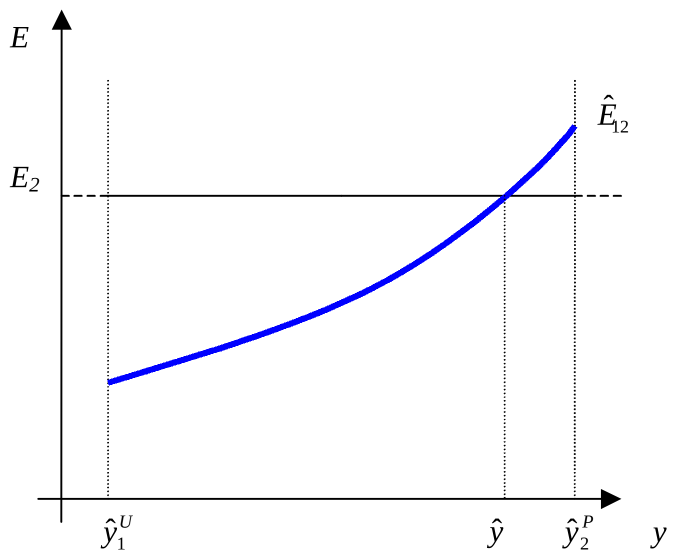

Figure 10

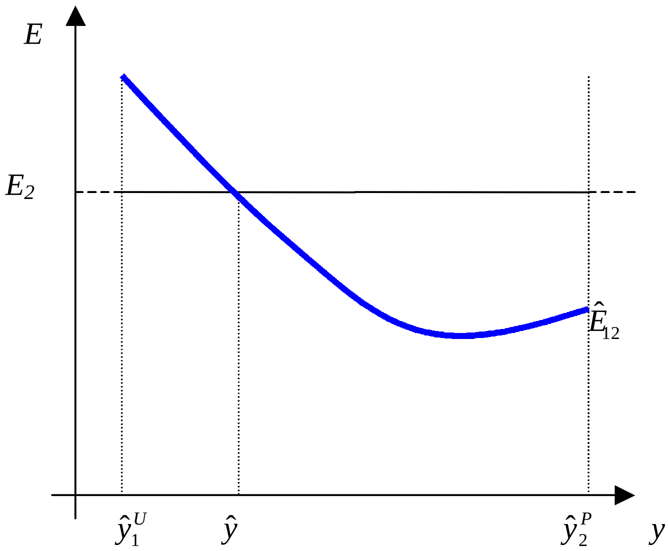

Figure 11

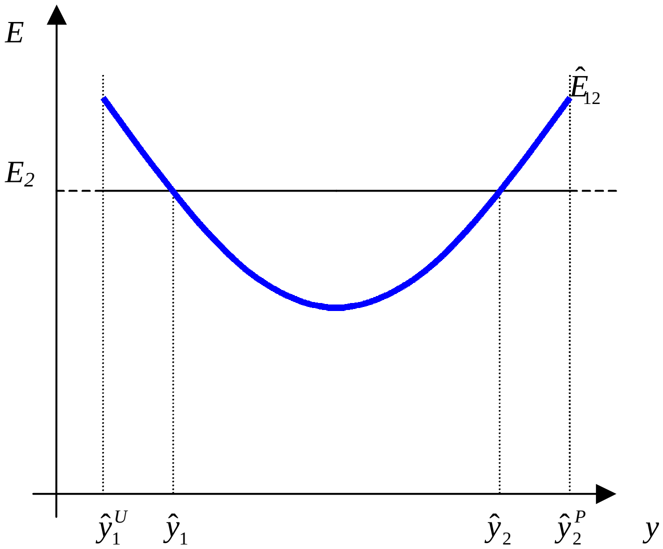

Figure 12

Appendix A2. Proof of proposition 3:

From proposition 7, for any $p_2>p_1$ no household chooses the option private education-community 2. Thus, we only need to consider the alternatives public education-community 1 (PUB1), public education-community 2 (PUB2) and private education-community 1 (PR1). Given the equilibrium vector $e^*= (E_1,p^1_h,t_1,E_2,p^2_h,t_2)$, and with some abuse of notation let $v_1(y)$ be $v(E_1,y-p_1)$, $v_2(y)$ be $v(E_2,y-p_2)$ and $w_1(y)$ be $w(y-p_1)$, where $p_i=p^i_h(1+t_i)$. From (12) we obtain:

$$v_i(y) = \frac{1}{1-\sigma}\left[(y-p_i)^{1-\sigma} + \delta E_i^{1-\sigma}\right], \qquad \text{(A1)}$$

$$x(y-p_1) = \frac{(y-p_1)\delta^{1/\sigma}}{1+\delta^{1/\sigma}}, \text{ and} \qquad \text{(A2)}$$

$$w_1(y) = \frac{1}{1-\sigma}(y-p_1)^{1-\sigma}\left(1+\delta^{1/\sigma}\right)^{\sigma} \qquad \text{(A3)}$$

Example 14:

(i) *Rational choices.*

(a) Consider alternatives PUB1 and PUB2. For an income level y=62.885, the following can be verified: $v_1(62.885)=v_2(62.885)$. By proposition 8, this implies that all households with income y<62.885 strictly prefer PUB1 to PUB2, while all households with income y>62.885 strictly prefer PUB2 to PUB1.

(b) Consider alternatives PUB1 and PR1. For y=59.88, v_1 and v_2 satisfy: $v_1(59.88)=w_1(59.88)$. By corollary 4, this implies that all households with income y<59.88 strictly prefer PUB1 to PR1, while all households with income y>59.88 strictly prefer PR1 to PUB1.

(c) Consider alternatives PR1 and PUB2. It can be verified that $w_1(71.96)=v_2(71.96)$, which implies $E_2 = \hat{E}_2(71.96,\cdot)$. By lemma 11 and given that $p_2-p_1=13.2-7.5=5.7>4.557=x(71.96-7.5)$ and that educational services are normal, all households with income y<71.96 satisfy $E_2 < \hat{E}_2(y',\cdot)$ and, hence, they strictly prefer PR1 to PUB2.

(d) Consider again alternatives PR1 and PUB2. Due to normality of educational services, the sign of $\partial\hat{E}_2(\cdot)/\partial y$ becomes negative for all y>88.128 (because x(88.128-7.5)=5.7). Moreover, for y=117.47, $w_1(117.47)= v_2(117.47)$ and, therefore, $E_2 = \hat{E}_2(117.47,\cdot)$. This result and (c) imply that households with income $y \in [71.96,117.47]$, strictly prefer PUB2 to PR1 (because for them $E_2 > \hat{E}_2(\cdot)$); while households with income *y>117.47* strictly prefer PR1 to PUB2 (because for them $E_2 < \hat{E}_2(\cdot)$).

From (a) and (b), households with income $y \in [8,59.88]$ maximise their utility choosing public education-community 1, while households with income $y \in [59.88,62.885]$ do so choosing private education-community 1. (b) and (c) establish that the optimal choice for households with income $y \in [62.885,71.96]$ is private education-community 1. Finally, (b) and (d) imply that the optimal choices for households with income $y \in [71.96, 117.47]$ and $y \in [117.47, 125]$ are, respectively, public education-community 2 and private education-community 1.

(ii) *Housing market equilibrium*:

The housing market is in equilibrium if all houses in every community are occupied. Community 1 capacity is 0.75. It is inhabited by households with income $y \in [8, 71.96]$ and $y \in [117.47, 125]$. It can be verified that:

$F_1(125)-F_1(117.47)=0.05$

$F_1(71.96)-F_1(8)=0.7$

which adds up to 0.75.

Community 2 capacity is 0.25. It is populated by households with income $y \in [71.96, 117.47]$. This income interval satisfies:

$F_1(117.47)-F_1(71.96)=0.25$

(iii) *Majority voting equilibrium*:

(a) *Community 1*: The income of the pivotal voter is implicitly defined by $F_1(59.88) - F_1(\tilde{y}_1) = 0.375$, which yields $\tilde{y}_1$=23.3679. Because households with this income choose public schooling, from (8), the median voter most preferred tax rate t_1^* is:

$$t_1^* \equiv \arg\max \frac{1}{1-\sigma}\left[\left(\tilde{y}_1 - p_1^h(1+t_1)\right)^{1-\sigma} + \delta\left(\frac{t_1 p_1^h N_1}{n_1}\right)^{1-\sigma}\right] =$$

$$= \frac{\tilde{y}_1 - p_1^h}{p_1^h\left(1+\left(\frac{\delta N_1}{n_1}\right)^{-1/\sigma}\left(\frac{N_1}{n_1}\right)\right)} \tag{A4}$$

which yields t_1^*=0.2712. The corresponding level of provision is:

$$E_1^* = \frac{t_1^* p_1^h 0.75}{0.6} = 2$$

(b) *Community 2*: The median (income) voter has income $\tilde{y}_2$=94.7001, which satisfies $F_1(117.47) - F_1(94.7001) = 0.125$. Since all households in community 2 choose public schooling, the median voter most preferred tax rate t_2^* is:

$$t_2^* \equiv \arg\max \frac{1}{1-\sigma}\left[\left(\tilde{y}_2 - p_2^h(1+t_2)\right)^{1-\sigma} + \delta\left(t_2 p_2^h\right)^{1-\sigma}\right] = \frac{\tilde{y}_2 - p_2^h}{p_2^h\left(1+\delta^{-1/\sigma}\right)} \tag{A5}$$

which leads to $t_2^* = 0.8857$. The associated level of provision is $E_2^* = t_2^* p_2^h = 6.2$

Example 15:

(i) *Rational choices.*

(a) Consider alternatives PUB1 and PUB2. For an income level y=53.18, the following can be verified: $v_1(53.18)=v_2(53.18)$. By proposition 8, this implies that all households with income y<53.18 strictly prefer PUB1 to PUB2, while all households with income y>53.18 strictly prefer PUB2 to PUB1.

(b) Consider alternatives PUB1 and PR1. For y=48.09, v_1 and v_2 satisfy: $v_1(48.09)=w_1(48.09)$. By corollary 4, this implies that all households with income y<48.09 strictly prefer PUB1 to PR1, while all households with income y>48.09 strictly prefer PR1 to PUB1.

(c) Consider alternatives PR1 and PUB2. It can be verified that $w_1(64.57)= v_2(64.57)$, which implies $E_2 = \hat{E}_2(64.57,\cdot)$. By lemma 11 and given that $p_2-p_1=14.2-8.8= 5.4>3.94= x(64.57-8.8)$ and that educational services are normal, all households with income y<64.57 satisfy $E_2 < \hat{E}_2(y,\cdot)$ and, hence, they strictly prefer PR1 to PUB2. For the same reason, households with income levels marginally larger than 64.57 satisfy $E_2 > \hat{E}_2(y,\cdot)$ and they prefer PUB2 to PR1. Moreover, for households with the highest income it can be checked that $w_1(120)<v_2(120)$.

From (a) and (b), households with income $y \in [8,48.09]$ maximise their utility choosing public education-community 1, while households with income $y \in [48.09,53.18]$ do so choosing private education-community 1. (b) and (c) entail that the optimal choice for households with income $y \in [53.18,64.57]$ is private education-community 1. Finally, (b) and (c) imply that the optimal choices for households with income $y \in [64.57,120]$ is public education-community 2.

(ii) *Housing market equilibrium*:

The housing market is in equilibrium if all houses in every community are occupied. Community 1 capacity is 0.8. It is inhabited by households with income $y \in [8,64.57]$. It can be verified that:

$F_2(64.57)-F_2(10)=0.8$

Community 2 capacity is 0.2. It is populated by households with income $y \in [64.57,125]$. This income interval satisfies:

$F_2(120)-F_2(64.57)=0.2$

(iii) *Majority voting equilibrium*:

(a) *Community 1*: The income of the pivotal voter is implicitly defined by $F_2(48.09) - F_2(\tilde{y}_1) = 0.4$, which yields $\tilde{y}_1$=22.9718. Because households with this income choose public schooling, the median voter most preferred solves (A4), yielding $t_1^*=0.1753$. This tax rate has an associated level of provision equal to:

$$E_1^* = \frac{t_1^* p_1^h 0.8}{0.7} = 1.5$$

(b) *Community 2*: The median (income) voter has income $\tilde{y}_2$=100.9582, satisfying $F_2(120) - F_2(100.9582) = 0.1$. Since all households in community 2 choose public schooling, the median voter most preferred tax rate t_2^* is given by (A5). This gives an equilibrium tax rate equal to 0.8, whose associated level of provision is $E_2^* = t_2^* p_2^h = 6.2$.

Example 16:

(i) *Rational choices.*

(a) Consider alternatives PUB1 and PUB2. For an income level y=54.52, the following can be verified: $v_1(54.52)=v_2(54.52)$. By proposition 8, this implies that all households with income y<54.52 strictly prefer PUB1 to PUB2, while all households with income y>54.52 strictly prefer PUB2 to PUB1.

(b) Consider alternatives PUB1 and PR1. For y=60.18, v_1 and v_2 satisfy: $v_1(60.18)=w_1(60.18)$. By corollary 4, this implies that all households with income y<60.18 strictly prefer PUB1 to PR1, while all households with income y>60.18 strictly prefer PR1 to PUB1.

(c) Consider alternatives PR1 and PUB2. It can be verified that $w_1(99.18)=v_2(99.18)$, which implies $E_2 = \hat{E}_2(99.18,\cdot)$. Given that p_2-p_1=11.2-7.8=3.4<6.46=x(99.18-7.5), lemma 11 implies that all households with income y>99.18 satisfy $E_2 < \hat{E}_2(y',\cdot)$ and, hence, that they strictly prefer PR1 to PUB2.

(d) Consider again alternatives PR1 and PUB2. The sign of $\partial\hat{E}_2(\cdot)/\partial y$ is positive for all y>55.9 (because x(55.9-7.8)=3.4). This result and (c) imply that households with income $y \in [55.9, 99.18]$, strictly prefer PUB2 to PR1 (because for them $E_2 > \hat{E}_2(\cdot)$).

(a) and (b) imply that households with income $y \in [8, 54.52]$ maximise their utility choosing public education-community 1 and that for households with income $y \in [54.52, 60.18]$ it is rational to choose public education-community 2. (b) and (c) establish that the optimal choice for households with income $y \in [99.18, 125]$ is private education-community 1. Finally, (b) and (d) imply that the optimal choices for households with income $y \in [60.18, 99.18]$ is public education-community 2.

(ii) *Housing market equilibrium*:

Community 1 capacity is 0.8. This community is populated by households with income $y \in [8, 54.52]$ and $y \in [99.18, 125]$. It can be checked that the cumulative income distribution function $F_3(y)$ satisfies:

$F_3(125)$-$F_3(99.18)$=0.1

$F_3(54.52)$-$F_3(8)$=0.7

which adds up to 0.8.

Community 2 capacity is 0.2. It is populated by households with income $y \in [54.52, 99.18]$. This income interval satisfies:

$F_3(99.18)$-$F_3(54.52)$=0.2

(iii) *Majority voting equilibrium*:

(a) *Community 1*: The income of the pivotal voter is implicitly defined by $F_3(54.52) - F_3(\tilde{y}_1) = 0.4$, which yields $\tilde{y}_1$=26.6958. Because households with this income choose public schooling, the median voter most preferred tax rate solves (A4), yielding t_1^*=0.2892. This tax rate has an associated level of provision equal to:

$$E_1^* = \frac{t_1^* p_1^h 0.8}{0.7} = 2$$

(b) *Community 2*: The median (income) voter has income $\tilde{y}_2$=70.3533, which satisfies $F_3(99.18) - F_3(54.52) = 0.1$. Households with this level of income choose public schooling and thus the median voter most preferred tax rate t_2^* is given by (A5). This leads to an equilibrium tax rate equal to 0.6716 and to a level of provision equal to $E_2^* = t_2^* p_2^h = 4.5$.