

# OPTIMAL ENDOWMENTS OF PUBLIC INVESTMENT: AN EMPIRICAL ANALYSIS FOR THE SPANISH REGIONS

Oscar Bajo-Rubio\*  
Carmen Díaz-Roldán\*  
M. Dolores Montáñez-Garcés\*\*

(\*Universidad de Castilla-La Mancha)  
(\*\*Universidad Pública de Navarra)

Preliminary version

October 2001

## Abstract

The aim of this paper is to analyse the degree of optimality of the endowments of public capital in the Spanish regions. To this end, we will estimate a growth equation derived from a simple production function, where the coefficients on the rates of investment in private and government capital would be their respective marginal products. By comparing the estimates of the marginal products for both factors, we would be able to infer whether the public capital stock in the Spanish regions is underprovided or not.

## Address for correspondence:

Department of Economics, Universidad Pública de Navarra, 31006 Pamplona, Spain

E-mail addresses: [obajo@cje-cr.uclm.es](mailto:obajo@cje-cr.uclm.es), [mdold@cje-cr.uclm.es](mailto:mdold@cje-cr.uclm.es), [dolores.montavez@unavarra.es](mailto:dolores.montavez@unavarra.es)

## 1. Introduction

Following Aschauer's (1989) influential contribution, the role of public investment has been stressed as a crucial factor leading to higher private capital productivity, which would lead in turn to higher growth rates. According to this author, the decline in productivity growth experienced by the US economy during the seventies, would be explained to a great extent by the decrease in the provision of public infrastructures during that period. In this way, the next years have witnessed the appearance of a great amount of empirical literature that analysed the impact of public investment on economic growth; a comprehensive survey of that literature can be found in Sturm, Kuper and de Haan (1998).

Although the first empirical studies made use of aggregate time series for countries, this approach has been also extended to a regional framework using panel data, obtaining results that were quantitatively lower than those found with aggregate data [see, e.g., Holtz-Eakin (1994)]. The reason would be the spillover effects related to the regional endowments of public capital, whose effect would extend not only the own region, but also to the neighbouring regions. In any case, public infrastructure seems to play an important role in the growth process of regions that should not be neglected (Button, 1998).

On the other hand, the issue of the optimal endowments of public infrastructure has been hardly discussed. In an empirical analysis of the Swedish case, Berndt and Hansson (1992) pointed out that, since, according to their estimates, public infrastructure capital would have been above its optimal level, this could help to explain the relatively weak effect found for the latter on productivity growth. More recently, Karras (1997) has developed a simple condition to assess whether public capital is optimally provided, namely, whether the marginal productivities of both private and public capital are equal or not. By estimating a simple growth equation for fifteen European countries during the period 1960-1992, he is unable to reject the null hypothesis that the marginal productivities of private and public capital are equal, so that government investment would be neither underprovided nor overprovided in the fifteen countries of his sample.

In this paper we try to address this issue (i.e., whether the endowments of public investment are optimal or not) in a regional framework, using Spanish data for the period 1967-91. The Spanish economy can provide an interesting case of study, since it has experienced a sustained period of growth in the last forty years, which has been accompanied by a strong process of

structural change. In particular, the establishment of new regional governments after the restoration of democracy in 1977, coupled with the strong increase experienced by public investment since then, are all of them elements that can justify the interest of the Spanish case for the objectives of this paper.

The paper is organised as follows. In section 2, the theoretical condition under which public capital would be optimally provided is derived from an optimization growth model. In section 3, we provide an empirical application of the model, for the case of the Spanish regions during the period 1967-1991. Finally, the main conclusions are presented in section 4.

## 2. Theoretical framework

In this section we will derive the condition that will allow us to assess whether public capital is optimally provided or not, following the approach of Karras (1997). The theoretical framework is based on Ramsey's optimization growth model [see Blanchard and Fischer (1989) for an overview], extended to incorporate the role of government capital into the production function.

We begin by assuming an aggregate production function such as:

$$Y_t = A_t F(K_t, KG_t, L_t) \quad (1)$$

where  $Y$  denotes real output, which depends on the amounts utilized of private capital,  $K$ , government capital,  $KG$ , and labour,  $L$ ;  $A$  is an index of the level of technology. The function  $F$  is assumed twice continuously differentiable, with  $F_x = \frac{\partial F}{\partial X_t} > 0$  and  $F_{xx} = \frac{\partial^2 F}{\partial X_t^2} < 0$  (for  $X = K, KG, L$ ), and homogeneous of degree one in all the productive factors. The last assumption allows us to write the production function in *per capita* terms:

$$y_t = A_t f(k_t, kg_t) \quad (2)$$

where  $x = X/L$  denote a variable in *per capita* terms (for  $X = Y, K, KG$ ), with  $f_x = \frac{\partial f}{\partial x_t} > 0$  and

$$f_{xx} = \frac{\partial^2 f}{\partial x_t^2} < 0 \quad (\text{for } x = k, kg).$$

The output is either consumed or invested, so that, in *per capita* terms:

$$\dot{k} = A_t f(k_t, kg_t) - c_t - (\delta + n)k_t - \tau_t \quad (3)$$

where  $\dot{k} = \frac{dk_t}{dt}$ ,  $c$  is *per capita* consumption,  $\delta$  is the rate of depreciation of private capital,  $n$  is the rate of population growth. The last term in equation (3),  $\tau$ , denotes taxes *per capita*, which are used to finance government capital's accumulation following the government budget constraint, also in *per capita* terms:

$$\dot{kg} = \tau_t - (\delta + n)kg_t \quad (4)$$

where  $\dot{kg} = \frac{dkg_t}{dt}$ , and government capital is assumed to depreciate at the same rate than private

capital.

On the other hand, the representative individual is assumed to maximize utility, which depends on *per capita* consumption, over an infinite planning horizon:

$$U = \int_0^{\infty} u(c_t) e^{-\rho t} dt \quad (5)$$

where  $\rho$  is the rate of time preference and  $u_c = \frac{du}{dc_t} > 0$ , subject to (3), (4), and  $k_0, k_{g0} > 0$ . This

optimization problem is solved by setting the Hamiltonian:

$$H_t = u(c_t) e^{-\rho t} + \lambda_1 (A_t f(k_t, k_{g_t}) - c_t - (\delta + n)k_t - \tau_t) + \lambda_2 (\tau_t - (\delta + n)k_{g_t})$$

from which the first-order conditions would be:

$$\frac{\partial H_t}{\partial c_t} = e^{-\rho t} u_c - \lambda_1 = 0$$

$$\frac{\partial H_t}{\partial \tau_t} = -\lambda_1 + \lambda_2 = 0$$

$$\frac{\partial H_t}{\partial k_t} = \lambda_1 A_t f_k - \lambda_1 (\delta + n) = -\dot{\lambda}_1$$

$$\frac{\partial H_t}{\partial k_{g_t}} = \lambda_1 A_t f_{k_g} - \lambda_2 (\delta + n) = -\dot{\lambda}_2$$

In this way, from the first three conditions we get:

$$-\frac{\dot{u}_c}{u_c} = [A_t f_k - (\delta + n)] - \rho \quad (6)$$

and, for the last three:

$$A_t f_k = A_t f_{k_g} \quad (7)$$

where  $A_t f_k$  and  $A_t f_{k_g}$  are the marginal products of private and government capital, respectively. Equation (6) is the Euler condition, which implies that, the higher the marginal product of private capital (net of depreciation and population growth) relative to the rate of time preference, the more it pays to depress the current level of consumption in order to enjoy higher consumption later. In turn, equation (7) states that optimal accumulation of private and government capital requires that their marginal products be equal. The latter condition would imply that, given the marginal product of

private capital, if the marginal product of government capital would be higher than that of private capital, it would be profitable for the government to raise public investment; in other words, and assuming that private capital is optimally provided, a marginal product of government capital above (below) the marginal product of private capital would mean that government capital is underprovided (overprovided), relative to private capital. In the next section we will provide an empirical test of equation (7), using Spanish regional data.

### 3. Empirical model and results

In order to test empirically equation (7), we start from the production function above, equation (1), with time subscripts omitted for simplicity:

$$Y = AF(K, KG, L)$$

which, after differentiating with respect to time, dividing by  $Y$ , and rearranging, becomes:

$$\frac{\dot{Y}}{Y} = \frac{\dot{A}}{A} + e_K \frac{\dot{K}}{K} + e_{KG} \frac{\dot{KG}}{KG} + e_L \frac{\dot{L}}{L} \quad (8)$$

where  $\dot{X} = \frac{dX}{dt}$  denotes the time derivative of variable  $X$  (for  $X = Y, A, K, KG, L$ ), and

$e_X = \frac{\partial Y}{\partial X} \frac{X}{Y}$  denotes the elasticity of real output with respect to variable  $X$  (for  $X = K, KG, L$ ).

On the other hand, the accumulation of private and government capital would be given by:

$$\dot{K} = s_K Y - \delta K \quad (9)$$

$$\dot{KG} = s_{KG} Y - \delta KG \quad (10)$$

where  $s_K$  and  $s_{KG}$  are the output shares of gross investment in private and government capital, respectively; and  $\delta$  is the depreciation rate (assumed to be the same for both types of capital).

Replacing the accumulation equations (9) and (10) in (8) above, we get:

$$g_Y = -\mathbf{d}(e_K + e_{KG}) + g_A + MPK s_K + MPKG s_{KG} + e_L g_L \quad (11)$$

where  $g_X = \frac{\dot{X}}{X}$  denotes the rate of growth of variable  $X$  (for  $X = Y, A, L$ ); and  $MPK = e_K \frac{Y}{K}$  and

$MPKG = e_{KG} \frac{Y}{KG}$  are the marginal products of private and government capital, respectively.

Finally, writing (11) in per capita terms, assuming constant returns to scale (so that  $e_K + e_{KG} + e_L = 1$ ), we get:

$$g_y = g_A + MPK s_K + MPKG s_{KG} - (e_K + e_{KG})(\mathbf{d} + g_L) \quad (12)$$

where  $g_y$  denotes the rate of growth of per capita output. In the rest of this section we will provide econometric estimates of equation (12), and then a test on the estimated coefficients on  $s_K$  and  $s_{KG}$  being equal will be performed.

The data used in the empirical part of the paper come from an earlier paper by the authors (Bajo-Rubio, Díaz-Roldán and Montávez-Garcés, 1999), and cover the 17 regions (“comunidades autónomas”) established after the approval of the current Spanish Constitution in 1978, along the period 1967-1991. In particular, real GDP is taken from Doménech, Escribá and Murgui (1999), the data on physical capital investment (both private and public) have been taken from Mas, Pérez and Uriel (1995), and those on human capital and population (see below) come from Mas, Pérez, Uriel and Serrano (1995). The exact definition of the data can be found in the Appendix.

There is some available evidence on the favourable effect of the public capital stock on the productivity of private capital for the Spanish case, both with aggregate data (e.g., Bajo-Rubio and Sosvilla-Rivero, 1993), and with regional data (e.g., Mas, Maudos, Pérez and Uriel, 1996). Regarding the evidence specifically addressed to the study of growth, Bajo-Rubio and Sosvilla-Rivero (1998) found a positive effect on growth for public investment as a percentage of GDP, with aggregate data for the whole Spanish economy, for the period 1964-93. Finally, in Bajo-Rubio, Díaz-Roldán and Montávez-Garcés (1999) the same result was obtained when estimating a convergence regression with regional data over the period 1967-91.

Some descriptive evidence is provided in Figures 1 and 2, which show the levels of per capita GDP (in real terms) and the GDP share of government investment, for the 17 Spanish regions in the first and last year of our sample period, respectively. As can be seen in Figure 1, per capita GDP would have experienced a significant increase between both dates, reaching twice its initial level in most regions. The growth in per capita GDP would have been somewhat stronger in the case of poorer regions, supporting previous findings on convergence [see, e.g., Raymond and García (1994)]. In turn, the evolution of the GDP share of government investment would have been also impressive, being this increase especially remarkable after the first eighties, when the first Socialist government took office.

Some econometric estimates of equation (12) are provided in Table 1, where the whole period of analysis has been divided into five-year spans in order to avoid the effect of cyclical fluctuations. The method of estimation is Ordinary Least Squares (OLS) including individual effects for each region, which would proxy the differential effect of technical progress among regions.



The results of the estimation for the whole set of regions are shown in column (1). We obtain the expected signs, together with significant coefficients for every variable. In particular, the output shares of gross investment in both private and government capital would affect positively per capita output growth; these results are not substantially modified when human capital (measured by the initial value of the share of working-age population with undergraduate studies) is introduced in column (2). In addition, according to the reported values of the F-statistic, the null hypothesis that the coefficients on  $s_K$  and  $s_{KG}$  are equal can be rejected at the 5% level. Therefore, government capital would have been productive (that is, it would have contributed to the growth of per capita GDP) in the Spanish regions during our period of analysis; and, since its estimated marginal product would be higher than that for private capital, it would have been still underprovided along the whole period.

Next, as in our previous paper, we have divided regions into two groups, i.e., those with per capita GDP above and below the Spanish average in 1967 (the first year of our sample). The results from estimating equation (12) for both groups of regions (defined in the Appendix) appear in columns (3) to (6). As can be seen, the basic results still hold, even though stronger for “poor” regions. On the contrary, in the case of “rich” regions the coefficient on government capital is significant just at the 20% significance level, and the null hypothesis of the coefficients on both types of capital being equal is not rejected, but only when human capital is not included into the regression.

#### 4. Conclusions

In this paper we have tried to find some evidence on the optimality of the provision of government capital in the Spanish regions. To this end, we have derived from an optimization growth model a condition allowing us to assess whether public capital would be under or overprovided. This theoretical condition consists in testing whether the marginal products of private and government capital are equal or not.

This condition has been tested empirically using regional Spanish data during the period 1967-1991, by estimating a growth equation derived from a simple production function. When the model was estimated for the whole set of regions, favourable results were obtained regarding the effect of both private and public capital on growth. In addition, the null hypothesis that the coefficients on both types of capital are equal could be rejected, and the estimated coefficient on government investment proved to be higher than that on private investment. The basic results were not substantially modified when regions were separated according to their initial per capita GDP, although the coefficient on government capital showed a lower significance in the case of richer regions. Therefore, according with these results, government capital would be still underprovided in the Spanish regions, despite the high increase experienced in last years, in particular for the poorer regions.

In spite of the caution with which our provisional results should be taken, the main conclusion leads to a clear policy implication. Government capital would have been a relevant factor behind the growth process experienced by Spanish regions in last years, but there is still room for higher levels of public investment, especially in poorer regions.

### Appendix: Definitions and data sources

We have used annual data for the period 1967-1991. The variables included in the tables are defined as follows:

- $g_y$ : rate of growth of per working-age person GDP at factor cost, at 1980 prices, for each subperiod. Source: Doménech, Escribá and Murgui (1999).
- $\delta$ : rate of depreciation, equal to 8.28 per cent, the average of those used in Mas, Pérez and Uriel (1995).
- $g_L$ : annual average of the rate of growth of working-age population for each subperiod. Source: Mas, Pérez, Uriel and Serrano (1995).
- $s_K$ : annual average of the share of private physical capital investment in total GDP for each subperiod. Source: Mas, Pérez and Uriel (1995).
- $s_{KG}$ : annual average of the share of public physical capital investment in total GDP for each subperiod. Source: Mas, Pérez and Uriel (1995).
- $s_H$ : initial value of the share of working-age population with undergraduate studies, for the first year of every time span (1967, 1972, 1977, 1982, 1987). Source: Mas, Pérez, Uriel and Serrano (1995).

The “richer” regions appearing in Table 1 are: Madrid, País Vasco, Cataluña, Baleares, Cantabria, Navarra, and Asturias; and the “poorer” regions appearing in that table are: La Rioja, Comunidad Valenciana, Aragón, Castilla-León, Canarias, Murcia, Andalucía, Galicia, Castilla-La Mancha, and Extremadura.

## References

Aschauer, D. (1989): "Is public expenditure productive?", *Journal of Monetary Economics* 23, 177-200.

Bajo-Rubio, O. and Sosvilla-Rivero, S. (1993): "Does public capital affect private sector performance? An analysis of the Spanish case, 1964-88", *Economic Modelling* 10, 179-185.

Bajo-Rubio, O. and Sosvilla-Rivero, S. (1998): "El crecimiento económico en España, 1964-1993: Algunas regularidades empíricas", in de la Torre, J. and García-Zúñiga, M. (eds.): *Hacienda y crecimiento económico*, Marcial Pons, Madrid, 213-248.

Bajo-Rubio, O., Díaz-Roldán, C. and Montávez-Garcés, M.D. (1999): "Fiscal policy and growth revisited: the case of the Spanish regions", Documento de Trabajo 9904, Departamento de Economía, Universidad Pública de Navarra.

Berndt, E.R. and Hansson, B. (1992): "Measuring the contribution of public infrastructure capital in Sweden", *Scandinavian Journal of Economics* 94, S151-S168.

Blanchard, O. and Fischer, S. (1989): *Lectures on Macroeconomics*, The MIT Press, Cambridge, Mass.

Button, K. (1998): "Infrastructure investment, endogenous growth and economic convergence", *Annals of Regional Science* 32, 145-162.

Doménech, R., Escribá, F. J. and Murgui, M. J. (1999): "Cambios en precios relativos y crecimiento regional", *Revista de Economía Aplicada* 19, 5-29.

Holtz-Eakin, D. (1994): "Public-sector capital and the productivity puzzle", *Review of Economics and Statistics* 76, 12-21.

Karras, G. (1997): "Is government investment underprovided in Europe? Evidence from a panel of fifteen countries", *Economia Internazionale* 50, 223-235.

Mas, M., Maudos, J., Pérez, F. and Uriel, E. (1996): “Infrastructures and productivity in the Spanish regions”, *Regional Studies* 30, 641-649.

Mas, M., Pérez, F. and Uriel, E. (1995): *El stock de capital en España y sus comunidades autónomas*, Fundación BBV, Bilbao.

Mas, M., Pérez, F., Uriel, E. and Serrano, L. (1995): *Capital humano, series históricas, 1964-1992*, Fundació Bancaixa, Valencia.

Raymond, J. L. and García, B. (1994): “Las disparidades en el PIB per capita entre comunidades autónomas y la hipótesis de convergencia”, *Papeles de Economía Española* 59, 37-58.

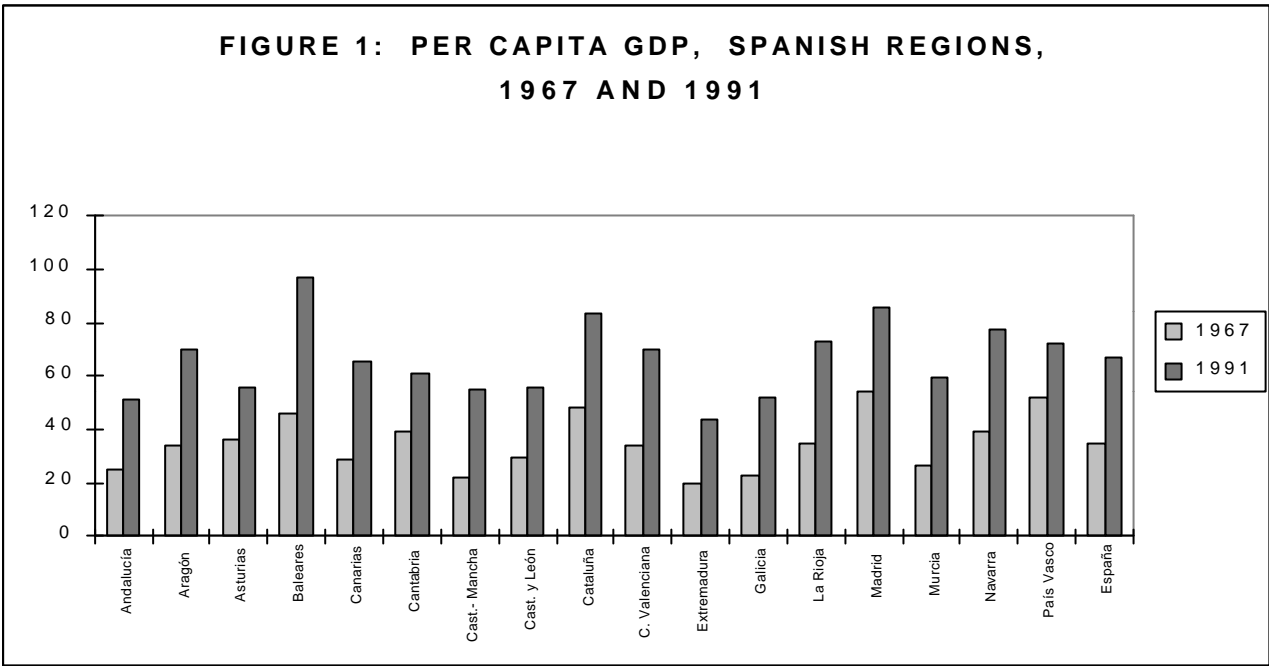
Sturm, J.-E., Kuper, G.H. and de Haan, J. (1998): “Modelling government investment and economic growth at the macro level: A review”, in Brakman, S., van Ees, H. and Kuipers, S. (eds.): *Market behaviour and macroeconomic modelling*, Macmillan/St. Martin's Press, London, 359-406.

**TABLE 1**Dependent variable:  $g_y$ 

	(1)	(2)	(3)	(4)	(5)	(6)
$(\delta + g_L)$ 2.4800 6.7932	-1.3103 (-3.9270)	-1.2720 (-3.9185)	-0.2641 (-0.4641)	-0.1588 (-0.2790)	-2.3484 (-5.9010)	- (-)
$S_K$ 0.3156 (8.2605)	0.2607 (6.7757)	0.2720 (7.2098)	0.1802 (2.7359)	0.1925 (2.9218)	0.3042 (7.2872)	
$S_{KG}$ 1.2525 (5.7251)	0.6718 (3.2453)	0.7538 (3.6860)	0.4884 (1.3763)	0.5236 (1.4872)	1.0623 (4.6277)	
$S_H$ 0.2179 (2.9396)	-	0.1707 (2.2075)	-	0.1849 (1.2499)	-	
F 4.2513 [0.0449]	4.9188 [0.0293]	6.9388 [0.0101]	0.9145 [0.3460]	5.6391 [0.0239]	13.2107 [0.0006]	
$R^2$ 0.7369	0.4979	0.5334	0.3487	0.3885	0.6738	

**Note:** t-statistics in parentheses. F is the F-statistic for the null hypothesis that the coefficients on  $S_K$  and  $S_{KG}$  are equal (significance levels in brackets).

**FIGURE 1: PER CAPITA GDP, SPANISH REGIONS, 1967 AND 1991**



**FIGURE 2: PUBLIC INVESTMENT AS A PERCENTAGE OF GDP, SPANISH REGIONS, 1967 AND 1991**

