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**Assessing Tax Reforms.**  
**Critical Comments and a Proposal: The Level and Distance Effects.**

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**Assessing Tax Reforms.  
Critical Comments and a proposal: The Level and Distance Effects<sup>1</sup>**

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## **1. Introduction**

An attempt to justify the “goodness” of tax reforms geared towards reducing the tax burden within the framework of Income Tax frequently leads its defenders to putting forward arguments based on equity and redistributive effects. They attempt to explain that such reforms improve how income is shared out among taxpayers. To support this kind of argument, analytical tools such as progressivity and redistribution indices tend to be employed. However, when such indices are used beyond the frameworks for which they were originally conceived, they lead to arguable and confusing results, although they may well be used to back a specific tax reform proposal.

This paper aims to invite us to reflect on the above-mentioned problem —certainly very frequent in the Social Sciences— of the biased use of analytical tools that were originally envisaged to study the effects of income maintenance and public expenditure policies on social welfare. Additionally, the study offers an alternative way to measure the effects of public policies that the authors believe could be more simple and direct when assessing such results.

In order to achieve these aims, the paper kicks off with a very simple review of the tools most commonly used to measure inequality, more specifically the indicators for progressivity and redistributive effects. A critical assessment on how such indicators are used and how their results are interpreted is made in the second section. The Level and Distance Effects are then introduced in the following section. These tools are put forward as an alternative analytical tool to the previously mentioned indices in order to assess public policies. Lastly, an attempt is made in the final section to apply these notions in order to assess reforms made to the main income tax elements. In addition to its conclusions, two appendixes are attached to the study. The first of these offers an analytical demonstration of the results of different income tax reform hypotheses starting off from the use of the Distance and Level Effects, which are contained in Table 1. The second appendix provides a series of examples on the already mentioned results.

## 2. Tools for Measuring Inequality, Progressivity and Redistribution

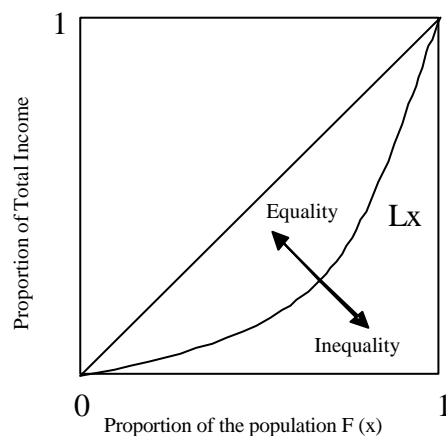
A preliminary approach to analyzing income distribution and the changes it undergoes (as well as the causes behind such changes and their effects on welfare) requires ways to properly measure distribution.

To do so, we need to rely on a ranking of the incomes to be studied. Such rankings in discrete terms ( $x_1, x_2, \dots, x_N$ ) tend to be represented mathematically by simulating the income that is distributed continuously throughout the income scale, thus enabling mathematical calculations to be made. This procedure allows us to employ the density frequency function,  $f(x)$  and equivalent distribution function  $F(x)$  which measures the proportion of people earning incomes lower than or equal to  $x$ . The equivalence between both functions, as it is well known, is specified by the expression:

$$f(x) = F'(x)$$

### INEQUALITY

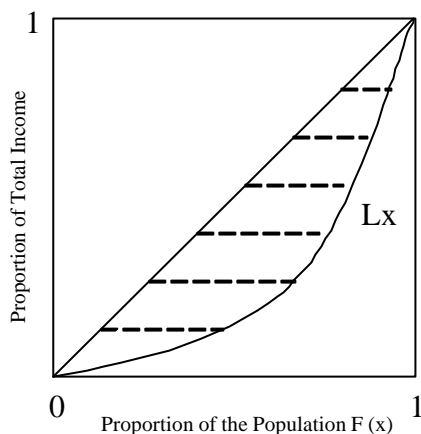
When assessing a specific income distribution, we should set out how we are going to measure the inequality existing in said income distribution. This is usually done graphically by mean of a Lorenz curve, which provides us with a standardized system of measuring percentage participations in total income. After ranking the population units according to their levels of income (from least to greatest), the proportion of accumulated income held by the different accumulated proportions of the population is represented. As long as inequality exists, a Lorenz curve below the diagonal line (known as the line of perfect equality) will be obtained.



The further away the Lorenz curve is from the diagonal line, the greater inequality there is. This method of measurement is only suitable if one is only interested in focusing on relative as opposed to absolute income differences. In addition, it is worth highlighting that **this method of measuring inequality is based on calculating the departure from proportionality while total income remains constant.** The 45° diagonal line represents an *ideal* way of sharing out total income in which each proportion of the population would have the same proportion of income (complete equality concerning total income). When we compare distributions with different levels of income, Lorenz

curves cannot show us in general terms which situation is preferable regarding welfare. To do so, we would have to compare generalized Lorenz curves, which are obtained by multiplying the original values of these curves based on some sort of indicator by the corresponding average income of each distribution.

Specific measurement indicators are usually used in order to summarize relative inequality by means of a single indicator. The most commonly used of these is the Gini Index, which measures the surface area between the diagonal line and the Lorenz curve compared to the total surface area below the diagonal line.



The Gini Index can be mathematically obtained for discrete income distributions by means of the following formula<sup>2</sup>:

$$(2.1) \quad G_x = \frac{\sum_{i=1}^N \sum_{j=1}^N |x_i - x_j|}{2N^2 \bar{x}}$$

This formula clearly reveals that the Gini Index is equivalent to half the average difference between income pairs divided by average income<sup>3</sup>.

## PROGRESSIVITY

Focusing now on the main topic broached by this piece of research; changes in distribution brought about governmental policies may come about as a result of tax reforms or the implementation of different spending policies. More specifically, we will focus on how progressivity is measured in order to evaluate the effects of these

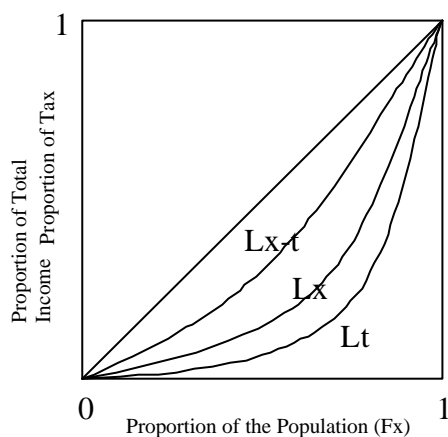
<sup>2</sup> This is the most commonly known formula for the Gini Index among the different variations available and has been shown for demonstration purposes.

<sup>3</sup> The Gini Index can therefore provide us with a complete ranking of the different situations in relation with income distribution. However, the reservation mentioned above must logically be kept when comparing distributions with different levels of income in order to interpret any conclusions in terms of their effects on welfare.

changes. Progressivity is commonly defined as a property of a tax structure in relation to a specific income distribution before the tax is actually applied and it is obtained through the use of indices known as effective or overall progression indices.

Two essential features of a progressive income tax are its departure from proportionality and the redistributive effect it generates. As we shall see below, both are necessary for progressivity to be considered as positive<sup>4</sup>.

Supposing that all the population units earning the same income bear the same tax burden (in other words, the tax burden solely depends on income), we can thus represent the distribution of the tax burden by using the same tool described for the Lorenz curves. A concentration curve  $L_t$  of taxes is therefore obtained as well as a concentration coefficient  $C_t$  (an index analogous to the Gini Index) associated to it. We could likewise obtain a concentration curve of after-tax income  $L_{x-t}$  as well as its corresponding concentration coefficient  $C_{x-t}$ <sup>5</sup>.



If a tax is progressive, the amounts of tax liability are systematically shifted away from proportionality in relation to pre-tax income. The distribution of the tax burden is therefore more uneven than income and is reflected in the concentration curve of tax liabilities, which is further away from the diagonal line than the Lorenz curve reflecting pre-tax income, hence  $L_x > L_t$ .

**In order to interpret the significance of the distance between curves  $L_x$  and  $L_t$ , it is helpful to consider  $L_x$  as the concentration curve of the tax liability that would be obtained with an equal-yield flat tax<sup>6</sup>.**

To sum up, the Kakwani Index – the most commonly used index to measure progressivity based on departure from proportionality (progressivity) – measures double the area between the Lorenz curve of pre-tax income ( $L_x$ ) and the concentration curve for tax liability ( $L_t$ ). Likewise, the Kakwani Index can also be expressed as the

<sup>4</sup> See Lambert (2001), p. 191.

<sup>5</sup> For simplicity's sake, let us suppose that the different incomes comprising the distribution are not re-ranked. Then,  $C_{x-t} = G_{x-t}$

<sup>6</sup> As expressed by Lambert (2001), p. 201

difference between the tax's concentration coefficient and the Gini Index for pre-tax income:  $K = C_t - G_x$ .

## REDISTRIBUTION

Concerning the redistributive effect, the leveling effect of a progressive income tax can be observed by noting that  $L_{x-t} > L_x$  (recalling that  $L_x$  also represents the after-tax Lorenz curve for a flat tax and supposing there is no re-ranking). It is likewise possible to quantify the distance separating both curves by means of the Reynolds-Smolensky Index  $RS = G_x - C_{x-t}$ , which measures double the area between the Lorenz curve for after-tax income and the concentration curve for pre-tax income.

It is worth underlining that **when we consider the Lorenz curve shift produced by a tax, we are implicitly making a comparison between the results after applying a tax that reduces inequality and the results that would be obtained after applying an equal-yield flat tax**. Such a tax would be neutral in terms of distribution and would maintain the relative pre-tax income differences. It is the natural term of reference to assess the redistributive effects of **a specific tax yield that reduces inequality**. The redistributive effect of progressivity is therefore measured in relation to the tax's proportionality. This justifies considering the "leveling effect" as a "redistributive effect" despite the fact that redistribution generally refers to a new distribution of a given amount of total income, which is now lower<sup>7</sup>.

It is therefore clear that departure from proportionality and the redistributive effect are two closely linked factors. To clarify their connection, a simple transformation is usually employed with the following result:

$$(2.2) \quad \begin{aligned} L_{x-t} - L_x &= (t/1-t) (L_x - L_t) \\ RS &= (t/1-t)K \end{aligned}$$

The term on the left represents the redistributive effect as a proportion of total income that is shifted down the income scale by the existence of progressivity, while the term on the right represents the relationships between the average tax rate ( $t$ ) and net income  $(1-t)$  on the one hand and the measurement of disproportionality measured as the proportion of the tax shifted up the income scale on the other. Hence, the redistributive effect is determined by disproportionality and a measurement of the tax burden.

It could be of interest to raise the question of how a progressive income tax can be considered as a positive phenomenon. **Reducing inequality in itself does not imply an improvement in terms of welfare. In fact, it reduces welfare, as does all taxation**. Nevertheless, we can affirm that **progressive tax rates are positive if they are compared to other ways of collecting the same amount of tax from a specific pre-tax income distribution**. As a matter of fact, according to Shorrocks, it can be proved that for individualistic, symmetric, additively separable and inequality-averse social welfare functions, income taxation reduces welfare. However, Lambert states that, "Progressive income taxation reduces social welfare by less than an equal-yield flat tax applied to the same pre-tax income distribution"<sup>8</sup>.

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<sup>7</sup> Further details of this topic can be seen in Lambert (2001), p 39. Its development can be consulted in pp. 208-209.

<sup>8</sup> Lambert (2001), Theorem 8.2, p. 191.

Lastly, regarding the assertion made in the paragraph above, it is worth asking whether any increase in a tax's progressivity is positive in terms of social welfare. The answer to this question is complex, especially if the tax yield is not constant, as is normally the case. The answer would depend on exactly who would benefit and who would be prejudiced, and to what extent they would be<sup>9</sup>. Nevertheless, as Lambert recalls, a progressive income tax is **generally** considered redistributive independently of what happens to the tax's total yield.

### 3. A Critical Assessment on Common Results Interpretations

The indices mentioned above are the ones most commonly used to describe income distribution changes and more specifically to assess the consequences resulting from a particular tax reform.

When the evolution of inequality in income distribution or its differences among countries are analyzed, most studies are aware of the fact that the tools used should be refined if one wishes to offer a normative assessment of the comparisons in terms of welfare. To be more precise, Lorenz curves show how total existing income is shared out, but they do not provide us with any information on the total amount of income or the number of individuals that make up the population, which would be summarized in average income.

As was mentioned previously, inequality measurements using Lorenz curves focus on departures from proportionality while total income remains constant. Moreover, the possibility that the curves could intersect exists even when average income remains constant, further complicating normative interpretations of such curves.

To overcome these problems, developments based on the works by Atkinson (1970) and Shorrocks (1983) are used by means of calculating the generalized Lorenz curves mentioned previously, ordinary Lorenz curves multiplied by average income. These tools enable normative assessments on income distribution changes or differences to be made for a wide variety of situations. However, a few particular cases still exist that are difficult to judge in terms of welfare<sup>10</sup>.

Nonetheless, using Lorenz and concentration curves in addition to the inequality, progressivity and redistribution indices associated with them is very common when tax reforms are evaluated. Their pre- and after-tax values are compared and normative conclusions are drawn based on the differences observed.

As an example, this occurs in the work published by the Spanish Ministry of Finance in 2001, which defends the new tax implemented in 1999<sup>11</sup>. After the tax-cutting reform of the previous year, the tax turned out to have greater redistributive effects based on an analysis of generalized Lorenz curves, mistaking its redistributive impact with its effects on welfare. Similarly, the papers written by García Vaquero, V. and Hernández

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<sup>9</sup> This question is broached by Lambert for some specific cases in Chapter 9. See Lambert (2001), pp. 219-236.

<sup>10</sup> See Lambert (2001), Chapter 3, pp. 44-83.

<sup>11</sup> See the Spanish Secretariat of State for Public Finance (2001), pp. 56-57.



de Cos, P. (2003), and Castañer, et. al. (2003) analysing the Spanish income tax reform of 2002, which among other aspects led to a reduction in the tax schedule as of 2003, reached the conclusion that the reform increased the tax's progressivity with hardly any redistributive effects once the results were analysed in terms of the Kakwani and Reynolds-Smolensky indices<sup>12</sup>.

Such comparisons and value judgments are correct if total tax yield does not change. If it does change however, obtaining greater progressivity (a larger K) or increased redistribution (a larger RS) after the reform do not in themselves have any normative implications<sup>13</sup>. As was set out in Section 2, the benefits of a tax's progressivity or its redistributive effect are justified by comparing it with an equal-yield flat tax.

Lambert asserts, "We cannot expect so robust a welfare recommendation for progression changes which affect the tax yield"<sup>14</sup>. He also mentions, however, that "all is not lost" because if the tax yield varies, the interchange of progressivity becomes explicit.

There are two ways to interpret and evaluate the results of studies geared towards assessing tax reforms. The first of these solutions consists of comparing after-tax income distributions by means of generalized Lorenz curves. This, in our view, is arguable in so far as the differences in available income levels clearly favor tax structures resulting in a tax cut. The differences in the tax yield should still be taken into account if one wishes to evaluate the welfare of individuals making up the population in as much as these differences have an affect on the benefits ensuing from public expenditure programs. If variations in the tax yield are offset with other taxes, their effects should likewise be taken into consideration.

Should arguments of efficiency be employed to alter the amounts of service provided or taxes collected within a dynamic perspective, such effects should also be verified. We are well aware of the complications these affirmations suppose for empirical studies. Nevertheless, we consider that obtaining inconsistent or incomplete results is more detrimental if such shortcomings are not explicitly set out.

A second way of assessing the effects of a tax reform that changes a tax's yield consists of taking advantage of the RS index decomposition described in the previous section, which differentiates the variation in the tax's redistributive capacity caused by:

- ? Changes in the taxes effective average rate ( $t/1-t$ )
- ? Changes in progressivity (K).

A reduction (increase) in the tax derived from a decrease (increase) in  $t$  will always have a negative (positive) effect on RS when the tax is progressive. The same effect would produce a reduction (increase) in progressivity as measured by K. Hence, in a tax reform that reduces  $t$ , one can only expect that the increase in progressivity (K) is sufficiently large to offset any change in the tax rates.

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<sup>12</sup> García-Vaquero, V. and Hernández de Cos, P. (2003), pp. 34-36. Castañer, et. al. (2003).

<sup>13</sup> It is worth adding that it does not seem correct to ascribe them with relevant positive content. This is so because the departure from proportionality they show happens in two different tax yield contexts. They are therefore not comparable.

<sup>14</sup> Lambert (2001), p.210.

This procedure seems to suggest that it is possible to make totally separate assessments about what happens to the tax yield and the reform's consequences regarding progressivity by making the interchange explicit. This would therefore permit a positive assessment to be made of an increase in progressivity measured by K—if it comes about—and explain the reduction or the lower pace of increase in the tax's redistributive effect solely on the amount of the tax reduction.

However, if we translate the RS variation (RS' represents the Reynolds-Smolensky Index's value after the tax reform in the formula resulting from decomposing the effective average rate and progressivity) into discrete terms, it becomes evident that **the variation in progressivity (K=Ct – Gx) is not independent of the tax rate reduction. A fall in the tax's yield (level of the tax) reduces the denominator of the tax's concentration coefficient by means of the average rate. This in turn increases the value of Ct and therefore contributes to making the value of K higher (increase in progressivity).** Furthermore, this effect would be independent of the tools used to implement the tax reform.

$$(3.1) \quad RS' = \frac{t(1 - \Delta t) \sum_{i=1}^N \sum_{j=1}^N |Ct'_i - Ct'_j|}{1 - t(1 - \Delta t) \sum_{i=1}^N \sum_{j=1}^N |Ct'_i - Ct'_j|} \cdot Gx$$

In the formula above, Δ represents the increase (Δ > 0; increase in the tax) or decrease (Δ < 0; reduction in the tax) of the average rate (t) as a result of the reform.

When tax yield changes as a result of a tax reform, decomposing the RS between changes in the tax yield's level and progressivity does not allow one to interpret that they can be handled independently. In other words, it cannot be affirmed that we could simply reduce the tax's level and maintain the gain in progressivity to achieve a greater redistributive effect. This reasoning would lead one to believe that the reform put into effect sets up a more progressive tax—and therefore better in terms of social welfare—and that the only negative feature lies in the amount of the tax cut. As we shall see further below, the specific tools (tax elements) used to implement the reform are essential to fully understand this connection.

An alternative method for separating the effects derived from a tax reform aimed at providing a deeper interpretation of the consequences of tax reforms is put forward in the following section.

#### 4. An Attempt at Decomposing the Results: The Level Effect and the Distance Effect

Our proposal is geared towards offering an additional interpretation of a tax reform's effects. We consider that the proposal could be of interest in order to come to a better understanding of tax reforms. To link up with the previous section, we will start off by applying the proposal to the variations observed in a tax's redistributive capacity before

and after the tax reform and then generalize the decomposition by applying it to an analysis of progressivity.

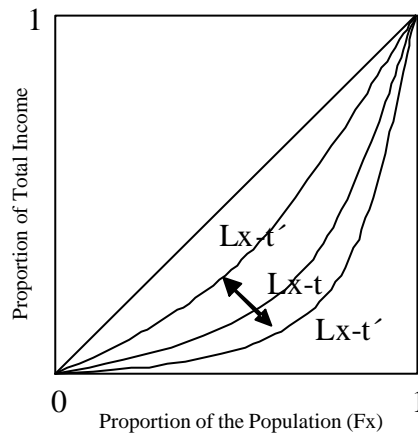
Given that tax level and progressivity are not separable in a tax reform process, we will make an attempt to differentiate level changes (understood as net income or average tax liability) and the distances separating net income or tax liabilities as elements that can be handled independently when a tax is reformed.

## REDISTRIBUTION

We can obtain the difference between  $RS'$  (after the tax reform) and  $RS$  (before the tax reform) through the formula set out below by calculating the formulas for the concentration or Gini Indices with discrete data applied to the redistributive effect's definition:

$RS' - RS > 0$ : Increase in redistributive capacity when going from  $t$  ( $Lx-t$ ) to  $t'$  ( $Lx-t'$ )

$RS' - RS < 0$ : Decrease in redistributive capacity when going from  $t$  to  $t'$



$$RS' - RS = (Gx - Cx-t') - (Gx - Cx-t) = Cx-t - Cx-t' =$$

$$(4.1) \quad \frac{\sum_{i=1}^N \sum_{j=1}^N |x_{t_i} - x_{t_j}|}{2N^2} - \frac{\sum_{i=1}^N \sum_{j=1}^N |x_{t'_i} - x_{t'_j}|}{2N^2(1 - \tau)}$$

$$= \frac{\sum_{i=1}^N \sum_{j=1}^N |x_{t_i} - x_{t_j}|}{2N^2} - \frac{\sum_{i=1}^N \sum_{j=1}^N |x_{t_i} - x_{t_j}| + \sum_{i=1}^N \sum_{j=1}^N |x_{t_i} - x_{t'_j}| + \sum_{i=1}^N \sum_{j=1}^N |x_{t'_i} - x_{t_j}|}{2N^2(1 - \tau)}$$

$$= \frac{\sum_{i=1}^N \sum_{j=1}^N |x_{t_i} - x_{t_j}|}{2N^2} - \frac{1}{1 - \tau} \frac{\sum_{i=1}^N \sum_{j=1}^N |x_{t_i} - x_{t_j}| + \sum_{i=1}^N \sum_{j=1}^N |x_{t'_i} - x_{t_j}|}{2N^2(1 - \tau)}$$

$$= Cx-t - \frac{1}{1 - \tau} \frac{D - D'}{2N^2(1 - \tau)}$$

Here  $\Delta$  represents the increase ( $\Delta > 0$ ) or decrease ( $\Delta < 0$ ) of average net income ( $\Delta$ ) as a result of the tax reform. To simplify the expression, we have denominated:

$$(4.2) \quad D = \sum_{i=1}^N \sum_{j=1}^N |x_{t_i} - x_{t_j}|$$

as the sum of the distances separating net incomes before the reform, and:

$$(4.3) \quad D' = \sum_{i=1}^N \sum_{j=1}^N |x'_{t_i} - x'_{t_j}|$$

as the sum of the distances separating net incomes after the reform. Lastly, we shall denominate the Level Effect (LE) as:

$$(4.4) \quad LE = Cx \Delta \left( \frac{1}{N} \right)$$

and the Distance Effect (DE) as:

$$(4.5) \quad DE = \frac{D - D'}{2N^2 \Delta}$$

Both the LE and DE can be positive (positive contribution to redistribution in the changes of level and distance), or negative (negative contribution).

If  $\Delta > 0$  ? LE > 0

If  $\Delta < 0$  ? LE < 0

If  $\Delta = 0$  ? LE = 0

For the Distance Effect:

If  $D > D'$  ? DE > 0

If  $D < D'$  ? DE < 0

If  $D = D'$  ? DE = 0

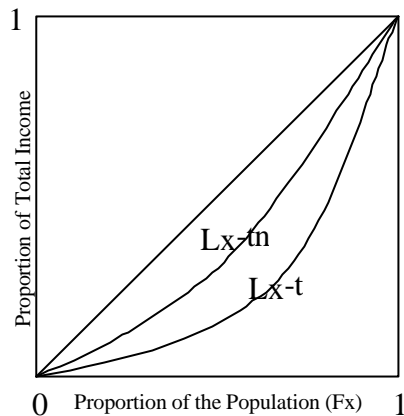
The interpretation of these effects can be conducted both in terms of curves as well as of concentration indices. Hence, the Level Effect would be equivalent to the difference between  $Lx-t$  and  $Lx-t_n$ .  $Lx-t$  represents the Lorenz curve for net income distribution before the reform, while  $Lx-t_n$  represents the Lorenz curve for net income distribution if a fixed per capita transfer<sup>15</sup> is added to ( $Lx-t$ ) whose total amount is equivalent to the variation in the net income level<sup>16</sup>:

- Tax cut: Positive transfer (+) of a fixed per capita amount

<sup>15</sup> Which would be positive should the tax be reduced, and negative should it be increased.

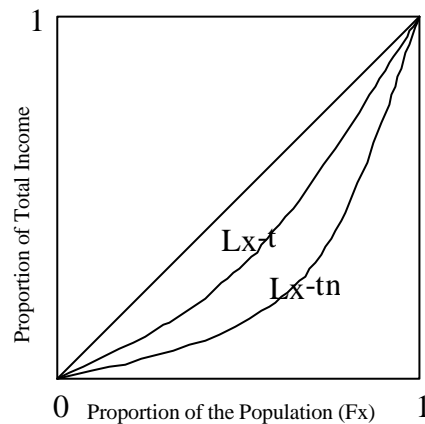
<sup>16</sup> Summarized by its corresponding concentration indices.

$$L_{x-t_n} - L_{x-t} ? \quad EN = C_{x-t} - C_{x-t_n} > 0$$



- Tax increase: Negative transfer (-) of a fixed per capita amount

$$L_{x-t_n} - L_{x-t} ? \quad EN = C_{x-t} - C_{x-t_n} < 0$$

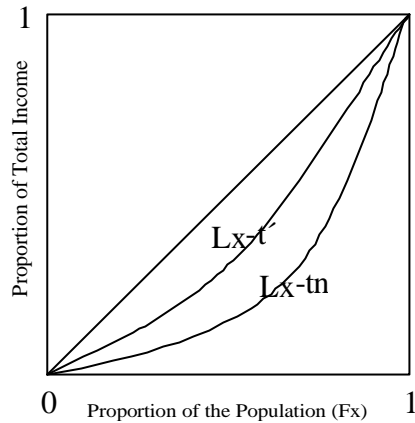


LE measures the effect on redistribution of a change in the tax level while the differences separating net incomes remain constant. It reflects the effects of a tax reform that does not change the distances but does change average net income levels through a modification in the tax yield.

As it was defined above, the Distance Effect represents the difference between  $L_{x-t_n}$  (mentioned previously) and  $L_{x-t'}$ , the Lorenz curve of net income distributions after the reform, summarized by their corresponding concentration indices.

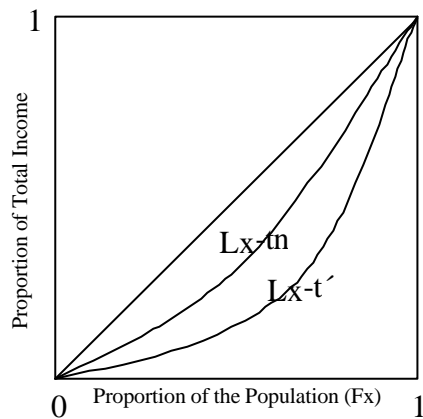
- Reduction of the differences between net incomes:

$$L_{x-t'} - L_{x-t_n} ? \quad ED = C_{x-t_n} - C_{x-t'} > 0$$



- Increase of the differences between net incomes:

$$Lx-t' - Lx-tn ? \quad ED = Cx-tn - Cx-t' < 0$$



DE measures the effect on redistribution of any changes in the distances separating net incomes if the amount of the tax's yield remains constant after the reform. It therefore measures variations in distribution separately through a normative assessment because it compares two income distributions depending on their departure from proportionality while average net income remains constant.

In short, an increase (reduction) in the average level of income that does not affect the distances would improve (worsen) the distribution measured by RS. While an increase (decrease) in the distances separating net incomes that does not affect average income would worsen (improve) the distribution<sup>17</sup>.

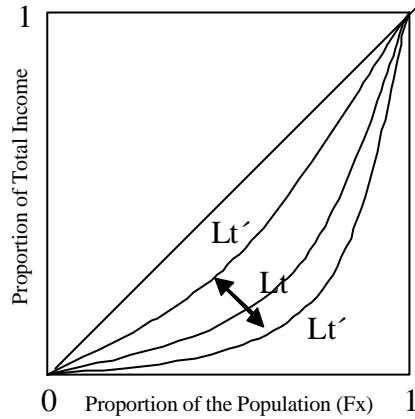
<sup>17</sup> It is worth highlighting that changes in progressivity are implicit when measuring both effects. Hence, this alternative decomposition of effects does not imply being able to separate progressivity from the tax level. As a matter of fact, all measurements of progressivity combine both the Distance and Level Effect.

PROGRESSIVITY

This same development can also be applied to progressivity by analyzing changes in the Kakwani index.

$K' - K > 0$ : Increase in progressivity when going from  $t$  ( $L_t$ ) to  $t'$  ( $L_{t'}$ ), the distance between the diagonal line and the concentration curve increases.

$K' - K < 0$ : Reduction in progressivity when going from  $t$  ( $L_t$ ) to  $t'$  ( $L_{t'}$ ), the distance between the diagonal line and the concentration curve decreases.



$$K' - K = (C_{t'} - G_x) - (C_t - G_x) = C_{t'} - C_t = -C_t + C_{t'}$$

$$(4.6) \quad \frac{\sum_{i=1}^N \sum_{j=1}^N |C_{t_i} - C_{t_j}|}{2N^2} - \frac{\sum_{i=1}^N \sum_{j=1}^N |C_{t'_i} - C_{t'_j}|}{2N^2(1 + \Delta)} = \frac{\sum_{i=1}^N \sum_{j=1}^N |C_{t_i} - C_{t_j}|}{2N^2} - \frac{\sum_{i=1}^N \sum_{j=1}^N |C_{t_i} - C_{t_j}| + \sum_{i=1}^N \sum_{j=1}^N |C_{t'_i} - C_{t'_j}|}{2N^2(1 + \Delta)}$$

$$= \frac{\sum_{i=1}^N \sum_{j=1}^N |C_{t_i} - C_{t_j}|}{2N^2} \left( \frac{1}{1 + \Delta} - 1 \right) - \frac{\sum_{i=1}^N \sum_{j=1}^N |C_{t'_i} - C_{t'_j}|}{2N^2(1 + \Delta)}$$

$$= \frac{\sum_{i=1}^N \sum_{j=1}^N |C_{t_i} - C_{t_j}|}{2N^2} \frac{1}{1 + \Delta} - \frac{D}{2N^2(1 + \Delta)}$$

Here  $\Delta$  represents the increase ( $\Delta > 0$ ; tax increase) or reduction ( $\Delta < 0$ ; tax cut) in average tax liability as a result of the reform. As before:

$$(4.7) \quad D = \sum_{i=1}^N \sum_{j=1}^N |C_{t_i} - C_{t_j}|$$

represents the sum of distances separating the amounts of tax liability before the reform and:

$$(4.8) \quad D' = \sum_{i=1}^N \sum_{j=1}^N |Ct'_i - Ct'_j|$$

represents the sum of distances separating the amounts of tax liability after the reform. The Level Effect (LE) would then be:

$$(4.9) \quad LE = \frac{1}{N} \sum_{i=1}^N Ct'_i - \frac{1}{N} \sum_{j=1}^N Ct_j$$

And the Distance Effect (DE) would be:

$$(4.10) \quad DE = \frac{D' - D}{2N^2}$$

Both LE and DE may be either positive (increase in progressivity) or negative (decrease in progressivity).

For the Level Effect:

If  $D' > D$  ?  $EN < 0$

If  $D' < D$  ?  $EN > 0$

If  $D' = D$  ?  $EN = 0$

For the Distance Effect

If  $D > D'$  ?  $ED < 0$

If  $D < D'$  ?  $ED > 0$

If  $D = D'$  ?  $ED = 0$

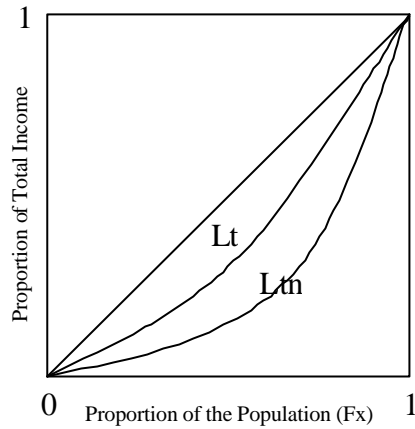
We should recall that positive LE and DE increase the progressivity measurement (Kakwani), while negative LE and DE values reduce it.

The interpretation of these effects may be carried out either by means of curves or concentration indices. Hence, the Level Effect would be the difference between  $L_t$  and  $L_{tn}$ . The former represents the Lorenz curve of tax liability distribution before the reform, and the latter the Lorenz curve of tax liability distribution if a fixed per capita amount equivalent to the total variation in the tax yield's level synthesized by its corresponding concentration indices were added (positive or negative, depending on whether the tax is increased or reduced) to the distribution of tax liabilities ( $L_t$ ).

- Fixed per capita tax reduction:

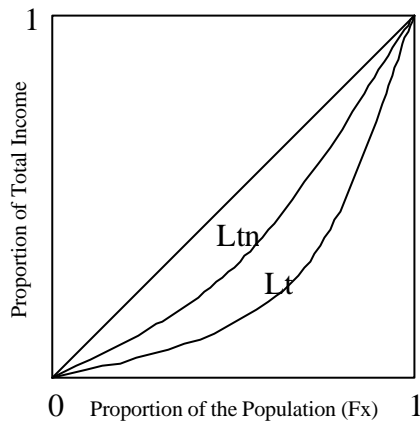
$$L_t - L_{tn} \quad EN = C_{tn} - C_t > 0$$





- Fixed per capita tax increase:

$$L_t - L_{tn} \quad ? \quad EN = C_{tn} - C_t < 0$$

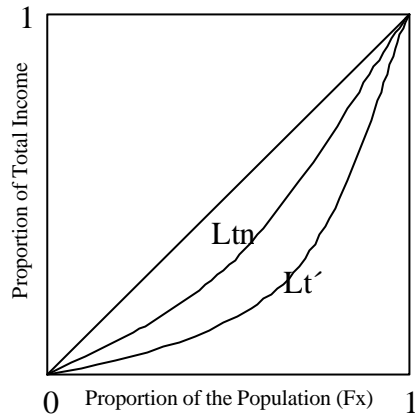


LE measures the effect on progressivity of changes in the tax yield's level while the differences between the amounts of tax liability remain constant in the pre-reform situation.

The Distance Effect would be the difference between  $L_{tn}$  (as defined above) and  $L_t'$ , representing the Lorenz curve of tax liability distribution after the reform synthesized by their corresponding concentration indices.

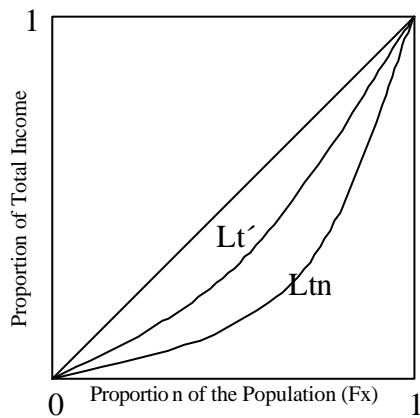
- Increase in the distances separating the amounts of tax liability:

$$L_{tn} - L_t' \quad ? \quad ED = C_t' - C_{tn} > 0$$



- Decrease in the distances separating the amounts of tax liability:

$$L_{tn} - L_{t'} \quad ED = C_{t'} - C_{tn} < 0$$



DE measures the effect on progressivity of changes in the distances separating the amounts of tax liability while the tax yield in the pre-reform situation remains constant. It therefore separately measures a variation in progressivity with a normative assessment because it compares two tax structures by their departure from proportionality while the tax yield remains constant.

To sum up, a tax reform leading to a reduction (increase) in tax liability that does not affect the distances would increase (decrease) progressivity as measured by K. While an increase (decrease) in the distances separating the amounts of tax liability that does not affect average tax liability would reduce (increase) progressivity<sup>18</sup>.

<sup>18</sup> As was mentioned before, progressivity combines both the Level and Distance Effects.

## COMPARING TAX REFORMS WITH DIFFERENT TAX COLLECTION COSTS

Lastly, it may be useful to have indicators that would allow us to make relative assessments on how tax reforms with different tax collection costs affect redistribution and progressivity. A decomposition of the Level Effect and the Distance Effect could be used to this end by designing what could be called the Distance-Level Coefficient (DLC).

The following formula would apply in the case of redistribution:

$$(4.11) \quad DLC(RS) = \frac{DE}{|LE|} \cdot \frac{\frac{D - D'}{2N^2(1 - \alpha)}}{\frac{D}{2N^2} - \frac{1}{1 - \alpha}} \cdot \frac{D - D'}{|D|} \cdot \frac{D' - D}{|D|}$$

*Variation rate of the distances between net incomes*  
*Variation rate of net income in absolute terms*

The progressivity indicator would be similar:

$$(4.12) \quad DLC(K) = \frac{DE}{|LE|} \cdot \frac{\frac{D' - D}{2N^2(1 - \alpha)}}{\frac{D}{2N^2} - \frac{1}{1 - \alpha}} \cdot \frac{D' - D}{|D|}$$

*Variation rate of the distances between tax liabilities*  
*Variation rate of the tax yield in absolute terms*

These indicators relate the effect a tax reform has on changing distances with the independent effect caused by changes in the tax yield's level and net income. By including LE as an absolute value, the sign of DLC would solely depend on whether the reform affects the distance positively or negatively. Thus, both indicators could be interpreted as follows:

$DLC > 0$  ? the tax reform is **progressive** in so far as it contributes to increasing progressivity (K) and redistribution (RS).

$DLC < 0$  ? the tax reform is **regressive** in so far as it contributes to reducing progressivity (K) and redistribution (RS)

Additionally, the higher the value of DLC is for a reform, the greater the reform's progressivity would be. Vice versa, the lower the value for DLC is, the lower would the reform's progressivity be.

Consequently, we would be in a position to compare tax reforms with different tax collection costs both with regard to amounts and well as direction (increases or decreases).

## EXTENSIONS

We consider that this alternative decomposition of a tax reform's effects allows one to make explicit the effects of changes in the distances separating net incomes and tax liabilities by using traditional tools based on a relative notion of inequality. This approach has a two-fold interest.

Firstly, those responsible for tax reforms and citizens may be interested in understanding the consequences of a tax reform in absolute terms (how much a specific decile saves, for instance) and what repercussions this would have on the distances separating individuals' incomes or amounts of tax liability. This increases the amount of information available to both groups.

Moreover, from a theoretical standpoint, this decomposition between the Level Effect and the Distance Effect allows us to approach the *relative income hypothesis*. According to this hypothesis, "individuals who live in a society are interested in the position they occupy within that society, their status or *relative position*"<sup>19</sup>. Social position in today's societies is very closely linked with "the relative available income level, the position occupied in the chain or hierarchy formed by the distribution of personal income" in so far as differences (inequality) are based on *positional distance*.

The hypothesis that an individual's utility not only depends on absolute income but that it also bears some connection with how much he/she has compared to the other members of society (not in relation to the total) is in keeping with the information provided by subjective welfare indicators, "in as much as the marginal utility of aggregated or per capita income can be very low or even negative (starting off from a certain threshold), the utility of *relative income* is always *positive* when individuals are considered in isolation"<sup>20</sup>.

A problem that is encountered when attempting to incorporate this perspective into the usual kind of income distribution studies is that, "obviously, the most common inequality indices (Gini, Atkinson, Theil indices, etc.) are not useful when attempting to reflect an individual's or a group's social position because their aim is to describe inequality from an aggregated viewpoint. They do not indicate the distance that separates individuals from those who are below them or the distance that separates them from those who are above"<sup>21</sup>. To overcome this difficulty, the criteria that must be met by *relative position indices* are described and different alternatives are put forward. These indices measure the relative position of each individual or group in relation to an *ideal* point of reference and measure the distance that separates each individual from those who are below and above him or her in terms of income.

The indicators described in this study cannot be considered to gather the effect of a tax reform on an individual's relative position. They do, however, gather a reform's

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<sup>19</sup> Esteve, F. (2001), p.377.

<sup>20</sup> Ibid., p.375.

<sup>21</sup> Ibid., p.375

aggregated effect on the set of relative positions by calculating changes in the distances separating incomes and tax liabilities.

A tax reform resulting in a positive LE regarding income distribution implies that the average distance between incomes is reduced. On average then, each individual is closer to the ones who are above and below him/her, resulting in a more egalitarian distribution of net income (and tax yield) that remains constant.

## 5. Tax Reforms and the Elements of a Tax Structure: Effects on Level and Distance

The decomposition of changes in the redistribution and progressivity indicators performed in the section above allows us to assess the effects of a tax reform by distinguishing between the amount of a tax cut and the effects of the different tax elements used to bring it about.

By differentiating what we have called the Level Effect (LE), the specific tax cut (or rise) component is effectively isolated. This LE will be the same for all tax reforms having the same effect on tax yield.

In this manner, we can observe the differential effects produced by the choice of specific tools used to carry out the reform, which are reflected in the Distance Effect (DE).

For instance, the table appearing below summarizes the effects of three possible measures to implement an income tax reform. The pre-reform tax schedule is assumed to be progressive with average rates (with growing marginal rates per income bracket)<sup>22</sup>.

<b>1. TAX CREDITS</b>		
Introduction of or increase in tax credits of an equivalent amount for all taxpayers		
PROGRESSIVITY	EN > 0 ED = 0	INCREASE IN K
REDISTRIBUTION	EN > 0 ED = 0	INCREASE IN RS
<b>2. REDUCTIONS IN TAX BASE</b>		
Introduction of or increase in tax base deduction of an equivalent amount for all taxpayers		
PROGRESSIVITY	EN > 0 ED < 0	K?
REDISTRIBUTION	EN > 0 ED < 0	RS?

<sup>22</sup> The formal demonstration of the conclusions contained in the table is to be found in Annex 1.

<b>3. TAX RATE CUT</b>		
Cut in the tax schedule's marginal tax rates (independently of which are reduced)		
PROGRESSIVITY	$EN > 0$	K?
	$ED < 0$	
REDISTRIBUTION	$EN > 0$	RS?
	$ED < 0$	

As was mentioned previously, the Distance Effect (DE) separately measures a change in the distribution (or progressivity) with a normative assessment because it compares two income distributions (or two tax structures) by their departure from proportionality while average net income (or tax yield) remains constant.

Thus, in as much as that the Level Effect (LE) is constant for a given amount of a tax cut, we can **assess the design (the tools used) of a reform in normative terms**. In the examples appearing in the table above, the design of **reform 1** (tax credits) would be **neutral** with regard to progressivity and redistribution ( $DE=0$ ), while the design of **reforms 2 and 3** would be **prejudicial** in terms of the tax's progressivity and redistributive capacity ( $DE<0$ ).

Assessing other tax reform alternatives<sup>23</sup>, as well as the joint effects of a combination of different measures would be more complex due to the fact that all the indicators involved would have to be calculated.

The measures adopted in the recent income tax reforms which have put into effect in Spain are essentially focused on:

- ? Eliminating tax credits on taxes payable
- ? Incorporating reductions into the tax base
- ? Reducing marginal tax rates

This is the reason why the design of the recent Spanish tax reforms has more than likely worsened the tax's progressivity and redistributive capacity, although it would be necessary to conduct a more in-depth analysis by means of micro-simulation exercises to affirm this definitively. Proving this hypothesis would provide us with a very different assessment from the ones offered by diverse studies. We consider that these studies are based on a mistaken interpretation of the progressivity and redistribution indices within a context of changes in total net income and tax yield.

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<sup>23</sup> Such as variable tax credits or reductions according to income levels; changes in the tax schedule's brackets; combining increases and reductions in tax credits, reductions or tax rates; changes in the way taxable income is calculated; etc.

## 6. Conclusions

To sum up, we think that the decomposition of the progressivity and redistribution indices proposed herein provides **greater clarity to the interpretation and analysis of tax reform processes.**

One of its benefits derived from using the Level Effect and Distance Effect indices is that it allows us to partly **recover the intuitive feel of notions like progressivity and redistribution.** This is essential in all cases, and particularly so for Spain due to the fact that Article 31 of the Spanish Constitution<sup>24</sup> sets forth that the tax system should be based on the principle of progressivity.

Determining who benefits most from a tax reform is very complicated and subject to value judgments. Traditional indicators like K, RS and the latter's decomposition provide an approach based on relative income or tax burden differences. They are very useful to conduct comparisons in a static context without reforms affecting a tax's yield. On the contrary, however, when yield-changing tax reforms are put into effect, the conclusions actually obtained are not at all intuitive.

For instance, how can a tax be considered as more progressive with a reform that cuts the taxes due from high-income earners much more than that of other taxpayers? How can a reform be progressive when high-income earners receive the bulk of a tax cut in both absolute (euros per taxpayer) as well as in relative terms (percentage of the total tax cut)? Should this be the case, is increasing progressivity really *beneficial*? Seen from another standpoint, would the majority of citizens vote for an electoral platform advocating this kind of tax reform if they were really aware of its consequences?

As we have attempted to show in this study, the misunderstanding is based on how the indicators used to assess the tax reforms are interpreted when the tax's yield varies. Departure from proportionality can only be assessed normatively in a context of constant income and tax yield. The options used to solve this problem (generalized Lorenz curves, decomposing the RS between tax collection capacity and progressivity) are false solutions that can be criticized on many points.

Our proposal puts forward a different option to assess tax reforms. The Level Effect isolates the effects that a tax reform (tax cut or increase) would have on taxpayers' income or tax burden percentages (on progressivity and redistribution indices) if the distances separating tax liabilities and income remain constant. The Distance Effect reflects the effects a tax reform's specific design (in other words, the tax elements modified) has on progressivity and redistribution for a total tax yield and income that remain constant.

The Distance Effect enables a normative assessment of the tax reform's design to be made. It tells us if a reform increases or decreases the differences between the amount high-income earners and low-income earners pay, and consequently the differences separating their net incomes. For instance, a tax reform that would contribute to

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<sup>24</sup> Article 31 of the Spanish Constitution states, "All individuals shall contribute towards paying for public expenditure in accordance with their means through a just tax system based on the principles of equity and progressivity, which shall on no accounts reach levels that could be considered as confiscatory."

diminishing the distance separating the amounts of tax due but would simultaneously increase the distances separating incomes (a reform by means of which the *rich* would obtain a larger cut in taxes in absolute terms) would obtain a negative Distance Effect. Consequently, it would contribute to diminishing the tax's progressivity and redistributive capacity compared to other possible tax reforms having the same tax collection costs. This procedure would allow us to reconcile intuitive interpretations and analytical studies.

Furthermore, the Distance-Level Coefficients (DLC) relate the effect produced by a tax reform's design on changes in the distances with regard to the independent effect caused by changes in the tax yield or net income levels. In this way, we could compare tax reform designs with different tax yield effects both with regard to amount and direction (increases or reductions) and therefore assess their contribution towards progressivity and redistribution.

Finally, it is worth underlining the fact that the design of the indicators used in this study should be refined further so that they could be applied to more complex situations. This would, for instance, enable us to enter into considerations regarding re-ranking. Similarly, only their empirical application would permit us to assess their relevance and usefulness for improving analyses of tax reforms.



## APPENDIX 1: Analytical Demonstration of Table 1

Proving the contents of the table analytically is fairly simple. In fact, one only has to find out whether DEs of a tax reform are either negative or positive, as a tax reform that reduces tax yield always has positive LEs. We shall begin by making the conditions explicit: income ranked by least to greatest and a progressive tax with growing marginal tax brackets.

- Ranked incomes:  $x_j \geq x_i$
- The incomes are taxed by brackets,  $I_t$ , so that  $x_i \in I_t$ . The last bracket is different for each income without it having to be necessarily identical to the tax rate brackets.
- Each income is divided into  $n_i$  brackets for taxation purposes, each being subjected to a different marginal tax rate ( $t'$ ).
- The marginal rates increase so that:

$$(1) \quad t'_j \geq t'_i$$

- Therefore, tax liability increases with income:  $C_j \geq C_i$

Each individual's tax liability can be calculated in the following manner:

$$(2) \quad C_i = \sum_{t=1}^{n_i} I_t t'_t - R_i t'_{i \max} - D_i$$

Here  $R$  represents the amount of tax base reductions,  $t'_{i \max}$  the marginal rate (or combination of marginal rates) the last units of individual  $I$  income are subjected to (recalling that a reduction in the tax base leads to a savings on the maximum marginal tax rate) and  $D$  represents the amount of tax credits.

We start off by analyzing the effects the proposed reforms would have on progressivity. To do so, we examine its consequences on the distances between tax liabilities by means of formula (3). Its effects on the DE are derived from formulas (4.6) to (4.10).

$$(C_j - C_i) \sum_{t=1}^{n_j} I_t' - R_j t_{j \max}' - D_j \sum_{t=1}^{n_i} I_t' - R_i t_{i \max}' - D_i$$

(3)  $\sum_{t=1}^{n_j} I_t' - \sum_{t=1}^{n_i} I_t' - R_i t_{i \max}' - R_j t_{j \max}' - D_i - D_j$

? A ? B ? C

The effect of the implementation (or increase in) of tax credits ( $D$ ) which is the same for all individuals is represented by:

$$\sum_j, i \quad D_j - D_i - (D_i - D_j) = 0 \quad \text{en (3) } C = 0 \quad (C_j - C_i) = 0 \quad ED = 0 \quad K = CTE$$

The tax credit does not affect the distances between tax liabilities. Hence, the Distance Effect is null, and does not in itself alter progressivity.

The effect of the implementation (or increase in) of tax base reductions ( $D$ ) which is the same for all individuals is represented by:

$$\sum_j, i \quad ; \text{ given (1) } R t_{j \max}' - R t_{i \max}' \quad \text{in (3) } C = 0 \quad (C_j - C_i) = 0 \quad ED = 0 \quad K = DIS$$

The tax base reduction reduces the distances<sup>25</sup> between tax liabilities. Hence, the Distance Effect is negative and contributes to reducing progressivity.

The effect of a cut in tax rates, independently of the amount of the cuts, is represented by:

$$\sum_j, i \quad n_j - n_i \quad \sum_{t=1}^{n_j} I_t' - \sum_{t=1}^{n_i} I_t' \quad \text{in (3) } \text{A lower or the same after the reform?}$$

$$\sum (C_j - C_i) = 0 \quad \text{aggregated } ED = 0 \quad K = DIS$$

The reduction of rates reduces the distances between tax liabilities. Hence, the Distance Effect is negative and contributes to diminishing progressivity. All of the above can be seen clearly if the highest marginal rate is reduced. For cases when lower marginal rates are reduced, one has to take into account that the rate cut not only affects taxpayers in the income bracket whose rate is reduced but also all other individuals earning more income<sup>26</sup>.

<sup>25</sup> The effect of the reduction for some individuals would be the same regarding nominal savings. Nonetheless, in all real cases there would be individuals subjected to different maximum marginal rates, the aggregated effect would therefore correspond to what has been mentioned in the text.

<sup>26</sup> A reduction in tax rates affecting all taxpayers in the same way (in other words, the first bracket with income above the bracket's upper limit being taxed, a highly improbable case) would be the only case in

All of the above means that the effects of the previously mentioned reforms on redistribution are simple to obtain. If we define net incomes (R) as follows:

$$(4) \quad R_i = x_i - C_i$$

we find that the distances between net incomes in absolute terms are expressed as:

$$(5) \quad R_j - R_i = (x_j - C_j) - (x_i - C_i) = (x_j - x_i) - (C_j - C_i)$$

Hence, tax reforms that increase the distances between tax liabilities ( $C_j - C_i$ ) will reduce the distance between incomes. This means that the Distance Effect is positive regarding Redistribution, as derived from formulas (4.1) to (4.5). On the other hand, reforms that reduce the distance between tax liabilities ( $C_j - C_i$ ) increase the distance separating incomes. This means that the Distance Effect is negative regarding redistribution.

Bearing in mind these results, we can prove that:

- Reforms leading to the introduction or increase in tax credits do not change the distances between either tax liabilities or net incomes.  $DE=0$ ; RS remains constant.
- Reforms leading to the implementation or increase in reductions in the tax base reduce the distance separating tax liabilities and increase the distances separating incomes.  $DE < 0$ ; reduction in RS.
- Reforms leading to a reduction in tax rates reduce the distances separating tax liabilities and increase the distances between incomes;  $DE < 0$ ; reduction in RS.

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which distances between tax liabilities would not be reduced and ED would be equivalent to 0. In all other cases, the distance between some pair of tax liabilities would decrease (and no distance would increase). Hence, the aggregated Distance Effect would be negative.

## APPENDIX 2: The Distance and Level Effects in the Face of Different Tax Reform Alternatives

The characteristics of seven possible tax reforms are shown in the table below in order to analyze their effects in terms of progressivity and redistribution. Both the population of taxpayers as well as the initial situation (A) —in relation to which all the alternatives (B, C, D, E, F, G and H) are to be assessed— will remain constant throughout the exercise.

<b>ASSESSMENT OF TAX REFORMS EXAMPLES</b>			
<b>POPULATION</b>			
<b>TAXPAYERS</b>		<b>INCOME</b>	
<i>FIRST</i>		<i>10,000</i>	
<i>SECOND</i>		<i>30,000</i>	
<i>THIRD</i>		<i>60,000</i>	
<b>TAXATION ALTERNATIVES</b>			
<b>NAME</b>	<b>TAX SCHEDULE RATES</b>	<b>REDUCTION OF TAX BASE</b>	<b>TAX CREDITS ON TAX LIABILITY</b>
<b>A. INITIAL SITUATION</b>	<i>TAX BRACKETS t'</i> <i>0-20,000 10%</i> <i>20,000-40,000 20%</i> <i>40,000-... 30%</i>	<i>NO</i>	<i>NO</i>
<b>B. <u>CONSTANT TAX CREDIT ON TAX LIABILITY</u></b>	<i>TAX SCHEDULE A</i>	<i>NO</i>	<i>1,000</i>
<b>C. <u>CONSTANT REDUCTION OF TAX BASE</u></b>	<i>TAX SCHEDULE A</i>	<i>5,000</i>	<i>NO</i>
<b>D. <u>REDUCTION OF FIRST TAX BRACKET RATE</u></b>	<i>TAX BRACKETS t'</i> <i>0-20,000 4%</i> <i>20,000-40,000 20%</i> <i>40,000-... 30%</i>	<i>NO</i>	<i>NO</i>
<b>E. <u>REDUCTION OF ALL TAX BRACKET RATES</u></b>	<i>TAX BRACKETS t'</i> <i>0-20,000 7%</i> <i>20,000-40,000 17%</i> <i>40,000-... 27%</i>	<i>NO</i>	<i>NO</i>
<b>F. <u>VARIABLE TAX CREDITS ON TAX LIABILITY</u></b>	<i>TAX SCHEDULE A</i>	<i>NO</i>	<i>FIRST 750</i> <i>SECOND 500</i> <i>THIRD 250</i>
<b>G. <u>FLAT TAX</u></b>	<i>TAX BRACKETS t'</i> <i>0-10,000 0%</i> <i>10,000- ... 20%</i>	<i>NO</i>	<i>NO</i>
<b>H. <u>QUASI-FLAT TAX RATE+ CONSTANT TAX CREDIT ON TAX LIABILITY</u></b>	<i>TAX BRACKETS t'</i> <i>0-40,000 20%</i> <i>40,000-... 30%</i>	<i>NO</i>	<i>2,000</i>

The effects are shown in the results table appearing below from three different perspectives. Firstly, the variation of each taxpayer's tax liability is shown (TXPYR; 1<sup>st</sup>, 2<sup>nd</sup> and 3<sup>rd</sup>), specifying the value of this variation in nominal terms (TOTAL); the percentage of the total tax reduction or increase for each taxpayer (% TOT); and the percentage variation compared with previous individual tax liability (% IND).

Secondly, the classical measurement indicators are shown, including the percentage yield variation (YLD); amount of the overall average rate ( $t^*$ ); percentage variation of Reynolds-Smolensky redistribution index (RS); percentage variation of the Kakwani progressivity index (K); and the variation in the tax's level ( $t/1-t$ ).

Lastly, an assessment of the various alternatives is shown in terms of redistribution and progressivity through the Level (LE) and Distance (DE) Effects summarized by the Distance-Level Coefficients (DLC).

The first three alternatives (B, C, D and E) allow us to explore the consequences ensuing from the simplest reforms, such as the introduction of constant tax credits or tax base reductions, or the reduction of one or all tax rates. To show the differential effects introduced by a reform's specific design, these four options have the same tax collection costs so that the effects cannot be attributed to the amount of the reduction by appealing to the decomposition of the Reynolds-Smolensky index.

All these reductions can on their own be presented as "positive". In order to do so, one could allege, on the one hand, that they increase progressivity (K) and, on the other, that percentage savings on the previous tax liability (%IND) are greater as income decreases. If one of them obtains negative results—as is the case for alternative E—concerning redistribution (reduction in RS), one could appeal to the decomposition of the RS ( $RS=(t/1-t)K$ ). Given that in this case K increases although  $t/1-t$  decreases, it could be argued that the reform improves the tax's progressivity and that the negative results concerning redistribution are solely due to the fact that the amount of the tax cut is excessive.

In so far as other tax cut alternatives are available (B, C and D) having the same tax collection costs, it seems obvious that the above-mentioned argument does not hold. Introducing either tax credits (B) or tax base reductions (C), as well as cutting only the first tax bracket rate (D) all offer better results in terms of progressivity and redistribution. Hence, reform E offers the worst results regarding progressivity and redistribution in terms of the Distance Effect and the Distance-Level Coefficient (negative in both cases). At the same time, we can also observe that both tax base reductions (C) and a cut in the first tax bracket rate (D) mean that the reform's design contributes negatively to progressivity and redistribution (negative Distance Effects and Distance-Level Coefficients). Only the tax reform design based on implementing tax credits on tax liability (B) is neutral in terms of its contribution to progressivity and redistribution.

**ASSESSMENT OF TAX REFORMS  
RESULTS**

	TAX YIELD VARIATIONS				CLASSICAL ASSESSMENTS					DISTANCE-LEVEL ASSESSMENT					
	TXPYR.	TOTAL	% TOT.	% IND.	YLD	t*	RS	K	t/1-t	REDISTRIBUTION			PROGRESSIVITY		
										LE	DE	DLC	LE	DE	DLC
A	---	---	---	---	17.000	17%	0,02	0,098	0,2	---	---	---	---	---	---
B	1 <sup>st</sup>	-1000	33.33	100	-17.6%	14%	+54.4%	+94.2%	-20.5%	+0.0109	0	0	+0.0924	0	0
	2 <sup>nd</sup>	-1000	33.33	25											
	3 <sup>rd</sup>	-1000	33.33	8.33											
C	1 <sup>st</sup>	-500	16.66	50	-17.6%	14%	+15.8%	+45.7%	-20.5%	+0.0109	-0.0077	-0.7094	+0.0924	-0.0476	-0.5151
	2 <sup>nd</sup>	-1000	33.33	25											
	3 <sup>rd</sup>	-1500	50	12.5											
D	1 <sup>st</sup>	-600	20	60	-17.6%	14%	+31.2%	+65.1%	-20.5%	+0.0109	-0.0046	-0.4256	+0.0924	-0.0285	-0.309
	2 <sup>nd</sup>	-1200	40	30											
	3 <sup>rd</sup>	-1200	40	10											
E	1 <sup>st</sup>	-300	10	42.85	-17.6%	14%	-3.48%	+21.42%	-20.5%	+0.0109	-0.0116	-1.0641	+0.0924	-0.0714	-0.7727
	2 <sup>nd</sup>	-900	30	29.03											
	3 <sup>rd</sup>	-1800	60	17.64											
F	1 <sup>st</sup>	-750	50	75	-8.8%	15.5%	+47.3%	+64.5%	-10.4%	+0.0055	+0.0039	+0.7094	+0.0417	+0.0215	+0.5151
	2 <sup>nd</sup>	-500	33.33	12.5											
	3 <sup>rd</sup>	-250	16.66	2.08											
G	1 <sup>st</sup>	-1000	-38.46	100	+15.2%	19.6%	+73.4%	+45.7%	+19%	-0.0101	+0.0248	+2.4556	-0.0572	+0.1020	+1.7832
	2 <sup>nd</sup>	+1600	+61.53	+40											
	3 <sup>rd</sup>	+2000	+76.92	+16.66											
H	1 <sup>st</sup>	-1000	100	100	-5.88%	16%	+58%	+70%	-7%	+0.0037	+0.0079	+2.1282	+0.0269	+0.0416	+1.5454
	2 <sup>nd</sup>	0	0	0											
	3 <sup>rd</sup>	0	0	0											

The remaining tax design alternatives shown (F, G and H) have positive implications in terms of their contribution to progressivity and redistribution.

Option F is based on introducing a variable tax credit on tax liability that decreases as income levels increase. This option offers better results than alternatives C, D and E with regard to all indicators and with half their tax collection costs. Its main disadvantage lies in its practical application in a real environment of this kind of measurement as it would introduce a substantial amount of complexity.

Option G puts forward a flat tax with a tax-free allowance, which gives the tax progressivity. This kind of design alternative has been the object of a wide-ranging academic debate and its simplicity must be highlighted. In this example, the reform would suppose an increase in taxation and its results in terms of DLC are better than all the other options. Nonetheless, reforms that tend to introduce a flat tax have several important disadvantages given the fact that they usually benefit both ends of the income distribution scale to the detriment of average incomes.

The last of the alternatives analyzed, H, reduces the number of tax brackets and introduces a single tax rate for low and middle-incomes, in addition to incorporating a tax credit on tax liability. The tax is simplified in this way at the same time as its progressivity and redistributive effect are increased with a relatively low tax collection cost.

Lastly, it is important to highlight that the use of the DLC allows us to rank the various design alternatives analyzed despite the fact that they imply different tax collection costs. We would thus obtain the following ranking from best to worst in terms of progressivity and redistribution: flat tax (G); quasi-flat tax + tax credit (H); variable tax credits (F); constant tax credit (B); reduction in the first bracket rate (D); tax base reduction (C); and reduction in all the tax rates (E).

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