Optimal Environmental Standards Under Asymmetric Information and Imperfect Enforcement*

Carmen Arguedas

Universidad Europea de Madrid

Abstract

We study optimal policies composed of pollution standards, probabilities of inspection and fines dependant on the degree of noncompliance with the standards, in a context where regulated firms own private information. In contrast with previous literature, we show that optimal policies, being either uniform or type-contingent, can imply violations to strictly positive standards. This result crucially depends on the monitoring costs, the types of firms and the regulator's degree of uncertainty.

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*Address for correspondence: Departamento de Economía, Universidad Europea de Madrid. Calle Tajo, s/n, 28670 Villaviciosa de Odón, Madrid, Spain. Tel. (+34) 91-211-5645. Fax. (+34) 91-616-8265. E-mail: mcarmen.arguedas@uem.es. I wish to thank the Environmental Economics Group leaded by Aart de Zeeuw at Tilburg University for their hospitality during my visit in 2005, where part of this paper was written. I also thank Erkki Kostela, Larry Kranich, Bernard Sinclair-Desgagné, Sjak Smulders and Cees Withagen for helpful comments and discussions on this issue.

1 Introduction

Very often, environmental regulators do not have perfect information (either *ex-ante* or *ex-post)* about polluting firms that are required to comply with recommended pollution limits or *standards*. *Ex-ante* concerns standard setting. Regulators are less informed than firms about their technological characteristics, sometimes needing mechanisms which elicit private information.[1] *Ex-post* refers to the behavior of firms in response to the standards already in place. Regulators may not observe the performance of firms unless they engage in costly monitoring. Therefore, they design *enforcement policies* composed of inspection frequencies and sanctions in case firms are discovered exceeding the standards.[2] Depending on the monitoring costs, the standards to be enforced and the information authorities own about the regulated firms, enforcement may be imperfect, that is, some firms may find it profitable to violate environmental standards.

Despite the relatively high frequency of this observation[3], surprisingly the theoretical literature on environmental regulation has not explored whether it is desirable from a normative point of view to set environmental policies which induce firms to deliberately violate the standards. In this paper, we explore this issue, that is, whether we can rationalize the setting of standards under imperfect enforcement situations. Our main finding is precisely to show that

[1] For example, under the US National Pollutant Discharge Elimination System (NPDES) Program, the Environmental Protection Agency (EPA) issues individual permits to facilities which discharge pollutants into waters of the US, based on reported information about their pollution control processes.

[2] The Civil Penalty Policy of the Clean Water Act establishes the factors that the EPA should consider when imposing sanctions for noncompliance. Among others, the degree of noncompliance is a key gravity factor.

[3] Consult www.epa.org for details on the EPA's enforcement actions against noncompliant facilities and the merits of the EPA's Audit Policy to increase compliance rates.

the *ex-ante* informational constraint plays a key role in the results.[4]

We consider a firm that owns private information about its benefits from pollution. The firm can be of two possible types, namely *clean* and *dirty*, based on its induced pollution level in response to a given policy. In principle, the regulator may decide to set a uniform policy (regardless of type) or an *incentive compatible* policy (contingent on type) where each type prefers the policy initially designed for it.[5]

The message of the paper is clear. Under ex-ante imperfect information, we can find situations where it is optimal to set positive standards that induce noncompliance. The result is independent of the optimal policy being uniform or type-contingent.

In the case of a uniform policy, we find that full compliance is never optimal. It is always worth to infinitesimally decrease the probability of inspection, since the savings in monitoring costs are larger than the decrease in welfare associated with both types' larger pollution levels. Under complete information, a positive standard is never optimal. However, a zero standard under incomplete information may result in over-enforcement of the clean type, with the corresponding negative effect on social welfare. This result is relevant and provides

[4] Arguedas and Hamoudi (2004) and Arguedas (2005a) have recently studied the characteristics of optimal policies composed of pollution standards, probabilities of inspection and fines, under perfect information *ex-ante* and imperfect enforcement. There, it is shown that optimal policies can induce noncompliance to zero standards only. The result is quite intuitive. Since fines depend on the degree of noncompliance, if a positive standard that induced noncompliance were set, the regulator would find it profitable to decrease the standard and the probability of inspection at the same time keeping the firm's pollution incentives unchanged and reducing monitoring costs. This is the spirit of Becker (1968)'s well known result of imposing maximal fines to keep enforcement costs at the minimum. Given a pollution level and a structure of fines dependant on the degree of noncompliance, a lower standard increases the fine for noncompliance and, therefore, it is possible to decrease the probability of inspection, then saving monitoring costs.

[5] Our approach differs from that in which, given the standard, the firm reports its emission level with the possibility of under-reporting, such as in Sandmo (2002). In our case, we have an added *ex-ante* informational asymmetry and, since we analyze optimality of the standards, we can restrict ourselves to incentive compatible policies. Also, once emissions have been released, we assume that they can be measured through costly monitoring.

an additional explanation to the literature in favor of non-maximal fines.[6]

In fact, we find violations to strictly positive standards under low monitoring costs, intermediate values of the clean type's profitability and large regulator's uncertainty. The explanation is clear, since there exists a trade-off between enforcement costs and the clean type's over-enforcement problem mentioned above. Given clean type's profitability, the larger the monitoring costs, the larger the enforcement costs, which favors a zero standard setting. Also, given a level of the monitoring costs, the smaller clean type's profitability, the larger the over-enforcement problem, which favors a positive standard setting. Finally, when uncertainty decreases (in favor of any type) , the solution approximates to the complete information outcome, which implies a zero standard.

By contrast, if the policy is type-contingent, incentive compatibility requires both smaller standard and inspection probability for the dirty type.[7] Also, the dirty type always finds it profitable to violate the standard, which again can be positive under low monitoring costs. However, as opposed to the uniform case, this result is more likely to be found when both clean type's profitability and likelihood are large, since in these two cases, the solution approximates to the clean type's compliance solution. Here, there exists a trade-off between enforcement costs and dirty type's under-enforcement. The larger the clean type's profitability and likelihood, the larger the dirty type's under-enforcement problem if its standard is zero.

The literature on standards and enforcement issues started with Downing

[6] After Becker (1968), several papers in the crime context have explained the reasons why fines are not maximal, such as risk aversion (Polinsky and Shavell (1979)), imperfect information about the regulatory policy (Bebchuk and Kaplow (1991), Kaplow (1990)), differences in wealth (Polinsky and Shavell (1991)) or marginal deterrence (Andreoni (1991), Shavell (1992), Heyes (1996)), among others. In all these papers, however, standards are exogenous.

[7] In the tax evasion literature, the optimal inspection probability is also a decreasing function of reported income. For example, see Reinganum and Wilde (1985).

and Watson (1974) and it is vast nowadays (Heyes (2000) provides a comprehensive survey in the environmental context). However, our approach has not been considered yet, namely combining standard-setting, endogenous imperfect enforcement and asymmetric information. This allows us to rationalize positive standard violations, a result that is not possible under alternative assumptions within the principal-agent framework. For example, Ellis (1992a) studies standard-setting under *ex-ante* incomplete information, but restricting attention to policies which induce compliance. There are papers which study incentive compatible optimal pollution taxes, such as Jebjerg and Lando (1997), which implicitly constrain to zero standards. Swierzsbinky (1994) consider optimal taxation also, relaxing the assumption of incentive compatibility, but they again restrict to zero standards. The only exception is Arguedas (2005b), which considers a bargaining context under complete information and assumes that the firm can choose the environmental technology as well. There, it may be beneficial for both the regulator and the firm to achieve a cooperative agreement where the firm chooses a cleaner technology in exchange for a relaxed regulation consisting of a positive standard and a reduced fine for noncompliance.

The remainder of the paper is organized as follows. In the next section, we present the model. In Section 3, we study the optimal behavior of the firm. In Section 4, we analyze optimal uniform policies and the likelihood of obtaining positive standards. In Section 5, we discuss the case of the optimal type-contingent policy. We conclude in Section 6. All the proofs are in the Appendix.

2 The Model

We consider a single firm that generates pollution as a by-product of its production activity. The firm obtains private benefits from pollution, which depend on the pollution level $e \geq 0$ and a parameter $\theta_i > 0$, $i = 1, 2$, $\theta_1 < \theta_2$, which refers to the firm's pollution profitability. Let $B(e, \theta_i) = \theta_i b(e)$ represent the firm's profits, where $b(e)$ is continuous and concave in the pollution level with an interior maximum at $\widetilde{e} > 0$, and such that $b(0) = 0$ and $b'''(e) \geq 0$.[8] Given a pollution level, the clean type (θ_1) obtains lower total and marginal benefits than the dirty type (θ_2). The firm knows its type but the regulator only knows the probability distribution of the types. Let γ_i denote the probability that the firm is of type θ_i, such that $\gamma_i \in [0, 1]$ and $\gamma_1 + \gamma_2 = 1$.

Pollution generates external damages measured by $d(e)$, which is continuous, strictly increasing and convex in the pollution level, and such that $d(0) = 0$.

In the absence of regulation, the firm does not internalize external damages and pollution is $\widetilde{e} = \arg\max_{e \geq 0} \theta_i b(e)$, for all i.

We assume there exists a regulator who sets a standard $s \in [0, \widetilde{e}]$, that is, a maximum level of permitted pollution.[9] The regulator knows the firm's pollution level in response to the standard only under costly (but perfectly accurate) monitoring. The cost *per* inspection is $c > 0$. Therefore, the regulator does not generally inspect the firm in every instance but only with probability $p \in [0, 1]$. Once inspected, if the firm is discovered exceeding the standard, then it is forced to pay a penalty which depends on the degree of noncompliance,

[8] This specification of profits simplifies the algebra without affecting the qualitative nature of the results.

[9] Obviously, the regulator is not interested in a standard larger than the pollution level chosen by the firm in the absence of regulation.

$e-s$. We assume that the sanction is represented by the function $F(e-s)$, which is strictly increasing and convex in $e-s \geq 0$, and such that $F(e-s)=0$ for all $e-s \leq 0$. For simplicity, we consider $F''' = 0$.[10] We assume that the sanction is fixed by a government entity other than the regulator, for example, the judiciary.[11]

We take a principal-agent approach where the regulator (principal) chooses the standard and the probability of inspection that maximizes social welfare, considering the optimal response of the firm (agent) to the policy.

Given $\{s,p\}$, a firm of type θ_i chooses the pollution level that maximizes its expected payoff, that is, private benefits minus expected penalties, as follows:

$$P(s,p,\theta_i) = \max_{e \geq 0} \{\theta_i b(e) - pF(e-s)\} \quad (1)$$

Let $e(s,p,\theta_i)$ be the pollution level chosen by type θ_i given the policy $\{s,p\}$, i.e., $e(s,p,\theta_i) = \arg\max_{e\geq 0} \{\theta_i b(e) - pF(e-s)\} \leq \widetilde{e}$.

Considering the firm's best response, the regulator now chooses the policy that maximizes social welfare. Since the regulator does not know the true type of the firm, the policy cannot be based upon it. There are two kind of policies the regulator may choose. The first is a *uniform policy* $\{s,p\}$, that is, the same

[10] Given these assumptions, we have $(F')^2 - FF'' > 0$ for all $e-s \geq 0$, a property that plays a key role in the results, as we will see later on.

[11] This assumption is common in the literature in this context except, for example, in Heyes (1996) or Arguedas (2005b). In other contexts, such as crime, there are several papers which determine optimal fines and inspection probabilities, such as Becker (1968), Polinsky and Shavell (1979, 1990) or Bebchuck and Kaplow (1991), but there the standard is exogenous. In the context of tax evasion, few papers consider endogenous fines. See, for instance, Mookherjee and Png (1989).

policy regardless of the type. In this case, social welfare is as follows:

$$SW(s,p) = \sum_{i=1}^{2} \gamma_i \left[P(s,p,\theta_i) - d(e(s,p,\theta_i)) + pF(e(s,p,\theta_i) - s) \right] - cp \quad (2)$$

The regulator is concerned about the firm's expected payoff, the generated damages, the expected collected fines and the expected monitoring costs. We assume that there are no social costs associated with collecting fines, and that fines are redistributed lump-sum. Also, we do not impose any budget requirement on the monitoring activity. Considering (1), (2) reduces to:

$$SW(s,p) = \sum_{i=1}^{2} \gamma_i \left[\theta_i b(e(s,p,\theta_i)) - d(e(s,p,\theta_i)) \right] - cp \quad (3)$$

The second type of policy is *type-contingent*. Here, the regulator has to design a mechanism to elicit the firm's private information. By the *revelation principle*, we can concentrate on direct mechanisms where the regulator asks the firm to report its type, $\widehat{\theta}_i$, and then, it sets the policy based on the report, $\left\{ s\left(\widehat{\theta}_i\right), p\left(\widehat{\theta}_i\right) \right\}$, such that it induces the firm to reveal its true type, $\widehat{\theta}_i = \theta_i$. This is the well known *incentive compatibility condition*, represented as follows:

$$\theta_i \in \arg\max P\left(s\left(\widehat{\theta}_i\right), p\left(\widehat{\theta}_i\right), \theta_i \right) \quad (4)$$

For convenience, we assume that if the firm is indifferent between announcing any of the two types, then it announces the true type.

Denoting $s_i = s\left(\widehat{\theta}_i\right)$ and $p_i = p\left(\widehat{\theta}_i\right)$, $i = 1, 2$, social welfare is now:

$$SW(s_1, s_2, p_1, p_2) = \sum_{i=1}^{2} \gamma_i \left[\theta_i b(e(s_i,p_i,\theta_i)) - d(e(s_i,p_i,\theta_i)) - cp_i \right] \quad (5)$$

where (s_1, s_2, p_1, p_2) satisfy (4). Note that a uniform policy is trivially incentive compatible.[12]

Throughout the paper, we assume that the regulator commits to the announced inspection probability. This can be justified considering that the regulator has to build up a reputation, that is, policy announcements must be credible to induce the desired behavior.[13]

In the next section, we study the firm's induced behavior with respect to the announced policy.

3 The Optimal Behavior of the Firm

Consider a feasible policy $\{s, p\}$. As explained in the previous section, the corresponding type θ_i's expected payoff is given by (1).

If type θ_i complies with the standard ($e \leq s$), it does not incur any penalty. Since $b(e)$ is strictly increasing in $e \leq \widetilde{e}$, the *optimal compliance decision* is s and its payoff is $\theta_i b(s)$.

If type θ_i exceeds the standard ($e > s$), then there is a chance of inspection. Consequently, the *optimal noncompliance decision* is $n_i = n(s, p, \theta_i) = \arg\max_{e>s} \{\theta_i b(e) - pF(e-s)\} > s$ and the corresponding payoff is $\pi(s, p, \theta_i)$. Since the maximand is strictly concave in e, the first order condition character-

[12] Besides incentive compatibility, the literature on economics of information considers participation constraints also, that is, feasible policies must be such that firms' payoffs are nonnegative. In our case, this additional requirement is trivially satisfied since $b(0) = 0$.

[13] A formal justification of this assumption would require to consider a dynamic model, which is beyond the scope of this paper. In static models such as ours, the assumption of commitment is common in the literature. Some exceptions in the environmental context are in Ellis (1992b), Grieson and Singh (1990) or Franckx (2002).

izes the interior noncompliance decision:

$$\theta_i b'(e) = pF'(e - s) \tag{6}$$

Implicitly differentiating (6), we obtain $n_{ip} = n_p(s, p, \theta_i) = \frac{F'}{\theta_i b'' - pF''}$ and $n_{is} = n_s(s, p, \theta_i) = -\frac{pF''}{\theta_i b'' - pF''}$. Observe that $n_{ip} < 0$ and $0 \leq n_{is} < 1$. That is, type θ_i's pollution level increases when the probability of inspection decreases and the standard increases. However, since $n_{is} < 1$, the degree of violation decreases when the standard increases.[14]

Given $\{s, p\}$, type θ_i chooses whether to comply or not depending on the expected payoff of each possibility. Thus, its optimal response is:

$$e(s, p, \theta_i) = \begin{cases} s & \text{if } \theta_i b(s) \geq \pi(s, p, \theta_i) \\ n(s, p, \theta_i) & \text{if } \theta_i b(s) < \pi(s, p, \theta_i) \end{cases} \tag{7}$$

and its expected payoff can be further expressed as:

$$P(s, p, \theta_i) = \max \{\theta_i b(s), \pi(s, p, \theta_i)\} \tag{8}$$

In the following lemma, we show the properties of the function $P(s, p, \theta_i)$:

Lemma 1 *The function $P(s, p, \theta_i)$ is non-decreasing and concave in s, non-increasing and convex in p, it has a nonnegative cross partial, and it is such that $P(s, p, \theta_2) > P(s, p, \theta_1)$. Moreover, $P(s, 0, \theta_i) = \theta_i b(\widetilde{e})$ for all i.*

We now characterize the set of policies for which each type is indifferent between complying and noncomplying with the standard. Since sanctions are

[14] Note that $n_{is} = 0$ when either $F'' = 0$ or $p = 0$.

continuous at $e = s$, the maximand in (1) is continuous for all s. Therefore, considering (6), type θ_i complies with the standard only if $\theta_i b'(s) \leq pF'(0)$. Thus, the minimum probability that induces type θ_i to comply with s is:

$$p^c(s, \theta_i) = \frac{\theta_i b'(s)}{F'(0)} \leq 1 \tag{9}$$

which is decreasing and convex in s, and such that $p^c(s, \theta_2) > p^c(s, \theta_1)$.[15] Since $p \leq 1$, there may exist a subset of nonenforceable standards for each θ_i.[16]

In Figure 1, we represent the functions $p^c(s, \theta_i)$ in the space of feasible policies. In the horizontal axis we measure the standard and in the vertical axis, we measure the probability of inspection. These functions divide the set of feasible policies into three regions, namely *compliance* (C), *partial compliance* (PC) and *noncompliance (NC)*. Therefore, all the policies on or above the function $p^c(s, \theta_2)$ induce both types to comply with the standard. The set of policies between $p^c(s, \theta_1)$ and $p^c(s, \theta_2)$ induce the clean type to comply only. Finally, the policies below the function $p^c(s, \theta_1)$ induce both types to violate the standard. Thus, θ_2's noncompliance region is larger than that of θ_1.

In the figure, we also include each type's indifference map, where each indifference curve is composed of the set of policies such that type θ_i´s expected payoff is constant. By Lemma 1, type θ_i's payoff increases to the southeast, i.e., whenever the standard is larger and the probability of inspection is smaller. And it obtains the maximum expected payoff at $s = \widetilde{e}$, $p \in [0, 1]$ and $s \geq 0$, $p = 0$. The shape of the indifference curves is now presented in the following:

Lemma 2 *If a policy $\{s, p\}$ induces type θ_i to comply with the standard, the*

[15] The assumptions on the penalty function ensure that $F'(0)$ is finite and strictly positive.
[16] If there exists $\widehat{s}_i > 0$ such that $p^c(\widehat{s}_i, \theta_i) = 1$, then $s \in [0, \widehat{s}_i)$ cannot be enforced for θ_i.

indifference curve at that policy is vertical. If it induces noncompliance, the indifference curve at that policy is strictly increasing and convex. At any $\{s, p\}$, the slope of θ_1's indifference curve is not smaller than that of θ_2.

In both the full noncompliance and the partial compliance regions, indifference curves satisfy the *single crossing property.* However, in the full compliance region, indifference curves do not cross.

The revelation principle allows us to restrict attention to incentive compatible policies. For example, a policy $\{s_1, p_1\}$ for $\widehat{\theta_1}$ and a policy $\{s_2, p_2\}$ in the shaded area of Figure 1 for $\widehat{\theta_2}$ is incentive compatible, i.e., no type has an incentive to misrepresent its type. Note that $s_2 \leq s_1$ and $p_2 \leq p_1$.

Having studied the firm's optimal response, we now analyze the features of the optimal policy. First, we consider the case of the uniform policy.

4 The Optimal Uniform Policy

In this section, we analyze the case in which the regulator sets the same policy regardless of the type. Here, the regulator maximizes social welfare given by (2), considering the firm's optimal behavior analyzed in the previous section. In the following proposition, we provide a useful preliminary result to characterize the optimal policy in this case.

Proposition 3 *Let (s^*, p^*) be the optimal uniform policy. Then, $p^* \leq p^c(s^*, \theta_1)$.*

A uniform policy which induces full compliance is never optimal, since, by (9), $p^c(s^*, \theta_1) < p^c(s^*, \theta_2)$. A policy as such would imply clean type's under-enforcement and dirty type's over-enforcement with respect to the complete information case (see Figure 2). Intuitively, full compliance is socially too expensive, and welfare increases if the regulator decreases the inspection probability, since clean type's incentives remain unchanged, and the savings in monitoring costs are larger than the decrease in efficiency due to the larger dirty type's induced pollution level.

Thus, if the optimal policy is uniform, at least it induces the dirty type to violate the standard. The clean type cannot strictly prefer to comply with the standard at the optimal policy. If a policy as such were set, welfare could be increased decreasing the probability of inspection, since incentives for the clean type would remain unchanged and we would overcome the dirty type's over-enforcement problem.

Consequently, the optimal uniform policy is obtained as follows:

$$\begin{aligned} & Max_{s,p} \quad \sum_{i=1}^{2} \gamma_i \left(\theta_i b\left(e_i\right) - d\left(e_i\right)\right) - cp \\ s.t. \quad & p \leq p^c\left(s, \theta_1\right) \\ & s \geq 0 \end{aligned} \tag{10}$$

where $e_i = e\left(s, p, \theta_i\right)$ is given by (7).[17] The optimality conditions are summarized in the following:

[17] For the sake of clarity, we assume that the optimal probability of inspection is included in the interval $[0, 1]$. This remains valid in Proposition 7.

Proposition 4 *The optimal uniform policy* (s^*, p^*) *is such that:*

$$\sum_{i=1}^{2} \gamma_i \left(\theta_i b'(n_i) - d'(n_i)\right) n_{ip} - c - \lambda = 0 \tag{11}$$

$$\sum_{i=1}^{2} \gamma_i \left(\theta_i b'(n_i) - d'(n_i)\right) n_{is} + \lambda \frac{\partial p^c(\theta_1)}{\partial s} + \eta = 0 \tag{12}$$

$$\lambda \geq 0,\ p^* \leq p^c(s^*, \theta_1),\ \lambda\left(p^* - p^c(s^*, \theta_1)\right) = 0$$

$$s^* \geq 0,\ \eta \geq 0,\ \eta s^* = 0$$

where (λ, η) *are the Lagrange multipliers associated with problem (10) and* $n_i = n(s^*, p^*, \theta_i)$*, given by* (6).

Figures 3 and 4 represent the cases of partial compliance and full noncompliance, respectively, both of them compatible with the solution. In the figures, we have included the social welfare contours, where each contour represents the set of policies (s, p) such that social welfare remains constant.

We cannot generally conclude that the optimal standard is zero in any case. If $p^* = p^c(s^*, \theta_1)$, the case of Figure 3, the optimal policy induces partial compliance and the optimal standard is generally positive (the contrary would require to enforce type θ_1 to comply with a zero standard, see footnote 16). Therefore, the dirty type violates a positive standard, a result that is not possible under complete information. In this case, combining (11) and (12), we obtain that the optimal standard and inspection probability are such that the marginal rate of substitution in terms of optimality of the induced pollution levels must equal the marginal rate of substitution to ensure type θ_1's compliance.

If $p^* < p^c(s^*, \theta_1)$, the case of Figure 4, the optimal policy induces full noncompliance. Here, the optimal standard need not be zero either, as opposed to the one type case.[18] Therefore, it is possible that both types violate positive standards. Since $\lambda = 0$, even a positive standard implies that $\theta_1 b'(n_1) - d'(n_1) > 0$, which means that type θ_1 is over-enforced.[19] Therefore, a zero standard could restrict type θ_1's pollution even more, with the corresponding welfare decrease. By contrast, type θ_2 is under-enforced.

In general, we can conclude that the most likely solution is that of Figure 4, except when monitoring costs are small or when the full noncompliance region is small (or equivalently, when θ_1 is small). By (11), it is easy to see that the monitoring costs and the optimal inspection probability are negatively related. Thus, the smaller the monitoring costs, the larger the inspection probability and, therefore, the more likely that the solution induces partial compliance. Also, the smaller the full noncompliance region, the larger the likelihood that the inspection probability induces partial compliance. The following corollary shows that, regardless of θ_1, the optimal uniform policy induces partial compliance if monitoring is costless.

Corollary 5 *If $c = 0$, the optimal uniform policy induces partial compliance.*

When the optimal policy induces full noncompliance, the standard is positive for some values of the parameters. As we have already pointed out, type θ_1 is over-enforced at the solution. On one hand, the smaller the standard, the larger the over-enforcement problem. On the other hand, the smaller the standard, the smaller the enforcement costs of inducing a particular pollution level.

[18] In the case of a unique type θ_i, by (11), we can easily see that $\theta_i b'(n_i) - d'(n_i) < 0$, since $n_{ip} < 0$, $c > 0$ and $\lambda = 0$. But then, $\eta > 0$, by (12), since $n_{is} > 0$, which implies that $s^* = 0$.

[19] If $s^* > 0$, then $\eta = 0$. By (12), we have $\gamma_1 A_1 n_{1s} = -\gamma_2 A_2 n_{2s}$, where $A_i = \theta_i b'(n_i) - d'(n_i)$, which implies that $c = \gamma_2 A_2 \left(n_{2p} - \frac{n_{2s}}{n_{1s}} n_{1p} \right)$. Since $n_{2p} - \frac{n_{2s}}{n_{1s}} n_{1p} = \frac{F'(n_2 - s) - F'(n_1 - s)}{\theta_2 b'' - pF''} < 0$ and $c > 0$, we then have $A_2 < 0$ and $A_1 > 0$.

Therefore, there exists a trade-off between the over-enforcement problem and the enforcement costs. Thus, the larger (smaller) the monitoring costs, the more likely the optimal standard is zero (positive). By contrast, the larger (smaller) type θ_1's profitability, the less (more) important the over-enforcement problem and the more likely the optimal standard is zero (positive). In consequence, the larger θ_1, the smaller the interval of the monitoring costs for which the standard is positive. Finally, a positive standard is more likely under large uncertainty, that is, when γ_1 takes intermediate values. This is so because under low uncertainty, the solution approximates to the complete information solution, where the optimal standard is zero (see Figure 4).

The following example illustrates all these features.

4.1 Example 1

Consider the specific functional forms:[20]

$$b(e) = \begin{cases} e, & e \leq 1 \\ 1-e, & e > 1 \end{cases}$$

$$\theta_2 = 1, \ \theta_1 < 1.$$

$$d(e) = e^2$$

$$F(e-s) = (e-s) + (e-s)^2$$

[20]Note that $b(e)$ is linear, which considerably simplifies the algebra without affecting the results. Here, $p^c(s, \theta_i) = \frac{\theta_i}{F'(0)}$, i.e., θ_i's threshold probability does not depend on s.

We compute the optimal uniform policy applying Proposition 4:

$$\gamma_1\theta_1\frac{p\left(\theta_1+1-2s\right)-\theta_1}{2p^3}+\left(1-\gamma_1\right)\frac{2p\left(1-s\right)-1}{2p^3}+c+\lambda=0$$
$$\gamma_1\frac{p\left(\theta_1+1-2s\right)-\theta_1}{p}+\left(1-\gamma_1\right)\frac{2p\left(1-s\right)-1}{p}+\eta=0$$
$$\lambda\left(p-\theta_1\right)=0,\ \lambda\geq 0,\ p\leq\theta_1$$
$$\eta s=0,\ s\geq 0,\ \eta\geq 0$$

We now explore the likelihood of obtaining optimal positive standards. Figure 5 illustrates the relationship between the optimal standards and the monitoring costs for different values of θ_1, in the case of large uncertainty ($\gamma_1 = 0.5$). If $\theta_1 = 0.8$ for example, the solution induces partial compliance to $s = 0.3875$ if $c \in [0, 0.002]$. If $c \in [0.002, 0.0024]$, the optimal solution induces full noncompliance to a positive standard, which decreases when the monitoring cost increase. Finally, if $c > 0.0024$, the optimal standard is zero. If θ_1 is lower, the solution induces partial compliance for a larger interval of the monitoring costs. This is intuitive since the lower θ_1, the lower the full noncompliance region, and therefore, the larger the restriction for the full noncompliance solution to exist. If $\theta_1 = 0.5$, we now obtain a larger interval of the monitoring costs for which the optimal standard is positive, $c \in [0.125, 0.227]$. For θ_1 sufficiently small, we obtain full noncompliance to a zero standard.

A similar picture can be obtained under alternative values of γ_1, that is, under different degrees of uncertainty. For example, if $\gamma_1 = 0.1$, the range of monitoring costs for which we obtain full noncompliance to a strictly positive standard is $c \in [0.0007, 0.007]$ if $\theta_1 = 0.8$ and $c \in [0.045, 0.049]$ if $\theta_1 = 0.5$. Alternatively, if $\gamma_1 = 0.9$, we have $c \in [0.0007, 0.01]$ and $c \in [0.045, 0.16]$ for

$\theta_1 = 0.8$ and $\theta_1 = 0.5$, respectively. In the limiting case of no uncertainty, the solution jumps from partial compliance to full noncompliance to a zero standard, with no possible violations of positive standards.

5 The Optimal Type-Contingent Policy

In this case, the regulator maximizes social welfare given in (5), considering the firm's optimal behavior and the incentive compatibility constraints given in (4). We first present a useful preliminary result:

Proposition 6 *Let* $(s_1^*, s_2^*, p_1^*, p_2^*)$ *be the optimal type-contingent policy. Then,* $p_1^* \leq p^c(s_1^*, \theta_1)$.

As in the uniform case, the optimal policy cannot induce full compliance. The regulator now solves the following problem:

$$\begin{aligned} & Max_{s_1,s_2,p_1,p_2} && \{\gamma_1(\theta_1 b(s_1) - d(s_1) - p_1 c) + \gamma_2(\theta_2 b(n_2) - d(n_2) - p_2 c)\} \\ & \text{s.t.} && p_1 \leq p^c(s_1, \theta_1) \\ & && P(s_i, p_i, \theta_i) \geq P(s_j, p_j, \theta_i) \\ & && s_i \geq 0 \end{aligned} \tag{13}$$

Proposition 7 *The optimal type-contingent policy is given by the following* *conditions:*

$$
\begin{aligned}
&\frac{\gamma_1\left(\theta_1 b'\left(n_1\right)-d'\left(n_1\right)\right)n_{1s}+\lambda\frac{\partial p^c(\theta_1)}{\partial s_1}}{\gamma_1\left(\left(\theta_1 b'\left(n_1\right)-d'\left(n_1\right)\right)n_{1p}-c\right)-\lambda}=\frac{\frac{\partial P\left(s_1^*,p_1^*,\theta_2\right)}{\partial s_1}}{\frac{\partial P\left(s_1^*,p_1^*,\theta_2\right)}{\partial p_1}}\\
&\frac{\gamma_1\left(\theta_1 b'\left(n_1\right)-d'\left(n_1\right)\right)n_{1s}+\lambda\frac{\partial p^c(\theta_1)}{\partial s_1}}{\gamma_2\left(\left(\theta_2 b'\left(n_2\right)-d'\left(n_2\right)\right)n_{2p}-c\right)}=-\frac{\frac{\partial P\left(s_1^*,p_1^*,\theta_2\right)}{\partial s_1}}{\frac{\partial P\left(s_2^*,p_2^*,\theta_2\right)}{\partial p_2}}\\
&\frac{\gamma_1\left(\theta_1 b'\left(n_1\right)-d'\left(n_1\right)\right)n_{1s}+\lambda\frac{\partial p^c(\theta_1)}{\partial s_1}}{\gamma_2\left(\theta_2 b'\left(n_2\right)-d'\left(n_2\right)\right)n_{2s}}=-\frac{\frac{\partial P\left(s_1^*,p_1^*,\theta_2\right)}{\partial s_1}}{\frac{\partial P\left(s_2^*,p_2^*,\theta_2\right)}{\partial s_2}}\\
&s_2^*\geq 0,\ \eta_2\geq 0,\ s_2^*\eta_2=0\\
&\lambda\geq 0,\ p_1^*\leq p^c\left(s_1^*,\theta_1\right),\ \lambda\left(p_1^*-p^c\left(s_1^*,\theta_1\right)\right)=0\\
&P\left(s_2^*,p_2^*,\theta_2\right)=P\left(s_1^*,p_1^*,\theta_2\right)
\end{aligned}
\qquad (14)
$$

where (λ, μ_i, η_i) *are the Lagrange multipliers associated with problem (13) and* n_i *is type* θ_i*'s optimal response to* (s^*, p^*) *given by* (6).

Note that (14) implies $s_1^* > s_2^* \geq 0$ and $p_1^* > p_2^*$. Thus, type θ_1 faces both a larger standard and a larger probability of inspection in order to preserve incentive compatibility. Here, the standard for type θ_2 could be zero but not necessarily, since $\eta_2 \geq 0$. By (14), type θ_2 is indifferent between (s_1^*, p_1^*) and (s_2^*, p_2^*). By Lemma 2, this means that type θ_1 strictly prefers (s_1^*, p_1^*). At the solution, type θ_1 is over-enforced and type θ_2 is under-enforced with respect to the complete information case. type θ_2 is under-enforced and type θ_1 is over-enforced with respect to the complete information case. If the regulator were to naively impose the complete information solution, type θ_2 would find it profitable to misreport its type, and this is why type θ_2's incentive compatibility constraint is binding.

The optimality conditions mean that, at the optimum, (s_1, s_2, p_1, p_2) are such that the marginal rate of substitution between each pair of variables in terms of efficiency of the induced pollution levels equals the marginal rate of substitution between that pair of variables to induce type θ_2's truthful revelation.

Observe that the optimal type-contingent policy can induce either partial compliance or full noncompliance (see Figure 6 for an illustration of the first case). In any event, the standard for type θ_2 need not be zero. In this case, type θ_2 is under-enforced with respect to the complete information case. Since the slope of θ_2's indifference curve is larger than the slope of the curve where n_2 is constant[21], moving along the indifference curve towards $s = 0$ means that n_2 increases, so the under-enforcement problem is worse. Similarly to the uniform case, the lower the standard, the lower the enforcement costs. Now there is a trade-off between these enforcement costs and the under-enforcement problem of type θ_2. Thus, the smaller (larger) the monitoring costs, the more (less) likely the optimal standard for type θ_2 is positive. By contrast with the uniform case, the larger (lower) θ_1's profitability, the larger (lower) type θ_2's under-enforcement problem associated with $s = 0$. Therefore, it is more likely to have $s_2 > 0$ ($s_2 = 0$) when θ_1 is large (small).

In the next example, we illustrate all these results.

[21] See footnote 24.

5.1 Example 2

We consider the same functions of Example 1. By (8), we have:

$$P(s,p,1)=\frac{1-2p(1-2s)+p^2}{4p} \tag{15}$$

since type θ_2 violates the standard.

Therefore, the incentive compatibility constraint of Proposition 7 reads:

$$\frac{1-2p_1(1-2s_1)+p_1^2}{4p_1}=\frac{1-2p_2(1-2s_2)+p_2^2}{4p_2} \tag{16}$$

From (15), we have $\frac{\partial P(s_i,p_i,\theta_2)}{\partial s_i}=1$ and $\frac{\partial P(s_i,p_i,\theta_2)}{\partial p_2}=-\frac{1-p^2}{4p^2}$. The following equations and (16) characterize the optimal separating policy:

$$\begin{aligned}
&\frac{\gamma_1\left(p_1\left(\theta_1+1-2s_1\right)-\theta_1\right)}{\gamma_1\left(\theta_1 p_1\left(\theta_1+1-2s_1\right)-\theta_1^2+2cp_1^3\right)+2\lambda p_1^3}=\frac{2}{1-p_1^2}\\
&\left(\frac{\gamma_1}{\gamma_1-1}\right)\left(\frac{p_2}{p_1}\right)\left(\frac{p_1\left(\theta_1+1-2s_1\right)-\theta_1}{2p_2\left(1-s_2\right)-1+2cp_2^3}\right)=\frac{2}{1-p_2^2}\\
&\gamma_1\frac{p_1\left(\theta_1+1-2s_1\right)-\theta_1}{p_1}=\left(\gamma_1-1\right)\frac{2p_2\left(1-s_2\right)-1}{p_2}-\eta_2\\
&s_2\geq 0,\ \eta_2\geq 0,\ \eta_2 s_2=0\\
&\lambda\geq 0,\ p_1\leq\theta_1,\ \lambda\left(p_1-\theta_1\right)=0
\end{aligned}$$

We have computed the results for different values of the parameters. While we can find solutions that induce partial compliance, however we cannot find a solution which induces full noncompliance for any feasible values of the parameters. Therefore, if the policy were to be separating, in this case it would induce partial compliance. Thus, $p_1=\theta_1$.

In Figure 7, we present the relationship between the optimal standards and the monitoring costs, for different values of θ_1 and $\gamma_1 = 0.5$, that is, when there is large uncertainty. If $\theta_1 = 0.5$, there does not exist a separating solution when $c \in [0, 0.3125]$. While type θ_1 always complies with s_1, type θ_2 violates a strictly positive standard s_2 when $c \in [0.3125, 0.58853]$. Finally, $s_2 = 0$ if $c > 0.58853$. Note that both types' standards decrease when monitoring costs increase. Also, both inspection probabilities decrease when monitoring costs increase. When $\theta_1 = 0.8$, we observe the same pattern, but here, the interval where type θ_2 violates a strictly positive standard is larger, i,e., $c \in [0.016, 0.70177]$. Therefore, it is more likely that we find noncompliance to strictly positive standards when θ_1 is large, since type θ_2's under-enforcement problem associated with $s = 0$ is worse in this case.

We find an analogous structure of the solution under different values of γ_1. However, it is interesting to see that, the smaller γ_1, the smaller the intervals of the monitoring costs where type θ_2 violates a positive standard. Thus, if $\gamma_1 = 0.1$, we find that type θ_2 violates a positive standard when $c \in [0.0625, 0.069524]$ if $\theta_1 = 0.5$ and when $c \in [0.0032, 0.0804]$ if $\theta_1 = 0.8$. Conversely, if $\gamma_1 = 0.9$, these intervals are, respectively, $c \in [0.5625, 2.304]$ and $c \in [0.028828, 2.8518]$.

Finally, comparing Figures 5 and 7, the interval of the monitoring costs for which we obtain violations to strictly positive standards under the uniform policy always contains lower values than the interval under the separating policy. Regarding social welfare, we have made some computations which show that a uniform policy may be preferred to a separating policy. For example, if $c = 0.58853$, $\theta_1 = 0.5$ and $\gamma_1 = 0.5$, we obtain $sw(pool) = -0.11822 > sw(sep) = -0.1478$. Alternatively, if $c = 0.069524$, $\theta_1 = 0.5$ and $\gamma_1 = 0.1$, we have $sw(pool) = 0.191169 > sw(sep) = 0.1909$. Therefore, this example shows that type-contingent policies are not always preferred to uniform policies, form a social viewpoint.

6 Conclusions

In this paper, we have studied optimal regulatory policies composed of pollution standards, probabilities of inspection and fines for noncompliance in a context of asymmetric information and imperfect enforcement, an approach different from that which has been studied in the literature. Our model is able to explain a salient feature of environmental regulation, namely violations to strictly positive standards, a result that is not possible under either complete information or incomplete information subject to perfect enforcement.

We have shown that violations to positive standards are more likely when monitoring costs are low. Since a positive standard implies that the fine for non-compliance is not maximum, this result is more likely when enforcement costs are less important than the costs associated with clean type's over-enforcement or dirty type's under-enforcement, depending on the policy being uniform or type-contingent, respectively. We have shown that positive standards are more likely under intermediate or large clean type's profitability, depending again on the policy being uniform or type-contingent, respectively. Finally, regulator's uncertainty also matters. On one hand, the larger the uncertainty, the more likely we obtain a positive standard in the uniform case. On the other hand, the larger the likelihood of the clean type, the more likely we have a positive standard in the type-contingent case.

There would be no substantial changes if we considered a continuum of types instead of the two types presented here. If the optimal policy induced some types to comply and others to violate the standards, the latter ones would be the dirtiest. In that case, the optimal policy would imply partial uniformity: the compliant types would be confronted to the same policy to avoid misreporting.

Alternatively, the model presented is also valid for a problem of several firms clasified into two subgroups, the clean and the dirty ones. All our results easily extend to this case, as long as we continue to assume risk neutrality.

Our results have three implications on the previous literature. First, we can rationalize violations to positive standards. This suggests that restricting attention to incentive compatible environmental taxation (where all the pollution levels are punishable) may be restrictive. Second, we have shown that the optimal policy never induces full compliance, which implies that concentrating on the subset of perfectly enforceable policies may be also restrictive. Finally, some computations have shown that separating policies may not always be socially preferred to pooling policies. This implies that, under some circumstances, information collection might be useless.

7 Appendix

Proof of Lemma 1.

When $P(s, p, \theta_i) = \theta_i b(s)$, the function is strictly increasing and concave in s, but it does not depend on p. Also, $\theta_2 b(s) > \theta_1 b(s)$. Conversely, when

$P(s,p,\theta_i) = \pi(s,p,\theta_i)$, we have:

$$\pi_s(s,p,\theta_i) = pF'(n_i - s) \geq 0 \tag{17}$$

$$\pi_{ss}(s,p,\theta_i) = pF''(n_i - s)(n_{is} - 1) \leq 0 \tag{18}$$

$$\pi_p(s,p,\theta_i) = -F(n_i - s) < 0 \tag{19}$$

$$\pi_{pp}(s,p,\theta_i) = -F'(n_i - s)\, n_{ip} > 0 \tag{20}$$

$$\pi_{sp}(s,p,\theta_i) = F'(n_i - s) + pF''(n_i - s)\, n_{ip} > 0 \tag{21}$$

where $n_i = n(s,p,\theta_i)$. Also, we trivially obtain that $\pi(s,p,\theta_2) > \pi(s,p,\theta_1)$. Summing up both possibilities we obtain the desired result.

Finally, $\pi(s,0,\theta_i) = \max_{e>0} \theta_i b(e) = \theta_i b(\widetilde{e})$, for all i. Thus, $P(s,0,\theta_i) = \theta_i b(\widetilde{e})$, as desired. ■

Proof of Lemma 2.

In θ_i's compliance region, the expected payoff is $\theta_i b(s)$, that is, it does not depend on the probability of inspection. Therefore, indifference curves have an infinite slope. In the noncompliance region, the expected payoff is $\pi(s,p,\theta_i) = b(n(s,p,\theta_i)) - pF(n(s,p,\theta_i) - s)$. Implicitly differentiating $\pi(s,p,\theta_i) = k$, we obtain:

$$\frac{dp}{ds}\Big|_{\pi=k} = \frac{pF'(n(s,p,\theta_i) - s)}{F(n(s,p,\theta_i) - s)} > 0 \tag{22}$$

Now, differentiating (22) with respect to s we have:

$$\frac{d^2p}{ds^2}\Big|_{\pi=k} = \frac{F'}{F}\frac{dp}{ds}\Big|_{\pi=k} + \frac{p}{F^2}\left(F''F - (F')^2\right)(n_{is} - 1) > 0 \tag{23}$$

since $n_{is} < 1$ and $F''F - (F')^2 < 0$.

(For analytical convenience, we prove the last part considering a continuum of types. The result is easily adapted to the case in which θ takes discrete values.)

In the compliance region, both types' indifference curves are vertical. In the partial compliance region, θ_1's are vertical and θ_2's are strictly increasing In the full noncompliance region, we differentiate (22) with respect to θ to obtain:

$$\frac{d^2p}{dsd\theta} \mid_{\pi=k} = \frac{F''F - (F')^2}{(F)^2} pn_\theta (s,p,\theta) \tag{24}$$

Since $F''F - (F')^2 < 0$, $\frac{d^2p}{dsd\theta} \mid_{\pi=k}$ and $n_\theta (s,p,\theta)$ have the opposite sign. Differentiating (6) with respect to θ, we obtain $n_\theta (s,p,\theta) = -\frac{b'}{\theta b'' - pF''} > 0$. Therefore, $\frac{d^2p}{dsd\theta} \mid_{\pi=k} < 0$, as desired. ■

Proof of Proposition 3.

Assume first that $p \geq p^c (s, \theta_2)$, that is, the pooling policy induces full compliance. The problem the regulator faces in this case is:

$$\begin{aligned} &Max_{s,p} \sum_{i=1}^{2} \gamma_i \left[\theta_i b\left(e\left(s,p,\theta_i\right)\right) - d\left(e\left(s,p,\theta_i\right)\right)\right] - cp \\ &s.t. \; p \geq p^c (s,\theta_2) \end{aligned} \tag{25}$$

Since $p \geq p^c (s, \theta_2)$, we then have $e(s,p,\theta_i) = s$ for all i. The Lagrangian of problem (25) is the following:

$$L(s,p,\lambda) = \sum_{i=1}^{2} \gamma_i \left[\theta_i b(s) - d(s)\right] - cp - \lambda \left(p^c (s,\theta_2) - p\right)$$

where $\lambda \geq 0$ is the corresponding Lagrange multiplier. The interior solution is given by the following Kuhn-Tucker conditions:[22]

$$\gamma_1 \left(\theta_1 b' (s) - d' (s)\right) + \gamma_2 \left(\theta_2 b' (s) - d' (s)\right) - \lambda \frac{dp^c (s, \theta_2)}{ds} = 0$$

$$c - \lambda = 0$$

$$\lambda \left(p^c (s, \theta_2) - p\right) = 0$$

Since $c > 0$, we have $\lambda > 0$ and $p^c (s, \theta_2) = p$, which leads to:

$$\sum_{i=1}^{2} \gamma_i \left(\theta_i b' (s^*) - d' (s^*) - c \frac{dp^c (s^*, \theta_2)}{ds}\right) = 0 \qquad (26)$$
$$p^* = p^c (s^*, \theta_2)$$

Since $\theta_1 < \theta_2$, (26) implies that $\theta_1 b' (s^*) - d' (s^*) - c \frac{dp^c(s,\theta_2)}{ds} < 0$ and $\theta_2 b' (s^*) - d' (s^*) - c \frac{dp^c(s,\theta_2)}{ds} > 0$. This last expression can be written as $\left(\theta_2 b' (s^*) - d' (s^*)\right) \frac{ds}{dp^c} - c < 0$, since $\frac{dp^c(s,\theta_2)}{ds} < 0$. By the continuity of the sanction at $e - s = 0$, we can infinitesimally decrease p to increase social welfare, without affecting θ_1's behavior. Therefore, a uniform policy which induces full compliance is never optimal.

We now consider the case in which $p^c (s, \theta_1) < p \leq p^c (s, \theta_2)$. This corresponds to the partial compliance region, where the clean type strictly prefers to comply with the standard. Now, the problem is:

$$\begin{aligned} Max_{s,p} \quad & \sum_{i=1}^{2} \gamma_i \left[\theta_i b \left(e (s, p, \theta_i)\right) - d \left(e (s, p, \theta_i)\right)\right] - cp \\ s.t. \quad & p > p^c (s, \theta_1) \end{aligned} \qquad (27)$$

[22] The assumptions of the model ensure that these conditions are necessary an sufficient for an interior optimum. This continues to hold for the remaining optimality results of the paper.

where $s = e(s, p, \theta_1)$ and $n_2 = e(s, p, \theta_2)$. The Kuhn-Tucker conditions are the following:

$$\gamma_1 (\theta_1 b'(s) - d'(s)) + \gamma_2 (\theta_2 b'(n_2) - d'(n_2)) n_{2s} - \lambda \frac{dp^c(s, \theta_1)}{ds} = 0$$

$$\gamma_2 (\theta_2 b'(n_2) - d'(n_2)) n_{2p} - c + \lambda = 0$$

$$\lambda (p^c(s, \theta_1) - p) = 0$$

where $\lambda \geq 0$ is the corresponding Lagrange multiplier of problem (27).

Observe that $\lambda = c - \gamma_2 (\theta_2 b'(n_2) - d'(n_2)) n_{2p} \geq 0$. Since $p > p^c(s, \theta_1)$, λ must be equal to 0. This implies that $c = \gamma_2 (\theta_2 b'(n_2) - d'(n_2)) n_{2p}$, which means that $c > \left(\theta^2 b'\left(n^2\right) - d'\left(n^2\right)\right) n_{2p}$, since $\gamma_2 < 1$. Therefore, welfare can increase if p decreases infinitesimally, since type θ_1 continues to comply with s. Therefore, the optimal uniform policy cannot be such that $p > p^c(s, \theta_1)$. ■

Proof of Corollary 5.

Assume, to the contrary, that $\lambda = 0$, that is, the optimal policy induces full noncompliance when $c = 0$. By (11), we have $\gamma_1 A_1 n_{1p} = -\gamma_2 A_2 n_{2p}$, where $A_i = \theta_i b'(n_i) - d'(n_i)$ for all i. Considering (12), it must be true that:

$$\eta = -\gamma_2 A_2 \left(\frac{n_{2p}}{n_{1p}} n_{1s} - n_{2s} \right)$$

Since $A_2 < 0$ and $\frac{n_{2p}}{n_{1p}} n_{1s} - n_{2s} > 0$ (see footnote 19), we then have $\eta < 0$, which contradicts the fact that the solution induces full noncompliance.

Proof of Proposition 6.

First, a type-contingent policy in the full compliance region is not possible since indifference curves do not cross. Thus, any attempt to set a type-contingent policy would induce misreporting of the type with the lowest assigned standard. Also, any policy that assigns the same standard but a different probability is incentive compatible but suboptimal, since both probabilities can be decreased till the boundary $p^c(s, \theta_2)$ without distorting incentives and reducing monitoring costs.

Next, consider the case of partial compliance where $p_1 > p^c(s_1, \theta_1)$. The problem is:

$$
\begin{aligned}
& Max_{s_1,s_2,p_1,p_2} && \{\gamma_1 (\theta_1 b(s_1) - d(s_1) - p_1 c) + \gamma_2 (\theta_2 b(n_2) - d(n_2) - p_2 c)\} \\
& s.t. && p_1 > p^c(s_1, \theta_1) \\
& && P(s_i, p_i, \theta_i) \geq P(s_j, p_j, \theta_i), \; i, j = 1, 2, \; i \neq j \\
& && s_i \geq 0, \; i = 1, 2
\end{aligned} \qquad (28)
$$

Considering $\lambda \geq 0$ to be the Lagrange multiplier associated with the first restriction in problem (28), the Kuhn-Tucker conditions associated with p_1 are:

$$
\begin{aligned}
& \lambda = \gamma_1 c \\
& \lambda \geq 0, \; \lambda (p^c(s_1, \theta_1) - p_1) = 0
\end{aligned}
$$

Since $p_1 > p^c(s_1, \theta_1)$, we then must have $\lambda = 0$. However, $\lambda = \gamma_1 c > 0$, which is a contradiction. Therefore, $p_1 \leq p^c(s_1, \theta_1)$, as desired. ■

Proof of Proposition 7.

The Kuhn-Tucker conditions of problem (13) are the following:

$$\gamma_1 \left(\left(\theta_1 b' \left(n_1 \right) - d' \left(n_1 \right) \right) n_{1p} - c \right) - \lambda + \mu_1 \frac{\partial P \left(s_1^*, p_1^*, \theta_1 \right)}{\partial p_1} - \mu_2 \frac{\partial P \left(s_1^*, p_1^*, \theta_2 \right)}{\partial p_1} = 0$$

$$\gamma_1 \left(\theta_1 b' \left(n_1 \right) - d' \left(n_1 \right) \right) n_{1s} + \lambda \frac{\partial p^c \left(\theta_1 \right)}{\partial s_1} + \mu_1 \frac{\partial P \left(s_1^*, p_1^*, \theta_1 \right)}{\partial s_1} - \mu_2 \frac{\partial P \left(s_1^*, p_1^*, \theta_2 \right)}{\partial s_1} = 0$$

$$\gamma_2 \left(\left(\theta_2 b' \left(n_2 \right) - d' \left(n_2 \right) \right) n_{2p} - c \right) - \mu_1 \frac{\partial P \left(s_2^*, p_2^*, \theta_1 \right)}{\partial p_2} + \mu_2 \frac{\partial P \left(s_2^*, p_2^*, \theta_2 \right)}{\partial p_2} = 0$$

$$\gamma_2 \left(\theta_2 b' \left(n_2 \right) - d' \left(n_2 \right) \right) n_{2s} - \mu_1 \frac{\partial P \left(s_2^*, p_2^*, \theta_1 \right)}{\partial s_2} + \mu_2 \frac{\partial P \left(s_2^*, p_2^*, \theta_2 \right)}{\partial s_2} = 0$$

$$s_1^* > 0, \ \eta_1 = 0$$

$$s_2^* \geq 0, \ \eta_2 \geq 0, \ s_2^* \eta_2 = 0$$

$$\lambda \geq 0, \ p_1^* \leq p^c \left(s_1^*, \theta_1 \right), \ \lambda \left(p_1^* - p^c \left(s_1^*, \theta_1 \right) \right) = 0$$

$$\mu_i \geq 0, \ P \left(s_i^*, p_i^*, \theta_i \right) \geq P \left(s_j^*, p_j^*, \theta_i \right), \ \mu_i \left(P \left(s_i^*, p_i^*, \theta_i \right) - P \left(s_j^*, p_j^*, \theta_i \right) \right) = 0$$

where $\lambda \geq 0$, $\mu_i \geq 0$, $\eta_i \geq 0$ are, respectively, the Lagrange multipliers associated with the restrictions in problem (13).

Assume first that $\mu_1 = \mu_2 = \eta_2 = 0$. This implies that $\theta_2 b' \left(n_2 \right) - d' \left(n_2 \right) = 0$ and $c = 0$, since $n_{2p} < 0$. Since $c > 0$, either one of the incentive compatibility constraints must be binding or $\eta_2 \geq 0$.[23] Assume first that $\mu_1 \geq 0$ and $\mu_2 = \eta_2 = 0$. However, n_2 can be kept constant decreasing both (s_2, p_2) through expression (6) without distorting the incentive compatibility constraints.[24] Therefore, $\mu_1 \geq 0$, $\mu_2 = \eta_2 = 0$ is not possible.

Now, consider $\mu_1 = 0$, $\mu_2 = 0, \eta_2 \geq 0$. In this case, first order conditions would reduce to $\theta_1 b' \left(s_1 \right) - d' \left(s_1 \right) = c \frac{dp^c(\theta_1)}{ds_1}$ and $\left(\theta_2 b' \left(n_2 \right) - d' \left(n_2 \right) \right) n_{2p} = c$, respectively, the optimal compliance solution for type θ_1 and the optimal

[23] It is easy to see that both incentive compatibility constraints cannot be binding except in the case of a pooling policy. Thus, $\mu_1 \geq 0$, $\mu_2 \geq 0$ is not possible if the policy is separating.

[24] To see this, consider (22) and $\frac{dp}{ds} \mid_{n_2} = -\frac{n_{2s}}{n_{2p}} = \frac{pF''}{F'}$ to conclude that $\frac{dp}{ds} \mid_{n_2} < \frac{dp}{ds} \mid_{P_2}$, since $\left(F' \right)^2 - FF'' > 0$. By Lemma 2, we then have $\frac{dp}{ds} \mid_{n_2} < \frac{dp}{ds} \mid_{P_1}$.

noncompliance solution for type θ_2 if information were complete. But, in this case, type θ_2 would prefer to misreport its type. Therefore, $\mu_1 = 0$, $\mu_2 = 0, \eta_2 \geq 0$ is not possible. For the same reason, $\mu_1 \geq 0$, $\mu_2 = 0, \eta_2 \geq 0$ is not possible either.

Therefore, $\mu_1 = 0$ and $\mu_2 > 0$. As for η_2, both $\eta_2 = 0$ and $\eta_2 \geq 0$ are compatible with the solution, thus obtaining the desired result. ■

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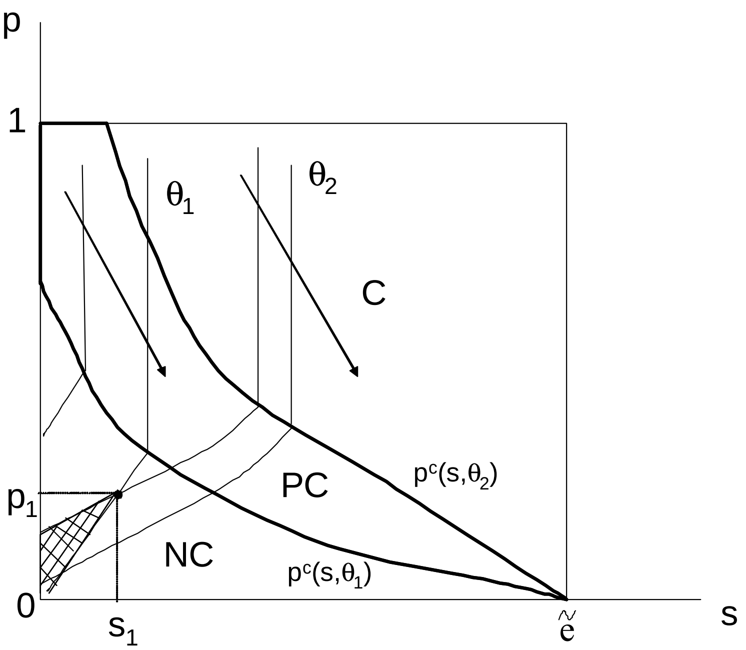

Figure 1: The compliance and noncompliance regions

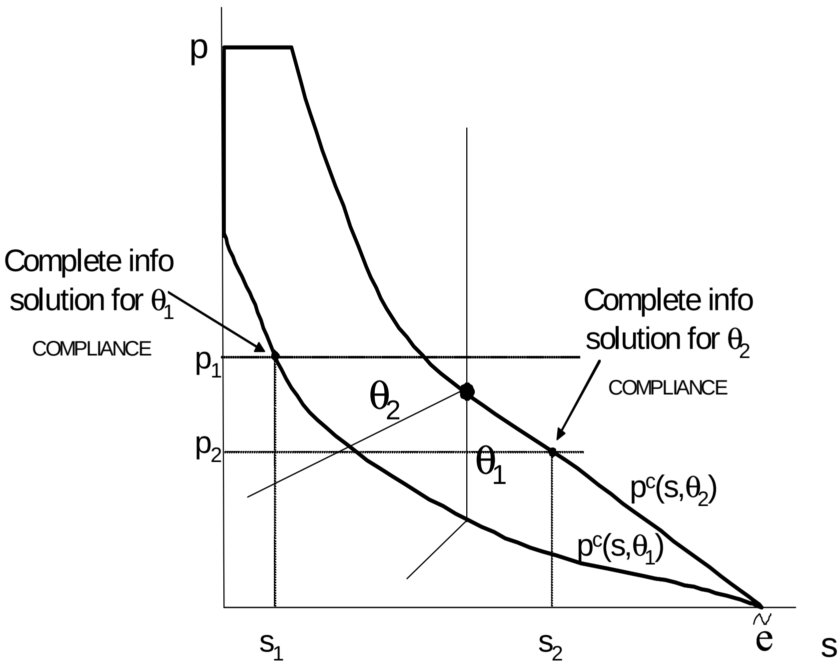

Figure 2: The uniform policy under full compliance

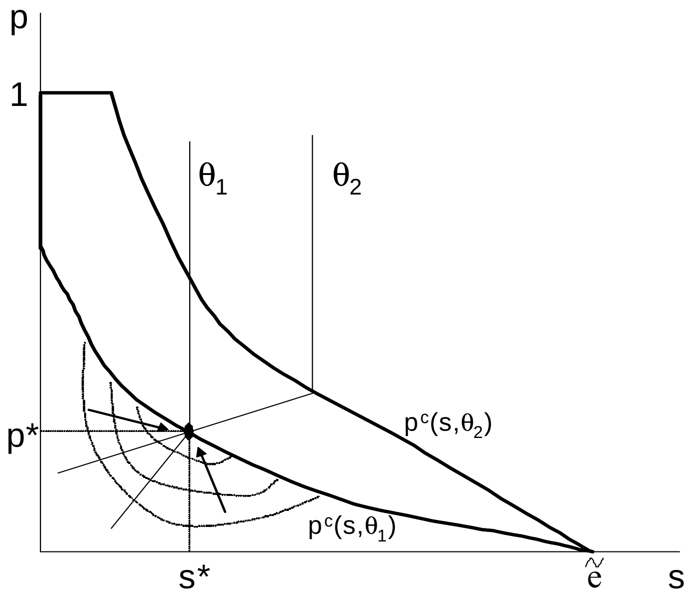

Figure 3: The optimal uniform policy inducing partial compliance

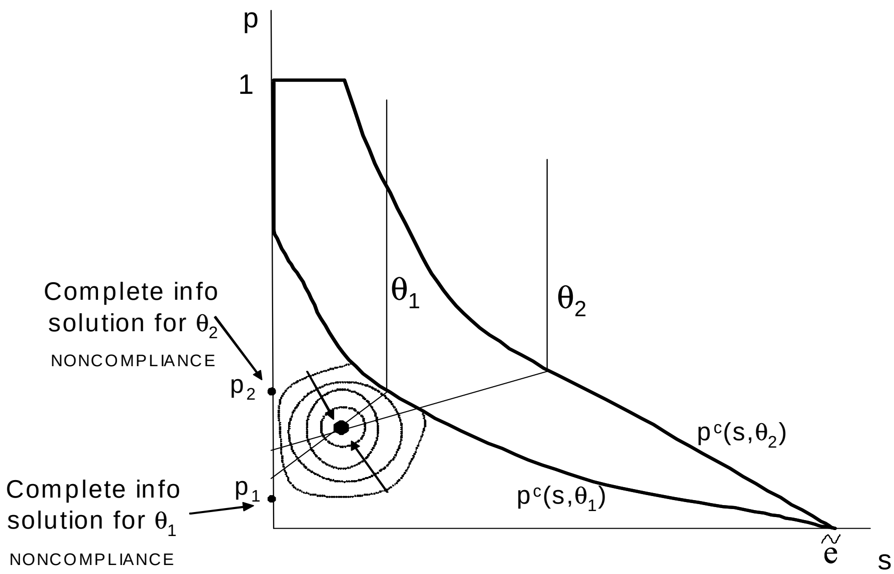

Figure 4: The optimal uniform policy inducing full noncompliance

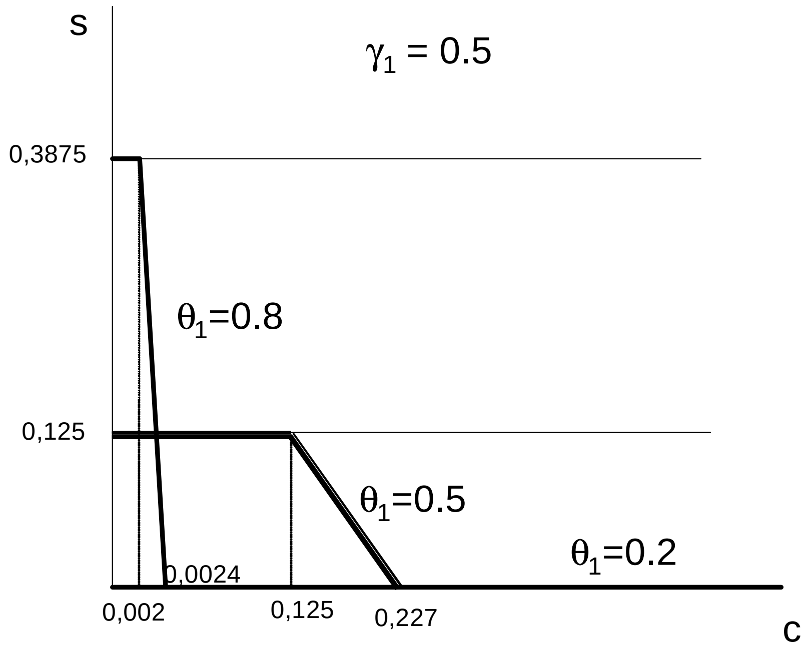

Figure 5: The optimal standard in the uniform case

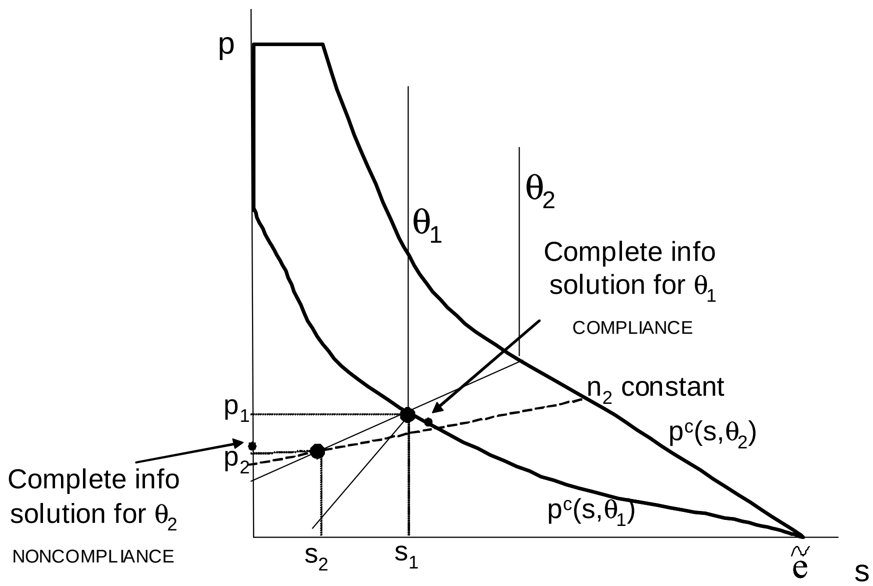

Figure 6: The optimal type-contingent policy when it induces partial compliance

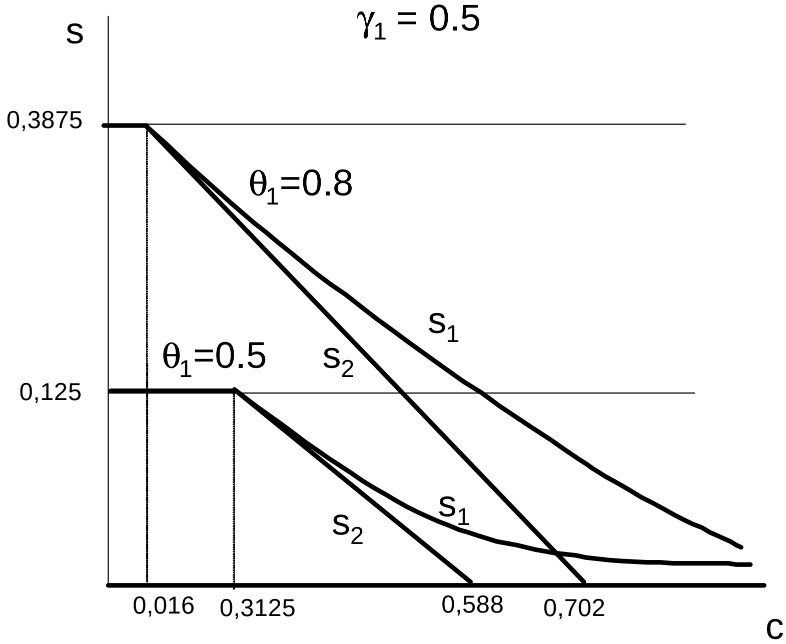

Figure 7: The optimal standards in the type-contingent case