

**Taxing deficits to restrain government spending and foster capital  
accumulation**

by

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**Abstract:**

In a dynamic model of fiscal policy, social polarization provokes a deficit bias. Policy advisors have recently proposed that governments running a deficit should be forced to generate additional tax revenue. We show that this deficit taxation reduces the deficit bias as it internalizes the externality different lobby groups impose on others. The mechanism described here is not due to the political risk of being elected out of office because the private sector dislikes taxation. Lower government spending and the resulting reduced deficit bias augment capital accumulation.

**Keywords:** fiscal rules, deficit taxation, polarization, capital accumulation.

**JEL-Classification:** H 61, H 62, H 63, E 62.

## Non-technical summary

Disagreement on the optimal government spending among different groups (social polarization) can result in excessive deficits and excessive spending if spending is financed from a common pool of resources. In this view, politicians and constituencies benefit from specific spending programs, while imposing the costs on a common pool. Owing to this negative externality, the individually rational strategies generate budgets that are sub-optimal from the perspective of the group. In an inter-temporal version of the model, current spending will be high and financed by deficits leading to even higher costs to future generations because current deficits reduce future spending potential.

Empirical studies and recent experience confirm that the higher the degree of social polarization, the higher deficit financing of public expenditures is. Strengthening of budget institutions is found to reduce the deficit bias caused by social polarization. Recent proposals that aim to strengthen budget institutions include deficit taxation, which implies that governments running a deficit are forced to generate additional tax revenues depending on the size of the deficit. The main argument why deficit taxation reduces the deficit bias made up to now is that, as the private sector dislikes taxation, higher deficit financing inducing higher taxes increases the likelihood of politicians being voted out of office. In order to prevent this, politicians are more reluctant to generate deficits whenever deficit taxation applies.

The present paper motivates deficit taxation differently: The reduction of the deficit bias is driven entirely by diminishing the externality of the common pool problem and is not connected to the political risk. Deficit taxation generates two effects. First, the spending incentive increases as higher deficits imply higher current government revenue that may be used for further spending. Second, whenever the additional revenue is not fully spent, only a fraction of the current deficit cuts future spending potential. Hence, deficit taxation directly influences the evolution of future resources available to the government positively. It can be shown that this increases the marginal costs of deficit financing, which reduces the spending incentive and, thus, reduces the incentive to impose costs on the common pool. To put it crudely: The perspective of higher future spending potential reduces the incentive for excessive current spending resulting from the common pool problem. As the latter effect dominates the former effect, deficit taxation reduces the deficit bias.

Further, deficit taxation fosters capital accumulation. The reason for this is that the government deficit is financed through the private sector by giving up capital investments. The decreases in the deficit bias owing to deficit taxation releases resources that can be used for capital accumulation overcompensating the additional tax payments due.

# Nicht-technische Zusammenfassung

Unterschiedliche Auffassung über die optimale Zusammensetzung staatlicher Ausgaben (soziale Polarisation) kann aufgrund eines *Common Pool* Problems zu übermässigen Defiziten und zu übermässigen staatlichen Ausgaben führen. Politiker oder Wahlbezirke profitieren von einem bestimmten Ausgabenprogramm, während sie die entsprechenden Kosten auf ein gemeinsames Budget (*common pool*) überwälzen. Aufgrund dieser negativen Externalität generieren die individuell rationalen Strategien Haushaltspläne, die aus gesamtwirtschaftlicher Sicht suboptimal sind. In einer intertemporalen Betrachtung führt dies zu hohen laufenden staatlichen Ausgaben, die durch Defizite finanziert werden. Dies führt zu einer Erhöhung der Finanzierungskosten für zukünftige Generationen, weil heutige Defizite zukünftiges Ausgabenpotential einschränken.

Empirische Studien und bisherige Erfahrungen zeigen, dass Defizitfinanzierung mit dem Grad der sozialen Polarisation steigt. Es ist bekannt, dass die Stärkung von Haushaltsinstitutionen den Hang zur Defizitfinanzierung verringert. Aktuelle Vorschläge, die das Ziel der Stärkung der Haushaltsinstitutionen haben, sehen deshalb vor, dass zusätzliche Steuereinnahmen generiert werden müssen, wenn Defizite zur Finanzierung staatlicher Ausgaben anfallen. Das gängigste Argument, warum eine solche "Defizitbesteuerung" zu Ausgabenzurückhaltung führt, kann wie folgt zusammengefasst werden. Weil der Privatsektor, der die zusätzliche Steuer aufbringen muss, Besteuerung ablehnt, steigt für einen Politiker, der übermässige Defizite verursacht, das Abwahlrisiko. Um dies zu vermeiden, wird er eine zurückhaltendere Ausgabenpolitik verfolgen.

In diesem Papier wird der Vorschlag einer Besteuerung von Defiziten anders begründet: Die Reduzierung der Defizitfinanzierung erfolgt hier ausschliesslich aufgrund einer Verringerung der Externalität, die durch das *Common Pool* Problem ausgelöst wird und ist unabhängig von dem politischen Abwahlrisiko. Defizitbesteuerung generiert zwei Effekte. Einerseits erhöht sie den Anreiz für übermässige Staatsausgaben, da zusätzliche defizitinduzierte Einnahmen kurzfristig zusätzliches Ausgabenpotential schaffen. Andererseits führt sie dazu, dass nur ein Teil des gegenwärtigen Defizits das zukünftige Ausgabenpotential reduziert, solange die zusätzlichen Einnahmen nicht vollständig ausgegeben werden.

Demzufolge beeinflusst Defizitbesteuerung direkt die Entwicklung von zukünftigen staatlichen Ressourcen positiv. Es kann gezeigt werden, dass dieser Einfluss die marginalen Kosten einer Defizitfinanzierung erhöht und somit den Anreiz für eine solche Politik verringert. Anders ausgedrückt: Die Aussicht auf grössere staatliche Ausgabenmöglichkeiten in der Zukunft reduzieren den Anreiz zu übermässigen Ausgaben heute, die durch das *Common Pool Problem* ausgelöst werden. Da der zuletzt beschriebene Effekt den ersten Effekt dominiert, reduziert eine Defizitbesteuerung die Defizitfinanzierung staatlicher Ausgaben.

Zusätzlich wird die Kapitalakkumulation belebt. Der Grund ist, dass staatliche Defizite von dem Privatsektor durch Verzicht von Kapitalinvestitionen finanziert werden müssen. Die Reduktion der Defizite führt dazu, dass mehr Ressourcen freigesetzt werden. Die Freisetzung ist grösser als die zu zahlende Steuerlast, sodass mehr Ressourcen zur Kapitalinvestition verwendet werden können und der Kapitalstock steigt.

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# Taxing deficits to restrain government spending and foster capital accumulation<sup>1</sup>

## 1 Introduction

The political economy literature reveals that the competition for government funds between a variety of political agents, all representing and seeking maximum benefit for various lobby groups, yields a deficit bias. This can be seen as a common pool problem (see Velasco, 1999, 2000, von Hagen, 1992 or Harden and von Hagen, 1994). Even if all political agents have agreed in principle to adhere to a specific budget target, it appears very difficult to actually meet this target. Each agent has an incentive to secure funding of expenditure important for its clientele and to consider only the financing burden that accrues to its lobby group. The overall government budget constraint is not internalized. This confronts the various political agents with a prisoner's dilemma, since it is those who secure supplementary funding who find most favor with their lobby group or the voters, provided the others honor the prearranged deficit ceiling. As a result, the incentive for each individual agent to deviate from the plan is extremely high (see Roubini and Sachs, 1989, Patinkin, 1993 or Krogstrup and Wyplosz, 2006).<sup>2</sup>

Indeed, the “old” EU member states’ government debt ratio has multiplied in recent decades and has now reached a level in excess of the 60% reference value stipulated in the EU Treaty. Unless the EU member states take measures to counter this development, the upward trend can be expected to continue in the future. Allowing for the change in demography which places a burden on public expenditure, the European Commission (2006) calculated that EU

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<sup>2</sup>The underlying mechanism can be explained by fiscal illusion of economic subjects (i.e. while citizens fully appreciate the benefits of credit-financed spending and/or tax relief, the same cannot always be said for the associated financing burden), intergenerational income distribution or that an incumbent government may also be motivated to raise the debt level so as to restrict the new government's leeway (see Alesina Tabellini, 1990). Those issues are, however, not addressed in more detail within this paper.

member states' average debt ratio would increase to 100% by 2050 even if they meet the medium-term budgetary targets set in their stability and convergence programmes. If we take the 2005 structural primary deficits as a starting point, things get worse. In this case, the Commission is forecasting an average debt ratio of more than 180% by the end of the reference period. Even though public finances seem to be improving owing to a sound cyclical upturn at the moment, overall experience indicates that tackling the deficit bias is an important task to be accomplished within the near future. Many studies find that strengthening of budget institutions helps to reduce the deficit bias (see Harden and von Hagen, 1994 or Hallerberg and Wolff, 2006). The German Council of Economic Experts (2007) proposes a rule that forces governments to create additional revenue whenever their deficits exceed a certain threshold value. The Swiss Debt Brake already contains such features (see Müller, 2006). Similar rules can be found in a variety of countries (see Joumard and Kongsrud, 2003 or Sutherland et al., 2005 for an overview).

The main argument why such rules prevent excessive government spending is that, as the private sector dislikes higher taxation, the likelihood of the government being voted out of office may increase. This, however, is not the only mechanism that produces a spending restraint owing to deficit taxation. In this paper, we show how – even in the absence of political risk – additional government revenue bound to deficits can reduce the deficit bias.

We build our argument on a model with social polarization presented by Woo (2005, 2006). The economy is populated by two groups who may disagree on the ideal composition of government spending. The degree of social polarization indicates the magnitude of this disagreement and, hence, contributes to a greater overall spending and a larger deficit than without polarization. Each group is represented by a government official, say, a minister, who determines the provision of public goods for his group. In doing so, each minister exerts an externality on the other minister by fixing his preferred amount of government spending. The reason for the externality is that each official having access to a common government resource fails to internalize the full cost of his own appropriation. To put it crudely, government spending of a single official today may contribute to a deficit that translates into debt tomorrow. Because interest payments on tomorrow's debt apply, tomorrow's spending potential decreases for all government officials. Each government official is aware that whatever resources he does not exploit today may not be available for future

government spending, depending on the other's spending decision today. The fear of the utility loss owing to the reduced spending potential tomorrow makes each official spend disproportionately more on his favorite public good today.

The introduction of deficit taxation generates two effects. First, the spending incentive increases as deficits imply higher revenue and, thus, higher spending potential today. Second, today's deficit does not fully translate into tomorrow's debt whenever deficit taxation applies. Thus, the externality each official exerts on the other is reduced and the spending incentive decreases as deficit taxation affects the marginal costs each minister faces from over-exploiting the common budget. This implies that the perspective of higher future spending potential owing to deficit taxation reduces the incentive for excessive current spending resulting from the common pool problem. It can be shown that the latter effect dominates the first effect. Hence, even though, at first sight, revenue related to the amount of deficits increases the deficit bias, this is offset by the reduction of the externality owing to social polarization.

Further, as already shown by Woo (2005), the deficit bias owing to social polarization leads to inefficient capital accumulation in the private sector and, hence, permanently reduces the level of capital stock in the economy. The reason for this is that policymakers waste government resources to maximize their own utility and, thus, overspend beyond the level that results without social polarization for a given tax revenue. This overspending is financed by the private sector by giving up capital investments (and, instead, investing in government bonds). Empirically, this is backed by Fischer (1993) or Woo (2003a, 2003b). Deficit taxation does indeed, on the one hand, reduce the amount the private sector can invest into capital accumulation. On the other hand, it decreases the deficit bias as described above and, thus, releases resources that can be used for capital accumulation. Again, the latter effect dominates the former effect, because the reduction in the deficit bias compensates the increased tax payment. The higher deficit taxation is, the higher the reduction of the deficit bias. Therefore, the higher deficit taxation is, the higher capital accumulation is.

The rest of the paper is organized as follows. Section 2 describes the benchmark model following Woo (2005). In section 3, we introduce deficit taxation and describe its implications. Section 4 concludes. A mathematical appendix is added.

## 2 The benchmark model

Before introducing deficit taxation, we briefly review the model introduced by Woo (2005). We derive overall government spending chosen by the government, the corresponding deficit bias and the level of national debt for any given moment in time. We compare this to the situation without social polarization which proves to be the optimum when spending levels are set by a strong finance minister who is able to internalize the externality caused by non-cooperative determination of the spending levels. This is helpful for two reasons. First, we can describe the externality resulting from social polarization in more detail. Second, we can use this as a benchmark to see how deficit taxation changes the results. In section 3 below, we then introduce deficit taxation.

We consider an economy populated by two groups, indexed by  $i = 1, 2$ . These two groups may represent two powerful vested interest (ethnic) groups, right-wing and left-wing parties or anything else that may capture social polarization within the economy. Each group consists of a large number of atomistic individuals. Further, there is a government representing these groups. The government and the private sector have perfect foresight, agents are infinitely lived, there is no population growth and no uncertainty.

### 2.1 The private sector

The representative agent in group  $i$  seeks to maximize his lifetime utility

$$J^i = \int_0^{\infty} [\log(c_i) + \lambda_i \log(g_1) + (1 - \lambda_i) \log(g_2)] e^{-\rho t} dt, \quad (1)$$

where  $c_i$  is private consumption and  $g_1$  and  $g_2$  are two different public goods provided by the government. Agents discount with the personal discount factor  $\rho$ . Being small, each member of group  $i$  has the same preferences for the two public goods within the group. But the two groups differ in their preferences for the public goods, captured by  $\lambda_i$ . We assume  $0 \leq \lambda_i \leq 1$  for  $i = 1, 2$ . Further, we assume that  $\lambda_2 \leq \frac{1}{2} \leq \lambda_1$ . This means that group 1 prefers  $g_1$  to  $g_2$  (the opposite holds true for group 2). We define  $\theta = \lambda_1 - \lambda_2$  as the degree of differences in the groups' preferences for public goods and call this social polarization (note that  $0 \leq \theta \leq 1$ ). For  $\theta = 1$ , we have complete disagreement, while  $\theta = 0$  implies total agreement in the groups' preferences and, hence, no social polarization within the economy.

An agent can hold his wealth in the form of government bonds,  $b$ , and capital  $k$ . Bonds are assumed to be perfect substitutes for capital and pay the same rate of real interest,  $r$ . The flow budget constraint of group  $i$ 's agent is

$$\dot{a}_{it} = ra_{it} - c_{it} - \tau_{it}, \quad (2)$$

for all  $t \geq 0$  and  $a_0 > 0$ ,<sup>3</sup> where  $a_{it} = k_{it} + b_{it}$ .  $\tau_{it}$  is the lump-sum tax collected by the government. Imposing the No-Ponzi-Game condition yields

$$\lim_{t \rightarrow \infty} a_{it} e^{-rt} \geq 0. \quad (3)$$

As long as the marginal utility is positive, this condition holds with equality (see Barro and Sala-i-Martin, 1995).

Agent  $i$ 's utility maximization with respect to consumption,  $c_{it}$ , subject to equations (2) to (3) yields  $(1/c)\dot{c} = (r - \rho)$ . Hence, given an initial consumption level  $c_0$  (which remains to be determined later), the optimal consumption path for each agent is

$$c_{it} = c_0 e^{(r-\rho)t}. \quad (4)$$

## 2.2 The government and the non-cooperative solution

We now turn to the endogenous fiscal policy controlled by two ministers who jointly represent the fiscal authority. The two ministers, indexed by  $i = 1, 2$ , represent the corresponding group  $i = 1, 2$  and their preferences. Minister  $i$  provides the public good  $g_i$  to the private sector which is financed by the government's revenue. Each minister  $i$  derives greater utility from the provision of his favored public good  $g_i$  than from the other. Since they have different preferences for the public goods and seek to maximize their own utility, they behave strategically in determining the amount of public goods provided. Each minister has the following objective function

$$V^i = \int_0^{\infty} [\lambda_i \log(g_1) + (1 - \lambda_i) \log(g_2)] e^{-rt} dt, \quad (5)$$

where again, different preferences are captured by  $\lambda_i$  by analogy with section 2.1. We assume that ministers discount with the market interest rate  $r$  for analytical convenience. Woo (2005) has shown that, when the ministers' discount rate differs from the market interest rate (for example, representing the

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<sup>3</sup>Note that, in what follows, we mean  $\frac{dx}{dt} = \dot{x}$  is the change over time  $t$  for any variable  $x$ .

ministers' time horizon because of (re)election uncertainty), any discount rate greater than the market interest rate will additionally increase the deficit bias derived below. For simplicity, we abstract from this issue. The government provides the public goods  $\tilde{g} = g_1 + g_2$  at each instant in time and collects lump-sum taxes  $\tau$  from the private sector. Additionally, the expenditures can be financed by issuing bonds at a constant real rate  $r$ . The government's budget constraint at each instant in time, therefore, is

$$\dot{b} = rb + g_1 + g_2 - \tau, \quad (6)$$

where  $b$  is the stock of national debt and  $\tau = \tau_1 + \tau_2$ , where we assume that  $\tau_1 = \tau_2$ . Note that we omit the time index whenever there is no confusion. The No-Ponzi-Game condition relevant for the government is  $\lim_{t \rightarrow \infty} be^{-rt} = 0$ .

Each minister  $i$  chooses his control variable,  $g_i$ , to maximize his utility, equation (5), subject to the government budget constraint, equation (6), and the No-Ponzi-Game condition for every possible choice of the other minister's control variable  $g_j$ ,  $j \neq i$ . Following Woo (2005) and employing the non-cooperative feedback Nash equilibrium concept (which allows each player to revise actions as time evolves) yields

$$g_1^* = \lambda_1[\tau - rb] \quad (7)$$

and

$$g_2^* = (1 - \lambda_2)[\tau - rb] \quad (8)$$

as the optimal provision of the two public goods from each minister's point of view (see Appendix A for the derivation and a brief description of the game). The total government spending can then be stated as  $\tilde{g}^* = g_1^* + g_2^*$ ,

$$\tilde{g}^* = (1 + \theta)[\tau - rb]. \quad (9)$$

Substituting the overall government spending, equation (9), into the government's budget constraint, equation (6), gives

$$\dot{b} = \theta(\tau - rb) \geq 0, \quad (10)$$

which, solving the first-order differential equation for any randomly chosen point in time  $t = T$ , yields

$$b_T = (1 - e^{-r\theta T}) \frac{\tau}{r} + b_0 e^{-r\theta T} \quad (11)$$

as the level of national debt in  $T$ , where the initial level of government debt,  $b_0$ , is given. Equation (10) states that, whenever the ministers have different preferences about the provision of public goods,  $\theta > 0$ , there occurs an endogenous fiscal deficit,  $\dot{b} > 0$ , owing to the strategic behavior. This is due to the fact that even though the ministers' preferences concerning the provision of public goods differ, they share the same government budget. Each minister therefore has an incentive to overexploit the common resource in each period and insist on a higher spending for the favored public good which is exerting a negative externality on the other minister.

The negative externality of a one-unit provision of  $g_j$  on minister  $i$ 's utility through the state variable  $b$  always dominates the positive effect that directly enters minister  $i$ 's utility function of  $g_j$  for  $\theta > 0$ . This is because the utility gain from a higher provision of  $g_j$  today cannot compensate for the utility loss of the lower provision of  $g_i$  tomorrow (resulting from the increase in  $b$ ; see also equations (7) and (8)) whenever  $\lambda_1 \neq \lambda_2$ . The larger social polarization,  $\theta$ , is, the bigger the incentive is for each minister to overexploit the common government budget, because one unit of the common resource devoted to the opponent's favorite public good then generates a bigger negative externality, which induces each minister to spend even more on his preferred public good. Therefore, the size of the current deficit, equation (10), is a positive function of the degree of polarization,  $\theta$  (see also Woo, 2005, pp. 1462-1463). Similar mechanisms apply in the common pool problem discussed in Alesina and Perotti (1995), Hallerberg and von Hagen (1999), Persson and Tabellini (1999) or Velasco (1999).

**Proposition 1.** *Assuming  $b_0 = 0$  for simplicity of the argument, the level of national debt for any randomly chosen point in time,  $t = T$ , increases (i) with increasing tax revenue  $\tau$ , (ii) with increasing social polarization  $\theta$  and (iii) the further ahead  $T$  lies for a positive level of social polarization,  $\theta > 0$ .*

*Proof.* (i)  $\frac{db_T}{d\tau} = (1 - e^{-r\theta T}) \frac{1}{r} > 0$ , (ii)  $\frac{db_T}{d\theta} = \tau T e^{-r\theta T} > 0$ , (iii)  $\frac{db_T}{dT} = \tau \theta e^{-r\theta T} > 0$ .  $\square$

In the present model, it is not the level of tax income (which is exogenously fixed) that causes the fiscal deficit but social polarization. However, any higher level of tax revenue  $\tau$  increases the deficit and the level of national debt for any point in time  $T$ . Whenever  $\tau$  increases, minister 1 will claim  $\lambda_1 \times \Delta\tau$  and minister 2  $(1 - \lambda_2) \times \Delta\tau$ . This implies an increase in tax revenue increases government spending by  $(1 + \theta) \times \Delta\tau$ . It is plain to see that the higher social

polarization, i.e.  $\theta$ , is, the higher is the higher the over-exploitation of the government income. As time evolves (i.e.  $T$  lies further ahead), continuous deficits accumulate. However, we can also state the following.

**Proposition 2.** *The growth of debt is not explosive.*

*Proof.* From equation (11), it becomes obvious that  $\lim_{T \rightarrow \infty} b_T e^{-rT} = (1 - e^{-r\theta T}) \frac{\tau}{r} e^{-rT} = 0$  and the No-Ponzi-Game condition is satisfied.  $\square$

As  $T$  approaches infinity,  $b$  approaches  $\tau/r$ . The reasoning behind this lies in the construction of the game between the two ministers, which implies that each minister spends less as time evolves (increase in  $T$ ) in order to comply with the No-Ponzi-Game condition. From equations (7) and (8) we know that the spending on each public good depends on the state variable  $(\tau - rb)$  which decreases as time evolves because national debt,  $b$ , accumulates (see equation (11) as well as Appendix A). As a result, government spending shrinks asymptotically to zero for  $\theta > 0$  owing to the lump-sum taxation (for  $\theta = 0$ , the budget is always balanced, and total government spending equals the lump-sum tax in each period - because then,  $b = 0$  (for  $b_0 > 0$ ,  $b = b_0$ ) - as becomes obvious in equation (9)). Summing up, greater social polarization generates higher current fiscal spending and deficits which, in turn, forces policymakers to cut tomorrow's spending by more than they would have to do with lower polarization. Hence, it can furthermore be shown that the larger the degree of polarization is, the greater are the changes in fiscal outcomes over time (for a proof, see Woo, 2005).

Additionally, it is worth noting that the non-explosiveness of debt is not associated with Ricardian equivalence. Ricardian equivalence implies that the timing of taxation does not matter as long as the present value of net government surplus is equal to the value of initial government debt for an exogenously given government expenditure path. Here, taxes are exogenous while government spending is endogenous. Thus, national debt acts like net wealth for the private sector ("common" models ditching Ricardian equivalence - with a slightly different perspective, however - in which public debt is interpreted as wealth can, for instance, be found in the Fiscal Theory of the Price Level, initiated by Sims, 1994 or Woodford, 1994; nevertheless, there, taxes are still endogenous and spending is exogenous). In a Ricardian world, higher bond holdings mean higher future taxes and, thus, make bonds irrelevant for the economy and consumption. In our model, however, higher initial bond hold-



ings reduce the present value of future government spending and, thus, can be interpreted as an externality which was mentioned above.

### 2.3 The cooperative solution

A finance minister who is strong enough to be able to optimize government spending by maximizing both minister's aggregated welfare, has the following objective function

$$W^i = \int_0^{\infty} [(\lambda_1 + \lambda_2)\log(g_1) + (2 - \lambda_1 - \lambda_2)\log(g_2)]e^{-rt} dt \quad (12)$$

(which is simply the sum of both ministers' utility functions). The finance minister also faces the budget constraint, equation (6), and the No-Ponzi game condition  $\lim_{t \rightarrow \infty} be^{-rt} = 0$ . Again, applying the linear Markov strategies and following Appendix A, we find that

$$g_1^F = \frac{\lambda_1 + \lambda_2}{2}[\tau - rb] \quad (13)$$

and

$$g_2^F = \frac{(2 - \lambda_1 - \lambda_2)}{2}[\tau - rb] \quad (14)$$

which yields

$$\tilde{g}^S = [\tau - rb], \quad (15)$$

where the superscript  $F$  indicates the optimum from a finance minister's point of view. Substituting this into equation (6) yields  $\dot{b} = 0$ , i.e. the optimum from a finance minister's point of view implies a balanced budget at each instant in time.

Comparing the finance minister's optimal spending levels to those chosen by the two ministers in a non-cooperative game, we find that the optimum is only reached for  $\lambda_1 = \lambda_2 = 1/2$  under decentralized spending decisions. This implies that, when ministers determine government spending individually, the optimal solution from a finance minister's point of view can be achieved only for no social polarization (i.e.  $\theta = 0$ ). The larger social polarization is, the further away the ministers' choice is from the optimum. The finance minister's choice of the provision of public goods can be interpreted as a coordinated behavior of each minister, which implies that coordinated behavior dominates uncoordinated behavior because the externality one minister exerts on the other is internalized. This implies that, presuming that the finance minister is strong enough to determine the spending levels for each minister (as implied by

equation (12)), he can internalize the externality caused by social polarization. But it seems appropriate to assume that a finance minister does not seem to have the incentive or the power to enforce this optimum. Therefore, we have to think about rules which may help in the enforcement. One possible rule, namely deficit taxation, will be discussed in section 3.

## 2.4 Capital accumulation under social polarization

Substituting the government budgets constraint, equation (6), into the private sector budget constraint, equation (2), we find that

$$\dot{k} = rk + (1 + \theta)[rb - \tau] - 2c_0e^{(r-\rho)t}, \quad (16)$$

where  $c = c_1 + c_2 = 2c_0e^{(r-\rho)t}$  results from the aggregated consumption path of both groups (see equation (4)). Additionally,  $k = k_1 + k_2$ ,  $b = b_1 + b_2$  and  $\tau = \tau_1 + \tau_2$ . Then, solving the first-order differential equation (16), we obtain

$$k_t = \frac{\tau - rb_0}{r}e^{-r\theta t} + \frac{2c_0}{\rho}e^{(r-\rho)t} \quad (17)$$

as the equilibrium capital stock in period  $t$ . It is straightforward to show that, in the presence of social polarization,  $\theta > 0$ , the capital stock will always be below the capital stock without polarization.<sup>4</sup> For  $t = 0$ , we can determine the initial level of consumption,  $c_0$ , from

$$k_0 = \frac{\tau - rb_0}{r} + \frac{2c_0}{\rho}, \quad (18)$$

where the initial level of government debt,  $b_0$ , and the initial stock of capital,  $k_0$ , are predetermined by earlier periods. For more details and some further interesting aspects concerning volatility of deficits and capital accumulation or uncertainty, see Woo (2005).

Having strained the reader's patience long enough (at least of those who are acquainted with the model), we will now actually turn to deficit taxation.

## 3 Deficit taxation

Assume now that, in addition to the lump-sum tax  $\tau$ , the government raises further taxes depending on the current deficit,  $\phi_b \dot{b}$ , where  $\phi_b > 0$  is the deficit tax rate applied. We again assume that  $\phi_b \dot{b}$  is levied lump-sum from the private

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<sup>4</sup>Note that for  $\theta = 0$ , equation (17) becomes  $k_t = \frac{\tau - rb_0}{r} + \frac{2c_0}{\rho}e^{(r-\rho)t}$  which is greater.

sector, which basically leaves its decision unchanged (some minor changes have to be made, though). We can, therefore, revert to section 2.1 for the analysis of the private sector later on. As agents have perfect foresight and there is no uncertainty, the current deficit can be considered to be known by agents. We will now describe the effects of deficit taxation on the government first and, then, focus on capital accumulation, which includes the private sector behavior.

### 3.1 The government

We maintain each minister's utility function, equation (5). However, the government's budget constraint, equation (6), re-writes to

$$\dot{b} = rb + g_1 + g_2 - \tau - \phi_b \dot{b}, \quad (19)$$

In principle, utility maximization is analogous to section 2.2 and the non-cooperative feedback Nash equilibrium concept is adopted. However, we have to bear in mind that, now, the expenditure decision directly influences revenues as higher deficits imply more tax income. Following Appendix A, we again focus on the linear strategies  $g_{it} = \chi_i R_t$ , where  $R_t$  is the government's net revenue and  $\chi_i \in [0, 1]$  is the fraction of this revenue spent on the public good  $i$ .<sup>5</sup> Government net revenue can now be expressed as  $R_t = \tau + \phi_b \dot{b} - rb$ , which, making use of equation (19) and  $g_{it} = \chi_i R_t$ , yields

$$R_t = \frac{\tau - rb}{1 + \phi_b(1 - \chi_1 - \chi_2)}. \quad (20)$$

Defining  $\psi_t = \log R_t$ , we can express each minister's utility function as

$$V^i(\chi_1, \chi_2) = \int_0^\infty [\lambda_i \log(\chi_1) + (1 - \lambda_i) \log(\chi_2) + \psi_t] e^{-rt} dt, \quad (21)$$

and the budget constraint as

$$\dot{\psi}_t = \frac{r[1 - \chi_1 - \chi_2]}{[1 + \phi_b(1 - \chi_1 - \chi_2)]} \quad (22)$$

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<sup>5</sup>Note that, by the restriction  $\chi_i \in [0, 1]$ , we exclude  $\chi_i > 1$  which is, as will become obvious later, a possible mathematical solution. Nevertheless, regarding the effects of deficit taxation, this solution has, in principle, the same implications as the one presented beneath, though at a higher overall spending level. However, then, an increase of the level of social polarization may decrease the deficit bias – a counterintuitive result contradicting the findings derived in section 2.2. Furthermore, from an intuitive point of view, it seems reasonable that ministers do not choose a spending rule that exploits more than the total government resources available. More details can be sent upon request.

because  $\dot{\psi}_t = \frac{\dot{R}_t}{R_t} = -\frac{r}{1+\phi_b(1-\chi_1-\chi_2)}\frac{\dot{b}}{R_t}$  yields equation (22) after making use of equation (19) and  $g_{it} = \chi_i R_t$ . Each minister's maximization of the utility function (21) with respect to his choice variable,  $\chi_i$ , subject to the budget constraint, equation (22), yields

$$\frac{\lambda_1}{\chi_1} = \frac{1}{[1 + \phi_b(1 - \chi_1 - \chi_2)]^2} \quad (23)$$

as minister 1's reaction function and

$$\frac{(1 - \lambda_2)}{\chi_2} = \frac{1}{[1 + \phi_b(1 - \chi_1 - \chi_2)]^2} \quad (24)$$

as minister 2's reaction function (see Appendix A for the derivation). The equations determine how each minister sets its fraction  $\chi_i$  depending on the other minister's choice  $\chi_j$ ,  $i \neq j$  and  $i, j = 1, 2$ . Equations (23) and (24) imply  $\chi_1 = \frac{\lambda_1}{(1-\lambda_2)}\chi_2$ . Substitution and bearing in mind that  $\lambda_1 - \lambda_2 = \theta$  and  $\chi_i \in [0, 1]$  yields

$$\chi_1^* = \frac{\lambda_1 \left[ 1 + 2(1 + \theta)\phi_b(1 + \phi_b) - \sqrt{1 + 4(1 + \theta)\phi_b(1 + \phi_b)} \right]}{2(1 + \theta)^2\phi_b^2} \quad (25)$$

and

$$\chi_2^* = \frac{(1 - \lambda_2) \left[ 1 + 2(1 + \theta)\phi_b(1 + \phi_b) - \sqrt{1 + 4(1 + \theta)\phi_b(1 + \phi_b)} \right]}{2(1 + \theta)^2\phi_b^2} \quad (26)$$

as the fractions of the revenue,  $R_t$ , spent on each public good  $i$ . Substitution in equation (20) yields

$$R_t = (\tau - rb) \frac{2(1 + \theta)\phi_b}{\sqrt{1 + 4(1 + \theta)\phi_b(1 + \phi_b)} - 1}. \quad (27)$$

Using  $g_{it} = \chi_i R_t$  and  $\tilde{g}^* = g_1^* + g_2^*$ , and substituting  $R_t$ , we can express total government expenditure as

$$\tilde{g}^* = F(\tau - rb), \quad (28)$$

where

$$F = \frac{\left[ 1 + 2(1 + \theta)\phi_b(1 + \phi_b) - \sqrt{1 + 4(1 + \theta)\phi_b(1 + \phi_b)} \right]}{\phi_b \left[ \sqrt{1 + 4(1 + \theta)\phi_b(1 + \phi_b)} - 1 \right]}. \quad (29)$$

Substituting equation (28) into equation (19) yields

$$\dot{b} = D(\tau - rb) \quad (30)$$

which, solving the first-order differential equation for any randomly chosen point in time  $t = T$ , yields

$$b_T = (1 - e^{-rDT}) \frac{\tau}{r} + b_0 e^{-rDT}, \quad (31)$$

where

$$D = [F - 1] = \frac{\sqrt{1 + 4(1 + \theta)\phi_b(1 + \phi_b)} - (1 + 2\phi_b)}{2\phi_b}. \quad (32)$$

Note that  $\theta > D > 0$  for  $\theta > 0$ , which implies  $(1 + \theta) > F > 1$  for  $\theta > 0$ .<sup>6</sup>

**Proposition 3.** *Again, assuming  $b_0 = 0$  for simplicity of the argument, the level of national debt for any randomly chosen point in time,  $t = T$ , increases (i) with increasing tax revenue  $\tau$ , (ii) with increasing social polarization  $\theta$  and (iii) the further ahead  $T$  lies for a positive level of social polarization,  $\theta > 0$ . (iv) The growth rate of debt is not explosive.*

*Proof.* From equations (31) and (32), we see that (i)  $\frac{db_T}{d\tau} = (1 - e^{-rDT}) \frac{1}{r} > 0$ , (ii)  $\frac{db_T}{d\theta} = \tau T \frac{\partial D}{\partial \theta} e^{-rDT} > 0$ , where  $\frac{\partial D}{\partial \theta} = \frac{(1 + \phi_b)}{\sqrt{1 + 4(1 + \theta)\phi_b(1 + \phi_b)}} > 0$ , (iii)  $\frac{db_T}{dT} = \tau D e^{-rDT} > 0$ , and (iv)  $\lim_{T \rightarrow \infty} b_T e^{-rT} = (1 - e^{-rDT}) \frac{\tau}{r} e^{-rT} = 0$ . It is furthermore a straightforward matter to show that  $D = 0$  and  $F = 1$  for  $\theta = 0$  and, thus, there is no deficit bias for no social polarization.  $\square$

This implies that the equilibrium with deficit taxation has the same qualitative properties as the equilibrium derived in section 2. Hence, the interpretation is analogous. However, in the presence of deficit taxation, we find by comparing equations (10) and (30) that the deficit bias is reduced as  $D < \theta$ . The reason for this is that the deficit taxation generates a spending reluctance as becomes obvious by comparing the rhs of equation (9) – the overall government spending in absence of deficit taxation – and the rhs of equation (28) – the overall government spending with deficit taxation, where  $F < (1 + \theta)$ . Intuitively, the fact that additional government revenue generates a spending reluctance certainly seems odd at first sight, especially as we have learned in section 2.2 that additional tax revenue increases the level of national debt at any randomly chosen moment in time. It is even more surprising because the higher the deficit is, the higher is the additional revenue for a given parameter  $\phi_b$ . Therefore, this issue certainly warrants further explanation.

The reason for the deficit bias derived in section 2 is that, whenever a minister restricts himself to not spending the desired amount today, he might

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<sup>6</sup>It is straightforward to show that  $D > 0$  as  $\sqrt{1 + 4(1 + \theta)\phi_b(1 + \phi_b)} > (1 + 2\phi_b)$ . However, as  $\sqrt{1 + 4(1 + \theta)\phi_b(1 + \phi_b)} < [1 + 2\phi_b(1 + \theta)]$ ,  $D < \theta$ .

not be able to spend these “savings” tomorrow, because the other minister may take advantage of this. Whenever the other minister does so (or spends even more), a deficit occurs, which implies a higher level of debt tomorrow. This actually reduces tomorrow’s spending potential for the minister who has refrained from spending today as higher interest payments apply. Hence, ministers are confronted with a prisoner’s dilemma. To see the argument more clearly, assume the most extreme case of disagreement,  $\lambda_1 = 1$  and  $\lambda_2 = 0$  (however, any other constellation that yields  $\theta > 0$  implies the same outcome). Then, whenever minister 1 does not spend the desired amount on public good  $g_1$  today (because he may want to save this for tomorrow), minister 2 may use this to spend more on  $g_2$ . However, the provision of  $g_2$  today creates no utility for minister 1. Still, the potentially higher spending of minister 2 may increase the level of national debt tomorrow. This reduces the amount minister 1 can spend on  $g_1$  tomorrow. Therefore, his savings may be eaten up by his opponent. In order to prevent this, minister 1 spends as much as is optimal from his perspective today, only partly bearing in mind future development of government spending. For  $\theta > 0$ , this is more than would be optimal from a finance minister’s point of view who internalizes the externality. Hence, minister 1 exerts an externality on minister 2 by reducing his future spending potential. The same holds true for minister 2.

Now, in the presence of deficit taxation, a certain part  $\phi_b$  of the deficit generated by the spending behavior of both ministers is refinanced through additional tax income. This implies that only a fraction  $\frac{1}{(1+\phi_b)} < 1$  of the deficit  $\hat{b}$  translates into tomorrow’s level of national debt. Therefore, the externality exerted by one minister on the other is reduced as the cut in tomorrow’s spending potential is diminished. Hence, each minister’s incentive to spend more today (because he fears lower spending potential tomorrow owing to the other minister’s behavior) decreases. This is captured by the spending restraint described in more detail by equation (28).

We can rephrase the more or less intuitive statement just made in a more technical manner as follows. We find that, through the existence of deficit taxation, each minister can increase government net revenue and, hence, the spending potential in the actual period  $t$  by increasing spending for his favorite good (captured by an increase of  $\chi_i$ ), which becomes easily obvious by differentiating equation (20) with respect to  $\chi_i$ . This augments each minister’s incentive to spend more on his favorite good. We term this “income effect”.

We also know, however, that government net revenue evolves over time according to equation (22). It is easy to see that an increase of the fraction  $\chi_i$  implies a reduction in tomorrow's spending potential by  $\frac{r}{[1+\phi_b(1-\chi_1-\chi_2)]^2}$  which, discounted to the actual period, reflects each minister's marginal cost displayed by the rhs of equations (23) and (24), respectively. Basically, in the absence of deficit taxation, a similar mechanism applies, where an increase in  $\chi_i$  reduces tomorrow's spending potential by  $r$  (see Appendix A). In the presence of deficit taxation, however, the magnitude of the reduction in tomorrow's spending potential is directly influenced by the choice of  $\chi_i$ . This is because deficit taxation now generates additional government revenue (as indicated by what we have termed income effect) which changes the evolution of tomorrow's spending potential. Therefore, in the presence of deficit taxation, marginal costs themselves depend directly on the choice of  $\chi_i$ .

In optimum, each minister chooses  $\chi_i$  such that marginal utility and marginal costs are equalized. In the absence of deficit taxation, this implies that marginal utility must equal one which yields  $\chi_1^{SP} = \lambda_1$  and  $\chi_2^{SP} = (1 - \lambda_2)$  in Figure 1. (See also section 2.2 and Appendix A for the derivation). In the presence of deficit taxation, however, marginal costs, the rhs of equations (25) and (26), increase with an increase of  $\chi_i$  as long as  $[1 + \phi_b(1 - \chi_1 - \chi_2)] > 0$  which will always hold true for  $(\chi_1 + \chi_2) \leq (\chi_1^* + \chi_2^*)$ . This is because, then, additional government revenue induced by deficit taxation does not exceed the resulting government spending. Therefore, whenever minister  $i$  augments spending for his favorite public good, he also increases marginal costs. Given that the evolution of marginal utility is the same for the situations with and without deficit taxation, higher marginal costs imply lower government spending. We term this “substitution effect” (even though, this is not precisely how this term is “commonly” used). In addition, minister  $j$ 's choice of his optimal fraction now also directly raises minister  $i$ 's marginal costs through an analogous channel. Therefore, the opponent's fraction  $\chi_j$  chosen further increases the marginal costs of minister  $i$  which gradually reduces each minister's optimal choice of  $\chi_i$ . These effects are depicted by an inward shift of the reaction functions (now truly depending on the other's choice) in Figure 1.

The equilibrium without deficit taxation is given by point  $A$  in Figure 1. The new equilibrium in the presence of deficit taxation is given by point  $B$ . The fractions of the revenue spent by each minister are smaller in the presence of deficit taxation, which can easily be approved formally by comparing equations

(25) and (26) with the solutions of section 2.2,  $\chi_1 = \lambda_1$  and  $\chi_2 = (1 - \lambda_2)$ , respectively. Hence, in total, we find that the substitution effect dominates the income effect because the aggregated fractions of the revenue spent on each public good decrease by more than the government net revenue increases. This becomes obvious as  $F < (1 + \theta)$ . Hence, deficit taxation generates a spending reluctance which reduces the deficit bias.

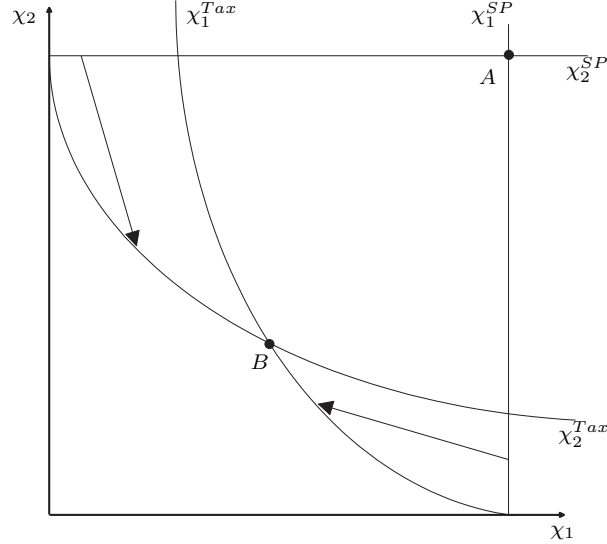


Figure 1: Reaction Functions

**Proposition 4.** (i) *The higher deficit taxation is, the lower is the deficit bias.* (ii) *The lower the deficit taxation is, the closer the deficit bias will be to the decentralized situation.* (iii) *In the extreme, deficit taxation does not fully prevent the deficit bias.*

*Proof.* (i) From equation (32) we obtain  $\frac{dD}{d\phi_b} = \frac{\sqrt{1+4(1+\theta)\phi_b(1+\phi_b)^2}-[1+2(1+\theta)\phi_b]}{2\phi_b^2\sqrt{1+4(1+\theta)\phi_b(1+\phi_b)^2}} < 0$ . Thus, from equation (31), we see unambiguously that the deficit bias is reduced whenever  $\phi_b$  is increased. (ii) Furthermore, we see from equation (32) that  $\lim_{\phi_b \rightarrow 0} D = \theta$ . (iii)  $\lim_{\phi_b \rightarrow \infty} D = \sqrt{(1+\theta)} > 1$ .  $\square$

We have shown in this section that, whenever governments must generate large enough additional revenue related to their deficit, social polarization is partly internalized. This yields a spending reluctance that reduces the deficit bias. However, deficit taxation cannot fully internalize the deficit bias because there is still the incentive to augment today's revenue.<sup>7</sup> As the deficit tax must

<sup>7</sup>Whenever the additional deficit-related revenues occur exogenously, one could also consider this as a solution for foreign aid, i.e. subsidizing developing countries' deficits rather than giving debt relief – a prominent theme in recent public debate, as pointed out by Krüger



be collected within the economy, this has a potential feedback on the private sector and, thus, capital accumulation, which we will analyze in the following section.

### 3.2 Capital accumulation with deficit taxation

As pointed out earlier, the utility function of the private sector stays the same as in section 2. Hence, as deficit taxes are levied lump-sum, the optimal consumption decision is unchanged. However, the aggregated budget constraint changes to

$$\dot{k} + \dot{b} = rk + rb - c - \tau - \phi_b \dot{b}, \quad (33)$$

which, substituting equation (19) and  $c = 2c_0 e^{(r-\rho)t}$ , yields

$$\dot{k} = rk - \tilde{g}^* - 2c_0 e^{(r-\rho)t}, \quad (34)$$

where  $\tilde{g}^*$  is given by equation (28). Solving this first-order differential equation, bearing in mind equation (31), yields

$$k_t = \underbrace{\frac{F}{(1+D)}}_{=1} \frac{\tau - rb_0}{r} e^{-rDt} + \frac{2c_0}{\rho} e^{(r-\rho)t}. \quad (35)$$

As  $e^{-rDt} > e^{-r\theta t}$  (because  $D < \theta$ ), we see directly that the capital stock in the presence of social polarization and deficit taxation is always larger than in the absence of deficit taxation (see also equation (17)).

**Proposition 5.** *The higher deficit taxation is, the lower is the (positive) gap to the capital stock without polarization.*

*Proof.* The optimal capital stock without social polarization can be calculated with the help of equation (17) for  $\theta = 0$ . This yields  $k_t = \frac{\tau - rb_0}{r} + \frac{2c_0}{\rho} e^{(r-\rho)t}$ . We know that  $\lim_{\phi_b \rightarrow 0} D = \theta$  and  $D < \theta$ ; see equation (32). This implies that the capital stock under deficit taxation approaches  $k_t = \frac{\tau - rb_0}{r} e^{-r\theta t} + \frac{2c_0}{\rho} e^{(r-\rho)t}$

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and Morath (2007), for example. Social polarization is considered to be one of the driving forces behind developing countries' unsound fiscal situation (see Fearon, 2003, Woo 2003a, 2003b, Annett, 2001 or Alesina et al., 1999, 2003, to mention just a few studies confirming this claim). Under such a premiss, deficit subsidization may indeed dominate debt relief. To clarify the argument, let us assume that a developing country is well depicted by the model described above. Then, debt relief would not really tackle the problem of the deficit bias. After the debt relief has taken place, the country would again start accumulating debt and create deficits, having permanently lower stocks of capital than is optimal (see section 2.4). Using the amount considered by the developed world for the debt relief to subsidize the developing countries' current deficits, however, can help to improve the fiscal situation and, as we will see below, foster capital accumulation.

as the deficit tax rate,  $\phi_b$  approaches zero, which is the capital stock of social polarization without deficit taxation (see equation (17)). For positive deficit taxation,  $\phi_b$ , the capital stock is larger because  $e^{-rDt} > e^{-r\theta t}$  (owing to  $D < \theta$ ). As  $\frac{dD}{d\phi_b} < 0$  (see the proof of Proposition 4), the capital stock increases with increasing deficit taxation.  $\square$

The reason for an increase in capital accumulation is that the reduced deficit bias releases resources that can be used for capital investment by the private sector. Indeed, additional taxation reduces the resources available. However, this reduction is overcompensated by the diminished need to finance government expenditures. Hence, in the presence of social polarization, the stock of capital is increased under deficit taxation.

## 4 Summary

In this paper, we have shown that, in the presence of social polarization, which captures disagreement on how government resources should be allocated, deficit taxation generates a spending reluctance and, thus, reduces the deficit bias and fosters capital accumulation.

Social polarization generates a deficit bias because policymakers are caught in a prisoner's dilemma. Whenever they cut their spending today in order to spend it tomorrow, they do not know if these "savings", depending on their opponents' behavior, will be available tomorrow. Hence, they overexploit the common resource and spend too much. This generates an externality on the others as today's deficit translates into tomorrow's debt, lessening tomorrow's overall spending potential. Deficit taxation implies that only a fraction of today's deficit will become tomorrow's debt as part of the deficit must be collected as additional revenue. Hence, the externality is diminished. This reduces the fear that "savings" for tomorrow's spending will disappear and induces a spending restraint on each policymaker. In total, the deficit bias decreases. Note that the effect described is not connected to the political risk of being voted out of office. Here, the reduction of the deficit bias is completely driven by diminishing the externality.

Additionally, a reduced deficit bias gives room for more capital accumulation because deficits have to be financed by the private sector by giving up capital investment. As deficits are decreased, resources are released that can be used for capital accumulation. The cut in resources available for capital

accumulation due to the additional tax collection falls short of the release due to a lower deficit bias, which yields an overall increase of the capital stock.

## A Mathematical Appendix

### The non-cooperative feedback Nash equilibrium

The calculation perfectly follows Woo (2006, pp. 18-19). To facilitate the computation of equilibrium, we define the government's net revenue as

$$R_t = \tau - rb. \quad (36)$$

In order to be able to find a closed-form solution, we will only focus our attention on linear Markov strategies depending on the current state. Considering linear strategies is common in differential game literature as, otherwise, one may not be able to find a closed-form solution. Thus, we conduct a transformation of the variables such that a game is constructed in which an open-loop strategy generates the same rate of public good provision as the feedback strategy at every point in time. This is called *synthesizing the feedback control* (see Fudenberg and Tirole, 1992 for a more detailed description). The linear strategies we will focus on are

$$g_{it} = \chi_i R_t, \quad (37)$$

where  $\chi_i$  is the endogenously determined fraction of the revenues spent on public good  $g_i$  in time  $t$ . We assume  $\chi_i \in [0, \infty)$ . Defining  $\psi_t = \log(R_t)$ , each minister's objective function, equation (5), is given by

$$V^i(\chi_1, \chi_2) = \int_0^\infty [\lambda_i \log(\chi_1) + (1 - \lambda_i) \log(\chi_2) + \psi_t] e^{-rt} dt, \quad (38)$$

and the budget constraint rewrites as

$$\dot{\psi}_t = r - r\chi_1 - r\chi_2. \quad (39)$$

This gives minister 1's Hamiltonian

$$H^1(\chi_1, \chi_2, \psi_t) = [\lambda_1 \log(\chi_1) + (1 - \lambda_1) \log(\chi_2) + \psi_t] e^{-rt} + \mu_{1t} [r - r\chi_1 - r\chi_2]. \quad (40)$$

Minister 2's Hamiltonian is given by

$$H^2(\chi_1, \chi_2, \psi_t) = [\lambda_2 \log(\chi_1) + (1 - \lambda_2) \log(\chi_2) + \psi_t] e^{-rt} + \mu_{2t} [r - r\chi_1 - r\chi_2]. \quad (41)$$

The corresponding first-order conditions are

$$H_{\chi_1}^1 = \frac{\lambda_1}{\chi_1} e^{-rt} = r\mu_1, \quad (42)$$

$$H_{\psi}^1 = e^{-rt} = -\dot{\mu}_1 \quad (43)$$

for minister 1 and

$$H_{\chi_1}^2 = \frac{(1 - \lambda_2)}{\chi_2} e^{-rt} = r\mu_2, \quad (44)$$

$$H_{\psi}^2 = e^{-rt} = -\dot{\mu}_2 \quad (45)$$

for minister 2. The transversality condition yields  $\lim_{t \rightarrow \infty} \mu_i(t) = 0$ , with  $i = 1, 2$ , for both ministers. Using this, we find from equations (43) and (45) that  $\mu_i(t) = e^{-rt}/r$ . Substitution into equations (42) and (44) yields  $\chi_1^* = \lambda_1$  and  $\chi_2^* = (1 - \lambda_2)$ . Substituting this into equation (37) and using equation (36) yields equations (7) and (8).

## Public good provision under deficit taxation

The maximization of equation (21) subject to the constraint (22) yields the Hamiltonians

$$\begin{aligned} H^1(\chi_1, \chi_2, \psi_t) &= [\lambda_1 \log(\chi_1) + (1 - \lambda_1) \log(\chi_2) + \psi_t] e^{-rt} \\ &+ \mu_{1t} \left[ \frac{r}{1 + \phi_b} \left\{ 1 - \frac{\chi_1 + \chi_2}{1 + \phi_b(1 - \chi_1 - \chi_2)} \right\} \right] \end{aligned} \quad (46)$$

for minister 1 and

$$\begin{aligned} H^2(\chi_1, \chi_2, \psi_t) &= [\lambda_2 \log(\chi_1) + (1 - \lambda_2) \log(\chi_2) + \psi_t] e^{-rt} \\ &+ \mu_{2t} \left[ \frac{r}{1 + \phi_b} \left\{ 1 - \frac{\chi_1 + \chi_2}{1 + \phi_b(1 - \chi_1 - \chi_2)} \right\} \right] \end{aligned} \quad (47)$$

for minister 2. The corresponding first-order conditions are

$$H_{\chi_1}^1 = \frac{\lambda_1}{\chi_1} e^{-rt} = \frac{r}{[1 + \phi_b(1 - \chi_1 - \chi_2)]^2} \mu_1, \quad (48)$$

$$H_{\psi}^1 = e^{-rt} = -\dot{\mu}_1 \quad (49)$$

and

$$H_{\chi_1}^2 = \frac{(1 - \lambda_2)}{\chi_2} e^{-rt} = \frac{r}{[1 + \phi_b(1 - \chi_1 - \chi_2)]^2} \mu_2, \quad (50)$$

$$H_{\psi}^2 = e^{-rt} = -\dot{\mu}_2. \quad (51)$$

Applying the transversality condition,  $\lim_{t \rightarrow \infty} \mu_i(t) = 0$ , with  $i = 1, 2$ , again for both ministers yields  $\mu_i(t) = e^{-rt}/r$  (see equations (49) and (51)) which, substituted into equations (48) and (50), yields each minister's reaction function, equations (23) and (24).

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