Income Redistribution under Sincere Lobbying Formation

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This paper develops a new model of endogenous lobbying formation to study the impact of lobbying on income redistribution policy. Individuals are assumed to behave sincerely while deciding on whether to take part in lobbying, modeled as menu-auction. We show that the equilibrium structure depends on the policymaker's identity, that is, less egalitarian is a policymaker, more extreme income levels have individuals joining lobbying activities. Moreover, no lobbies are formed in case of the policymaker's favoring just one of the special interest groups. We analyze then a proportional representation example to conclude that minority groups profit with the introduction of sincere lobbying technology while majority group has enough political power to affect policy outcome through other democratic rights such as voting and enrolling in political parties.

JEL classification: D72, H24.

Key Words: Sincere lobbying formation; Endogenous lobbying; Menu-auction; Income redistribution.

1. INTRODUCTION

Lobbies play an important role in the policymaking process. There is a huge variety of special interest groups, that organize about economic, occupational, ideological or social issues.[3] To what extent lobbying affects policy outcomes has been the researchers main concern for decades. The literature has addressed this question in the context of menu-auction model, analyzed by Bernheim and Whinston (1986) and applied to lobbying by Grossman and Helpman (1994). In this approach lobbying is modeled as "menu-auction", where lobbies confront a policymaker with contribution schedules that maps any possible policy into a contribution payment. Several authors have adapted the menu-auction model of lobbying to study trade policy, commodity taxation, provision of local public goods and other policies (see Dixit, Grossman and Helpman (1997), Grossman and Helpman (1996), Helpman and Persson (2001), Persson (1998)). In their turn, Besley and Coate (2001) have integrated the menu-auction model of lobbying with the citizen-candidate model of representative democracy to show that "on the positive side lobbying need have little or no effect on policy outcomes because voters can restrict the influence of lobbyists by supporting candidates with offsetting policy preferences".

However, this rich literature lacks an important aspect that can shed some light on all the other concerns about lobbying, namely, how and by whom lobbies are

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[3]See Introduction in Grossman and Helpman (2001) for the detailed overview.

organized. To our knowledge, just few papers make an attempt to endogenize lobbies formation. Mitra (1999) analyzes endogenous formation of lobbies within Grossman and Helpman (1994) framework for trade policy, introducing a first stage in which exogenously given special interest groups decide whether to form corresponding lobbies with fixed and sunk costs. Analogous approach is used by Laussel (2006), who adds a first stage of fixed cost endogenous lobbying formation in Persson (1998) model of local public goods provision. In particular, his results indicate that "the larger groups become organized in equilibrium while the smaller ones remain unorganized". In their turn, Felli and Merlo (2006), (2007) interpret endogenous lobbying in quite different terms. They assume that "given the set of existing lobbies, the elected candidate chooses the coalition of lobbies he will bargain with over policy in exchange for transfers". However, the literature focuses mainly on the formation of lobbies from exogenously given special interest groups, but not on the agents' choice to enter a lobby. Damania and Fredriksson (2000), (2003) addressed this question in some way, analyzing incentives for two polluting firms to organize into a single industry lobby to affect the government's decision on pollution tax policy. They have shown, in particular, that industries with higher collusive profits as well as more polluting industries have a greater incentive to form a lobby group. In his turn, Magee (2002) studies an analogous problem faced by n identical firms to form an industry lobby to bargain with a policymaker over tariff levels. As far as we know, there is no more work dealing with agent's choice to participate in special interest politics. This paper makes an attempt to fill the gap.

We develop a new model of special interest politics analyzing individual's decision to join a lobby, representing her interests. Individuals are assumed to be heterogeneous in income, where low-income individuals and high-income ones will be referred as workers and entrepreneurs, respectively. In the economy there exists a possibility of income redistribution through the proportional tax levied on every individual with the tax revenues redistributed as a lump-sum subsidy. The incumbent policymaker cares about the individuals welfare and her own well-being. Moreover, the policymaker may favor one of the interest groups more than another one. The political decision the individuals are to take is whether to participate in special interest politics namely to join a lobby, where we introduce a new concept of sincere lobbying formation. To be more specific, we analyze a lobby formation stage where each individual can join a lobby corresponding to her interests, that is workers can enter a labor union and entrepreneurs can join an entrepreneurs association. Once formed, lobbies confront the policymaker with contribution schedules as in menu-auction model of lobbying. Entering a lobby each individual faces a trade-off: affecting a tax rate through special interest politics and paying contribution versus enjoying the outcome of lobbying activities of other individuals without contribution paying. We assume that individuals behave sincerely, that is, they join a lobby if they are strictly better off from the lobby activities and they abstain if they prefer the tax rate implemented by the policymaker, when there does not exist any lobby corresponding to their interests. We call the above concept sincere lobbying formation.

Why individuals will behave sincerely deciding to enter a lobby? One possible explanation could be that individuals simply enjoy to participate in special interest politics, unless they cannot afford it or rationally expect that lobbying would not change a policy outcome. In the light of this, individuals may gain some personal satisfaction from showing allegiance to their special interest group. Another possible answer captures the idea of a social norm individual behavior, that is, individuals

take part in lobbying activities (unless it is too costly), because it is a social norm of the society.[4] Moreover, a norm of this kind could be enforced at the level of the interest group, so lobbying becomes a special interest group norm.

To explain the paradox of large lobbying groups, like "the union laborers, the farmers, and the doctors", Olson (1971) developed a theory of "by-product" lobby, where only an organization, performing some other function except lobbying, has enough capacity to mobilize its members to participate in special interest politics. "The lobby is then a by-product of whatever function this organization performs..."[5]. In fact, workers and entrepreneurs groups can be defined as organizations that perform some other non-political functions except lobbying, thus Olson's theory can serve as one more reasoning for sincere lobbying formation concept we introduce here.

The analysis yields a number of results that can be summarized as follows.

First, no lobbies will be formed when the society is ruled by a policymaker that favors too much one of the special interest groups. The policymakers with such "extreme" preferences demand so high contribution for a policy change in favor of the other group, that the latter simply cannot afford it and, therefore, is deterred from organizing into a lobby by the rational expectation that is would not change a final policy.

If a policymaker has more egalitarian preferences, she is ready to change a policy for reasonable contribution payments, therefore one lobby will be formed depending on the policymaker's identity: when the policymaker favors more a group of workers, an entrepreneurs association is organized, while a labor union is formed under the policymaker favoring more a group of entrepreneurs. Moreover, less egalitarian is a policymaker, higher contribution she is demanding for a policy change, so more extreme income levels have individuals who are able to afford this contribution, since they profit a lot with this policy change. The model predicts the formation of large lobbies that goes in line with results of Laussel (2006) and Olson (1971) who has attributed large pressure groups existence to the "by-product" nature of these groups.

Finally, the model predicts utility waste due to non-optimal policy choice, but no welfare waste due to rent-seeking, since contribution donations are just transfers from lobbies to a policymaker.

Our framework can be applied to study an important question of why individuals prefer lobbying to other ways of affecting policy outcomes (e.g. voting, enrolling in political parties, running for elections, etc.), that is a question of why lobbying exists in the representative democracy world. To be more specific, we develop a simple proportional representation example to show that minority groups profit with the introduction of sincere lobbying technology while majority group favors other ways (like voting and enrolling in political party) to affect policy outcomes. This result is due to the nature of policymaking process where majority has enough political power to get its preferable policy outcome. In this case lobbying is an opportunity for minority groups to change final outcome in their favor.

The remainder of the paper is organized as follows. Section 2 lays out the model combining the economy, the polity and the lobbying description. Section 3 develops the characterization of political equilibria and Section 4 provides welfare analysis. Section 5 outlines the proportional representation example. Finally,

[4] Similar explanations have been suggested to resolve the voting paradox. See Chapter on Turnout in Grossman and Helpman (2001) for the overview of the topic.

[5] Mancur Olson (1971), pp. 132-133.

Section 6 concludes.

2. THE MODEL

2.1. The Economy

The economy is inhabited with a large number of individuals that differ in their income x. The individuals number is normalized to one and a generic individual's income x is assumed to be described by a smooth, at least thrice differentiable cumulative distribution function $\mathcal{F}(\cdot)$ with mean $\widehat{x}$ and support $[0,\infty)$. The corresponding density function $f(\cdot)$ is assumed to be strictly quasi-concave. Assume further that x is skewed to the right, since incomes are necessarily skewed to the right. Moreover, $\mathcal{F}(\cdot)$ is assumed to satisfy the monotone hazard rate condition, that is $\frac{d}{dx}\left(\frac{f(x)}{1-\mathcal{F}(x)}\right) \geq 0$.[6]

The proportional tax $t \in [0,1]$ is levied on every individual and the tax revenue is redistributed equally among the individuals as a lump-sum subsidy $s(t)$, therefore

$$s(t) = \int_0^\infty s(t) f(x)\, dx = \int_0^\infty xtf(x)\, dx = \widehat{x}t.$$

Then the individual utility is given by

$$u(x,t) = x(1-t) + \widehat{x}t.$$

The tax rate t_x preferred by the individual with income x is simply

$$t_x = \begin{cases} 1 & \text{if} \quad x < \widehat{x} \\ 0 & \text{if} \quad x \geq \widehat{x}. \end{cases}$$

Let us refer to the group of low-income individuals with $x < \widehat{x}$ as workers and to the group of high-income individuals with $x \geq \widehat{x}$ as entrepreneurs.[7]

2.2. The Polity

In the economy there exists a possibility of income redistribution through the proportional taxation. The society is ruled by a policymaker, who cares about the individuals welfare and her own welfare.[8] To be more specific, we assume that the policymaker may favor one of the special interest groups, that is her utility is given

[6] This assumption is essentially technical and holds for a variety of distributions (e.g. uniform, normal, logistic, chi-squared, exponential, Laplace, and, under some restrictions on the parameters, Weibull, gamma, or beta). See Bagnoli and Bergstrom (1989) for further details.

[7] This occupational division is introduced to reflect the tendency observed in the reality: workers are paid moderate wages while entrepreneurs gain high profits; workers fight for income redistribution while entrepreneurs prefer minimum taxation.

[8] The analysis of a policymaker's coming to power is beyond the scope of the paper.

by

$$\begin{aligned} U(t)+m &= \lambda\int_0^{\widehat{x}} u(x,t)\,f(x)\,dx+(1-\lambda)\int_{\widehat{x}}^{\infty} u(x,t)\,f(x)\,dx+m \\ &= \lambda\left(\mathcal{F}(\widehat{x})\,E[x|x\leq\widehat{x}]+t\mathcal{F}(\widehat{x})\left(\widehat{x}-E[x|x\leq\widehat{x}]\right)\right)+ \\ &\quad (1-\lambda)\left(\widehat{x}-\mathcal{F}(\widehat{x})\,E[x|x\leq\widehat{x}]-t\mathcal{F}(\widehat{x})\left(\widehat{x}-E[x|x\leq\widehat{x}]\right)\right)+m \\ &= (2\lambda-1)\,\mathcal{F}(\widehat{x})\,E[x|x\leq\widehat{x}]+(1-\lambda)\,\widehat{x}+ \\ &\quad t(2\lambda-1)\,\mathcal{F}(\widehat{x})\left(\widehat{x}-E[x|x\leq\widehat{x}]\right)+m, \end{aligned}$$

where $\lambda\in[0,1]$ stands for the weight given to workers welfare and m defines the policymaker's private welfare.[9,10] If $\lambda=\frac{1}{2}$, the policymakers is benevolent. In the extreme cases $\lambda=0$ and $\lambda=1$, the policymaker is completely pro-entrepreneurs or pro-workers, respectively. Introducing such preferences of the policymaker allows us to analyze different political regimes that can occur as a result of the policymaker's coming to power (e.g. election) without concentrating on the process itself. The policymaker will implement a tax rate maximizing her utility. If policymaker has no private welfare ($m=0$), then the final tax rate is

$$t^*=\begin{cases} 1 & \text{if} \quad \lambda>\frac{1}{2} \\ 0 & \text{if} \quad \lambda\leq\frac{1}{2}. \end{cases}$$

If a policymaker favors an entrepreneurs group $\left(\lambda\leq\frac{1}{2}\right)$, there is no income redistribution that is a tax regime preferred by entrepreneurs.[11] In case of a pro-workers policymaker $\left(\lambda>\frac{1}{2}\right)$, the full taxation regime is implemented.

2.3. The Lobbying

The policymaker is ready to impose a tax rate maximizing her utility. However, special interest politics are feasible. Formally, each individual can join a lobby corresponding to her interests, that is, workers can enter a labor union W and entrepreneurs can join an entrepreneurs association E. We introduce a new concept of *sincere lobbying formation*. Once formed, a lobby maximizes the joint net-of-contribution welfare of its members and confronts the policymaker with a contribution schedule that maps any possible tax rate into a contribution payment. The policymaker implements the tax rate maximizing her utility considering donations from the lobbies if there are any. We proceed backwards to analyze sincere lobbying formation and contribution game stages.

Lobbies Contribution Game. Proceeding backwards we assume that a labor union W and an entrepreneurs association E have been formed. The lobbies objective is to maximize the joint welfare of their members net of contribution payments in case there are any. Let us denote by $U^W(t)$ and $U^E(t)$ the joint welfare of the

[9] We assume that the policymaker is indifferent about the source of her income.

[10] The conditional expectation of x given that $x\leq\widehat{x}$ is $E[x|x\leq\widehat{x}]=\frac{\int_0^{\widehat{x}}xf(x)dx}{\mathcal{F}(\widehat{x})}$.

[11] In case of a benevolent policymaker $\left(\lambda=\frac{1}{2}\right)$ the individuals joint welfare is not affected by a tax rate, that is $t^*\in[0,1]$. Here we assume the laissez-faire outcome $t^*=0$.

union W and the association E, respectively. Then they are simply given by

$$\begin{aligned} U^W(t) &= \int\limits_{x\in W} u(x,t)\, f(x)\, dx \\ U^E(t) &= \int\limits_{x\in E} u(x,t)\, f(x)\, dx. \end{aligned}$$

A lobby develops a contribution schedule $C(t)$ that maps any tax rate the policymaker could impose into a payment the lobby is willing to contribute. There are a lot of ways to elaborate feasible $C(t)$, but according to the standard approach we will narrow the wide set of possible contribution schedules a lobby can use in the equilibrium. To be more specific, let us concentrate on truthful (or compensating) contribution schedules.[12] According to Bernheim and Whinston (1986) it is not costly for a lobby to design this kind of schedule, while equilibria of the game with lobbies using truthful contribution schedules happen to be jointly efficient for the policymaker and the lobbies and coalition-proof.[13]

More formally, lobby i $(i = W, E)$ sets a contribution schedule of the following form

$$C^i(t) = \max\left[U^i(t) - r^i, 0\right],$$

where r^i is chosen optimally by lobby i.

Finally, let us describe the problem faced by the policymaker. She is willing to maximize her utility, taking into account the contribution payments from the lobbies, that is

$$U(t) + C^W(t) + C^E(t).$$

DEFINITION 1. A vector of feasible truthful contribution schedules $\left(C^W(t), C^E(t)\right)$ and a tax rate t^{WE} constitute a *contribution game equilibrium* if and only if

1. t^{WE} is in the policymaker's best response set to $\left(C^W(t), C^E(t)\right)$, that is $t^{WE} = \arg\max\left\{U(t) + C^W(t) + C^E(t)\right\}$;

2. for every lobby i $(i = W, E)$, $C^i(t)$ and t^{WE} are in the set of best responses to $C^{-i}(t)$, that is

   (i) t^{WE} belongs to the policymaker's best response set to $\left(C^W(t), C^E(t)\right)$;

   (ii) there do not exist a contribution amount $\widehat{c}^i \geq 0$ and a tax rate $\widehat{t}^i$ such that $U^i\left(\widehat{t}^i\right) - \widehat{c}^i > U^i\left(t^{WE}\right) - C^i\left(t^{WE}\right)$ and $U\left(\widehat{t}^i\right) + \widehat{c}^i + C^{-i}\left(\widehat{t}^i\right) \geq U\left(t^{-i}\right) + C^{-i}\left(t^{-i}\right)$, where $t^{-i} = \arg\max\left\{U(t) + C^{-i}(t)\right\}$.[14]

Let us examine the contribution game in detail. Choosing optimal constants r^i lobby i $(i = W, E)$ has to take into account that in equilibrium the policymaker must weakly prefer to enjoy lobby i's contribution, that is at least get the same

[12] Grossman and Helpman (2001) define a truthful contribution schedule to be the one that coincides with a lobby's indifference curve, where contributions are positive, and returns zero otherwise (p. 232).

[13] Coalition-proof equilibrium occurs only if there is no coalition of the players that prefer to engage in private and costless communication before the play of a game in order to suggest the strategies to pursue. For further details see Bernheim and Whinston (1986).

[14] For further details see Grossman and Helpman (2001), pp. 250-252.

utility as in the case of only lobby $-i$ contributing. Formally, for the policymaker to consider lobby i's offer the following should hold:

$$U\left(t^{WE}\right)+\left[U^{i}\left(t^{WE}\right)-r^{i}\right]+\left[U^{-i}\left(t^{WE}\right)-r^{-i}\right]=U\left(t^{-i}\right)+\left[U^{-i}\left(t^{-i}\right)-r^{-i}\right].$$

Therefore the truthful contribution schedules are given by

$$\begin{aligned} C^{W}(t) &= \max\left[U^{W}(t)-\left[\begin{array}{l} U^{W}\left(t^{WE}\right)-\left(U\left(t^{E}\right)-U\left(t^{WE}\right)\right)- \\ \left(U^{E}\left(t^{E}\right)-U^{E}\left(t^{WE}\right)\right) \end{array}\right],0\right] \\ C^{E}(t) &= \max\left[U^{E}(t)-\left[\begin{array}{l} U^{E}\left(t^{WE}\right)-\left(U\left(t^{W}\right)-U\left(t^{WE}\right)\right)- \\ \left(U^{W}\left(t^{W}\right)-U^{W}\left(t^{WE}\right)\right) \end{array}\right],0\right]. \end{aligned}$$

Sincere Lobbying Formation. Each individual decides either to join the corresponding lobby and to pay contribution or to abstain. If an individual enters the lobby, her welfare is taken into account when the lobby develops a contribution schedule, but she should bear a contribution burden which is the same for all the lobby members. So the individuals have incentives to free-ride, that is to leave lobbying activities and contribution payments for others. To rule out the free-riding problem we assume that individuals behave sincerely, that is they join the lobby in case they strictly prefer the outcomes of the lobby activities and they abstain if they are not worse off without the corresponding lobby. We call this new concept *sincere lobbying formation*.

Formally, let us denote by $\varphi_x \in \{0,1\}$ a pure strategy of a generic individual with income x, where $\varphi_x = 1$ denotes entry to the corresponding lobby, and by φ the profile of entry decisions. Then given the entry profile φ, $W(\varphi)$ and $E(\varphi)$ are the sets of members of the labor union and the entrepreneurs association, respectively.

DEFINITION 2. Given the truthful contribution schedules $\left(C^{W(\cdot)}(t), C^{E(\cdot)}(t)\right)$ and the tax rate $t^{WE}(\cdot)$, a *sincere lobbying formation equilibrium* is a pure strategy profile φ^* such that

1. for each individual x, φ_x^* is a "sincere" best response to φ_{-x}^*, that is

$$\varphi_x^* \in \arg\max \left\{ \begin{array}{l} \varphi_x \left[u\left(x, t^{WE}\left(\varphi_x, \varphi_{-x}^*\right)\right) - \dfrac{C^{W\left(\varphi_x,\varphi_{-x}^*\right)}\left(t^{WE}\left(\varphi_x,\varphi_{-x}^*\right)\right)}{\int\limits_{\varkappa \in W\left(\varphi_x,\varphi_{-x}^*\right)} d\varkappa} \right] + \\ (1-\varphi_x)\, u\left(x, t^{E}\left(\varphi_x, \varphi_{-x}^*\right)\right) \end{array} \right\} \quad \text{if} \quad x < \widehat{x}$$

$$\varphi_x^* \in \arg\max \left\{ \begin{array}{l} \varphi_x \left[u\left(x, t^{WE}\left(\varphi_x, \varphi_{-x}^*\right)\right) - \dfrac{C^{E\left(\varphi_x,\varphi_{-x}^*\right)}\left(t^{WE}\left(\varphi_x,\varphi_{-x}^*\right)\right)}{\int\limits_{\varkappa \in E\left(\varphi_x,\varphi_{-x}^*\right)} d\varkappa} \right] + \\ (1-\varphi_x)\, u\left(x, t^{W}\left(\varphi_x, \varphi_{-x}^*\right)\right) \end{array} \right\} \quad \text{if} \quad x \geq \widehat{x}.$$

2. for each individual x, φ_x^* is not a weakly dominated strategy.

Thus the timing of the game is as follows. In the sincere lobbying formation stage each individual behaves sincerely to decide on joining a lobby corresponding

to her interests. Then the lobbies (if there are any) develop truthful contribution schedules to confront the policymaker with. At the final stage the policymaker imposes the tax rate to maximize her utility considering donations from the lobbies if there are any. For the equilibrium concept we use subgame perfect Nash.

Combining the analysis of all the stages, we define a *political equilibrium* as follows.

DEFINITION 3. A vector of lobbying entry decisions φ^*, a vector of truthful contribution schedules $\left(C^W(t), C^E(t)\right)$ and a tax rate t^{WE} constitute a *political equilibrium* if and only if

1. φ^* is a sincere lobbying formation equilibrium given $\left(C^W(t), C^E(t)\right)$ and t^{WE};

2. for all nonempty lobbies W and E, $\left(C^W(t), C^E(t)\right)$ and t^{WE} constitute a contribution game equilibrium.

In what follows we analyze political equilibria and provide normative analysis for different political scenarios depending on λ.

3. CHARACTERIZATION OF POLITICAL EQUILIBRIA

When lobbying is feasible the policymaker's private welfare includes the lobbies contribution payments if there are any, that is $m = C^W(t) + C^E(t)$. Let us remind that depending on λ, the policymaker prefers either no taxation or maximum taxation. As for the special interest groups, all entrepreneurs agree on no taxation, while workers prefer maximum taxation. We can then establish:[15]

LEMMA 1. *There exist worker with income $\xi \in [0, \widehat{x})$ and entrepreneur with income $\eta \in [\widehat{x}, \infty)$ such that*

1. *(i) worker ξ is indifferent in the "sincere" sense between joining a labor union and not forming any union at all, and*

   (ii) each worker $x < \xi$ strictly prefers to join a labor union and each worker $x > \xi$ is better off when there is no union;

2. *(i) entrepreneur η is "sincerely" indifferent between entering an entrepreneurs association and not organizing any association, and*

   (ii) each entrepreneur $x > \eta$ is strictly better off to join an association, while each entrepreneur $x < \eta$ prefers no lobbying activities.

According to the lemma, a labor union $W = \{x \mid x \in [0, \xi)\}$ with $\xi \in [0, \widehat{x})$ and an entrepreneurs association $E = \{x \mid x \in (\eta, \infty)\}$ with $\eta \in [\widehat{x}, \infty)$ can be formed.

[15] The formal proofs of this and all the following results are in the Appendix.

In this case the lobbies joint welfare becomes

$$U^W(t) = \int_0^{\xi} u(x,t) f(x)\, dx = \mathcal{F}(\xi) E[x|x \le \xi] + t\mathcal{F}(\xi)(\widehat{x} - E[x|x \le \xi])$$

$$U^E(t) = \int_{\eta}^{\infty} u(x,t) f(x)\, dx = (1-\mathcal{F}(\eta)) E[x|x > \eta] + t(1-\mathcal{F}(\eta))(\widehat{x} - E[x|x > \eta]).$$

The lobbies develop the truthful contribution schedules $C^i(t) = \max\left[U^i(t) - r^i, 0\right]$ $(i = W, E)$ to propose to the policymaker, who imposes the final tax rate depending on what contributions she is offered:

$$t^{WE} = \begin{cases} 1 & \text{if} \quad \lambda > \lambda^{WE} \equiv \frac{1}{2} - \frac{\mathcal{F}(\xi)(\widehat{x} - E[x|x \le \xi]) + (1-\mathcal{F}(\eta))(\widehat{x} - E[x|x > \eta])}{2\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \le \widehat{x}])} \\ 0 & \text{if} \quad \lambda \le \lambda^{WE}; \end{cases}$$

$$t^{W} = \begin{cases} 1 & \text{if} \quad \lambda > \lambda^{W} \equiv \frac{1}{2} - \frac{\mathcal{F}(\xi)(\widehat{x} - E[x|x \le \xi])}{2\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \le \widehat{x}])} \\ 0 & \text{if} \quad \lambda \le \lambda^{W}; \end{cases}$$

$$t^{E} = \begin{cases} 1 & \text{if} \quad \lambda > \lambda^{E} \equiv \frac{1}{2} - \frac{(1-\mathcal{F}(\eta))(\widehat{x} - E[x|x > \eta])}{2\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \le \widehat{x}])} \\ 0 & \text{if} \quad \lambda \le \lambda^{E}. \end{cases}$$

Note that a pro-entrepreneurs policymaker with sufficiently low λ is not affected by lobbying activities and always implements null tax rate. By analogy, a pro-workers policymaker with sufficiently high λ always imposes maximum taxation independently on lobbying contribution payments. The reason for this is just that they so badly want to implement their preferable tax rate that no lobby could propose enough contribution for them to give up. As a result only policymakers with more egalitarian preferences are influenced by lobbying activities. The following proposition describes political equilibria.

PROPOSITION 1. *There exists a political equilibrium such that*

1. *if* $\lambda \in [0, \frac{1}{2} - \frac{(\widehat{x}-\underline{\xi})\mathcal{F}(\underline{\xi})}{2\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \le \widehat{x}])})$, *where* $\underline{\xi}$ *satisfies* $\frac{\mathcal{F}(\underline{\xi})}{f(\underline{\xi})} = \widehat{x} - \underline{\xi}$, *there are no lobbies formed, and the final tax rate is* $t^* = 0$;

2. *if* $\lambda \in \left[\frac{1}{2} - \frac{(\widehat{x}-\underline{\xi})\mathcal{F}(\underline{\xi})}{2\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \le \widehat{x}])}, \frac{1}{2}\right]$, *a labor union*

$$W = \{x \mid x \in [0, \xi^*)\}$$

is formed, where ξ^* *is such that*

$$(\widehat{x} - \xi^*)\mathcal{F}(\xi^*) = (1-2\lambda)\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \le \widehat{x}]) \;\; \text{and} \;\; \xi^* > \underline{\xi},$$

and the final tax rate is $t^* = 1$;[16]

[16] Note, that $\frac{d\xi^*}{d\lambda} \ge 0$.

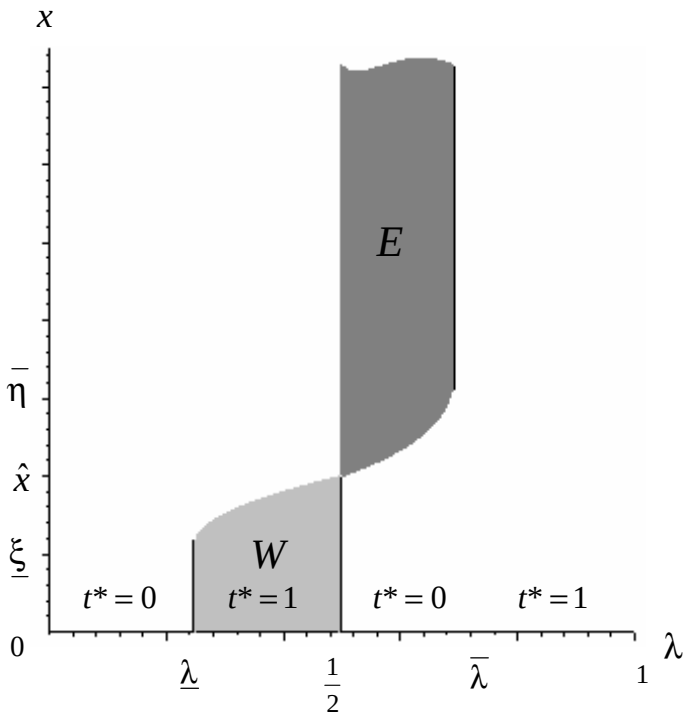

FIG. 1 Political equilibria.

3. *if* $\lambda \in (\frac{1}{2}, \frac{1}{2} + \frac{(\overline{\eta}-\widehat{x})(1-\mathcal{F}(\overline{\eta}))}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])}]$, *where* $\overline{\eta}$ *satisfies* $\frac{1-\mathcal{F}(\overline{\eta})}{f(\overline{\eta})} = \overline{\eta} - \widehat{x}$, *an entrepreneurs association*

$$E = \{x \mid x \in (\eta^*, \infty)\}$$

is formed, where η^* *is such that*

$$(\eta^* - \widehat{x})(1 - \mathcal{F}(\eta^*)) = (2\lambda - 1)\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \leq \widehat{x}]) \text{ and } \eta^* < \overline{\eta},$$

and the final tax rate is $t^* = 0$;[17]

4. *if* $\lambda \in (\frac{1}{2} + \frac{(\overline{\eta}-\widehat{x})(1-\mathcal{F}(\overline{\eta}))}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])}, 1]$, *there are no lobbies formed, and the final tax rate is* $t^* = 1$.

Figure 1 illustrates the proposition above.[18] As we mentioned before a policymaker with extreme preferences (pro-workers or pro-entrepreneurs policymaker)

[17] Note that $\frac{d\eta^*}{d\lambda} > 0$ and $\frac{d^2\eta^*}{d\lambda^2} > 0$.

[18] Figure 1 is made for Chi squared distribution which is a special case of the gamma distribution with *pdf* $f(x|p) = \frac{1}{\Gamma(\frac{p}{2})2^{\frac{p}{2}}} x^{\frac{p}{2}-1} e^{-\frac{x}{2}}$, $0 \leq x < \infty$, $p = 1, 2, ...$, $EX = p$, $VarX = 2p$, where $\Gamma(\alpha) = \int_0^\infty t^{\alpha-1} e^{-t} dt$. Here $p = 4$. Then $f(x) = \frac{1}{4} e^{-\frac{x}{2}} x$ and $\mathcal{F}(x) = 1 - \frac{1}{2} e^{-\frac{x}{2}} (2 + x)$, and it is skewed to the right, satisfies the monotone hazard rate condition and the den-

needs so much contribution to change her preferred tax policy that no lobby is willing to provide this payment since it is too costly. Therefore for low or high λ there is no lobby formed. As for the policymakers with more egalitarian preferences, they are less demanding and ready to give up their preferred tax rate for satisfactory contribution payments that lobbies could manage. Note that threshold functions $\xi^*(\lambda)$ and $\eta^*(\lambda)$ are increasing, that is more extreme are the policymaker's preferences, the individuals with more extreme income levels are organized into lobbies. This is due to the fact that once a lobby has been organized, the tax outcome is the same, while contribution payment depends on the policymaker's identity. More pro-workers is a policymaker, higher contribution she is demanding to implement null taxation, so higher income levels have entrepreneurs who are willing to provide this contribution since they profit a lot with a tax change and are ready to pay high donation for this change. By analogy, more pro-entrepreneurs is a policymaker, higher contribution she wants to impose maximum taxation, so poorer are workers who are ready to pay this contribution because they gain a lot with change to income redistribution regime. Finally, at some point there will be no individuals willing to pay too high contribution for a change in tax policy. Still, if formed, lobbies will comprise a sufficiently high part of the groups population, since individuals behave sincerely deciding whether to participate in special interest politics.

Our model does not predict equilibria with both lobbies organized due to the policy nature. The policymaker can favor only one group with a tax change, while another group is necessarily loosing, since the groups have opposite views on the policy issue. One can think of real situations, where a potential lobby could be deterred from organizing by the rational expectation that it would not change a political outcome.

4. NORMATIVE ANALYSIS

Lobbying is recognized as one of the rent-seeking activities, where the resources are wasted to influence political outcomes. However, in our model lobbying contributions are just transfers from lobbies to a policymaker. Let us denote by $\Omega(t)$ the joint welfare of all individuals:

$$\Omega(t) = \int_0^\infty u(x,t) f(x)\, dx = \widehat{x},$$

that does not depend on a tax rate that serves as an income redistribution instrument. Then the social welfare, defined as the sum of the individuals joint net-of-contribution welfare and the policymaker's welfare m (equal to lobbies contributions if there are any), is simply the same as $\Omega(t) = \widehat{x}$. So there is no welfare waste with lobbying activities either due to rent-seeking or due to non optimal taxation regime.

However, there is an utility loss, since a policymaker values both her own welfare m and the individuals welfare. With lobbying activities a policymaker is gaining in

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sity is strictly quasi-concave. In Figure 1 $\overline{\eta} = 3 + \sqrt{13}$, $\underline{\xi}$ satisfies $\frac{1-\frac{1}{2}e^{-\frac{\underline{\xi}}{2}}(2+\underline{\xi})}{\frac{1}{4}e^{-\frac{\underline{\xi}}{2}}\underline{\xi}} = 4 - \underline{\xi}$, $\overline{\lambda} = \frac{1}{8}\left(4 + \left(2+\sqrt{13}\right)e^{\frac{1-\sqrt{13}}{2}}\right)$ and $\underline{\lambda} = \frac{1}{32}\left(16 - e^2\left(4-\underline{\xi}\right)\left(2-e^{-\frac{\underline{\xi}}{2}}\left(2+\underline{\xi}\right)\right)\right)$.

her private welfare, receiving contribution payments, but loosing in her "ideological" utility due to non optimal choice of tax rate. At the end a policymaker is totally reimbursed and receives the same utility both with and without lobbying activities. As for the individuals, their joint welfare does not change with a tax rate, but there is a loss due to contribution payments.

To be more specific, let us analyze the joint utility losses due to lobbying activities.[19] As we know, the individuals joint welfare without lobbying is given by $\Omega(t) = \widehat{x}$. Let us denote by $\Omega^W(t)$ and by $\Omega^E(t)$ the joint welfare of workers and entrepreneurs, respectively. Then

$$\begin{aligned}
\Omega^W(t) &= \int_0^{\widehat{x}} u(x,t) f(x)\, dx = \mathcal{F}(\widehat{x})\, E[x|x \le \widehat{x}] + t\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \le \widehat{x}]) \\
\Omega^E(t) &= \int_{\widehat{x}}^{\infty} u(x,t) f(x)\, dx = \widehat{x} - \mathcal{F}(\widehat{x})\, E[x|x \le \widehat{x}] - t\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \le \widehat{x}]).
\end{aligned}$$

If there is no lobbying, they become

$$\begin{aligned}
\Omega^W(t) &= \begin{cases} \widehat{x}\mathcal{F}(\widehat{x}) & \text{if} \quad \lambda > \frac{1}{2} \\ E[x|x \le \widehat{x}]\,\mathcal{F}(\widehat{x}) & \text{if} \quad \lambda \le \frac{1}{2}; \end{cases} \\
\Omega^E(t) &= \begin{cases} \widehat{x}\,(1 - \mathcal{F}(\widehat{x})) & \text{if} \quad \lambda > \frac{1}{2} \\ \widehat{x} - E[x|x \le \widehat{x}]\,\mathcal{F}(\widehat{x}) & \text{if} \quad \lambda \le \frac{1}{2}. \end{cases}
\end{aligned}$$

For $\lambda \in \left[\frac{1}{2} - \frac{(\widehat{x}-\underline{\xi})\mathcal{F}(\underline{\xi})}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\le\widehat{x}])}, \frac{1}{2}\right]$ a labor union $W = \{x \mid x \in [0, \xi^*)\}$ is formed and the final tax rate is $t^* = 1$. The labor union compensates the policymaker with a tax change, paying contribution $C^W(t^*) = (1-2\lambda)\,\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \le \widehat{x}])$. Therefore, the difference in the joint utility with lobbying intervention is given by:

$$\begin{aligned}
\triangle\Omega &= \Omega(1) - C^W(1) - \Omega(0) = -(1-2\lambda)\,\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \le \widehat{x}]) \le 0, \\
\triangle\Omega^W &= \Omega^W(1) - C^W(1) - \Omega^W(0) = 2\lambda\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \le \widehat{x}]) > 0, \\
\triangle\Omega^E &= \Omega^E(1) - \Omega^E(0) = -\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \le \widehat{x}]) < 0.
\end{aligned}$$

Note, that the society as a whole is loosing with lobbying due to non-optimal tax rate. The labor union represents the workers interests, so workers are better off with the labor union activities. As for the entrepreneurs, they are obviously loosing with the labor union formation.

If $\lambda \in (\frac{1}{2}, \frac{1}{2} + \frac{(\overline{\eta}-\widehat{x})(1-\mathcal{F}(\overline{\eta}))}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\le\widehat{x}])}]$, an entrepreneurs association $E = \{x \mid x \in (\eta, \infty)\}$ is formed and the final tax rate is $t^* = 0$. The entrepreneurs association provides a policymaker with contribution $C^E(t^*) = (2\lambda - 1)\,\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \le \widehat{x}])$, so the utility change becomes:

$$\begin{aligned}
\triangle\Omega &= \Omega(0) - C^E(0) - \Omega(1) = -(2\lambda - 1)\,\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \le \widehat{x}]) < 0, \\
\triangle\Omega^W &= \Omega^W(0) - \Omega^W(1) = -\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \le \widehat{x}]) < 0, \\
\triangle\Omega^E &= \Omega^E(0) - C^E(0) - \Omega^E(1) = 2(1-\lambda)\,\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \le \widehat{x}]) > 0.
\end{aligned}$$

[19] We do not take into account the policymaker's utility, since she gets the same utility with and without lobbying activities.

As before, the joint utility decreases with lobbying due to non-optimal tax level. Entrepreneurs are strictly better off with lobbying, since they prefer no income redistribution. The labor joint welfare has dropped with entrepreneurs lobbying.

Our model does not predict welfare waste from lobbying due to rent-seeking, but utility waste due to non-optimal tax choice, since contribution payments are just transfers from lobbies to a policymaker. Still these transfers may create "rents from holding office", that can be analyzed with introducing an election stage.[20] However, our primary goal is to build an analytically tractable model of lobbying formation, not concentrating on election procedures.

5. WHY LOBBYING?

Lobbying is a necessary part of the policymaking process nowadays. Citizens have a lot of ways to affect policy outcomes in the representative democracy world through voting, entering political parties or running for elections. And still it seems to be not enough since lobbying exists. This section develops a simple proportional representation example in the framework of sincere lobbying formation to shed some light on this question.

Let us consider an economy described in Section 2. There are two political parties PW and PE, that represent the interests of workers and entrepreneurs, respectively. It is not costly to enter a political party, so we assume that all workers belong to party PW and all entrepreneurs belong to party PE. Each party maximizes the joint welfare of its members, therefore the party objective functions $U^{PW}(t)$ and $U^{PE}(t)$ are given by

$$\begin{aligned} U^{PW}(t) &= \int_0^{\widehat{x}} u(x,t) f(x)\, dx = \mathcal{F}(\widehat{x})\, E[x|x \le \widehat{x}] + t\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \le \widehat{x}]) \\ U^{PE}(t) &= \int_{\widehat{x}}^{\infty} u(x,t) f(x)\, dx = \widehat{x} - \mathcal{F}(\widehat{x})\, E[x|x \le \widehat{x}] - t\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \le \widehat{x}]). \end{aligned}$$

The society operates through strict proportional representation, that is the number of seats in the legislature each party is awarded with, is proportional to its share in the aggregate vote. We assume that voting is not costly, and each citizen ballots her vote. Moreover, we restrict citizens to vote for a party they belong to. Therefore, all workers vote for party PW, while all entrepreneurs vote for party PE. Thus, the sizes of party PW and PE delegation in the legislature are $\mathcal{F}(\widehat{x})$ and $1 - \mathcal{F}(\widehat{x})$, respectively.

Once in the legislature, the parties bargain over policy outcome in the following way: they maximize the weighted sum of their utilities with the weights proportional to the sizes of each party's delegation. So the bargaining rule objective is the following one:

$$\mathcal{F}(\widehat{x})\, U^{PW}(t) + (1 - \mathcal{F}(\widehat{x}))\, U^{PE}(t).$$

Note that this example goes in line with the benchmark model of Section 2: the size $\mathcal{F}(\widehat{x})$ of party PW delegation in the legislature corresponds to the weight λ the policymaker gives to workers welfare. In the case lobbying is feasible, pressure

[20] The complete analysis of exogenous lobbying in citizen-candidate framework can be found in Besley and Coate (2001).

groups offer contribution schedules to the legislature that maximizes the sum of its objective function and contributions. Therefore, we can apply the results of the previous sections here. Taking into account that $\mathcal{F}(\widehat{x}) > \frac{1}{2}$ (since incomes are skewed to the right), we can establish:

LEMMA 2. *Under strict proportional representation*

1. *if sincere lobbying is not feasible, the final tax rate is* $t^* = 1$*;*

2. *if sincere lobbying is feasible and*

   (*i*) *if* $\mathcal{F}(\widehat{x}) \leq \frac{1}{2} + \frac{(\overline{\eta}-\widehat{x})(1-\mathcal{F}(\overline{\eta}))}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])}$*, where* $\overline{\eta}$ *satisfies* $\frac{1-\mathcal{F}(\overline{\eta})}{f(\overline{\eta})} = \overline{\eta} - \widehat{x}$*, an entrepreneurs association*

$$E = \{x \mid x \in (\eta^*, \infty)\}$$

   is formed, where η^* *is such that*

$$(\eta^* - \widehat{x})(1 - \mathcal{F}(\eta^*)) = (2\lambda - 1)\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \leq \widehat{x}]) \text{ and } \eta^* < \overline{\eta},$$

   and the final tax rate is $t^* = 0$*;*

   (*ii*) *if* $\mathcal{F}(\widehat{x}) > \frac{1}{2} + \frac{(\overline{\eta}-\widehat{x})(1-\mathcal{F}(\overline{\eta}))}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])}$*, there are no lobbies formed, and the final tax rate is* $t^* = 1$*.*

There exist always distributions such that $\mathcal{F}(\widehat{x}) \leq \frac{1}{2} + \frac{(\overline{\eta}-\widehat{x})(1-\mathcal{F}(\overline{\eta}))}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])}$.[21] Therefore, there are economies where lobbying technology gives a "second chance" to certain special interest groups to affect a policy outcome. In fact, entrepreneurs cannot get their favorable outcome through voting and enrolling in political party, while lobbying gives them opportunity to change a tax rate in their favor. Note, that the key assumption that leads to this result is the skewness of the distribution to the right. If the distribution were skewed to the left, we would come to the opposite result: workers would profit with the introduction of lobbying technology. In general, one can conclude that minority groups have more chances to affect policy outcomes through lobbying activities, while majority has enough political power to get its favorable outcome through other democratic processes due to its nature (of majority).

6. CONCLUSION

This paper has developed a new theory of endogenous lobbying formation, namely, sincere lobbying formation, where individuals are assumed to behave sincerely making decision on whether to participate in lobbying, modeled as menu-auction. The model predicts the formation of large lobbies, that is due to the nature of sincere lobbying formation concept. Moreover, lobbies structure depends on the policymaker's identity in the following way: less egalitarian is a policymaker, more extreme income levels have individuals joining lobbying activities. In case a policymaker favors just one of the interest groups, no lobbies are formed, since it becomes too costly. On the normative side, the model predicts no welfare waste

[21] Consider distributions that are not "too much" skewed to the right, so $\mathcal{F}(\widehat{x}) - \frac{1}{2}$ is not sufficiently high.

due to rent-seeking. Still, lobbies pressure results in a non-optimal policy choice, that leads to utility losses.

We study then a proportional representation example in the sincere lobbying formation framework to show that minority groups profit with the introduction of lobbying technology while majority group can get its preferable policy outcomes through other democratic rights such as voting and enrolling in political parties.

The sincere lobbying formation concept introduced in this paper can be readily applied to study the impact of endogenous lobbying on election outcomes, where lobbies provide potential policymakers with campaign contributions that enhance their election chances. Moreover, the model can be easily extended to analyze the process of the policymaker's coming to power either in the current framework or in the citizen-candidate approach, pioneered by Besley and Coate (1997) and Osborne and Slivinski (1996). Finally, our framework can be used to study endogenous lobbying formation, when lobbying activities involve the dissemination of information.[22]

Appendix

Proof of Lemma 1. Each worker $x \in [0, \widehat{x})$ rationally expects that without labor union the policymaker $\lambda \in [0,1]$ confronted by an entrepreneurs association E will implement some tax rate $t^E \in [0,1]$. All workers prefer maximum taxation regime, therefore, in case a labor union is organized, workers predict that a final tax rate $t^{WE} \in [0,1]$ will go up that is $t^{WE} \geq t^E$. Workers with very low income gain a lot with income redistribution, so they can afford to pay quite high contributions to the policymaker for a tax increase, therefore, low-income workers in general prefer lobbying activities (unless contributions become too high). As for workers with high income (close to $\widehat{x}$), they cannot pay even low contributions since their gain from a tax change is a tiny one, so they are better off without any lobbying activities. Then there exists a worker with income $\xi \in [0, \widehat{x})$ which is indifferent between joining a union and not having any union activities at all, that is

$$u\left(\xi, t^{WE}\right) - \overline{C}^W = u\left(\xi, t^E\right),$$

where $\overline{C}^W$ denotes an average contribution paid by a labor union member. Moreover, all workers $x < \xi$ are strictly better off to enter the union, while all workers $x > \xi$ prefer to abstain in the "sincere" sense, since

$$\begin{aligned} & u\left(x, t^{WE}\right) - \overline{C}^W - u\left(x, t^E\right) \\ = \; & u\left(\xi, t^{WE}\right) - (\xi - x)\left(1 - t^{WE}\right) - \overline{C}^W - \left[u\left(\xi, t^E\right) - (\xi - x)\left(1 - t^E\right)\right] \\ = \; & (\xi - x)\left(t^{WE} - t^E\right). \end{aligned}$$

By analogy, each entrepreneur $x \in [\widehat{x}, \infty)$ perfectly predicts that in case of only a labor union formed some tax rate $t^W \in [0,1]$ is to be implemented. However, by organizing an entrepreneurs association entrepreneurs could affect the policymaker's choice and get some $t^{WE} \leq t^W$, since all entrepreneurs prefer no taxation at all. Entrepreneurs with very high income can afford to pay contributions because their gain with a tax decrease is huge. Thus, high-income entrepreneurs will enter a

[22] See Lohmann (1995) for a model of this type.

corresponding lobby (unless contribution are too high). As for entrepreneurs with moderate income (close to $\widehat{x}$), they prefer to abstain since a welfare gain from a tax decrease is not enough to cover the contribution payment. Therefore there exists an entrepreneur $\eta \in [\widehat{x}, \infty)$ indifferent between entering an association and not forming any association at all, that is

$$u\left(\eta, t^{WE}\right) - \overline{C}^{E} = u\left(\eta, t^{W}\right),$$

where $\overline{C}^{E}$ stands for an average contribution of an association member. Finally, each entrepreneur $x > \eta$ will join the entrepreneurs association, while each entrepreneur $x < \eta$ prefers to have no lobby corresponding to her interests, since

$$\begin{aligned} & u\left(x, t^{WE}\right) - \overline{C}^{E} - u\left(x, t^{W}\right) \\ = \; & u\left(\eta, t^{WE}\right) - (\eta - x)\left(1 - t^{WE}\right) - \overline{C}^{E} - \left[u\left(\eta, t^{W}\right) - (\eta - x)\left(1 - t^{W}\right)\right] \\ = \; & (\eta - x)\left(t^{WE} - t^{W}\right). \end{aligned}$$

■

Proof of Proposition 1. The two following conditions must hold for a "sincere" indifferent worker ξ and a "sincere" indifferent entrepreneur η:

$$\xi\left(1 - t^{WE}\right) + \widehat{x}t^{WE} - \frac{1}{\int_{0}^{\xi} f(x)\,dx} C^{W}\left(t^{WE}\right) = \xi\left(1 - t^{E}\right) + \widehat{x}t^{E} \qquad (1)$$

$$\eta\left(1 - t^{WE}\right) + \widehat{x}t^{WE} - \frac{1}{\int_{\eta}^{\infty} f(x)\,dx} C^{E}\left(t^{WE}\right) = \eta\left(1 - t^{W}\right) + \widehat{x}t^{W}, \qquad (2)$$

where the contribution payments are

$$\begin{aligned} C^{W}\left(t^{WE}\right) &= \left(U\left(t^{E}\right) - U\left(t^{WE}\right)\right) + \left(U^{E}\left(t^{E}\right) - U^{E}\left(t^{WE}\right)\right) \\ C^{E}\left(t^{WE}\right) &= \left(U\left(t^{W}\right) - U\left(t^{WE}\right)\right) + \left(U^{W}\left(t^{W}\right) - U^{W}\left(t^{WE}\right)\right). \end{aligned}$$

For $\xi \in [0, \widehat{x})$ and $\eta \in [\widehat{x}, \infty)$, $\lambda^{W} \leq \lambda^{WE} \leq \lambda^{E}$.

First, if $\lambda \leq \lambda^{W}$ the policymaker imposes $t^{WE} = t^{W} = t^{E} = 0$, therefore all individuals are indifferent between lobbying or not, so no lobbies are formed.

In case $\lambda^{W} < \lambda \leq \lambda^{WE}$ the tax rates become $t^{W} = 1$, $t^{WE} = t^{E} = 0$. Then the contribution payments are

$$\begin{aligned} C^{W}\left(t^{WE}\right) &= 0 \\ C^{E}\left(t^{WE}\right) &= (2\lambda - 1)\,\mathcal{F}(\widehat{x})\left(\widehat{x} - E\left[x | x \leq \widehat{x}\right]\right) + \mathcal{F}(\xi)\left(\widehat{x} - E\left[x | x \leq \xi\right]\right). \end{aligned}$$

The condition (1) holds for any $\xi \in [0, \widehat{x})$, that is all workers are "sincere" indifferent between lobbying or not, hence there is no labor union and $\xi = 0$. The condition (2) turns to

$$\eta - \frac{1}{1 - \mathcal{F}(\eta)}(2\lambda - 1)\,\mathcal{F}(\widehat{x})\left(\widehat{x} - E\left[x | x \leq \widehat{x}\right]\right) = \widehat{x},$$

where $\frac{1}{2} < \lambda \leq \lambda^E$. Making trivial calculations yields

$$\lambda = \frac{1}{2} + \frac{(\eta - \widehat{x})(1 - \mathcal{F}(\eta))}{2\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \leq \widehat{x}])}.$$

Note that $\lambda^E - \lambda = \frac{(1-\mathcal{F}(\eta))(E[x|x>\eta]-\eta)}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])} \geq 0$, therefore condition $\frac{1}{2} < \lambda \leq \lambda^E$ holds for each $\eta \in [\widehat{x}, \infty)$.

The first derivative is given by

$$\frac{d}{d\eta}\left(\frac{1}{2} + \frac{(\eta - \widehat{x})(1 - \mathcal{F}(\eta))}{2\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \leq \widehat{x}])}\right) = \frac{(1 - \mathcal{F}(\eta)) - (\eta - \widehat{x}) f(\eta)}{2\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \leq \widehat{x}])},$$

that has the same sigh as $\frac{1-\mathcal{F}(\eta)}{f(\eta)} - (\eta - \widehat{x})$. Then the function takes its extremum value in $\overline{\eta}$, such that $\frac{1-\mathcal{F}(\overline{\eta})}{f(\overline{\eta})} = \overline{\eta} - \widehat{x}$. Let us show that $\overline{\eta}$ exists and that it is unique. By the monotone hazard rate condition $\frac{1-\mathcal{F}(\eta)}{f(\eta)}$ is decreasing in η, while $\eta - \widehat{x}$ is strictly increasing. Moreover, $\lim\limits_{\eta\to\infty} \frac{1-\mathcal{F}(\eta)}{f(\eta)} = 0 < \infty = \lim\limits_{\eta\to\infty} (\eta - \widehat{x})$ and $\frac{1-\mathcal{F}(\eta)}{f(\eta)}|_{\eta=\widehat{x}} > (\eta - \widehat{x})|_{\eta=\widehat{x}}$. Therefore, there exists a unique $\overline{\eta} \in [\widehat{x}, \infty)$ such that $\frac{d\lambda}{d\eta}|_{\eta=\overline{\eta}} = 0$, $\frac{d\lambda}{d\eta}|_{\eta<\overline{\eta}} > 0$ and $\frac{d\lambda}{d\eta}|_{\eta>\overline{\eta}} < 0$. So we proved that there exists a unique $\overline{\eta} = \arg\max\limits_{\eta\in[\widehat{x},\infty)} \left[\frac{1}{2} + \frac{(\eta-\widehat{x})(1-\mathcal{F}(\eta))}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])}\right]$.

Next, for each $\lambda \in \left(\frac{1}{2}, \frac{1}{2} + \frac{(\overline{\eta}-\widehat{x})(1-\mathcal{F}(\overline{\eta}))}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])}\right)$ there exist 2 arguments η, such that $\lambda = \frac{1}{2} + \frac{(\eta-\widehat{x})(1-\mathcal{F}(\eta))}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])}$. Then the threshold for an entrepreneurs association formation η^* is the one that is to the left of the maximum argument $\overline{\eta}$, that is, η^* is such that $\lambda = \frac{1}{2} + \frac{(\eta^*-\widehat{x})(1-\mathcal{F}(\eta^*))}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])}$ and $\eta^* < \overline{\eta}$, since each entrepreneur with income $x \in (\eta^*, \infty)$ is "sincerely" strictly better off to join a corresponding lobby. The second derivative can be expressed in the following way:

$$\begin{aligned} &\frac{d^2}{d\eta^2}\left(\frac{1}{2} + \frac{(\eta - \widehat{x})(1 - \mathcal{F}(\eta))}{2\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \leq \widehat{x}])}\right) \\ = \quad &\frac{1}{2\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \leq \widehat{x}])}\left[\begin{array}{c}\left(\frac{d}{d\eta}\left(\frac{1-\mathcal{F}(\eta)}{f(\eta)}\right) - 1\right) f(\eta) + \\ \left(\frac{1-\mathcal{F}(\eta)}{f(\eta)} - (\eta - \widehat{x})\right) f'(\eta)\end{array}\right], \end{aligned}$$

that is negative for $\widehat{x} \leq \eta < \overline{\eta}$.

Finally, we conclude that for $\lambda \in (\frac{1}{2}, \frac{1}{2} + \frac{(\overline{\eta}-\widehat{x})(1-\mathcal{F}(\overline{\eta}))}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])}]$, where $\overline{\eta}$ satisfies $\frac{1-\mathcal{F}(\overline{\eta})}{f(\overline{\eta})} = \overline{\eta} - \widehat{x}$, an entrepreneurs association $E = \{x \mid x \in (\eta^*, \infty)\}$ is formed, where η^* is such that $(\eta^* - \widehat{x})(1 - \mathcal{F}(\eta^*)) = (2\lambda - 1)\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \leq \widehat{x}])$ and $\eta^* < \overline{\eta}$. Note that $\frac{d\eta^*}{d\lambda} > 0$ and $\frac{d^2\eta^*}{d\lambda^2} > 0$.

Consider the case $\lambda^{WE} < \lambda \leq \lambda^E$, where $t^W = t^{WE} = 1$ and $t^E = 0$. The lobbies contributions are given by

$$\begin{aligned} C^W(t^{WE}) &= -(2\lambda - 1)\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \leq \widehat{x}]) - (1 - \mathcal{F}(\eta))(\widehat{x} - E[x|x > \eta]) \\ C^E(t^{WE}) &= 0. \end{aligned}$$

So (2) is satisfied for any $\eta \in [\widehat{x}, \infty)$, therefore no entrepreneurs association is formed and $\eta = \infty$. An indifferent worker condition (1) becomes

$$\widehat{x} + \frac{1}{\mathcal{F}(\xi)}(2\lambda - 1)\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \leq \widehat{x}]) = \xi,$$

where $\lambda^W < \lambda \leq \frac{1}{2}$. Making calculations gives

$$\lambda = \frac{1}{2} - \frac{(\widehat{x} - \xi)\,\mathcal{F}(\xi)}{2\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \leq \widehat{x}])}.$$

Condition $\lambda^W < \lambda \leq \frac{1}{2}$ is satisfied for each $\xi \in [0, \widehat{x})$ since $\lambda - \lambda^W = \frac{\mathcal{F}(\xi)(\xi - E[x|x \leq \xi])}{2\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \leq \widehat{x}])} \geq 0$.

The first derivative is

$$\frac{d}{d\xi}\left(\frac{1}{2} - \frac{(\widehat{x} - \xi)\,\mathcal{F}(\xi)}{2\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \leq \widehat{x}])}\right) = \frac{\mathcal{F}(\xi) - (\widehat{x} - \xi)\,f(\xi)}{2\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \leq \widehat{x}])},$$

that has the same sigh as $\xi - \widehat{x} + \frac{1}{f(\xi)} - \frac{1-\mathcal{F}(\xi)}{f(\xi)}$. Then the extremum is reached in $\underline{\xi}$, such that $\frac{\mathcal{F}(\underline{\xi})}{f(\underline{\xi})} = \widehat{x} - \underline{\xi}$. Let us prove that $\underline{\xi}$ exists and that it is unique. At the bounds of the interval $[0, \widehat{x})$ the following holds: $\left(\xi - \widehat{x} + \frac{1}{f(\xi)}\right)|_{\xi=0} = \frac{1}{f(0)} - \widehat{x} < \frac{1}{f(0)} = \frac{1-\mathcal{F}(\xi)}{f(\xi)}|_{\xi=0}$ and $\left(\xi - \widehat{x} + \frac{1}{f(\xi)}\right)|_{\xi=\widehat{x}} = \frac{1}{f(\widehat{x})} > \frac{1-\mathcal{F}(\xi)}{f(\xi)}|_{\xi=\widehat{x}}$. Next, by the monotone hazard rate condition $\frac{d}{d\xi}\left(\frac{1-\mathcal{F}(\xi)}{f(\xi)}\right) \leq 0$, while $\frac{d}{d\xi}\left(\xi - \widehat{x} + \frac{1}{f(\xi)}\right) = 1 - \frac{f'(\xi)}{f^2(\xi)}$, that is positive if $f'(\xi) \leq 0$ that holds for each $\xi \geq \operatorname{mod}\mathcal{F}(x)$ (the mode of $\mathcal{F}(x)$). Moreover, $\frac{d^2}{d\xi^2}\left(\xi - \widehat{x} + \frac{1}{f(\xi)}\right) = \frac{2\left(f'(\xi)\right)^2 - f''(\xi)f(\xi)}{f^3(\xi)} > 0$ for $\xi \in [0, \widehat{x})$. So $\xi - \widehat{x} + \frac{1}{f(\xi)}$ is increasing for $\xi \geq \operatorname{mod}\mathcal{F}(x)$ and convex for $\xi \in [0, \widehat{x})$, therefore, it is either increasing for each $\xi \in [0, \widehat{x})$ or at first decreasing and then increasing at $[0, \widehat{x})$. So in both cases there exists a unique $\underline{\xi} \in [0, \widehat{x})$ such that $\frac{d\lambda}{d\xi}|_{\xi=\underline{\xi}} = 0$, $\frac{d\lambda}{d\xi}|_{\xi<\underline{\xi}} < 0$ and $\frac{d\lambda}{d\xi}|_{\xi>\underline{\xi}} > 0$. Thus there exists a unique $\underline{\xi} = \arg\min_{\xi \in [0,\widehat{x})}\left[\frac{1}{2} - \frac{(\widehat{x}-\xi)\mathcal{F}(\xi)}{2\mathcal{F}(\widehat{x})(\widehat{x} - E[x|x \leq \widehat{x}])}\right]$.

For each $\lambda \in (\frac{1}{2} - \frac{(\widehat{x}-\underline{\xi})\mathcal{F}(\underline{\xi})}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])}, \frac{1}{2}]$ there are 2 arguments ξ such that $\lambda = \frac{1}{2} - \frac{(\widehat{x}-\xi)\mathcal{F}(\xi)}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])}$. Then the threshold for a labor union formation ξ^* is the one that is to the right of minimum argument $\underline{\xi}$, that is ξ^* such that $\lambda = \frac{1}{2} - \frac{(\widehat{x}-\xi^*)\mathcal{F}(\xi^*)}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])}$ and $\xi^* > \underline{\xi}$, because each worker $x \in [0, \xi^*)$ "sincerely" prefers enter a labor union. Therefore, for $\lambda \in \left[\frac{1}{2} - \frac{(\widehat{x}-\underline{\xi})\mathcal{F}(\underline{\xi})}{2\mathcal{F}(\widehat{x})(\widehat{x}-E[x|x\leq\widehat{x}])}, \frac{1}{2}\right]$, where $\underline{\xi}$ satisfies $\frac{\mathcal{F}(\underline{\xi})}{f(\underline{\xi})} = \widehat{x} - \underline{\xi}$, a labor union $W = \{x \mid x \in [0, \xi^*)\}$ is formed, where ξ^* is such that $(\widehat{x} - \xi^*)\,\mathcal{F}(\xi^*) = (1 - 2\lambda)\,\mathcal{F}(\widehat{x})\,(\widehat{x} - E[x|x \leq \widehat{x}])$ and $\xi^* > \underline{\xi}$. Note, that $\frac{d\xi^*}{d\lambda} \geq 0$.

Finally, if $\lambda > \lambda^E$ the tax rates are $t^{WE} = t^W = t^E = 1$. The policymaker's choice is not affected by any lobbying activities, thus there are no lobbies formed.■

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