TAX EVASION IN INTERRELATED TAXES

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ABSTRACT: In 1969, Shoup postulated that the presence of interrelated taxes in a tax system would reinforce the tax penalty system ("self-reinforcing penalty system of taxes"). In this paper, we have tried to formally develop this idea. We find that in order for tax reinforcement to be maintained, it is necessary for interrelated taxes to be administered by a single tax administration, or if they are administered by different tax administrations, the level of collaboration between them has to be sufficiently high. If so, tax evasion in interrelated taxes might be considered as an alternative explanation for the gap between the levels of tax evasion that can be guessed in practice and the much higher levels predicted by the classical tax evasion theory (Allingham and Sandmo, 1972; Yitzhaki, 1974). Otherwise, the result anticipated by Shoup may even be reversed. Moreover, as long as collaboration is imperfect, the classical results of the comparative statics might change, since in some cases, although global tax compliance increases when faced with a variation in a tax parameter, it can decrease in a single tax.

JEL Code: H2, H26

Keywords: Tax Evasion, Self-reinforcing penalty system of taxes

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1. Introduction

Kaldor (1956) argued that in a tax system in which a capital gains tax, a personal income tax, an expenditure tax, a wealth tax and an inheritance and donations tax were present, with a single tax return audited, the extent of tax evasion could be checked comprehensively. This is the so-called "self-checking system of taxes". His argument is based on the obvious relationships between the tax bases of all five taxes. The sum of the amount of tax base declared in expenditure tax and in capital gains tax should therefore be congruent with the tax base declared in personal income tax. If it does not match, this might be because the taxpayer has either consumed part of her initial stock of wealth (or she has made a donation), or because some of the tax bases have been under-declared. In the first case, this should be compatible with a decrease in the tax base of the wealth tax (or with an increase in the tax base of the recipient's donations tax) once capital gains have also been taken into account, while in the second case, it should be an useful hint for starting a tax auditing process.

This certainly seems a powerful system to ease the tasks of the tax auditors. However, note that congruity between tax bases does not necessarily imply no tax evasion. That is why Shoup (1969) suggested renaming the tax system proposed by Kaldor as a "self-reinforcing penalty system of taxes". This means that when taxpayers face the decision about how much tax base to evade, they should bear in mind that as long as tax bases are crosschecked, their decision might not only have consequences on that tax, but also on other interrelated taxes. Having increased the expected cost of tax evasion, such type of tax systems should therefore be useful a priori in promoting tax compliance. In our paper, we will try to confirm that supposition by means of formally developing the original ideas of Kaldor (1956) and Shoup (1969), and we will do so by focusing our analysis on the interrelation between a wealth tax and a personal income tax. However, this analysis cannot only be applied to personal taxes, neither only in a static setting[1].

For example, Das-Gupta and Gang (2001) recently developed a similar model of tax evasion applied to Value Added Tax (VAT) (see also the analysis of interrelated tax evasion in the VAT by Fedeli and Forte, 1999). The reason for analyzing VAT arises from the tax administration's ability to match sales invoices with purchase invoices. These authors find that although crosschecking might distort purchase and sale decisions, a sufficiently high level of crosschecking can encourage truthful reporting. The interest in analyzing the taxpayer's behavior taking into account how the tax administration might process all the information provided by the taxpayer's returns is also recognized by Andreoni et al. (1998), who argue that while the existing theory generally assumes that taxpayers report only a single piece of information, in practice "they make rather detailed reports about their sources of income and deductions, providing the tax agency with multiple signals of their true tax liability" (p. 833).

To a certain extent, Engel and Hines (1999) also applied Shoup's (1969) idea in a dynamic setting (in fact, the seminal work by Allingham and Sandmo, 1972, section 5, also considered tax evasion within a dynamic setting). In their work, these authors find that for rational taxpayers, current evasion is a decreasing function of prior evasion, since, if audited for tax evasion in the current year, they may incur penalties for past

[1] We have decided to restrict our analysis to only two of the taxes that ideally compose a "self-checking system of taxes" in order to keep the analysis as simple as possible. In particular, the choice of those two taxes is partly caused by the recent evidence for the Spanish case of the utilization by the National Tax Administration of the interrelation between both taxes to design its tax auditing processes (see fn. 8).

evasions as well. Within this framework, they estimate that tax evasion is 42% lower than it would be if taxpayers were not concerned about retrospective audits. These results are certainly very interesting, and make the need for expansion of the classical analysis of tax evasion (Allingham and Sandmo, 1972; Yitzhaki, 1974) clear, taking into account the interrelation between tax bases, and therefore the possibility of crosschecking by the tax administration.

In our paper, when incongruity between tax returns is detected by the tax administration, the tax audit probability tends to increase above its "normal" level. This means that as long as both taxes are administered by a single tax administration, congruity is the optimal choice for the taxpayer. However, in some cases (typically, in federal systems) taxes are not always administered by the same layer of government. As long as collaboration between tax administrations is not perfect, it is therefore possible that crosschecking is not sufficient to induce congruity between tax returns. By imperfect collaboration, we refer to the situation in which when one tax administration is carrying out an audit, it does not put too much effort into detecting tax evasion on behalf of the other[2]. In fact, imperfect collaboration implies that the level of tax compliance might be even lower than that predicted by the classical analysis! In any case, as long as collaboration is perfect, or is not too low if imperfect, tax evasion in interrelated taxes might be considered as a partial explanation for the paradox of tax

[2] Niepelt (2002) analyzed tax evasion in a dynamic setting, and like us, considered the possibility that tax evasion is not fully discovered during an audit. However, he justified this assumption without referring to a lack of collaboration between tax administrations, but simply as a handicap of a tax administration. In any event, in his words, it leads to an "uncorrelated detection risk", which calls for analyzing tax evasion focusing on the many sources of taxpayer's tax base, instead of on the taxpayer. This conclusion very much resembles our differential analysis, depending on whether congruity or incongruity is optimal from the taxpayer's point of view. In the former, the unit of analysis will be the taxpayer, since differences in the level of tax compliance between tax bases do not arise, while in the latter, the unit of analysis is each tax base, since tax compliance is not homogenous across taxes.

evasion. The paradox of tax evasion comes about when the observed (or guessed) levels of tax compliance and those predicted by the classical analysis are compared. In order to achieve the observed levels of tax compliance from the classical analysis, the degree of risk-aversion and/or the level of the tax enforcement parameters have to be abnormally high. In order to overcome this paradox, the literature has proposed the existence of both economic and non-economic factors (e.g., see the clear and detailed review of this literature by Alm, 1999). In general, the conclusion of the literature is that the original model of gambling applied to tax evasion might be too simple to take into account the numerous factors that affect the reporting decisions of individuals. In this regard, interrelated tax evasion might be considered as another factor to be taken into account, and as we will show in numerical simulations, on some occasions this factor alone can solve the paradox of tax evasion.

In the context of interrelated tax evasion, we have also performed a comparative static analysis. In the classical analysis, a reinforcement of any tax parameter tends to promote tax compliance (see the review by Andreoni et al., 1998)[3]. In our analysis, as long as collaboration between tax administrations is perfect, those results remain unchanged. Nonetheless, when collaboration is imperfect, the results might change. Due to the ambiguity of the theoretical analysis in this latter situation, we have had to make use of the methodology of numerical simulations. All the results of numerical simulation confirm those obtained by the classical analysis with respect to overall tax compliance, or at least when tax compliance in each tax is weighted by the importance of their

[3] However, some authors have recently shown that the signs of the comparative statics can reverse by slightly modifying the original framework of the classical analysis. For instance, Boadway et al. (2002) or Borck (2002) have demonstrated that in certain circumstances an increase in the penalty per unit of tax evaded can decrease tax compliance; while Lee (2001) has shown the same, but for the case of an increase in the tax rate.

respective tax burdens. However, this result no longer holds when tax compliance is analyzed tax by tax[4]. This result is extremely important once we take into account that the policy decisions of one tax administration (i.e., level of government) will have consequences not only for its tax base, but also for the tax base of the other tax administration (level of government). A tax externality stemming from the tasks of tax administration therefore arises as long as different layers of government are responsible for each tax. Cremer and Gahvari (2000) considered the audit rate as an additional strategic tax parameter between sub-national governments within a federal system, while Baccheta and Espinosa (1995) analyzed the incentives for sharing information between national governments in an open economy, although they did not include the possibility of tax evasion in their model. The identification of a potential tax externality in the context of tax administration is thus not totally new. Nevertheless, this confirms Andreoni et al. (1998)'s statement in the sense that how to integrate tax enforcement across different levels of government may be one of the tax compliance issues that merits further research (p. 835). In any event, this line of research is not dealt with in this paper.

In the following section, we formulate the theoretical model and our assumptions, especially those referring to the tax audit probability in the presence of interrelated tax evasion. The taxpayer's decision over tax evasion is characterized, and a comparative static analysis is performed. This analysis crucially depends on the degree of collaboration between the tax administrations responsible for each tax. In section 3, we carry out a numerical simulation exercise, which enables us to complement the results

[4] This result is in accordance with Gordon and Slemrod (1998)'s remark: "A literature has developed analyzing the effects of tax rates [or any tax enforcement parameter] on tax evasion, but even here *the evasion considered primarily involves nonreporting rather than a shift in reporting between one tax base and another*" (p. 4).

of the theoretical model. Thus, given a simple parameterization, we can ascertain to what extent interrelated tax evasion can solve the paradox of tax evasion; in which circumstances it is more likely that the tax bases declared in each tax return are incongruent; and finally, we can examine some of the ambiguities detected in the analytical comparative statics. We conclude in section 4.

2. Theoretical Model

In this section, we will first establish how the presence of interrelated taxes changes tax enforcement parameters, and in particular, tax audit probability. Obviously, this is the key to all the theoretical analysis carried out in the paper. We will then analyze the behavior of the taxpayer in this context of interrelated tax evasion, including a comparative statics analysis. This analysis will be carried out in the presence of both perfect and imperfect collaboration between tax administrations.

Assumptions about the tax audit probability

We assume that the tax administration obtains valuable information from crosschecking the taxpayers' tax returns. In particular, the tax administration considers the following budget constraint for each taxpayer:

$$Y = C + S = \beta Y + S \qquad (1)$$

where Y is income obtained by a taxpayer during the fiscal year, which can be either consumed, C, or saved, S (i.e., S is the increase in the stock of wealth obtained during that fiscal year[5]), and β is the marginal propensity to consume. Income is taxed by personal income tax, while savings are taxed by wealth tax. Given the level of income declared in the personal income tax, Y_D, the level of wealth declared in the wealth tax, S_D, and supposing a certain marginal propensity to consume, β [6], the tax administration can infer whether the relationship given by expression (1) holds, i.e.

$$Y_D \underset{>}{\overset{\leq}{}} C' + S_D = \beta Y_D + S_D \tag{2}$$

where C' is the level of taxpayer's consumption inferred by the tax administration, and $Y_D \leq Y$ and $S_D \leq S$. As long as expression (2) holds with equality, the tax administration will not notice any incongruity between the tax bases declared in each tax return, and will not increase the tax audit probability above the "normal" level, that is, when there is no incongruity[7]. Otherwise, an incongruity will be an "alarm bell" for the tax

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[5] The budget constraint (1) could have been expressed within an inter-temporal framework, although for simplification we have left aside such possibility. For instance, Y could have been considered as the personal income obtained during a certain period of time, which has made the accumulation of a certain stock of wealth, S, and a certain level of inter-temporal consumption, C, possible. Otherwise, given that the tax base of the wealth tax is the stock of wealth and not the increase in wealth, for our analytical purposes, expression (1) has to be considered in such a way that at the beginning of the fiscal year (i.e., at the beginning of the only period of our static analysis), the taxpayer's stock of wealth is nil. As a consequence, there is no difference between stock of wealth and increase in the stock of wealth in our theoretical analysis, but that will be properly taken into account in the numerical simulations of section 3.

[6] From now on, we will suppose that the marginal propensity to consume adopted by the tax administration (see expression (2) next) and the real one coincide. As we will see, this assumption will make the interpretation of the results of comparative statics easier.

[7] Expression (2) could be modified in order to incorporate a margin of error, $|\varepsilon| > 0$, i.e., $Y_D \underset{>}{\overset{\leq}{}} \beta Y_D + S_D + \varepsilon$, which seems reasonable given the prediction the tax administration has to make with respect to each taxpayer's β. For example, suppose that according to the personal income tax return Y_D= 100. Assuming $\beta = 0{,}8$, the stock of wealth declared in the wealth tax should then be 20, but in fact, considering a certain margin of error of ±10%, S_D should be between 18 and 22. Otherwise, if S_D were above (below) 22 (18), the probability of auditing

administration to audit the taxpayer's tax returns[8]. Put in graphic terms,

[FIGURE 1]

On the left hand side, the graph shows the tax audit probability in personal income tax, p^Y. As long as $Y_D \geq S_D/(1-\beta)$, the tax audit probability remains at its "normal" level, $\bar{p}^Y$. Otherwise, the audit probability is increasing in the value of the incongruity, $S_D - Y_D(1-\beta) > 0$. Similarly, the graph on the right hand side shows the tax audit probability in wealth tax, p^S. In fact, by summing both functions of probability of tax auditing, we obtain the following function:

[FIGURE 2]

where we have assumed that $\bar{p}^Y = \bar{p}^S$. Keeping S_D unchanged, for values of Y_D below (above) S_D+C' an increase in Y_D therefore decreases (increases) the probability of being audited in personal income tax (wealth tax), and so $p^Y + p^S$.

In our model the tax audit probability is thus endogenous, since it depends on the level of tax bases declared, S_D and Y_D. In the next section, we will try to identify those

personal income tax (wealth tax) would increase above its "normal" level. However, the results of our marginal analysis are independent of the inclusion of a margin of error.

[8] For instance, in Spain during the summer of 2002, the National Tax Administration (the *Agencia Estatal de la Administración Tributaria*, AEAT), which is responsible for the auditing of personal income tax (while the responsibility for auditing wealth tax is shared with the regional governments), extensively crosschecked personal income tax and wealth tax returns. According to the Director of the AEAT, the reason for this massive crosschecking was that the price of new houses and luxury cars purchased (in our terms, an increase in the monetary value of the stock of wealth) did not match the income declared by taxpayers in personal income tax. See the information given by the newspaper *La Vanguardia*, 10/3/2002.

situations in which the taxpayer might find it optimal to deviate from the strategy that involves the minimization of the probability of being audited, $\bar{p}^Y + \bar{p}^S$, i.e. being congruent ($S_D = Y_D(1-\beta)$).

Definition: the tax audit probability, *p*, is a function $p(S_D, Y_D) = p^S + p^Y$, such that for $S_D > Y_D(1-\beta)$, $\partial p^Y / \partial S_D > 0$ and $\partial p^Y / \partial Y_D < 0$, while p^S is constant; for $S_D < Y_D(1-\beta)$, $\partial p^S / \partial S_D < 0$ and $\partial p^S / \partial Y_D > 0$, with p^Y being constant; finally, for $S_D = Y_D(1-\beta)$, *p* remains constant.

... When collaboration between tax administrations is perfect.

In this section, we will analyze a situation in which either both tax returns (personal income tax and wealth tax) are administered by a single tax administration or they are administered by two different tax administrations (i.e., each is a part of a different layer of government) but collaboration between them is perfect. An example of perfect collaboration is a situation in which when wealth tax return is audited, not only is tax evasion in wealth tax fully discovered, but also evasion in personal income tax. To the extent that there exists only one tax administration, it is perfectly understandable that tax evasion in both taxes will be fully discovered independently of which tax return is audited, since the total amount of tax revenue collected will remain in hands of that single tax administration. However, when there are two independent tax administrations, the situation is different: perfect collaboration implies that the tax administration that is carrying out an audit (e.g., in wealth tax) will make an additional effort to discover tax fraud (e.g., in personal income tax) that will only benefit the other tax administration, or simply will have enough incentives to communicate it to the other tax administration. In any case, we will suppose for now that even in the case that there were two (institutionally independent) tax administrations, each one of them would have an incentive to fully discover evasion of both taxes.

Characterization of the Taxpayer's Behavior

According to the classical analysis by Allingham and Sandmo (1972), the taxpayer aims at maximizing expected utility, which might imply evading a certain amount of taxes. However, such a decision is not without risk, since this tax fraud might be detected by the tax administration, depending on the tax auditing probability. If taxpayers are audited, they will then be fined in proportion to the amount of taxes evaded. We assume that the taxpayer is risk-averse, $U'(Y) > 0 > U''(Y)$, where *U(Y)* is the utility that the taxpayer derives from income[9], and *Y* is an exogenous variable in our model[10]. Analytically, the objective function of the taxpayer, *W*, is as follows:

$$W \equiv (p^Y + p^S)U\left[Y - t_R Y_D - t_P S_D - Ft_R(Y - Y_D) - Ft_P(S - S_D)\right] + \\ + (1 - p^Y - p^S)U\left[Y - t_R Y_D - t_P S_D\right] \qquad (3)$$

The first summand in square brackets (hereafter denoted by *A*) is income at the taxpayer's disposal net of paying taxes after auditing. In that case, taxpayer pay taxes on personal income tax according to the tax rate t_R ($0 \leq t_R \leq 1$), but as long as they have evaded taxes ($Y > Y_D$), they will also have to pay *F* per each unit of tax evaded in

[9] Partial derivatives of functions of only one variable will be denoted by a prime, while for functions of more than one variable, a subscript will indicate the variable of the corresponding partial derivative.

[10] See Pencavel (1979), for a model of tax evasion in which *Y* (labor supply) is considered as an endogenous variable; and other references cited in Andreoni et al. (1998), p. 824.

personal income tax, $t_R(Y\text{-}Y_D)$, where $F \geq 1$. The same reasoning applies to the case of wealth tax, where the tax rate in that case is t_P $(0 \leq t_P \leq 1)$. The second summand in square brackets (from now on, denoted by *B*) is income at the taxpayer's disposal when none of the tax returns are audited. Given the presence of perfect collaboration between tax administrations, only these two states can occur: *A* or *B*. The probability of occurrence of the first is $p^S + p^Y$, i.e. it occurs when either of the two tax administrations audits[11], while state *B* occurs when neither of them audits, with $1 - p^S - p^Y$ being the probability of occurrence of that state.

According to the taxpayer's objective previously stated, she will choose Y_D and S_D such that expression (3) is maximized[12]. Nevertheless, we know that as long as they are incongruent with respect to the amount of tax bases declared ($Y_D(1-\beta) \overset{\leq}{>} S_D$), the tax audit probabilities of expression (3) are not parameters. Before solving the maximization problem of the taxpayer, we therefore need to know whether (in)congruity could be an optimal strategy for them.

Is it optimal to be congruent in the tax bases declared?

We are assuming that regardless of which tax return is originally audited, both tax evasion in personal income tax and in wealth tax are fully discovered by the tax

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[11] Although it does not modify the results of the present analysis, the possibility that both tax administrations simultaneously carry out a tax audit can be reasonably ruled out, i.e., $p^Y \times p^S = 0$., These two events can therefore be considered as mutually exclusive.

[12] This characterization of a rational taxpayer is consistent with the following description given by Cowell (1990): "(he) is "predisposed to dishonesty" because the taxpayer does not put responsibility to the State before his own interests" (p. 50).

administration. Moreover, in order to simplify the analysis, we suppose that both functions of tax auditing are symmetric, i.e., $\left|p^Y_{S_D}\right| = \left|p^Y_{Y_D}\right| = \left|p^S_{S_D}\right| = \left|p^S_{Y_D}\right|$, and $\overline{p}^Y = \overline{p}^S$. From now on, unless necessary, we will therefore not distinguish between p^Y and p^S, and will simply refer to *p*, which is $p^Y + p^S$. Under these assumptions, we wonder whether under certain circumstances it will be optimal for the taxpayer to be incongruous.

For instance, we wonder whether it could be optimal that $S_D > Y_D(1-\beta)$. In that case, and keeping Y_D constant, the following conditions should hold:

$$\left.\frac{\partial W}{\partial S_D}\right|_{S_D+C'<Y_D} = p^1_{S_D}\left[U(A_1)-U(B_1)\right]+t_P\left[p_1(F-1)U'(A_1)-(1-p_1)U'(B_1)\right]>0 \qquad (4)$$

where the index 1 is necessary since the (marginal) utility of income is obviously not the same for all levels of S_D and Y_D, and the tax audit probability might also vary according to those two variables. Hence, in expression (4), the index 1 is referring to *a* situation under which $S_D < Y_D(1-\beta)$. The next condition – which implies that $S_D = Y_D(1-\beta)$ is not an optimal strategy for the taxpayer - should also hold

$$\left.\frac{\partial W}{\partial S_D}\right|_{S_D+C'=Y_D} = p_2(F-1)U'(A_2)-(1-p_2)U'(B_2)>0 \qquad (5)$$

where $p^1_{S_D} < 0$, $p_1 > p_2$, $U(A_i) < U(B_i)$ and $U'(A_i) > U'(B_i)$ $\forall$ *i*; while at the (supposed) optimum,

$$\left.\frac{\partial W}{\partial S_D}\right|_{S_D+C'>Y_D} = p^3_{S_D}\left[U(A_3)-U(B_3)\right]+t_P\left[p_3(F-1)U'(A_3)-(1-p_3)U'(B_3)\right]=0 \qquad (6)$$

where $p^3_{S_D} > 0$, $p_3 > p_2$, and $p_3 \underset{>}{\overset{\leq}{}} p_1$.

Additionally, in order to guarantee an interior solution (S>S_D), we need the following condition to hold:

$$\left.\frac{\partial W}{\partial S_D}\right|_{S_D=S} < 0 \qquad (7)$$

The next Lemma states the non-optimality of incongruity from a rational taxpayer's point of view.

Lemma: *As long as* $\left|p_{S_D}\right| > 0$ *and* $\left|p_{Y_D}\right| > 0$*, it will always be optimal for the taxpayer to be congruous. Otherwise, the optimal strategy for the taxpayer is indeterminate.*

Proof: the reasoning is as follows. Expression (6) must hold both for S_D and Y_D, i.e. at the optimum,

$$\left.\frac{\partial W}{\partial Y_D}\right|_{S_D+C'>Y_D} = p^3_{Y_D}\left[U(A_3)-U(B_3)\right]+t_R\left[p_3(F-1)U'(A_3)-(1-p_3)U'(B_3)\right]=0 \qquad (8)$$

Given that $p^3_{Y_D} < 0$ and $U(A_3) < U(B_3)$, in order for expression (8) to hold, it is therefore necessary that $p_3(F-1)U'(A_3)-(1-p_3)U'(B_3)<0$. However, according to expression (6), and given that $p^3_{S_D}>0$, $p_3(F-1)U'(A_3)-(1-p_3)U'(B_3)>0$. Therefore, expressions (6) and (8) cannot hold simultaneously, and incongruity cannot be an optimum. Given that the same reasoning is applicable for the case in which $S_D<Y_D(1-\beta)$, congruity is the only possible solution as long as $|p_{S_D}|>0$ and $|p_{Y_D}|>0$. In the case that the probability of auditing is independent of incongruity, i.e. $|p_{S_D}|=0$ and $|p_{Y_D}|=0$, the solution to the maximization of expression (3) is indeterminate, with congruity being one of the many solutions $\blacksquare$

We have shown that incongruity can never be an optimal strategy for a rational taxpayer in the event that the tax audit probability is determined by incongruity between tax returns, and by collaboration between tax administrations being perfect. The reason for this is that in order for $S_D>Y_D(1-\beta)$ to be optimal, for example, the taxpayer's welfare reduction due to a higher probability of tax auditing compared to its "normal" level, $p^Y>\bar{p}^Y$, must be exceeded by the net expected gains of increasing tax compliance in wealth tax keeping the tax audit probability constant. Nevertheless, given that the sign of the latter expected gains is independent of the tax rate, there would still be gains by increasing Y_D. Moreover, in that case, increasing Y_D would also lead to a decrease in the tax auditing probability (through a reduction in p^Y). The strategy under which $S_D>Y_D(1-\beta)$ can thus never be optimal, since in that situation it would always be welfare-enhancing to increase Y_D [13]. From the taxpayer's point of view, this means that

[13] To a certain extent, the result provided by the Lemma is similar to the one obtained under a "cut-off" rule (see Border and Sobel, 1987; Reinganum and Wilde, 1985; Sánchez and Sobel, 1993). Under such a rule, the tax administration establishes a threshold below which all taxpayers are audited, while above it all taxpayers are unaudited. Assuming that taxpayers are

both tax bases are perfect substitutes, and so they simply aim to minimize the tax auditing probability, *p*.

Optimal level of declared tax base

We have previously described the taxpayer as a rational individual predisposed to dishonesty (see fn. 12), i.e. an individual who aims at maximizing his or her own welfare by means of deciding how much tax base to declare independently of the consequences of this decision on the rest of society[14]. The social consequences of these actions are basically the loss of tax revenues for the government (and thus public welfare provision) due to erosion of the tax base. Given congruity in the taxpayer's returns, analytically, the taxpayer solves the following maximization problem:

$$\max_{S_D, Y_D} \; W \quad s.t. \;\; Y_D = S_D/(1-\beta)$$

Once Y_D has been substituted into W (which has been previously defined by means of expression (3)), the only decision variable of the taxpayer is therefore S_D. This is the

risk-neutral and that the tax administration can commit itself to such an audit rule, all those taxpayers with a tax base above the threshold declare only the amount fixed by the threshold. In our case, the threshold is endogenous. From the point of view of the tax administration responsible for the personal income tax, for instance, the relevant threshold is S_D+*C'*, with S_D being endogenous from the point of view of the taxpayer. In our model, despite the assumption of risk-aversion, the taxpayer thus finds it optimal to declare only the amount set by the threshold, or in our terms, finds it optimal to be congruent.

[14] See, e.g., Bordignon (1993) for a model that takes moral issues into account when describing the taxpayer's behavior; or Cowell and Gordon (1988), and Alm et al. (1992), who consider the taxpayer's evaluation of the activity of the public sector; see also the references cited by Andreoni et al. (1998), section 8; and the complete review by Alm (1999), already cited in the introduction.

decision we will consider. The FOC of the maximization problem with respect to S_D is thus the following:

$$pU'(A)(F-1) = (1-p)U'(B) \tag{9}$$

i.e. at the optimum, the marginal cost of evading taxes (the left-hand side of expression (9)) equals the marginal benefit of evading taxes (the right-hand side of expression (9)). Finally, in order to guarantee that full tax compliance (S=S_D) is not an optimal strategy for the taxpayer, and using expression (7), we obtain the classical condition that $pF < 1$ (see Yitzhaki, 1974, expression (6')*)[15]. From now on, we will assume that such condition holds, and so at the optimum $S > S_D$.

Comparative statics

As we have shown above, in the case of perfect collaboration between tax administrations, congruity is the only optimal strategy. In order to perform an exercise of comparative statics, we will therefore only analyze the way in which reported wealth, S_D, depends on the parameters of the model F, t_P, t_R, p, since congruity implies that Y_D can be directly obtained from $Y_D = S_D/(1-\beta)$.

In order to obtain dS_D/dF, we first totally differentiate expression (9), Φ,

$$\begin{aligned} d\Phi = 0 = (t_R' + t_P)\left[pU'(A) + pU'(A)R(A)(t_R(Y-Y_D) + t_P(S-S_D))(F-1)\right]dF - \\ -(t_R' + t_P)^2\left[pU'(A)R(A)(F-1)^2 + (1-p)U'(B)R(B)\right]dS_D \end{aligned} \tag{10}$$

in which we have employed the Arrow-Pratt measure of absolute risk aversion, $R(A) = \left(-U''(A)/U'(A)\right) \geq 0$, and the same for state *B*. In expression (10), t_R'$=t_R/(1-\beta)$[16]. Operating on expression (10), we then have

$$\frac{dS_D}{dF} = \frac{pU'(A)(1+R(A)(F-1)(Y-Y_D)(1-\beta)(t_R'+t_P))}{(t_R'+t_P)\left[pU'(A)R(A)(F-1)^2+(1-p)U'(B)R(B)\right]} \geq 0 \qquad (11)$$

An increase in the penalty per unit of tax evaded thus reduces the level of tax evasion in wealth tax, and given congruity, also in personal income tax[17]. The numerator of expression (11) can be disintegrated into an income effect and a substitution effect. On the one hand, this latter effect, *pU'(A)*, includes the increase in the profitability of tax compliance due to the increase in the fine per unit of tax evaded; on the other hand, an income effect, $pU'(A)R(A)(F-1)(Y-Y_D)(1-\beta)(t_R'+t_P)$, is also positive, since the increase in *F* reduces net income of the taxpayer both in state *A* and *B*, and given the assumption of decreasing risk-aversion, this tends to increase the valuation of the marginal cost of tax evasion more than the valuation of the marginal benefit, and thereby increases tax compliance.

In the case of an increase in t_P, operating as above but also making use of the FOC (expression (9)), we obtain the following reaction:

[15] Although in this case, remember that $p=p^Y+p^S$.

[16] This alternative definition of t_R arises from expression (2) when it holds with equality, $Y_D(1-\beta)=S_D$. An increase in S_D (and consequently in Y_D by $1/(1-\beta)$) therefore makes the tax rate of the personal income tax borne by the taxpayer $t_R/(1-\beta)$, and not only t_R, such that $t_R' > t_R$.

[17] Note that, on the one hand, $dY_D/dF=(1/(1-\beta))(dS_D/dF)$, so the total effect of increasing *F* on the amount of tax bases declared is $dS_D/dF(1+(1/(1-\beta)))$. On the other hand, in terms of

$$\frac{dS_D}{dt_P} = \frac{S_D\left[R(A)-R(B)\right]+R(A)F(S-S_D)}{(t_R'+t_P)\left[R(A)(F-1)+R(B)\right]} \geq 0 \qquad (12)$$

since according to the usual assumption about decreasing absolute risk aversion, $R(A) > R(B)$; while in the case of an increase in t_R,

$$\frac{dS_D}{dt_R} = \frac{Y_D\left[R(A)-R(B)\right]+R(A)F(Y-Y_D)}{(t_R'+t_P)\left[R(A)(F-1)+R(B)\right]} \geq 0 \qquad (13)$$

Faced with an increase in any of the tax rates, only an income effect is present, since a rise in the tax rate simultaneously increases the penalty per unit of tax evaded, and thus the substitution effect vanishes (see Yitzhaki, 1974). Note that as long as wealth tax is assigned to one government and personal income tax to another, considering tax evasion in interrelated taxes permits the detection of a tax externality between governments. For instance, according to expression (13), an increase in the tax rate of personal income tax will not only affect the amount of tax base declared in that tax (and so the amount of tax revenue collected), but also that declared in wealth tax. Finally, by comparing (12) and (13), it is easy to verify that $dS_D/dt_R > dS_D/dt_P$ as long as β>0.

Finally, faced with an increase in p,

$$\frac{dS_D}{dp} = \frac{U'(A)(F-1)+U'(B)}{(t_R'+t_P)(1-p)U'(B)\left[R(A)(F-1)+R(B)\right]} \geq 0 \qquad (14)$$

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elasticity, ε, there is no difference between the variation in S_D and the variation in Y_D, i.e., $\varepsilon_{S_D,F} = \varepsilon_{Y_D,F}$.

where only a substitution effect is at work.

From the results of this section, we can conclude that when collaboration between tax administrations is perfect, the results of the comparative statics do not differ from the original results from Allingham and Sandmo (1972) and Yitzhaki (1974). However, as suggested above, it is important to note that as long as tax evasion in interrelated taxes is considered, the statutory tax parameters of either or both taxes (t_R or t_P) or those instruments set by a tax administration (F, p^Y or p^S)[18] simultaneously affect the taxpayer's behavior in both taxes, i.e. we have been able to identify a tax externality. From a social point of view, it therefore seems necessary that those parameters are decided taking into account their effects on both taxes, otherwise their level will not be optimal with respect to the situation in which the tax administration is fully integrated and the power to change the statutory tax parameters is in the hands of just one government. On the whole, to use the terminology of Shoup (1969), the interrelation between tax bases creates a "self-reinforcing penalty system of taxes"[19]. In the numerical simulations of Section 3, we will analyze these issues in more detail in the sense of checking how this system of taxes raises the level of tax compliance.

... When collaboration between tax administrations is imperfect.

If collaboration between tax administrations is imperfect, when the taxpayer is caught

[18] It is true that the value of F is legally set by the political power. However, a tax inspector might vary its value discretionally depending on the development of the tax auditing process. In this regard, see OECD (1990) for a comparison between OECD countries of the divergence between the legal value of F and the real one set by tax auditors.

[19] Note that, as suggested in the introduction, this is quite different from a "self-checking system of taxes" (Kaldor, 1956), since it is not possible to be fully certain that the inferred level of wealth (income) is such that tax evasion is null from the level of income (wealth) declared.

evading taxes only a share of the tax revenue due to the other tax administration is discovered. This might be understood as a low-powered incentive of the tax administration that has audited to collect tax revenue on behalf of the other tax administration[20].

In the case of imperfect collaboration, net income at the disposal of the taxpayer when the tax administration responsible for the personal income tax audits is thus

$$A \equiv Y - Y_D t_R - S_D t_P - F t_R (Y - Y_D) - F t_P \alpha_Y (S - S_D) \tag{15}$$

where α_Y is the percentage of tax evasion discovered in wealth tax, such that $0 \le \alpha_Y < 1$. In the case of perfect collaboration between tax administrations, $\alpha_Y = 1$[21]. Similarly, when the tax administration responsible for the wealth tax audits, net income is

$$D \equiv Y - Y_D t_R - S_D t_P - F t_R \alpha_S (Y - Y_D) - F t_P (S - S_D) \tag{16}$$

where again $0 \le \alpha_S < 1$. Finally, when neither tax administration carries out a tax audit, net income is

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[20] Obviously, if there exists only one tax administration, there might also exist internal inefficiencies within that tax administration. For instance, it could be the case that different departments within the same tax administration - each one of them in charge of a tax or of a group of taxes - might not fully cooperate between them. Then, it would not be necessary to consider the possibility that there were two imperfectly coordinated tax administrations in order that in some occasions the percentage of tax evasion discovered is less than 100%. In any case, although it is not relevant for our analysis, the non-cooperative possibility seems less likely within a tax administration than between two institutionally independent tax administrations.

[21] Given that the percentage of tax fraud discovered depends on the effort carried out by the tax administration (that is, the level of collaboration between tax administrations), α_Y could also be interpreted as the effort of the tax administration in discovering tax evasion on behalf of the other tax administration. For $\alpha_Y = 1$, for example, that level of effort is thus maximum.

$$E \equiv Y - Y_D t_R - S_D t_P \qquad (17)$$

For instance, from (16), as long as $\alpha_S < 1$, it is thus not clear whether an increase in the amount of tax base declared in personal income tax, Y_D, increases the amount of net income at the taxpayer's disposal, since

$$D_{S_D} = t_R'(F\alpha_S - 1) + t_P(F - 1) \gtrless 0 \qquad (18)$$

where it will be remembered that $t_R' = t_R/(1-\beta)$ (see footnote 16). Only as long as $F > [(t_R' + t_P)/(t_R'\alpha_S + t_P)] > 1$, will expression (18) be positive, as in the case in which collaboration is perfect and the tax administration responsible for the wealth tax is auditing. Unlike that situation, in order for marginal net income to increase as a consequence of having reduced the level of tax evasion, it is therefore no longer sufficient that $F > 1$ if $\alpha_S < 1$. Otherwise, as long as $F < [(t_R' + t_P)/(t_R'\alpha_S + t_P)]$, although one of the two tax administrations were auditing, the taxpayer would still obtain marginal increases in net income by evading taxes[22]. As we will confirm, that possibility will partly make the results of the comparative statics ambiguous when collaboration between tax administrations is imperfect. However, before performing the exercise of comparative statics, we again previously need to know whether incongruity or merely congruity, as before, is an optimal strategy for the taxpayer.

[22] An alternative explanation for the negative sign of expression (18) is that the level of t_R' (with respect to t_P) is relatively high, while the level of α_S is low enough and in any case it is not compensated by a large value of *F*.

Is it optimal to be congruent in the declared tax bases?

In the case of imperfect collaboration between tax administrations incongruity might be an optimal strategy for the taxpayer. In order to show such a result, let us analyze the possibility in which $S_D < Y_D(1-\beta)$. The following FOC's should thus hold:

$$\left.\frac{\partial W}{\partial Y_D}\right|_{S_D+C'>Y_D} = p^1_{Y_D}\left[U(A_1)-U(E_1)\right]+t_R\left[p^Y_1(F-1)U'(A_1)+p^S_1(F\alpha_S-1)U'(D_1)-(1-p^Y_1-p^S_1)U'(E_1)\right]>0 \tag{19}$$

such that $U(A_i)<U(E_i)$, and $U'(A_i)>U'(E_i)\ \forall i$, and $p^1_{Y_D}<0$; but also

$$\left.\frac{\partial W}{\partial Y_D}\right|_{S_D+C'=Y_D} = t_R\left[p^Y_2U'(A_2)(F-1)+p^S_2U'(D_2)(F\alpha_S-1)-(1-p^Y_2-p^S_2)U'(E_2)\right]>0 \tag{20}$$

while at the (supposed) optimum, the following two conditions should hold:

$$\left.\frac{\partial W}{\partial Y_D}\right|_{S_D+C'<Y_D} = p^3_{Y_D}\left[U(D_3)-U(E_3)\right]+t_R\left[p^Y_3U'(A_3)(F-1)+p^S_3(F\alpha_S-1)U'(D_3)-(1-p^Y_3-p^S_3)U'(E_3)\right]=0 \tag{21}$$

such that $U(E_i)>U(D_i)$, and $U'(D_i)>U'(E_i)\ \forall i$, and $p^3_{Y_D}>0$; and

$$\left.\frac{\partial W}{\partial S_D}\right|_{S_D+C'<Y_D} = p^3_{S_D}\left[U(D_3)-U(E_3)\right]+t_P\left[p^Y_3U'(A_3)(F\alpha_Y-1)+p^S_3U'(D_3)(F-1)-(1-p^Y_3-p^S_3)U'(E_3)\right]=0 \tag{22}$$

where $p^3_{S_D} < 0$. According to expression (21), at equilibrium, the welfare cost of marginally increasing Y_D caused by a higher level of p, $p^3_{Y_D}[U(D_3)-U(E_3)]<0$, is exactly compensated for by the welfare benefit of increasing tax compliance keeping the tax audit probability constant. However, according to expression (22), the welfare benefit that leads to an increase in S_D due to a lower level of p is compensated by the welfare cost when the tax audit probability remains constant. In any case, note that as long as collaboration between tax administrations is imperfect (see again fn. 20), nothing prevents incongruity from being an optimal strategy for the taxpayer.

In order to ascertain under what circumstances it is more likely that $S_D < Y_D(1-\beta)$ is an optimal strategy from the taxpayer's point of view, we can obtain the following necessary condition using expressions (21) and (22):

$$p^Y_3U'(A_3)(F\alpha_Y-1)+p^S_3U'(D_3)(F-1)<(1-p^Y_3-p^S_3)U'(E_3)<p^Y_3U'(A_3)(F-1)+p^S_3U'(D_3)(F\alpha_S-1) \quad (23)$$

so

$$p^S_3U'(D_3)(1-\alpha_S)<p^Y_3U'(A_3)(1-\alpha_Y) \quad (24)$$

Note that if the tax audit probability functions are symmetric, for $S_D < Y_D(1-\beta)$, $p^S_3 > p^Y_3$. In general, expression (24) implies that all the tax parameters referring to personal income tax have to be more stringent than those referring to wealth tax, i.e. $\alpha_S > \alpha_Y$ and $t_R > t_P$ (and, leaving aside the assumption of symmetry, also $p^Y > p^S$). For instance, if we suppose that $\alpha_Y, \alpha_S < 1$, but $\alpha_Y = \alpha_S$, it can be shown that expression (24) necessarily implies $t_R' > t_P$, since only then $U'(A_3) > U'(D_3)$. Thus, given $p_3^S > p_3^Y$ and supposing $\alpha_Y = \alpha_S$, a necessary condition for it to be optimal for a taxpayer to evade less taxes in personal income tax than in wealth tax is simply that the tax rate of the former is lower than the tax rate of the latter, weighted by the marginal propensity to save. The reduction in disposable income thus has to be relatively greater when the tax administration responsible for personal income tax audits than when the other tax administration does[23]. On the whole, unlike the case of perfect collaboration, S_D and Y_D are no longer perfect substitutes with respect to the optimal decision over tax evasion.

a) … *when it is optimal to be congruent*

Optimal level of tax base declared

In the case of congruity and imperfect collaboration, the FOC is obtained from the following maximization problem:

$$\max_{S_D, Y_D} W' \quad s.t. \quad Y_D = S_D/(1-\beta)$$

where $W' = p^Y A + p^S D + (1 - p^Y - p^S)E$, and *A*, *D* and *E* have been previously defined by

[23] In fact, in our static model where the initial stock of wealth is null (see fn. 5), this seems to be the most plausible assumption, since the tax rates of wealth tax tend to be much lower than those of personal income tax, and in addition, the tax base of the former is just a percentage (1-β) of the tax base of the latter. However, as long as we were dealing with a dynamic model which had made possible the accumulation of a stock of wealth, although the tax rate of wealth tax were lower than the tax rate of personal income tax, the tax base of wealth tax (now a real stock of wealth) could be large enough in terms of current personal income as to make wealth tax a greater burden than personal income tax, and so S_D+C'>Y_D could equally be a plausible optimal strategy for the taxpayer.

expressions (15), (16) and (17), respectively. Now, unlike the case of perfect collaboration, net income is not the same after having audited each tax administration, i.e., $A \neq D$. Once we have substituted Y_D into *W'*, we obtain the following FOC with respect to S_D:

$$\begin{aligned} p^Y U'(A)\{t_R'(F-1)+t_P(F\alpha_Y-1)\}+p^S U'(D)\{t_R'(F\alpha_S-1)+t_P(F-1)\}= \\ =(1-p^Y-p^S)U'(E)(t_R'+t_P) \end{aligned} \quad (25)$$

As long as marginal income is positive in states *A* and *D*, the left-hand side of the equation (first row) can be defined as the marginal cost of evading taxes (or marginal benefit of tax compliance), while the right-hand side (second row) is the marginal benefit of evading taxes (or marginal cost of tax compliance). Nevertheless, as we already know (see, e.g., expression (18)), marginal income is not always positive (i.e., $A_{S_D}, D_{S_D} \lesseqgtr 0$). As long as one of the summands of the first row has a negative sign, it should therefore be considered as a marginal benefit of tax evasion and not as a marginal cost of tax evasion[24].

From now on, we will assume that $S_D<S$, which requires that expression (7) holds, in this case applied to the case of imperfect collaboration between tax administrations,

$$t_R'\{F(p^Y+p^S\alpha_S)\}+t_P\{F(p^S+p^Y\alpha_Y)\}<t_R'+t_P \quad (26)$$

Expression (26) therefore has the same interpretation as the classical condition that

[24] Obviously, it cannot be the case that those two summands are negative at the same time, since then there would not be a solution to the maximization problem.

guarantees an inner solution, i.e. the summation of the penalties expected when the taxpayer decreases tax compliance and is caught evading taxes (the left-hand side of the inequality) is smaller than the certain amount of taxes due when the taxpayer increases tax compliance (the right-hand side of the inequality)[25].

Comparative statics

We will first analyze how the tax base declared, S_D, varies in the face of an increase in F. Nevertheless, given that in this case the exercise of comparative statics is much more cumbersome than when collaboration between tax administrations is perfect, and given that we are only interested in the sign of each reaction, we will make use of the fact that

$$\frac{dS_D}{dF} = \frac{\Phi_F}{-\Phi_{S_D}} \tag{27}$$

where Φ is the FOC of the taxpayer's maximization problem (expression (25)). Since the SOC of the maximization problem does indeed hold[26], $\Phi_{S_D} < 0$, $sign\{dS_D/dF\} = sign\{\partial\Phi/\partial F\}$, we will therefore simply have to calculate the partial derivative $\partial\Phi/\partial F$, and the same for the other parameters of the model. Hence,

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[25] In any case, as expected, the condition given by expression (26) is less stringent than the classical one, $pF<1$. This can be easily shown once expression (26) is re-written as follows:

$$pF - \frac{1}{t_R'+t_P}\left\{t_R' Fp^S(1-\alpha_S) + t_P Fp^Y(1-\alpha_Y)\right\} < 1 \tag{26'}$$

[26] $$\Phi_{S_D} = -p^Y U'(A)R(A)\{t_R'(F-1) + t_P(F\alpha_Y - 1)\}^2 - \\ - p^S U'(D)R(D)\{t_R'(F\alpha_S - 1) + t_P(F-1)\}^2 - (1-p^S-p^Y)U'(E)R(E)(t_R'+t_P)^2 < 0$$

$$\Phi_F = p^S U'(D)\left[t_R'(F\alpha_S - 1) + t_P(F-1)\right]\left(Y - Y_D\right)(1-\beta)\left[R(D)\{t_R'\alpha_S + t_P\} - R(A)\{t_R' + t_P\alpha_Y\}\right] +$$
$$+ (1 - p^Y - p^S)U'(E)(t_R' + t_P)(Y - Y_D)(1-\beta)R(A)\{(t_R' + t_P\alpha_Y\} +$$
$$+ \left\{\frac{(1 - p^Y - p^S)U'(E)(t_R' + t_P) - p^S U'(D)\left[t_R'(F\alpha_S - 1) + t_P(F-1)\right]}{t_R'(F-1) + t_P(F\alpha_Y - 1)}\right\}(t_R' + t_P\alpha_Y) +$$
$$+ p^S U'(D)(t_P + t_R'\alpha_S) \geq 0 \qquad (28)$$

Both a substitution and an income effect mean that the optimal reaction of the taxpayer is unambiguously positive. The first two rows show the latter effect, which is always positive, and thus in favor of increasing S_D. This is so since, on the one hand, in the case in which $t_R'(F\alpha_S - 1) + t_P(F-1) < 0$, $R(A) > R(D)$ and $t_R' + t_P\alpha_Y > t_P + t_R'\alpha_S$, while the reverse is true when $t_R'(F-1) + t_P(F\alpha_Y - 1) < 0 < t_R'(F\alpha_S - 1) + t_P(F-1)$, which ensures the positive sign of the first row. On the other hand, when $t_R'(F\alpha_S - 1) + t_P(F-1) > 0$ and $t_R'(F-1) + t_P(F\alpha_Y - 1) > 0$, the income effect is also positive, which can be easily checked if $\partial\Phi/\partial F$ is analyzed without making use of the FOC. A substitution effect in favor of increasing S_D is shown in the last two rows of expression (28)[27].

In the case of an increase in t_R,

$$\Phi_{t_R} = -p^S U'(D)\left[t_R'(F\alpha_S - 1) + t_P(F-1)\right]\left\{\left[R(A) - R(D)\right]Y_D + \left[R(A) - R(D)\alpha_S\right]F(Y - Y_D)\right\} +$$
$$+ (1 - p^Y - p^S)U'(E)(t_R' + t_P)\left\{\left[R(A) - R(E)\right]Y_D + R(A)F(Y - Y_D)\right\} +$$
$$+ \frac{Ft_P}{t_R'(F-1) + t_P(F\alpha_Y - 1)}\left\{(1 - p^Y - p^S)U'(E)(1 - \alpha_Y) + p^S U'(D)\left[2 - (\alpha_Y + \alpha_S) - F(1 - \alpha_Y\alpha_S)\right]\right\} \overset{<}{\geq} 0 \qquad (29)$$

---

[27] In the third row of expression (28), note that the fraction that appears in brackets is simply $p^Y U'(A)$, which confirms the positive sign of a substitution effect regardless of the sign of marginal income in state *A* and state *D*.

In the first two rows, an income effect appears, while in the third row a substitution effect appears. On the one hand, as long as $\alpha_Y = 1$, a substitution effect always stimulates a decrease in S_D, while when $\alpha_Y < 1$, the sign of this effect is ambiguous. The reason is as follows: if $\alpha_Y = 1$, $\alpha_Y > \alpha_S$ (given the hypothesis of imperfect collaboration), and then faced with an increase in t_R the relative benefit of evading taxes when collaboration is imperfect increases, since in that case tax evasion in personal income tax is not fully discovered in state *D*, while it is in wealth tax. However, if $\alpha_Y < 1$, it is not possible to ascertain the sign of the substitution effect, since $\alpha_Y \overset{\leq}{\underset{>}{=}} \alpha_S$, and the final net effect will also depend on the marginal utility of income in each one of the three possible states (*A*, *D* and *E*)[28]. On the whole, it can be concluded that the lower (higher) the level of collaboration of the tax administration responsible for the wealth tax with regard to the level of collaboration of the other tax administration, the more likely that the increase in t_R will tend to promote tax evasion (compliance). Analysis of tax evasion in interrelated taxes has therefore enabled the identification of a situation (imperfect collaboration between tax administrations) in which the theoretical classical results on tax evasion might fail, i.e. an increase in the tax rate might not produce an increase in tax

[28] In fact, the profitability of diminishing the amount of tax base declared, $r(-S_D)$, can de defined as $r(-S_D) \equiv 1 - F\left\{\frac{t_R'(p^Y + \alpha_S p^S) + t_P(\alpha_Y p^Y + p^S)}{t_R' + t_P}\right\}$, i.e., assuming that the taxpayer is risk-neutral, it is calculated as the marginal income that would be obtained by increasing tax evasion compared to a situation in which tax evasion is null. As a result, $r_{t_R} = \frac{F t_P p}{(1-\beta)(t_R' + t_P)^2}(\alpha_Y - \alpha_S)$, where for simplification we have supposed that $p = p^Y = p^S$. It is therefore clear that as long as $\alpha_S < \alpha_Y$, faced with an increase in t_R, the profitability of increasing tax evasion (i.e., reducing S_D) has increased, $r_{t_R} > 0$, while the reverse happens when $\alpha_S > \alpha_Y$. As in the classical analysis, if $\alpha_S = \alpha_Y$, the substitution effect vanishes regardless of whether or not the degree of collaboration between tax administrations is perfect. Faced with an increase in the tax rate, imperfect collaboration therefore only modifies the profitability of tax evasion as long as both tax administrations do not exert the same level of effort in auditing on behalf of the other tax administration.

compliance.

On the other hand, the net impact of the income effect is more difficult to ascertain due to the ambiguity of the sign of the first row of expression (29) when $t_R'(F-1)+t_P(F\alpha_Y-1)<0$, while the sign of the second row is clearly positive, i.e. in favor of increasing S_D. The reason for this ambiguity is the following: an increase in t_R will certainly diminish net income both in state *A* and in state *D*, and so the valuation of marginal income will have increased. However, as long as $t_R'(F-1)+t_P(F\alpha_Y-1)<0$, marginal increases in S_D under state *A* have to be considered as a marginal cost of tax compliance. Unlike the traditional case, given that the marginal impact of the increase in t_R is greater under state *A* than under state *D*, i.e., $|A_{t_R}|>|D_{t_R}|$, the valuation of the marginal cost of tax compliance has therefore increased more than the valuation of the marginal benefit of tax compliance (i.e., net income in state *D*). Under these circumstances, for $t_R'(F-1)+t_P(F\alpha_Y-1)<0$, a sufficient condition for preventing such an ambiguity is that the valuation of the marginal benefit of tax compliance is large enough with respect to the valuation of the marginal cost of tax compliance such that $\alpha_S R(D)>R(A)$.

On the whole, the sign of expression (29) is not clear-cut, since a substitution and an income effect might have contradictory signs. We are thus back at the ambiguity originally observed by Allingham and Sandmo (1972). Note for instance that if $\alpha_Y=1$, a substitution effect stimulates a decrease in S_D, while an income effect stimulates in the opposite direction, since for $\alpha_Y=1$, there is no ambiguity with regard to the income effect.

In the case of an increase in t_P,

$$\Phi_{t_P} = -p^S U'(D)\left[t_R'(F\alpha_S - 1) + t_P(F-1)\right]\left\{\left[R(A) - R(D)\right]S_D + \left[R(A)\alpha_Y - R(D)\right](F(S - S_D))\right\} + \\ + (1 - p^Y - p^S)U'(E)(t_R' + t_P)\left\{\left[R(A) - R(E)\right]S_D + R(A)F\alpha_Y(S - S_D)\right\} - \\ - \frac{Ft_R'}{t_R'(F-1) + t_P(F\alpha_Y - 1)}\left\{(1 - p^Y - p^S)U'(E)(1 - \alpha_Y) + p^S U'(D)\left[2 - (\alpha_Y + \alpha_S) - F(1 - \alpha_Y\alpha_S)\right]\right\} \lesseqgtr 0 \tag{30}$$

As long as $\alpha_Y = 1$, a substitution effect always provides incentives for increasing S_D, while when $\alpha_Y < 1$, the sign of the substitution effect is ambiguous. The reasoning is identical to the one given above with respect to Φ_{t_R}, although the signs are obviously reversed[29]. In the first two rows, an income effect appears, the sign of which is again ambiguous. In this case, the ambiguity comes from those situations in which $t_P(F-1) + t_R'(F\alpha_S - 1) < 0$, with $\alpha_Y R(A) > R(D)$ being sufficient condition to avoid it.

In the case of an increase in p^Y,

$$\Phi_{p^Y} = U'(A)\left\{t_R'(F-1) + t_P(F\alpha_Y - 1)\right\} + U'(E)(t_R + t_P) \lesseqgtr 0 \tag{31}$$

Only a substitution effect is at work. As long as $t_R'(F-1) + t_P(F\alpha_Y - 1) > 0$, an increase in p^Y always leads to an increase in S_D. Otherwise, the sign is ambiguous. Paradoxically, an increase in p^Y might be welcome by the taxpayer as long as the tax administration dealing with personal income tax does not collaborate to a great extent

[29] Following the methodology used in the previous footnote, $r_{t_P} = \frac{Ft_R' p}{(t_R' + t_P)^2}(\alpha_S - \alpha_Y) \lesseqgtr 0$.

with the other tax administration, and then given the value of the rest of relevant parameters, $t_R'(F-1)+t_P(F\alpha_Y-1)<0$. In that case, the expected profitability of evading taxes will have increased, since the rise in p^Y has made more likely a state in which, even though one tax administration is auditing, the taxpayer can still obtain increases in net income by evading taxes. This is certainly a curious result that stems directly from the absence of perfect collaboration between tax administrations.

Similarly in the case of an increase in p^S,

$$\Phi_{p^S} = U'(D)\{t_R'(F\alpha_S-1)+t_P(F-1)\}+U'(E)(t_R+t_P) \underset{>}{\overset{\leq}{}} 0 \qquad (32)$$

Again, as long as $t_R'(F\alpha_S-1)+t_P(F\alpha-1)>0$, the sign is unambiguously positive. Otherwise, with only a substitution effect being at work, the reason for the ambiguity is the same as the one given above with respect to expression (31).

Finally, we are interested in showing how a reinforcement of the collaboration between tax administrations varies the level of S_D. Firstly, when the tax administration responsible for personal income tax increases its auditing effort with respect to the wealth tax:

$$\Phi_{\alpha_Y} = p^Y U'(A)Ft_P\{1+R(A)(S-S_D)[t_R'(F-1)+t_P(F\alpha_Y-1)]\} \underset{>}{\overset{\leq}{}} 0 \qquad (33)$$

As long as $t_R'(F-1)+t_P(F\alpha_Y-1)>0$, $\Phi_{\alpha_Y}>0$, as otherwise, the sign is ambiguous. A substitution effect always stimulates an increase in tax compliance, $p^Y U'(A)Ft_P>0$, while the sign of an income effect can go either way, depending on the sign of marginal income in state *A*. In the event that $t_R'(F-1)+t_P(F\alpha_Y-1)<0$, a reinforcement of collaboration by the tax administration responsible for personal income tax certainly reduces net income in state *A*, which increases the valuation of a marginal cost of tax compliance. As a consequence of the increase in that marginal valuation, there is an incentive to decrease the level of tax compliance.

Secondly, we analyze the variation in S_D when the tax administration responsible for the wealth tax increases its auditing effort with respect to personal income tax:

$$\Phi_{\alpha_S} = p^S U'(D) F t_R' \{1 + R(D)(S - S_D)[t_R'(F\alpha_S - 1) + t_P(F-1)]\} \begin{matrix} \le \\ > \end{matrix} 0 \qquad (34)$$

If $t_R'(F\alpha_S-1)+t_P(F-1)>0$, $\Phi_{\alpha_S}>0$, as otherwise, the sign is ambiguous. The reason for this ambiguity is identical to that given above with respect to expression (33).

Undoubtedly, the results of the comparative statics concerning collaboration between tax administrations are quite interesting. An increase in collaboration between tax administrations is always a good thing in the sense that it promotes higher levels of tax compliance only as long as marginal net income is positive in all those states where one tax administration is auditing (so, note that it is not strictly necessary that α_i=1). Otherwise, paradoxically, an increase in collaboration between tax administrations might produce a lower level of tax compliance! In Figure 3, on the left-hand side, there is the level of α_S from which an increase in α_S creates a substitution and an income effect that unambiguously promotes tax compliance. Similarly, on the right-hand side, there is the threshold with respect to the level of collaboration of the tax administration responsible for personal income tax, α_Y.

[FIGURE 3]

b) … *when it is optimal to be incongruent*

When collaboration between tax administrations is not perfect and incongruity between tax returns conditions the tax audit probability, the taxpayer might find it optimal not to be congruent. In this section, we will simply try to sketch how this strategy affects the results of the comparative statics, while the methodology of numerical simulation will complement this initial analysis.

Optimal level of tax base declared

We will analyze the case in which $Y_D > S_D/(1-\beta)$. The objective function of the taxpayer does not vary with respect to the previous case. There are thus still three possible states: *A*, *D* and *E* (expressions (15), (16) and (17), respectively), but now the tax audit probability for each of those three states is endogenous to the maximization problem of the taxpayer. Moreover, there are two decision variables: S_D and Y_D. Taking all this into account, the FOC's of the maximization problem are the following:

$$Y_D: \; p_{Y_D}\left[U(D)-U(E)\right]+t_R\left\{p^Y U'(A)(F-1)+p^S U'(D)(F\alpha_S-1)-(1-p^Y-p^S)U'(E)\right\}=0 \qquad (35)$$

$$S_D : -p_{Y_D}\left[U(D)-U(E)\right]+t_P\left\{p^Y U'(A)(F\alpha_Y-1)+p^S U'(D)(F-1)-(1-p^Y-p^S)U'(E)\right\}=0$$

(36)

where $p_{Y_D} > 0$. In fact, given that $Y_D>S_D/(1-\beta)$, an increase in Y_D provokes a greater tax audit probability in wealth tax, $p^S_{Y_D} > 0$; while in the same situation, an increase in S_D brings about a smaller tax audit probability in that tax, $p^S_{S_D} < 0$. However, given the assumption of symmetry, $\left|p^S_{Y_D}\right| = \left|p^S_{S_D}\right|$. This is why, in expression (36), we have used $-p_{Y_D}(<0)$ instead of p_{S_D}, while the super-index *s* has been suppressed for clarity of exposition.

Expression (35) can be rewritten as follows:

$$t_R\left\{p^Y U'(A)(F-1)+p^S U'(D)(F\alpha_S-1)\right\}=-p_{Y_D}\left[U(D)-U(E)\right]+t_R(1-p^Y-p^S)U'(E) \quad (35')$$

On the left-hand side, the marginal benefit of tax compliance appears, and on the right-hand side, the marginal cost of tax compliance. The new feature compared to the case in which congruity is optimal is the additional marginal cost incurred by the taxpayer when tax compliance increases, $-p_{Y_D}\left[U(D)-U(E)\right]>0$. Incongruity implies that an increase in Y_D causes a higher level of *p*, and so a loss of welfare since *U(E)>U(D)*. However, in expression [36], an increase in S_D brings about a higher level of welfare due to the decrease in *p*. Finally, note that as long as $\alpha_S < (1/F)$, the second summand of the left-hand side in expression (35') must be considered to be a marginal cost of tax compliance, and equally in expression (36) for $\alpha_Y < (1/F)$.

Incongruity implicitly prevents $S_D=S$ and $Y_D=Y$ from being an optimal strategy for the taxpayer. In order to obtain an inner solution, the following conditions should thus hold:

$$\left.\frac{\partial W}{\partial S_D}\right|_{S_D=S, Y_D<Y} < 0; \qquad \left.\frac{\partial W}{\partial Y_D}\right|_{S_D=S, Y_D<Y} < 0 \tag{37a}$$

$$\left.\frac{\partial W}{\partial S_D}\right|_{S_D<S, Y_D=Y} < 0; \qquad \left.\frac{\partial W}{\partial Y_D}\right|_{S_D<S, Y_D=Y} < 0 \tag{37b}$$

and from now on, we assume that they hold, meaning that an interior solution is obtained from the taxpayer's maximization problem.

Comparative statics

We will skip the comparative statics analysis corresponding for situation of incongruity due to its difficulty, which is mainly caused by the cross-effects between declared tax bases (Y_D and S_D). Instead, we will carry it out by means of a numerical simulations exercise. However, before that, it might be useful to briefly analyze the main difference with respect to the situation in which congruity is optimal. From expression (35), we thus define the cost of incongruity, K, as

$$K \equiv p_{Y_D}\left[U(E)-U(D)\right] > 0 \tag{38}$$

From this definition, it is easily verifiable that an increase either in α_S, t_R, t_P, F, p_S or in the sensitiveness of this latter variable with respect to incongruity[30] will lead to an increase in the cost of incongruity. Leaving aside the corresponding income and substitution effects, and with the initial situation therefore being one in which $Y_D>S_D/(1-\beta)$, the rise in K should lead to a reduction in Y_D and/or an increase in S_D, i.e., a decrease in the level of incongruity. Another new effect at work is a substitution effect between the tax bases declared. Given that they are independently decided, as long as one parameter exclusively affects one tax base (e.g., the statutory tax rate), it will tend to encourage an increase/decrease in tax compliance with that tax base compared to the other one. When interpreting the results of the numerical simulations, these two new effects will therefore have to be taken into account join with the income and substitution effects already identified in the theoretical analysis.

3. Numerical simulations

The methodology of numerical simulations must be helpful in addressing some key issues that were not totally solved by means of the theoretical analysis. Among those issues are the following:

- Does the approach of considering tax evasion in interrelated taxes overcome, at least partially, the paradox of tax evasion?

- Given this theoretical approach and considering the possibility of imperfect collaboration between tax administrations, in which circumstances is incongruity between tax returns an optimal choice for the taxpayer?; and finally,

[30] Later, in the numerical simulation exercises, such sensitivity will be denoted by h.

- The numerical simulations should be helpful in solving the inconclusive results of the analytical comparative statics.

In order to carry out the numerical simulations, we will employ the following well-known iso-elastic utility function:

$$U(Y_N) = \frac{Y_N^{1-\sigma}}{1-\sigma}, \sigma \neq 1 \tag{39}$$

where $\sigma(>0)$ is the coefficient of relative risk-aversion, and Y_N is net income after paying taxes, and in the presence of tax evasion is also the corresponding fine per unit of tax evaded. The greater the value of σ, the greater the degree of risk-aversion. According to the economic literature, a reasonable value of this parameter is 1.8 (see Karni and Schmeidler, 1990; Epstein, 1992).

The remaining values given to the basic parameters of the model are the following:

$$Y = 1; \beta = 0.8; S = 0.2; t_R = 0.5; \ t_P = 0.005; F = 2$$

The aim of these numerical simulations is not to replicate any real situation. That is why the values of the above parameters do not necessarily reflect those of any potentially average taxpayer. However, the value of the tax audit probability will be obtained from the model in such a way that the equilibrium values of tax evasion (Y_D/Y and S_D/S) range within a reasonable interval, as we will confirm below.

In the presence of incongruity, e.g. $Y_D(1-\beta) > S_D$, the tax audit probability of the wealth tax will adopt the following function:

$$p^S = \bar{p}^S \times exp\left[h \times (Y_D(1-\beta) - S_D)\right] \qquad (40)$$

where h>0. In the presence of incongruity, the higher the value of h, the higher the value of p^S above its normal level ($\bar{p}^S$). Similarly, in the case that $Y_D(1-\beta) < S_D$,

$$p^Y = \bar{p}^Y \times exp\left[h \times (S_D - Y_D(1-\beta))\right] \qquad (40')$$

In order to make the impact of wealth tax on net income significant in money terms, apart from the increase in wealth due to annual savings (S), we have assumed that at the beginning of the fiscal year the taxpayer owned an initial amount of wealth, S_0 (>0) (see fn. 5). Therefore, leaving tax evasion aside, the budget constraint becomes as follows

$$Y - Y_D t_R - (S + S_0)t_P \qquad (41)$$

Throughout all the numerical simulations, we will suppose that S_0=2[31]. Moreover, in order to keep things as simple as possible and thus focusing exclusively on the relationship between S and Y, we will assume that the taxpayer always declares the whole amount of the initial stock of wealth[32].

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[31] This implies that the initial stock of wealth subject to taxation is double current income. That seems a reasonable assumption, once we take into account that the tax law usually allows the deduction of a certain sum of money in the calculus of the tax base. S_0 must thus be considered as the initial stock of wealth after this sum of money has already been deducted.

[32] This assumption will prove extremely useful in the numerical simulations in order to isolate an income effect.

Paradox of tax evasion

The traditional analysis of tax evasion predicts very low levels of tax compliance, a situation that does not seem to hold true in practice. As we said in the introduction, in order to try to overcome this paradox, the literature on tax evasion has proposed several alternative explanations. It is in this context that we propose a new one. We therefore postulate that considering tax evasion in interrelated taxes can at least partially, help to solve this paradox. In fact, intuitively it seems that as long as the tax instruments of the interrelated taxes are (relatively) coordinated, they should positively interact with each other, making tax evasion less attractive (as we know, this is the idea which bases the so-called "self-reinforcing penalty system of taxes" due to Shoup (1969)).

[TABLE 1a]

In Table 1a, we show the first results of this exercise of numerical simulation. We have characterized a situation with tax evasion both in the personal income tax, Y_D=0.8 given Y=1, and in the wealth tax, S_D=0.16 given S=0.2, and in order to facilitate comparison with previous results of the literature, declared tax bases are congruous for $\beta = 0.8$. Next, we obtained the value of the audit probability compatible with this level of tax evasion, supposing that the taxpayer aims at maximizing the utility function (39). However, the results of the numerical simulation certainly depend on the assumptions regarding the context of tax evasion. Under the label *Classical analysis*, the model of tax evasion employed coincides with the original model by Allingham and Sandmo (1972), and thus each decision of tax evasion is considered separately. The maximization problem is therefore solved for each tax, such that p^Y and p^S are obtained given the values of the basic parameters of the model[33]. Since each decision is considered separately, there is no reason to treat both events (auditing of the personal income tax return and auditing of the wealth tax return) as mutually exclusive. That is why, the probability of occurrence of either event is calculated as $(p^Y+p^S)-(p^Y\times p^S)$. In the other simulated situations, tax evasion in interrelated taxes is the behavior under analysis (*Interrelated Evasion*), having analyzed, firstly, the situation in which collaboration between tax administrations is perfect; and secondly, the situations in which collaboration is imperfect.

In the *Classical analysis*, in order to ensure the aforementioned levels of tax compliance, the sum of tax audit probabilities has to be as high as 0.6625, while in the case of *Interrelated Evasion* and perfect collaboration, that level is "only" 0.3212[34]. Nonetheless, as long as collaboration is imperfect, the tax audit probability might be higher or lower than the value obtained in the *Classical analysis*. Taxes might therefore interact negatively with each other, leading to a decrease in the level of tax compliance as long as collaboration is imperfect. This prevents Shoup (1969)'s idea of the "self-reinforcing penalty system of taxes" from being universal, since it depends on the degree of collaboration between tax administrations. This negative possibility does not therefore come about when either tax administration is carrying out the maximum level of collaboration ($\alpha_i = 1$), and becomes a case that is identical to one of perfect collaboration. Table 2a illustrates the same cases as Table 1a, but for a situation of full tax compliance (i.e., $Y_D/Y = 1$ and $S_D/S = 1$).

[33] The method used to solve the system of non-linear equations is the so-called "Gauss-Newton".

[34] As we know from the theoretical analysis, when collaboration between tax administrations is perfect, we only have *p*, and so from the numerical simulations it is not possible to ascertain the

[TABLE 2a]

The values obtained for tax audit probabilities that are shown in both tables are certainly very high, and are obviously highest in Table 2a. For example, Bernasconi (1998), pp. 127-6, argues that in order to be in line with those in force in many countries, the individual tax audit probabilities should range from 0.01 to 0.03, whereas 0.09 might be the average for USA taxpayers (Harris, 1987). From the results of our numerical simulations, we should therefore conclude that *Interrelated Evasion* does not solve the paradox of tax evasion, since in order to ensure full tax compliance, p (defined as p^Y+p^S) has to be as high as 0.5 when collaboration is perfect (Table 2a), and 0.3212 in order to guarantee a level of tax compliance of 80% (Table 1a). Nevertheless, there is another way to read our results, which is by comparing those absolute values with those obtained in the *Classical analysis*, 0.75 and 0.66, respectively. Our approach might thus be considered as a partial explanation to the paradox of tax evasion, since our tax audit probabilities are around half those predicted by the *Classical analysis*.

[TABLE 1b]

[TABLE 2b]

Bernasconi (1998) also carried out a numerical simulation exercise in order to check whether his theory of over-weighted tax audit probabilities for taxpayers - which can be justified once different orders of risk aversion are distinguished - was able to overcome the paradox of tax evasion. In order to compare his results with ours, in Table 1b and

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value of p^Y and p^S, but only p^Y+p^S.

Table 2b, we have modified the value of the basic parameters of the previous numerical simulation. Now, $t_R = 0.3$; $t_P = 0.002$ (which might be considered as a low bound of the range of reasonable values of t_P), and F=4, which are the same values as those used by Bernasconi (1998) with the obvious exception of t_P[35]. In this case, the equilibrium values of the tax auditing probabilities are much lower. For instance, if we merely pay attention to the value of p^Y, for levels of tax compliance of 80%, when collaboration (between taxes or tax administrations) is symmetric and above 0.5, we can see that it lies within a relatively reasonable interval (0.096 to 0.0755), and is in any case much lower than in the *Classical analysis* (0.1441)[36]. Moreover, in this latter analysis, p^S should be as high as 0.2499, while in the former it should be between 0.1398 and 0.0820. In fact, although this result does not appear in Table 1b, in the case of *Interrelated Evasion*, a level of tax compliance of 60% is compatible with auditing probabilities in each tax as low as 0.03. Additionally, the necessary values of tax auditing probabilities are decreasing in S_0 (in our case, this is exclusively due to an income effect), so for large fortunes (i.e., those taxpayers with a high S_0/Y ratio), the tax

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[35] In fact, Bernasconi (1998) set F=3. However, he expressed net income when the taxpayer is audited as

$$Y - Yt_R - F' t_R (Y - Y_D)$$

Given our different way of expressing net income, it is therefore obvious that in our case $F=F'+1$.That is why, using F=4, and given the rest of values of the basic parameters, we are exactly replicating Bernasconi (1998)'s simulations.

[36] Note that the tax audit probabilities rated by Bernasconi (1998) as reasonable, which range from 0.01 to 0.03, are an average for taxpayers as a whole. These average values should thus be perfectly compatible with much higher (and lower) point values. In this sense, it could be the case that those taxpayers that submit a wealth tax return were audited in personal income tax more often than any other taxpayer, i.e. it could be the case that their "normal" tax audit probability (before considering the possibility of incongruity between tax bases) were above those average values. Once we take this possibility into account, a tax auditing probability of around 0.07 or even slightly above might not be too far from reality.

auditing probabilities should be even lower than the values shown in tables[37].

On the whole, from the results of our numerical simulations, it should be concluded that considering tax evasion in interrelated taxes permits the paradox of tax evasion to be partially overcome, since reasonable levels of tax evasion are compatible with relatively low values of the tax auditing probabilities. However, it is very important to note that this result is only valid as long as there is a significant degree of collaboration between tax administrations.

Incongruity

The consideration of tax evasion in interrelated taxes can produce an interesting result. As long as collaboration between the tax auditors responsible for each tax is not perfect, the tax bases declared in each tax return might not be congruous. This result has already been shown in the theoretical part of the paper. However, the numerical simulations should still provide us more information. In particular, they should first be useful in indicating which situation is more likely, $Y_D(1-\beta) \lessgtr S_D$, and secondly, which combination of values of the basic parameters of the model can produce incongruity.

The benchmark case will be that of Table 1a in which for $\alpha_Y = \alpha_S = 0.75$, $p^Y = 0.1848$

[37] For instance, for a taxpayer which initial stock of wealth (S_0) is 100, maintaining the rest of values equal to those in Table 1b, p^Y+p^S=0.1083. However, given that the wealth tax is a progressive tax, we would expect t_P to be higher than 0.002. Then, for example, for t_P=0.005, p^Y+p^S=0.0028!

and $p^S = 0.2296$ (i.e., p^Y/p^S=0.8049)[38], and tax bases are congruous for β=0.8. Based on these initial values, we then ask which new combination of tax audit probabilities should hold in order for incongruity to become an optimal choice for the taxpayer. In Table 3, first we find those situations under which $Y_D(1-\beta)>S_D$. As expected, while maintaining the rest of parameters constant, that type of situation is only compatible with a level of tax enforcement by the tax administration responsible for personal income tax that is much higher in relative terms (remember that in this situation p^S has to be calculated by means of expression (40)). Moreover, the greater the level of *h*, the greater this difference in the relative degree of tax enforcement has to be. Secondly, as also shown in Table 3, in the reverse situation, $Y_D(1-\beta)<S_D$, those differences – now, in favor of p^S, while p^Y has to be calculated by means of expression (40') - have to be even much more acute. In order for the taxpayer to find it optimal to be incongruent, keeping the rest of tax parameters unchanged, there must therefore be great differences in the relative level of tax enforcement, especially in the situation in which $Y_D(1-\beta)<S_D$.

[TABLE 3]

Next, we carried out the same exercise in Table 4, but this time to detect likely differences in the degree of collaboration. Again, in order for $Y_D(1-\beta)>S_D$ to become an optimal strategy for the taxpayer, the endogenous parameter for personal income tax, α_S, has to be greater than the one for wealth tax, α_Y. This difference must also increase

[38] As we already know, these values seem quite high in comparison with those in force in many countries. However, for the purposes of this section, this is not an important issue, since what we are really interested in is in the relative differences of tax enforcement necessary in order for incongruity to become an optimal strategy for the taxpayer. In any case, note that as long as we set *F*=4, as Bernasconi (1998) did (see also fn. 35), the tax audit probabilities would be much lower, p^Y=0.0821 and p^S=0.1001 (see Table 1b).

as the value of *h* increases. Curiously, the smaller the value of α_Y, the greater the value of α_S. This result, which might seem counterintuitive, can be easily understood from expression (24). In this expression, while keeping the other parameters unchanged, a decrease in α_Y should certainly permit a smaller value of α_S such that the sign of the inequality could still hold. However, note that as long as α_Y decreases, marginal utility in state *A* decreases as well, while marginal utility in state *D* remains unchanged. The combination of those facts might make thus an even greater value of α_S necessary. Finally, in Table 4, we can check that the situation $Y_D(1-\beta)<S_D$ is not compatible with differences in the degree of tax enforcement that remain within the boundaries of α_Y, since both for α_S=0.75 and α_S=0.25, the value of α_Y in those cases should be above 1.

[TABLE 4]

In conclusion, the results of these numerical simulations confirm those already obtained in the theoretical analysis i.e. incongruity as an optimal strategy for the taxpayer is only possible as long as the level of tax parameters of each tax is sufficiently different. Moreover, these differences must be more acute as the tax audit probabilities become more sensitive to the degree of incongruity. From the analysis of the numerical simulations, it is possible to infer that incongruity is much more likely as long as there are differences in the degree of collaboration (which not only produces a substitution effect between states, but also an income effect), since otherwise the differences in the tax audit probabilities probably have to be too sharp to hold in practice. In any case, note that if we focus either on the ratio p^Y/p^S or on the ratio α^S/α^Y, the most likely situation is one in which $Y_D(1-\beta)>S_D$. It is precisely this situation that will be analyzed in the following exercises of comparative statics.

Comparative statics

The numerical simulation exercise will prove extremely useful for analyzing the results of the comparative statics in the case of incongruity. However, before that exercise, and despite all the signs of the comparative statics being perfectly clear from the theoretical analysis, in Table 5, we show the results in the case of *Interrelated Evasion* and perfect collaboration such that $Y_D(1-\beta)=S_D$. In the first column, the values of the variable on which the exercise of comparative statics is based appear; in the second and third column, the equilibrium values of the tax bases declared; in the fourth, the percentage of tax compliance, while in the fifth column this percentage is expressed in money terms with respect to the amount of money that would be collected in the presence of full tax compliance[39]. Finally, in the sixth column, we have calculated expected net income, after paying taxes and, in the presence of tax evasion, the corresponding fine per unit of tax evaded[40].

[TABLE 5]

In fact, the only new results that appear in Table 5 are the comparative statics with respect to σ and S_0. In both cases, the sign is also positive. An increase in S_0 produces a reduction in net income in all the states, which given the assumption of decreasing risk aversion forces the taxpayer to increase tax compliance. Obviously, an increase in σ

[39] That is, $(((S_D+S_0)\times t_P)+(Y_D\times t_R))/(((S+S_0)\times t_P)+(Y\times t_R))$.

[40] Note that due to $S_0>0$, in some cases nothing prevents net income from being negative.

automatically generates the same reaction by the taxpayer.

In the Appendix, we have included the whole set of numerical simulations carried out for a situation in which $Y_D(1-\beta)>S_D$. The structure of the tables is the same as in Table 5, with the exception of two new definitions, which are the ratio between p_S and the "normal" level of p^S, $\bar{p}^S$; and the level of incongruity, $((1-\beta)-(S_D/Y_D))/(1-\beta)$.

The values of the basic parameters used in the numerical simulations make marginal income in states *A* and *D* always positive. On the one hand, the income effect detected in the theoretical analysis will therefore always favor an increase in tax compliance. On the other hand, a substitution effect always had a clear impact in favor of increasing tax compliance with the exception of those cases in which t_R or t_P varied. This latter ambiguity was caused by potential discrepancies in the level of collaboration between tax administrations (see, e.g., fn. 28). In the numerical simulations, we will see to what extent that potential situation can produce a decrease in tax compliance. These basic results apply in the case of congruity. Nonetheless, in the case of incongruity, because of the cross-effects between S_D and Y_D, it may be the case that despite global tax compliance increasing, either of the declared tax bases decreases. In any case, it should be recalled that the effect that occurs through variations in the cost of incongruity, *K*, also has to be taken into account as long as $h>0$.

As can be seen from tables A.5 and A.4, an increase in *h* or in p^S, respectively, causes a higher level of unweighted tax compliance through a small reduction in Y_D and an important increase in S_D. In both cases, the cost of incongruity has augmented, calling for an increase in the ratio S_D/Y_D. Moreover, the increase in that cost has occurred by means of an increase in the effective tax audit probability of the tax administration responsible for the wealth tax, making tax compliance in that tax relatively more attractive. Both effects therefore stimulate increasing S_D over Y_D. Obviously, this increase is greater, as the value of α_Y increases.

In Table A.10, it is interesting to analyze the consequences of an increase in S_0. Given that S_0 is fully declared, only an income effect is at work. Such an income effect causes an increase in the level of tax compliance weighted by the relative importance of the tax burden of each tax (in the table, denoted by $(S_D+Y_D)r$). This increase in the level of tax compliance is achieved by means of a decrease in the ratio S_D/Y_D. This result will therefore be useful in comparative static analysis where an income effect arises.

An increase in α_Y (Table A.8) or in α_S (Table A.7) provokes the same effect both on total tax compliance and on each one of the tax bases declared than in the case of an increase in h or in p^S. However, the reasoning is not exactly the same as the one given above. On the one hand, an increase in α_Y does not modify the cost of incongruity, since it does not change the value of marginal income neither in state D nor in state E. However, it certainly makes it more attractive to increase S_D compared to Y_D, while at the same time generating an income effect in favor of increasing total tax compliance (in particular, as we already know that occurs through increases in Y_D). In this case, a substitution effect between tax bases therefore prevails over an income effect. On the other hand, an increase in α_S makes incongruity more costly, thereby stimulating an increase in the ratio $S_D/Y_{D.}$ But, at the same time, both an income effect and a substitution effect between tax bases provide incentives for in the contrary direction. In this case the increase in the cost of incongruity therefore overcomes the impact of the latter two effects.

In the remaining cases, the results of the comparative statics show an incentive to increase Y_D and to decrease S_D. In Table A.3, we can see how an increase in p^Y causes a substitution effect in favor of increasing tax compliance, and in particular decreasing the ratio S_D/Y_D, while the cost of incongruity remains unchanged. The results concerning F are shown in Table A.9. An increase in F intensifies the cost of incongruity, thereby promoting a rise in the ratio S_D/Y_D. However, a substitution and an income effect in favor of increasing tax compliance predominate in the sense that this increase in tax compliance is achieved by a reduction in the ratio S_D/Y_D.

Finally, an increase in any one of the statutory tax rates generates an income effect, which as we know favors a decrease in S_D/Y_D, although total weighted tax compliance increases (tables A.1 and A.2). Moreover, in both cases, the cost of incongruity raises, thereby promoting an increase in S_D/Y_D. Given that the degree of collaboration between tax administrations is quite similar, we do not expect a substitution effect in favor of decreasing total tax compliance (see expressions (29) and (30)), but only a relative increase in the level of tax compliance of the tax base that has suffered an increase in its tax burden (i.e., a substitution effect between tax bases declared). In the particular case of t_R, the decrease in S_D/Y_D is therefore a consequence of the predomination of an income effect and a substitution effect between tax bases over the increase in the cost of incongruity; while in the case of t_P an income effect prevails over the other two effects.

4. Conclusions

The objective of this paper has been to analyze the consequences of considering the decision of tax evasion as a decision in which interrelated taxes (e.g., at the individual level, personal income tax and wealth tax, but a similar analysis could also be applied to the case of corporations, given the evident relationship between the VAT and corporate tax) interact with each other, and so the optimal level of tax compliance from the point of view of the taxpayer might differ with respect to those analyses in which tax evasion is considered tax by tax. In particular, the source of interaction arises from the information available to the tax administration as long as it compares the tax bases declared in each tax return. Given the evident relationship between tax bases (e.g., the increase in the wealth tax base with respect to the previous fiscal year should be a reasonable proportion of current income), that comparison should make any incongruity evident, and so it could be a hint to start an auditing process. In the paper, such an incongruity produces an increase in the auditing probability of the tax with a tax base that has supposedly been under-declared. However, given the possibility that those interrelated taxes were audited by different tax administrations or within a tax administration by different departments, we have also analyzed those situations in which collaboration between different tax administrations or between different departments of the same tax administration is imperfect. In the extreme case in which collaboration is null, this means that each tax administration might certainly obtain valuable information from comparing tax returns, but it does not make any effort to enforce tax obligations on behalf of the other tax administration/department when carrying out its own tax audits. On the contrary, perfect collaboration exactly replicates that situation in which there is a single tax administration or just one department responsible for both taxes.

The idea of interaction between taxes as a means of reinforcing tax compliance was suggested by Shoup (1969), but it had never been formally developed. Moreover, taking the possibility of imperfect collaboration into account can certainly affect the originally expected results. In the case of perfect collaboration between tax administrations, our theoretical analysis shows that congruity between tax bases is the only optimal decision for the taxpayer, while the results of the comparative statics do not vary from the classical analysis (Allingham and Sandmo, 1972; Yitzhaki, 1974). In that context, as expected, interrelated taxes slightly reinforce each other, making it possible to achieve higher levels of tax compliance maintaining the level of tax enforcement constant[41]. However, that result – which has been obtained from a numerical simulation exercise - might be reversed for low levels of collaboration. This type of analysis has also enabled us to shed some new light on the paradox of tax evasion. By also employing the methodology of numerical simulations, we have shown that using values of the penalty per unit of tax evaded equal to those previously used by the literature (Bernasconi, 1998), it is possible to obtain relatively reasonable values of the levels of tax enforcement compatible with reasonable values of tax compliance. Nonetheless, that result is again crucially dependent on the existence of high levels of collaboration between tax administrations. Interrelated tax evasion might be at least considered as a partial explanation to the paradox of tax evasion.

Indeed, the degree of collaboration between tax administrations becomes crucial in the

[41] For example, note that in Table 1a, 1b, 2a and 2b in the presence of perfect collaboration, lower levels of tax enforcement are compatible with equal levels of tax compliance in the situation of interrelated tax evasion and in the classical analysis. Keeping the same level of tax enforcement, in the former situation tax compliance will thus be greater than in the latter one.

results concerning tax evasion in interrelated taxes. In fact, when collaboration is imperfect, our theoretical analysis has shown that incongruity might be an optimal choice for a rational taxpayer. The direction of the incongruity depends on the relative importance of the tax parameters of each tax. We should thus expect relatively lower levels of tax compliance in those taxes in which the tax parameters (including the tax auditing probability and the level of collaboration of the other tax administration) are relatively less important. For example, in the case of personal income tax and wealth tax, we expect the level of tax compliance to be more important in the former tax than in the latter[42].

The results of the comparative statics are also crucially affected by the degree of collaboration. In the case of imperfect collaboration, theoretical analysis does not provide clear-cut results, and the methodology of numerical simulations becomes fundamental in ascertaining its effects. Moreover, this analysis depends on whether congruity or incongruity is the optimal decision of the taxpayer. Our numerical simulations have been performed for what is probably the most interesting case, i.e. the one in which incongruity is optimal. As a general result, it is worth mentioning that there is no tax policy that promotes tax compliance with both taxes at the same time. Additionally, although as stated in the introduction, our aim is not to characterize the optimal policies of a tax administration, it is also interesting to stress that on some occasions the incentives for a tax administration to carry out certain policies are null. For instance, we have found that as long as the tax administration responsible for personal income tax strengthens its collaboration with the other tax administration, the

[42] For example, although it does not appear in Table 4, the impossibility of $Y_D(1-\beta)<S_D$ is independent of the level of S_0., i.e. even for very big values of S_0, α_Y is still above 1.

level of tax compliance in that tax decreases, while it increases in wealth tax (see Table A.8). We should therefore see very low levels of collaboration on that tax administration's side. Precisely the reverse incentives hold for the other tax administration (see Table A.7). This analysis thus predicts an asymmetric degree of collaboration, which is null for the tax administration responsible for personal income tax and maximum for the tax administration responsible for wealth tax. Finally, the presence of imperfect collaboration, unlike the case of perfect collaboration, might produce negative externalities between tax administrations. For instance, an increase in the statutory tax rate of personal income tax promotes a higher level of tax compliance in that tax, but at the same time a decrease in the level of tax compliance in wealth tax. On the whole, all these results call for an integration of all the tax auditing processes, or as long as the responsibilities of auditing different taxes are assigned to different layers of government, they call for a high level of mutual collaboration between tax administrations.

5. Appendix

COMPARATIVE STATICS
(*imperfect collaboration*)

Table A.1.

t_R	S_D	Y_D	S_D+Y_D	(S_D+Y_D)r	$p_S / \bar{p}_S$	Incongruity	E(Y_N)
				$\alpha_Y = 0.73$			
0.41	0.1415	0.7101	0.7097	0.7169	1.0007	0.0035	0.6316
0.46	0.0936	0.7647	0.7153	0.7691	1.0931	0.3881	0.5778
0.50	0.0499	0.8006	0.7088	0.8034	1.1799	0.6887	0.5348
0.54	0.0011	0.8312	0.6935	0.8328	1.2812	0.9937	0.4918
0.57	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.61	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0.75$			
0.41	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.46	0.1438	0.7641	0.7566	0.7690	1.0135	0.0587	0.5778
0.50	**0.1000**	**0.8000**	**0.7500**	**0.8033**	**1.0942**	**0.3749**	**0.5349**
0.54	0.0511	0.8306	0.7348	0.8327	1.1884	0.6925	0.4919
0.57	0.0037	0.8540	0.7148	0.8550	1.2848	0.9782	0.4543
0.61	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0.77$			
0.41	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.46	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.50	0.1544	0.7993	0.7948	0.8032	1.0082	0.0340	0.5349
0.54	0.1054	0.8300	0.7795	0.8326	1.0952	0.3653	0.4919
0.57	0.0579	0.8534	0.7594	0.8550	1.1844	0.6609	0.4543
0.61	0.0056	0.8741	0.7332	0.8748	1.2890	0.9679	0.4167

n.s.: no solution

Table A.2.

t_P	S_D	Y_D	S_D+Y_D	(S_D+Y_D)r	$p_S / \bar{p}_S$	Incongruity	E(Y_N)
				$\alpha_Y = 0.73$			
0.0043	0.1485	0.7987	0.7893	0.8020	1.0170	0.0703	0.5351
0.0046	0.1043	0.7995	0.7532	0.8026	1.0870	0.3476	0.5350
0.0050	0.0512	0.8006	0.7098	0.8034	1.1776	0.6805	0.5348
0.0053	0.0119	0.8014	0.6778	0.8040	1.2492	0.9256	0.5347
0.0058	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.0063	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0.75$			
0.0043	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.0046	0.1546	0.7989	0.7946	0.8025	1.0078	0.0324	0.5350
0.0050	**0.1000**	**0.8000**	**0.7500**	**0.8033**	**1.0942**	**0.3749**	**0.5349**
0.0053	0.1013	0.8000	0.7511	0.8033	1.0920	0.3667	0.5349
0.0058	0.0620	0.8008	0.7190	0.8039	1.1587	0.6130	0.5347
0.0063	0.0034	0.8022	0.6713	0.8049	1.2657	0.9789	0.5345
				$\alpha_Y = 0.77$			
0.0043	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.0046	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.0050	0.1557	0.7993	0.7958	0.8032	1.0062	0.0258	0.5349
0.0053	0.1163	0.8001	0.7637	0.8038	1.0678	0.2733	0.5348
0.0058	0.0575	0.8015	0.7158	0.8048	1.1667	0.6413	0.5345
0.0063	0.0038	0.8028	0.6723	0.8058	1.2650	0.9760	0.5343

n.s.: no solution

Table A.3.

p_Y	S_D	Y_D	S_D+Y_D	$(S_D+Y_D)r$	$p_S\ /\ \bar{p}_S$	*Incongruity*	$E(Y_N)$
			$\alpha_Y = 0.73$				
0.288	0.1524	0.7628	0.7627	0.7674	1.0003	0.0011	0.5485
0.302	0.1058	0.7807	0.7387	0.7845	1.0785	0.3226	0.5418
0.318	0.0492	0.8008	0.7083	0.8036	1.1810	0.6926	0.5347
0.330	0.0040	0.8158	0.6832	0.8178	1.2696	0.9754	0.5299
0.343	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.355	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
			$\alpha_Y = 0.75$				
0.288	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.302	0.1559	0.7801	0.7800	0.7844	1.0001	0.0005	0.5418
0.318	0.0994	0.8002	0.7497	0.8035	1.0953	0.3790	0.5348
0.330	0.0542	0.8151	0.7244	0.8177	1.1774	0.6677	0.5299
0.343	0.0019	0.8312	0.6943	0.8329	1.2796	0.9886	0.5251
0.355	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
			$\alpha_Y = 0.77$				
0.288	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.302	0.1538	0.7995	0.7944	0.8034	1.0092	0.0382	0.5348
0.318	0.1086	0.8145	0.7692	0.8176	1.0849	0.3334	0.5300
0.330	0.0563	0.8305	0.7390	0.8327	1.1790	0.6609	0.5252
0.343	0.0044	0.8452	0.7080	0.8466	1.2802	0.9741	0.5211
0.355	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.

n.s.: no solution

Table A.4.

p_S	S_D	Y_D	S_D+Y_D	$(S_D+Y_D)r$	$p_S\ /\ \bar{p}_S$	*Incongruity*	$E(Y_N)$
			$\alpha_Y = 0.73$				
0.0041	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.0044	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.0048	0.0080	0.8009	0.6741	0.8033	1.2563	0.9498	0.5349
0.0051	0.0473	0.8006	0.7066	0.8034	1.1844	0.7046	0.5348
0.0056	0.1079	0.8002	0.7568	0.8036	1.0814	0.3259	0.5347
0.0059	0.1417	0.8000	0.7848	0.8037	1.0279	0.1145	0.5347
			$\alpha_Y = 0.75$				
0.0041	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.0044	0.0017	0.8007	0.6687	0.8030	1.2682	0.9891	0.5350
0.0048	0.0582	0.8003	0.7153	0.8032	1.1652	0.6367	0.5349
0.0051	0.0975	0.8000	0.7479	0.8033	1.0984	0.3909	0.5349
0.0056	0.1581	0.7996	0.7981	0.8035	1.0027	0.0114	0.5348
0.0059	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
			$\alpha_Y = 0.77$				
0.0041	0.0102	0.8004	0.6755	0.8028	1.2520	0.9361	0.5351
0.0044	0.0561	0.8000	0.7134	0.8029	1.1687	0.6496	0.5350
0.0048	0.1125	0.7996	0.7602	0.8031	1.0737	0.2964	0.5349
0.0051	0.1519	0.7993	0.7927	0.8032	1.0121	0.0500	0.5349
0.0056	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.0059	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.

n.s.: no solution

Table A.5.

h	S_D	Y_D	S_D+Y_D	$(S_D+Y_D)r$	$p_S / \bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_Y = 0.73$			
1.24	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1.33	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1.41	0.0007	0.8012	0.6683	0.8035	1.2523	0.9956	0.5348
1.51	0.0548	0.8005	0.7128	0.8034	1.1723	0.6576	0.5348
1.64	0.1117	0.7998	0.7597	0.8033	1.0823	0.3016	0.5349
1.77	0.1570	0.7992	0.7969	0.8031	1.0050	0.0177	0.5349
				$\alpha_Y = 0.75$			
1.24	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1.33	0.0055	0.8012	0.6722	0.8035	1.2285	0.9659	0.5348
1.41	0.0540	0.8006	0.7121	0.8034	1.1615	0.6630	0.5348
1.51	0.1046	0.7999	0.7538	0.8033	1.0872	0.3459	0.5349
1.64	0.1577	0.7993	0.7975	0.8032	1.0035	0.0134	0.5349
1.77	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0.77$			
1.24	0.0058	0.8012	0.6724	0.8035	1.2111	0.9641	0.5348
1.33	0.0666	0.8004	0.7225	0.8034	1.1324	0.5839	0.5348
1.41	0.1117	0.7999	0.7597	0.8033	1.0704	0.3016	0.5349
1.51	0.1587	0.7993	0.7983	0.8032	1.0017	0.0071	0.5349
1.64	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1.77	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.

n.s.: no solution

Table A.6.

σ	S_D	Y_D	S_D+Y_D	$(S_D+Y_D)r$	$p_S / \bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_Y = 0.73$			
1.50	0.1518	0.7605	0.7603	0.7652	1.0005	0.0020	0.5418
1.64	0.1022	0.7809	0.7359	0.7847	1.0843	0.3456	0.5383
1.79	0.0530	0.7995	0.7104	0.8024	1.1739	0.6685	0.5350
1.96	0.0015	0.8172	0.6823	0.8192	1.2749	0.9907	0.5319
2.14	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
2.35	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0.75$			
1.50	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1.64	0.1525	0.7803	0.7773	0.7846	1.0054	0.0230	0.5383
1.79	0.1032	0.7989	0.7517	0.8022	1.0886	0.3543	0.5350
1.96	0.0516	0.8166	0.7235	0.8191	1.1825	0.6843	0.5320
2.14	0.0012	0.8325	0.6948	0.8341	1.2813	0.9927	0.5292
2.35	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0.77$			
1.50	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1.64	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1.79	0.1576	0.7982	0.7965	0.8021	1.0031	0.0129	0.5351
1.96	0.1059	0.8160	0.7682	0.8190	1.0898	0.3514	0.5320
2.14	0.0554	0.8318	0.7393	0.8340	1.1811	0.6671	0.5292
2.35	0.0013	0.8474	0.7073	0.8487	1.2869	0.9922	0.5265

n.s.: no solution

Table A.7.

α_S	S_D	Y_D	S_D+Y_D	(S_D+Y_D)r	p_S / $\bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_Y = 0.73$			
0.62	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.67	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.71	0.0064	0.8008	0.6727	0.8032	1.2594	0.9599	0.5349
0.75	0.0499	0.8006	0.7088	0.8034	1.1799	0.6887	0.5348
0.80	0.1024	0.8004	0.7523	0.8037	1.0904	0.3606	0.5347
0.85	0.1531	0.8002	0.7944	0.8040	1.0105	0.0434	0.5346
				$\alpha_Y = 0.75$			
0.62	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.67	0.0113	0.8004	0.6764	0.8029	1.2500	0.9294	0.5350
0.71	0.0564	0.8002	0.7138	0.8031	1.1682	0.6477	0.5349
0.75	**0.1000**	**0.8000**	**0.7500**	**0.8033**	**1.0942**	**0.3749**	**0.5349**
0.80	0.1527	0.7997	0.7938	0.8036	1.0109	0.0450	0.5348
0.85	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_Y = 0.77$			
0.62	0.0064	0.8001	0.6721	0.8025	1.2592	0.9602	0.5351
0.67	0.0653	0.7998	0.7209	0.8028	1.1526	0.5919	0.5350
0.71	0.1106	0.7996	0.7584	0.8030	1.0768	0.3085	0.5350
0.75	0.1544	0.7993	0.7948	0.8032	1.0082	0.0340	0.5349
0.80	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.85	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.

n.s.: no solution

Table A.8.

α_Y	S_D	Y_D	S_D+Y_D	(S_D+Y_D)r	p_S / $\bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_S = 0.73$			
0.700	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.710	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.720	0.0047	0.8010	0.6714	0.8034	1.2627	0.9708	0.5348
0.764	0.1159	0.7996	0.7629	0.8031	1.0683	0.2754	0.5349
0.770	0.1327	0.7994	0.7768	0.8031	1.0417	0.1702	0.5349
0.777	0.1528	0.7992	0.7933	0.8031	1.0106	0.0439	0.5349
				$\alpha_S = 0.75$			
0.700	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.710	0.0034	0.8012	0.6704	0.8035	1.2653	0.9791	0.5348
0.720	0.0262	0.8009	0.6893	0.8035	1.2226	0.8366	0.5348
0.764	0.1376	0.7995	0.7809	0.8032	1.0340	0.1394	0.5349
0.770	0.1544	0.7993	0.7948	0.8032	1.0082	0.0340	0.5349
0.777	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_S = 0.77$			
0.700	0.0024	0.8014	0.6698	0.8037	1.2672	0.9850	0.5347
0.710	0.0245	0.8011	0.6880	0.8036	1.2258	0.8472	0.5347
0.720	0.0474	0.8008	0.7068	0.8036	1.1844	0.7043	0.5348
0.764	0.1590	0.7994	0.7987	0.8033	1.0013	0.0055	0.5348
0.770	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
0.777	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.

n.s.: no solution

Table A.9.

F	S_D	Y_D	S_D+Y_D	$(S_D+Y_D)r$	$p_S\ /\ \bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_S = 0.73$			
1.86	0.1444	0.7373	0.7347	0.7424	1.0046	0.0210	0.5519
1.93	0.0984	0.7712	0.7246	0.7751	1.0874	0.3622	0.5426
2.00	0.0499	0.8006	0.7088	0.8034	1.1799	0.6887	0.5348
2.06	0.0061	0.8229	0.6909	0.8248	1.2683	0.9627	0.5292
2.13	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
2.19	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_S = 0.75$			
1.86	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1.93	0.1486	0.7706	0.7660	0.7750	1.0083	0.0359	0.5426
2.00	**0.1000**	**0.8000**	**0.7500**	**0.8033**	**1.0942**	**0.3749**	**0.5349**
2.06	0.0563	0.8223	0.7322	0.8247	1.1762	0.6580	0.5292
2.13	0.0025	0.8455	0.7067	0.8469	1.2839	0.9853	0.5236
2.19	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_S = 0.77$			
1.86	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
1.93	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
2.00	0.1544	0.7993	0.7948	0.8032	1.0082	0.0340	0.5349
2.06	0.1106	0.8216	0.7769	0.8246	1.0839	0.3268	0.5292
2.13	0.0568	0.8448	0.7514	0.8468	1.1832	0.6637	0.5236
2.19	0.0081	0.8627	0.7257	0.8638	1.2797	0.9529	0.5196

n.s.: no solution

Table A.10.

$\boldsymbol{S_0}$	S_D	Y_D	S_D+Y_D	$(S_D+Y_D)r$	$p_S\ /\ \bar{p}_S$	*Incongruity*	$E(Y_N)$
				$\alpha_S = 0.73$			
0	0.0905	0.7958	0.7386	0.7951	1.1085	0.4314	0.5643
5	0.0597	0.8065	0.7218	0.8065	1.1647	0.6299	0.5500
8	0.0404	0.8129	0.7111	0.8135	1.2012	0.7515	0.5402
13	0.0068	0.8236	0.6920	0.8246	1.2673	0.9587	0.5250
16	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
19	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_S = 0.75$			
0	0.1122	0.7957	0.7566	0.7952	1.0729	0.2950	0.5712
5	0.0812	0.8064	0.7397	0.8066	1.1276	0.4965	0.5569
8	0.0618	0.8128	0.7288	0.8137	1.1631	0.6198	0.5470
13	0.0281	0.8235	0.7097	0.8248	1.2274	0.8294	0.5318
16	0.0069	0.8299	0.6973	0.8317	1.2695	0.9584	0.5214
19	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.	n.s.
				$\alpha_S = 0.77$			
0	0.1336	0.7956	0.7743	0.7953	1.0391	0.1604	0.5781
5	0.1025	0.8063	0.7573	0.8067	1.0922	0.3644	0.5637
8	0.0830	0.8127	0.7464	0.8138	1.1267	0.4894	0.5538
13	0.0697	0.8170	0.7389	0.8188	1.1509	0.5734	0.5463
16	0.0278	0.8298	0.7147	0.8318	1.2302	0.8325	0.5281
19	0.0058	0.8363	0.7018	0.8387	1.2741	0.9653	0.5174

n.s.: no solution

6. Acknowledgements

The author would like to express his sincere thanks to Antoni Castells, Quique López, Diego Martínez, Carlos Monasterio, Alexandre Pedrós, Pilar Soribas, the participants in the Workshop on Fiscal Federalism organized by CentrA in Seville (Spain), and in the 60th Congress of the IIPF celebrated in Milan (Italy) for the helpful comments received - the usual disclaimer applies. Research support from SEC2003-01388 (Mº de Ciencia y Tecnología) and 2001SGR-30 (Generalitat de Catalunya) is gratefully acknowledged.

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FIGURE 2

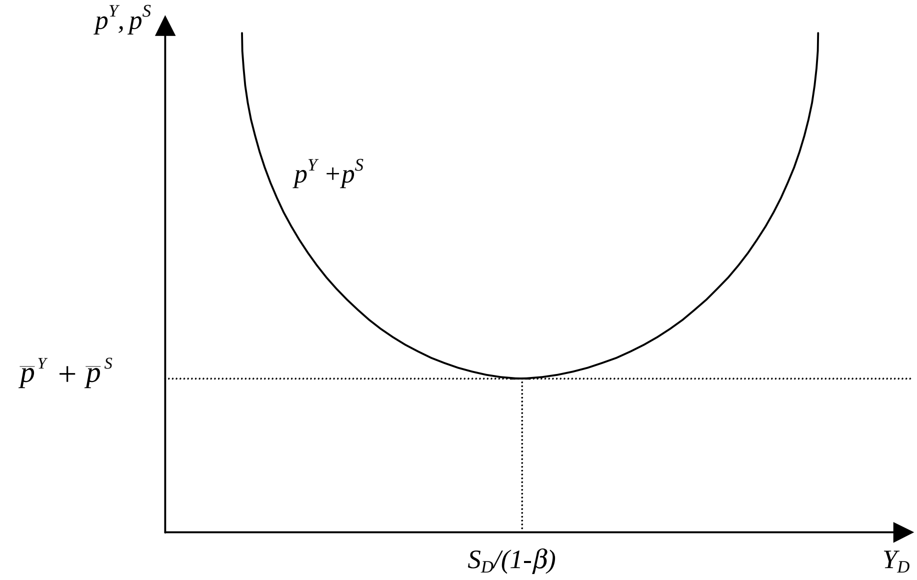


FIGURE 1

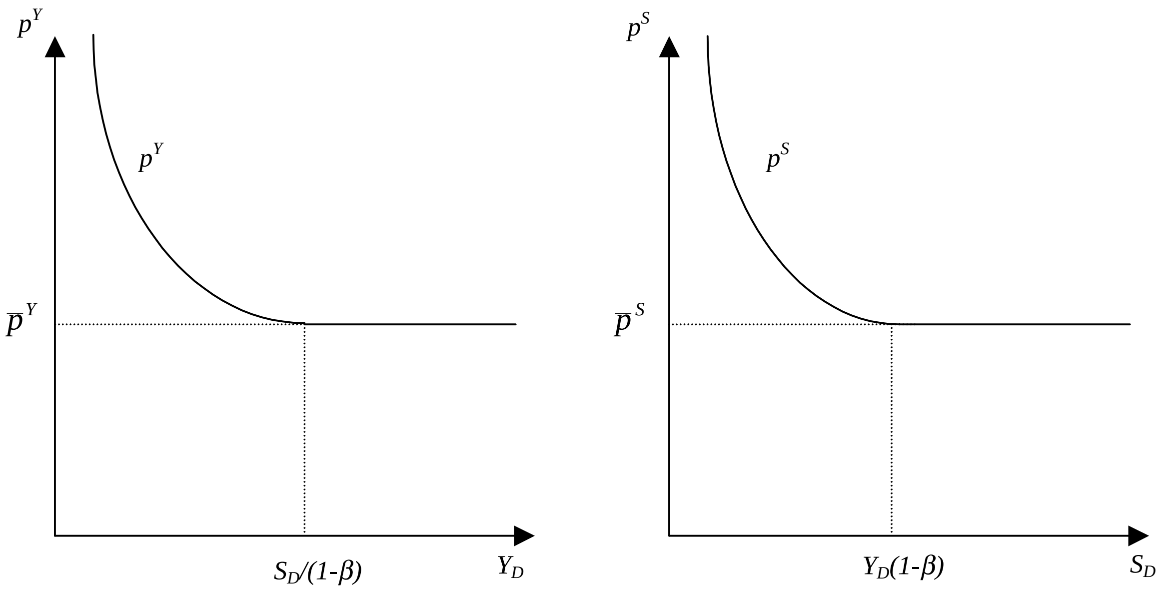
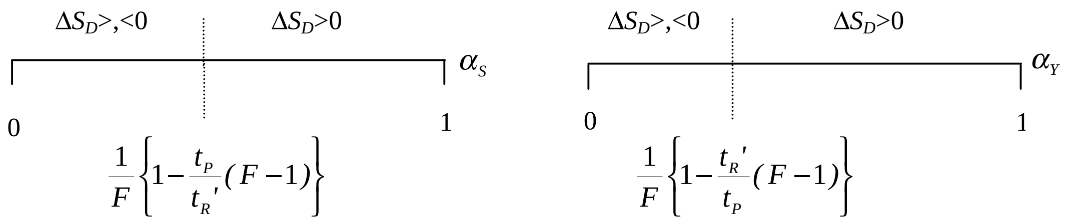


FIGURE 3



ΔS_D>,<0
ΔS_D>0
α_S
0
1
$\frac{1}{F}\left\{1-\frac{t_P}{t_R'}(F-1)\right\}$
ΔS_D>,<0
ΔS_D>0
α_Y
0
1
$\frac{1}{F}\left\{1-\frac{t_R'}{t_P}(F-1)\right\}$

Table 1a

Y_D=0.8; S_D=0.16			
	p^Y	p^S	$p^Y \cup p^S$
Classical analysis	0.3252	0.4998	0.6625
Interrelated Evasion			
	p^Y	p^S	$p^Y \cup p^S$
Perfect collaboration	n.a.	n.a.	0.3212
Imperfect collaboration			
	Symmetric Collaboration		
$\alpha_Y=\alpha_S=0$	0.3219	0.6781	1
$\alpha_Y=\alpha_S=0.25$	0.2596	0.4664	0.7260
$\alpha_Y=\alpha_S=0.5$	0.2163	0.3261	0.5424
$\alpha_Y=\alpha_S=0.75$	0.1848	0.2296	0.4144
$\alpha_Y=\alpha_S=0.9$	0.1696	0.1855	0.3551
	Asymmetric Collaboration		
$\alpha_Y=1$; $\alpha_S<1$	0.3212	0	0.3212
$\alpha_S=1$; $\alpha_Y<1$	0	0.3212	0.3212

n.a.: not available

Table 2a

Y_D=1; S_D=0.2			
	p^Y	p^S	$p^Y \cup p^S$
Classical analysis	0.5	0.5	0.75
Interrelated Evasion			
	p^Y	p^S	$p^Y \cup p^S$
Perfect collaboration	n.a.	n.a.	0.5
Imperfect collaboration			
	Symmetric Collaboration		
$\alpha_Y=\alpha_S=0$	0.5	0.5	1
$\alpha_Y=\alpha_S=0.25$	0.4	0.4	0.8
$\alpha_Y=\alpha_S=0.5$	0.3331	0.3331	0.6662
$\alpha_Y=\alpha_S=0.75$	0.2857	0.2857	0.5714
$\alpha_Y=\alpha_S=0.9$	0.2632	0.2632	0.5264
	Asymmetric Collaboration		
$\alpha_Y=1$; $\alpha_S<1$	0.5	0	0.5
$\alpha_S=1$; $\alpha_Y<1$	0	0.5	0.5

n.a.: not available

PARADOX OF TAX EVASION
(Replica of Bernasconi's analysis: F=4; t_R=0.3; t_P=0.002)

Table 1b

	Y_D=0.8; S_D=0.16		
	p^Y	p^S	$p^Y \cup p^S$
Classical analysis	0.1441	0.2499	0.3580
Interrelated Evasion			
	p^Y	p^S	$p^Y \cup p^S$
Perfect collaboration	n.a.	n.a.	0.1434
Imperfect collaboration			
	Symmetric Collaboration		
α_Y=α_S=0	0.1435	0.2853	0.4288
α_Y=α_S=0.25	0.1152	0.1973	0.3125
α_Y=α_S=0.5	0.0960	0.1398	0.2358
α_Y=α_S=0.75	0.0821	0.1001	0.1822
α_Y=α_S=0.9	0.0755	0.0820	0.1575
	Asymmetric Collaboration		
α_Y=1; α_S<1	0.1434	0	0.1434
α_S=1; α_Y<1	0	0.1434	0.1434

n.a.: not available

Table 2b

	Y_D=1; S_D=0.2		
	p^Y	p^S	$p^Y \cup p^S$
Classical analysis	0.2500	0.2500	0.4375
Interrelated Evasion			
	p^Y	p^S	$p^Y \cup p^S$
Perfect collaboration	n.a.	n.a.	0.25
Imperfect collaboration			
	Symmetric Collaboration		
α_Y=α_S=0	0.2500	0.2500	0.5000
α_Y=α_S=0.25	0.2000	0.2000	0.4000
α_Y=α_S=0.5	0.1667	0.1667	0.3334
α_Y=α_S=0.75	0.1429	0.1429	0.2858
α_Y=α_S=0.9	0.1316	0.1316	0.2632
	Asymmetric Collaboration		
α_Y=1; α_S<1	0.2500	0	0.2500
α_S=1; α_Y<1	0	0.2500	0.2500

n.a.: not available

Table 3
(*Benchmark*: p^Y= 0.1848; p^S=0.2296)

	$Y_D(1-\beta) > S_D$			
	Y_D=0.8; S_D=0.14; α_Y=0.75; α_S=0.75			
	p^Y	p^S	p^{SF}	p^Y/p^{SF}
h=0.5	0.3118	0.0159	0.0160	19.422
h=1.5	0.3181	0.0054	0.0056	56.604
h=10	0.3209	0.0007	0.0008	372.824
	Y_D=0.8; S_D=0.12; α_Y=0.75; α_S=0.75			
	p^Y	p^S	p^{SF}	p^Y/p^{SF}
h=0.5	0.3117	0.0157	0.0160	19.426
h=1.5	0.3179	0.0053	0.0056	56.668
h=10	0.3208	0.0006	0.0009	373.249
	Y_D=0.8; S_D=0.10; α_Y=0.75; α_S=0.75			
	p^Y	p^S	p^{SF}	p^Y/p^{SF}
h=0.5	0.3116	0.0155	0.0160	19.448
h=1.5	0.3178	0.0051	0.0056	56.732
h=10	0.3207	0.0005	0.0009	373.730
	$Y_D(1-\beta) < S_D$			
	Y_D=0.7; S_D=0.16; α_Y=0.75; α_S=0.75			
	p^Y	p^{YF}	p^S	p^S/p^{YF}
h=0.5	0.0091	0.0092	0.4590	49.842
h=1.5	0.0031	0.0032	0.4708	146.661
h=10	0.0004	0.0005	0.4762	968.198
	Y_D=0.6; S_D=0.16; α_Y=0.75; α_S=0.75			
	p_Y	p_{YF}	p_S	p_S/p_{YF}
h=0.5	0.0062	0.0063	0.4008	63.613
h=1.5	0.0021	0.0022	0.4107	187.425
h=10	0.0002	0.0003	0.4152	1239.826
	Y_D=0.5; S_D=0.16; α_Y=0.75; α_S=0.75			
	p^Y	p^{YF}	p^S	p^S/p^{YF}
h=0.5	0.0043	0.0044	0.3442	78.031
h=1.5	0.0014	0.0015	0.3530	229.826
h=10	0.0001	0.0002	0.3571	1520.698

p^{YF} and p^{SF} were calculated using expressions [40'] and [40], respectively.

Table 4
(*Benchmark*: α_Y=0.75; α_S =0.75)

	$Y_D(1-\beta) > S_D$			
	Y_D=0.8; S_D=0.14; p^Y= 0.1848; p^S=0.2296		Y_D=0.8; S_D=0.12; p^Y= 0.1848; p^S=0.2296	
	α_Y=0.75	α_Y=0.25	α_Y=0.75	α_Y=0.25
	α_S	α_S	α_S	α_S
h=0.5	0.7549	0.7555	0.7500	0.7508
h=1.5	0.7657	0.7663	0.7520	0.7527
h=10	0.8919	0.8925	0.8098	0.8105
	$Y_D(1-\beta) < S_D$			
	Y_D=0.7; S_D=0.16; p^Y= 0.1848; p^S=0.2296		Y_D=0.6; S_D=0.16; p^Y= 0.1848; p^S=0.2296	
	α_S=0.75	α_S=0.25	α_S=0.75	α_S=0.25
	α_Y	α_Y	α_Y	α_Y
h=0.5	>1	>1	>1	>1
h=1.5	>1	>1	>1	>1
h=10	>1	>1	>1	>1

COMPARATIVE STATICS

Table 5
(*perfect collaboration*)

	Y_D	S_D	S_D+Y_D	$(S_D+Y_D)r$	$E(Y_N)$
			t_R		
0.1675	0.0010	0.0002	0.0010	0.0570	0.8820
0.3000	0.5310	0.1062	0.5309	0.5460	0.8415
0.5000	**0.8000**	**0.1600**	**0.8000**	**0.8039**	**0.5757**
0.6000	0.8674	0.1735	0.8673	0.8695	0.4169
0.7000	0.9155	0.1831	0.9155	0.9167	0.2508
0.9890	1.0000	0.2000	1.0000	1.0000	-0.2505
			t_P		
0.0001	0.7952	0.1590	0.7952	0.7953	0.5875
0.0050	**0.8000**	**0.1600**	**0.8000**	**0.8039**	**0.5757**
0.0100	0.8049	0.1610	0.8049	0.8124	0.5637
0.0500	0.8433	0.1687	0.8433	0.8690	0.4673
0.1500	0.9343	0.1869	0.9343	0.9580	0.2245
0.2273	1.0000	0.2000	1.0000	1.0000	0.0350
			p		
0.0010	0.0651	0.0130	0.0652	0.0834	0.9564
0.0100	0.1650	0.0330	0.1650	0.1813	0.8993
0.0400	0.3091	0.0618	0.3092	0.3227	0.8099
0.0700	0.3988	0.0798	0.3988	0.4105	0.7536
0.1300	0.5277	0.1055	0.5277	0.5369	0.6777
0.5000	1.0000	0.2000	1.0000	1.0000	0.5880
			F		
1.310	0.0217	0.0043	0.0218	0.0409	0.7767
1.500	0.4333	0.0867	0.4333	0.4444	0.6986
2.000	**0.8000**	**0.1600**	**0.8000**	**0.8039**	**0.5757**
2.250	0.8763	0.1753	0.8763	0.8787	0.5267
2.500	0.9273	0.1855	0.9273	0.9287	0.4805
3.113	1.0000	0.2000	1.0000	1.0000	0.3734
			σ		
0.001	0.0240	0.0048	0.0239	0.0278	0.6654
0.751	0.5504	0.1101	0.5503	0.8642	0.6046
1.800	**0.8000**	**0.1600**	**0.8000**	**0.9614**	**0.5757**
3.000	0.8789	0.1758	0.8789	0.9812	0.5666
4.500	0.9190	0.1838	0.9190	0.9891	0.5620
12.000	0.9696	0.1939	0.9696	0.9967	0.5561
			S_0		
0	0.7959	0.1592	0.7959	0.7959	0.5862
5	0.8061	0.1612	0.8062	0.8153	0.5600
10	0.8164	0.1633	0.8163	0.8330	0.5338
20	0.8368	0.1674	0.8368	0.8640	0.4815
60	0.9186	0.1837	0.9186	0.9491	0.2720
99.8	1.0000	0.2000	1.0000	1.0000	0.0636