Income Tax Inertia

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JEL Classification H31
Keywords: Income (direct) taxation, commodity (indirect) taxation.

November 9, 2006

Abstract

In 1953, Friedman asserted that the Borgatta-Joseph proposition is not valid in the case of several heterogeneous consumers and/or general equilibrium economy with production. In this paper I show a counter-example to this assertion in a general equilibrium set-up with heterogeneous consumers and constants returns to scale. Moreover, as a non-convexity is assumed to justify tax revenue, our several-heterogeneous-consumers Borgatta-Joseph equilibrium turns into a marginal cost pricing solution which yields a Pareto optimal allocation.

1 Introduction

The comparison between income (direct) and commodity (indirect) taxation is one of the oldest issues in Public Economics. In this trend, Barone (1912) showed that, keeping constant the utility of the taxpayer, the Exchequer could obtain a larger revenue from an income tax as opposed to a commodity tax. Nowadays this dominance of the income taxation is taught in several microeconomics textbooks. In particular, Varian (1992) shows the version provided by Borgatta (1921) and Joseph (1939). That is, a given tax revenue yield would leave the taxpayer better off under an income tax than under a commodity tax. In other words, if the consumer is asked about what tax she would pay to bear a given tax revenue, she would choose the income taxation. Nevertheless, Friedman (1953) asserted that despite the analysis being valid in partial equilibrium with a unique individual, its application is not valid in the case of several heterogenous individuals or under general equilibrium set-up.

This note discuss the Friedman´s assertion by providing a version of the Borgatta-Joseph proposition for several heterogeneous individuals and general equilibrium economy with constant returns to scale in production. The model conceives of two goods produced competitively from a primary input and a fixed quantity k which has to be taken by the government. The primary input is supplied inelastically by n different consumers. Our version of the Borgatta-Joseph proposition with n heterogeneous consumers consists in a game where the government gives each consumer two possible tax regimes: a commodity taxation or an income taxation. In case of the consumer decides the income income taxation, she has to bear a constant tax rate on her income (given by the ratio between k and the total income of the economy), and paying marginal cost prices for all the goods. Whereas if she decides the commodity taxation, she has to bear an excise tax added to the marginal cost of one of the goods. The tax revenue is the sum of both commodity and income taxation and the government keeps budgetary equilibrium all time. As the income tax rate is constant, the commodity tax depends on the number of consumers who are bearing it. Thus, strategic interdependence comes from the number of consumers who are paying the commodity tax. As we will see, in the unique Nash equilibrium of this game everyone end up choosing the income taxation regime. In terms of a general equilibrium with production economy this means that the Nash equilibrium of our game yields to a marginal cost pricing equilibrium, that is, to a Pareto optimal allocation.

The paper is structured as follows. Section 2 presents the model, the so-called n-heterogeneous-consumers Borgatta-Joseph game and its equilibrium. In this section prices are considered exogenous and production matters are ignored because the result depends only on the heterogeneity of consumers. In section 3, we interpret the model as a general equilibrium with constant returns to scale in production model to show the Pareto optimality of the equilibrium. Finally, section 4 is devoted to the conclusion.

2 The Model

The economy consists in a set $I = \{1, 2, ..., n\}$ of consumers, denoted by $i \in I$, who consume two private goods X, Y produced from a quantity W of primary input which is the numerarie. The i-th consumer is characterized by a preference-indifference relation $\succsim_i$ rational, monotonic, continuous and strictly convex, and is endowed with a quantity w_i of primary input which represents her income, where $W = \sum_I w_i$. There is also a quantity $0 < k < W$ of numerarie that has to be collect by a government, and (p, q) are the prices of goods X, Y respectively.

For collecting the quantity k the government gives two options to consumers. On the one hand a excise tax on the consumption of good X. In this case the i-th consumer budget constraint is

$$B_i(t) = \left\{(x_i, y_i) \in R_+^2 : w_i \geq (p + t)x_i + qy_i\right\}, \tag{1}$$

being $(x_i(t), y_i(t)) = \arg\max\{\succsim_i \text{ s.t. } (x_i, y_i) \in B_i(t)\}$ her optimal consumption bundle.

On the other hand, the consumer can choose bearing a proportional tax rate T on her income. In this case her budget constraint is

$$B_i(T) = \left\{(x_i, y_i) \in R_+^2 : (1-T)w_i \geq px_i + qy_i\right\}, \tag{2}$$

and $(x_i(T), y_i(T)) = \arg\max\{\succsim_i \text{ s.t. } (x_i, y_i) \in B_i(T)\}$ her optimal consumption bundle.

These two strategies are excluyents, that is, the consumer who chooses to pay one is waiver to pay he other.

Government's tax policy is as follows, it sets a constant income tax rate, given by $T^* = k/W$, to those consumers who choose *to bear income taxation.* Whereas for those consumers who choose *to bear commodity taxation* the tax rate is determined by fulfilling the budgetary equilibrium. More precisely, calling $S \subseteq I$ the set of consumers who choose *to bear commodity taxation* and $I - S$ (complementary of S) the set of consumers who choose *to bear income taxation*, the commodity tax t_S is determined by the following equation

$$t \sum_{i \in S} x_i(t) + T^* \sum_{i \in I-S} w_i = k,$$

taking into account that $W = \sum_{i \in S} w_i + \sum_{i \in I-S} w_i$ and clearing, we can express t_S as the commodity tax which fulfills the following equation which depends on the cardinality of S

$$t_S = T^* \frac{\sum_{i \in S} w_i}{\sum_{i \in S} x_i(t_S)}. \tag{3}$$

3 The Equilibrium

Given the tax policy (T^*, t_S) each consumer has to solve a two-stage problem. In the first one she has to choose between *to bear commodity taxation* and *to bear income taxation*, this is equivalent to choose one of the budget constraints between (1) and (2). In the second stage, she has to solve her usual consumption problem given her budget constraint. Let us call to the first stage game the n-heterogeneous-consumers Borgatta-Joseph game in which $\{B_i(t_S), B_i(T^*)\}$ is the set of strategies of each consumer. The following propositions drive us to the Nash equilibrium of this game.

Proposition 1

Given the tax policy (T^*, t_S) and assuming that $S \neq \emptyset$, at least the consumer $h \in S$ with the largest ratio between consumption of good X and income prefers *to bear income taxation.*

Proof: Let be $h \in S$ and $(x_h(t_S), y_h(t_S))$ her consumption bundle which, due to the monotonicity, exhausts the bundle constraint given by (1), that is

$$w_h = (p + t_S)x_h(t_S) + qy_h(t_S). \tag{4}$$

Let us find out the conditions for which this consumption bundle is also affordable under the income tax regime. Thus, plugging $(x_h(t_S), y_h(t_S))$ in constraint (2) with $T = T^*$ and operating we hold

$$w_h \geq \left[p + T^* \frac{w_h}{x_h(t_S)}\right] x_h(t_S) + qy_h(t_S). \tag{5}$$

Comparing (4) con (5) and taking into account (3), $(x_h(t_S), y_h(t_S))$ is affordable under the income tax regime if and only if

$$\frac{x_h(t_S)}{w_h} \geq \frac{\sum_{i \in S} x_i(t_S)}{\sum_{i \in S} w_i}, \tag{6}$$

condition which is held for at least the consumer in S with largest ratio between

her consumption of good X and her income. In fact, if $h \in S$ is the consumer so that $\frac{x_h(t_S)}{w_h} \geq \frac{x_i(t_S)}{w_i} \forall i \in S$, we can write it as $w_i x_h(t_S) \geq w_h x_i(t_S)$ and adding for $i \in S$ we hold $x_h(t_S) \sum_{i \in S} w_i \geq w_h \sum_{i \in S} x_i(t_S)$ which is just the equation (6).■

To remark that, on the one hand, proposition 1 finds conditions for which the bundle chosen under commodity taxation is also affordable under income taxation. Our assumptions about consumers' preferences allow the weak axiom of revealed preference to ensure that, in such a case, the bundle chosen under income taxation is at least preferred to the bundle chosen under commodity taxation. On the other hand, a weaker drafting of proposition is possible due to condition (6) can be fulfilled by other consumers without the largest ratio consumption of good X-income. Nevertheless, I have choose the current draft for sake of simplicity. Finally, proposition 1 is true for every cardinal of S ($card(S)$) this leads us to the following proposition.

Proposition 2
In the Nash equilibrium of the n-heterogeneous-consumers Borgatta-Joseph game every consumer chooses *to bear income taxation*, that is, $S = \emptyset$.

Proof: Let us suppose that this is not true, i. e., $S \neq \emptyset$ or $card(S) = k \in [2, n]$. But, according with proposition 1 this is not an equilibrium because there is at least one individual in S who prefers *to bear income taxation.* But proposition 1 applies for any size of set S, hence, the same argument works for $k-1, k-2, ..., 2$. For $card(S) = 1$ proposition 1 is just the Borgatta-Joseph single-individual proposition. Finally, the weak axiom of revealed preference ensures that when $S = \emptyset$ nobody has incentives *to bear commodity taxation* because, according to proposition 1, for every consumer the bundle chosen under commodity taxation is affordable under income taxation. Thus, in the Nash equilibrium $S = \emptyset$.■

3.0.1 An example

Let us consider two Cobb-Douglas consumers $u_i(x_i, y_i) = x_i^{\alpha_i} y_i^{1-\alpha_i}$ where $\alpha_1 = 1/3, w_1 = 1, \alpha_2 = 2/3, w_2 = 9$ and $k = p = q = 1$. The tax rate on income is $T^* = 1/10$, and the commodity tax rates for the pairs (*to bear commodity taxation, to bear commodity taxation),* (*to bear commodity taxation, to bear income taxation*) and (*to bear income taxation*, *to bear commodity taxation*) are $3/16, 3/7$ and $9/51$ respectively. The payoffs matrix is

	commodity tax	*income tax*
commodity tax	$0.4996, 4.2467$	$0.4698, 4.2859$
income tax	$0.4762, 4.2732$	$0.4762, 4.2859$

as we see *to bear income taxation* is a dominant strategy for consumer 2, because if both consumers would bear commodity taxation, that is, if $card(S) = 2$, $\frac{x_2(t_S)}{w_2} > \frac{x_1(t_S)}{w_1}$. Given this dominant strategy for consumer 2, the best response for consumer 1 is *to bear income taxation.*

4 A general equilibrium with production case

As we have seen, the previous section has been devoted to show a version of the Borgatta-Joseph proposition for the case of n heterogeneous consumers in partial equilibrium. A brief reinterpretation of the same model allows us to illustrate a general equilibrium with production case which entails a counterexample to the Friedman´s (1953) assertion. In fact, let us assume that our economy is formed by the same individuals as before but commodities are produced under constant returns to scale from the numerarie W, which is a primary input as labor for example. That is, being x and y the total production of commodities X and Y, $C_x(x) = px$ and $C_y(y) = qy$ represent the absorption of labor for production of each commodity respectively. The total endowment W of labor has also to finance the quantity k, introducing a non-convexity problem in the economy. In this trend, w_i is the quantity of labor inelastically supplied by the i-th consumer[1] and her income as well. Assuming that commodities X, Y are produced in competitive industries, profits in equilibrium are zero (due to the constant returns in production) and the aggregated supply of goods is perfectly elastic, allowing equilibrium output to be determined by total demand. Therefore, the following proposition holds.

Proposition 3
For the economy described above, the equilibrium of the n-heterogeneous consumers Borgata-Joseph game yields to a Pareto optimal allocation.

[1]This allow the income tax to be non distortionary, otherwise opposite results could be reached (Little, 1951).

Proof: If every individual pays T^* the quantity k is financed through a non-distorting tax instrument. Thus, the equilibrium is just a Marginal Cost Pricing Equilibrium and, as it is well-known, this is enough for Pareto optimality.■

This result was provided by Guesnerie (1975) for more general economies and further extended by several authors as Bonnisseau and Cornet (1988) or Kahn and Vohra (1987). The insight is that for non-convex technologies any Pareto optimal allocation can be sustained as a marginal cost pricing equilibrium by means of a lump-sum redistribution of the losses.

5 Conclusion

As we have seen a version of the Borgatta-Joseph proposition has been proven for the case of n different individuals and a particular case of general equilibrium with constant returns in production. The proof is similar to that used for the individual case: the consumption bundle chosen under the commodity tax belongs to the feasible set defined by the income tax. The key of the proof for this set-up is the fact that while the income tax rate is always constant (the quotient between the fixed cost and total income), the commodity tax rate adjusts depending on the number of individuals who choose to pay it. In this way, I show that an equilibrium with consumers bearing the commodity taxation it is not possible because, if it would exist, those consumers whose ratio between consumption of the good assessed for tax relative to its income is larger would have incentives to change to the income tax. This inertia occurs for every size of the set of individuals who are bearing the commodity tax. Finally, if all the individuals except one decide to pay the income tax the best option left is also to pay that tax. The final conclusion is that all individuals prefer to pay the income tax which yields to a Pareto optimal allocation by means of a marginal cost pricing equilibrium.

6 References

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