Optimal Provision of Public Inputs in a Second Best Scenario: a numerical simulation*

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Abstract

This paper studies the optimal provision of public inputs under two different tax settings: with lump-sum taxes and with taxes on labor. After obtaining optimal rules for the provision of public inputs, a numerical simulation is carried out to compute the level of public spending in each scenario. We find that the level of public input provided under a second best scenario is higher than that corresponding to a first best outcome, with a bigger social welfare too.

JEL Classification: H21, H3, H41, H43

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1 Introduction

Conventional rule for the optimal provision of public goods claims that the sum of marginal rates of substitution over all individuals must be equal to marginal rate of transformation. However, a seminal contribution by Pigou (1947) questioned the above statement when lump-sum taxation is not available for the government. In fact, he stressed on the risk of overprovision if the deadweight loss from distortionary taxes is not taken into account. Atkinson and Stern (1974) found another relevant issue in this debate: as long as public good provision incentives the consumption of taxed goods, government intervention can decrease the provision cost of public spending. Discussion continues nowadays under different scenarios: arbitrary distorting taxation (Wildasin, 1984), heterogenous agents (Konishi, 1993), or non competitive labor markets (Aronsson and Sjogren, 2001).

A part of the debate around the public goods provision deals with the optimal *level* of public good provided. Indeed, the controversy here is rather on the quantity of public good than on the optimal *rules* from first order conditions. Papers such as Wilson (1991), Chang (2002) or Gaube (2000) highlight this topic using in many cases numerical examples (and counterexamples). The underlying idea of these papers is that using distorting taxes means an optimal level of public goods below its first best level.

Optimal provision of public inputs has received less attention. Feehan and Matsumoto (2000) study the use of benefit taxation to provide productive public spending. Also Feehan and Matsumoto (2002) show the differences between the first best and the second best rules in the provision of public inputs. But nothing is said about the optimal *level* of public input to be provided.

This paper aims to go beyond the paper by Feehan and Matsumoto (2002) by obtaining some insights about the levels of public input provided under different tax settings. We believe that particular features of productive public spending deserve a specific treatment. Both economic growth implications and social welfare consequences derived from the provision of public inputs justify this interest.

We use a simple general equilibrium model where public spending yields productive services to firms. Two different tax settings are available for government: one is a lump-sum tax and other a per unit tax on labor. After obtaining optimal rules for the provision of public inputs under each scenario, a numerical simulation is carried out to compute the level of public spending provided. The numerical procedure we follow, described by Martinez and Sanchez (2004), allows us not only to achieve the optimal values for public input and tax rate, but also to qualify these results according to the degree of precision we desire. At this point, our contribution takes into consideration the restrictions facing real governments by choosing the fiscal menu.

In the first part of the paper, we solve the model and discuss the optimality conditions for provision of public inputs. In such a way, we find that social cost of providing public spending must consider not only the cost of the public funds due to the existence of distortionary taxes, but also the tax revenue effect that this kind of public expenditure generates. Moreover, we prove that the production efficiency condition is satisfied in both cases. In the second part of the paper, we find that the level of public input provided under a second best scenario is bigger than that corresponding to a first best outcome. A battery of results relaxing the precision requirements is also offered so that several combinations of fiscal menus can be used by the government.

The structure of the paper is as follows. Section 2 presents the basic characteristics of the model (adapted from Boadway and Keen, 1996). Next section obtains the conditions for the optimal provision of public inputs. Section 4 explains the intuition of the numerical procedure used in the simulation. Section 5 discusses the results. Finally, section 6 concludes.

2 The model

Let an economy be populated by one representative household whose utility function is given by the form:

$$u(x, l), \tag{1}$$

where x is a private good used as numeraire and l is labor supplied. Properties of $u(x, l)$ are the standard ones to ensure a well-behaved function: strictly monotone, quasiconcave and twice differentiable. Weak separably in x and l is to be assumed[1]. Representative household faces the following budget constraint:

$$x = (\omega - \tau) l - T, \tag{2}$$

[1]Non-separability would add complexity into analysis without new relevant insights.

where ω is the wage rate, τ the per unit tax on labor, and T is a lump-sum tax. Household's optimization problem consists of maximizing (1) subject to (2) to yield labor supply $l(\omega-\tau,T)$, and indirect utility function $V(\omega-\tau,T)$. It is assumed that $l_{\omega_N}>0$ and $l_T>0$, where the net wage rate is denoted by $\omega_N=\omega-\tau$ [2].

Output in the economy is produced using labor services and a public input g according to the following aggregate production function:

$$F(l,g) \tag{3}$$

This function satisfies the usual assumptions: increasing in its arguments and strictly concave. Output can be used costlessly as x or g. Labor market is perfectly competitive so that wage rate is linked to marginal productivity of labor:

$$\omega=F_l\left[l(\omega-\tau,T),g\right], \tag{4}$$

where firms take g as given. It allows us to achieve a wage function such as $\omega(g,\tau,T)$. Some results of comparative statics can be found now; they will be used later:

$$\omega_g=\frac{F_{\mathrm{lg}}}{1-F_{ll}l_{\omega_N}}>0 \tag{5}$$

$$\omega_\tau=\frac{-F_{ll}l_{\omega_N}}{1-F_{ll}l_{\omega_N}}>0 \tag{6}$$

$$\omega_T=\frac{F_{ll}l_T}{1-F_{ll}l_{\omega_N}}<0 \tag{7}$$

The economic profits generated is equal to:

$$\pi(g,\tau,T)=F\left[l(\omega(g,\tau,T)-\tau),g\right]-l\left[\omega(g,\tau,T)-\tau\right]\omega(g,\tau,T) \tag{8}$$

Again, it is useful to obtain some results for later use:

$$\pi_g=F_g-\left(F_{ll}l_{\omega_N}\omega_g+F_{\mathrm{lg}}\right)l\lesseqgtr 0 \tag{9}$$

$$\pi_\tau=(1-\omega_\tau)F_{ll}l_{\omega_N}<0 \tag{10}$$

[2]Hereafter, a subscript is used for partial derivatives.

$$\pi_T = -\frac{lF_{ll}l_T}{1 - F_{ll}l_{\omega_N}} > 0, \tag{11}$$

where equilibrium condition in the labor market (4) and expression for ω_T have been used for obtaining the last equation. Note that the effect of the public input on rents is ambiguous because g increases output (and hence, the economic profits) but this type of productive public expenditure also exerts a positive impact upon wage rate, reducing rents.

Revenue raised by government to finance public expenditure is:

$$g = \tau l\left(\omega\left(g, \tau, T\right) - \tau\right) + \pi\left(g, \tau, T\right) + T \tag{12}$$

Note that all economic profits are taxed away by government because they are efficient resources for public sector[3]. In such a way, we do not need to qualify the public input as a factor-augmenting (using the nomenklature of Feehan and Matsumoto, 2002), so long as the effects of rents arising by constant returns to scale in all factors (including public input) have no consequences on the indirect utility function.

3 First order conditions with lump-sum and distorting taxes

In this section, we obtain the first order conditions for optimal provision of public inputs in two different cases: with a lump-sum tax ($\tau = 0$) and with a distorting tax on labor ($T = 0$). The optimization problem of the government for these two cases is as follows:

$$\begin{aligned} & Max \quad V\left(\omega - \tau, T\right) \\ s.t. \quad : \quad & g = \tau l\left(\omega\left(g, \tau, T\right) - \tau\right) + \pi\left(g, \tau, T\right) + T, \end{aligned} \tag{13}$$

that is, the government chooses the values of g and T or τ to maximize the representative household's utility subject to budget constraint[4].

[3] We establish here that the country is under-populated in order to avoid that a tax on rents may suffice to finance a first-best level of public good (Wildasin, 1986).

[4] Wildasin (1986) demonstrates that it is relevant to distinguish between to maximize the per capita utility or the total utility.

First order conditions in the scenario with only lump-sum taxes are the following ones:

$$V_{\omega_N}\omega_g - \mu + \mu\pi_g = 0 \tag{14}$$

$$V_{\omega_N}\omega_T + V_T + \mu\pi_T + \mu = 0, \tag{15}$$

where μ is the Lagrange multiplier. Since the tax is lump-sum, social and private marginal utility of income coincide ($V_T = -\ \mu = -\lambda$, where λ is the private marginal utility of income). Using Roy's identity as well as the expressions (7) and (11), optimal rule for the provision of the public input can be written as follows:

$$\frac{V_{\omega_N}\omega_g}{\lambda} = 1 - \pi_g \tag{16}$$

LHS of (16) are the benefits from one aditional unit of public input, while RHS is the marginal cost of providing the public input. At this point, note that marginal production cost of g is reduced by the revenue effect that the provision of the public input yields through profit taxes. This a feature key of the provision of public inputs, respect to the case where a pure consumption public good is considered.

On the basis of (16), if (5), (9) and Roy's identity are taken into consideration, an important condition containing efficiency implications can be derived:

$$F_g = 1 \tag{17}$$

This is the production efficiency condition for the provision of public inputs. It means that the production effects of the public input (its marginal productivity) are equal to its marginal production cost.

Next we focus on the optimal provision of public inputs when distorting taxes are used. In such a case, first order conditions for g and τ derived from (13) are:

$$V_{\omega_N}\omega_g - \mu + \mu\tau l_{\omega_N}\omega_g + \mu\pi_g = 0 \tag{18}$$

$$(\omega_\tau - 1)\, V_{\omega_N} + \mu l + \mu\,(\omega_\tau - 1)\,\tau l_{\omega_N} + \mu\pi_\tau = 0, \tag{19}$$

where μ is the Lagrange's multiplier. After some manipulation with equations (18) and (19), using again Roy's identity, and (6) and (10), second best condition for the optimal provision of g can be written as follows:

$$\frac{V_{\omega_N}\omega_g}{\lambda} = \frac{1}{1 - \frac{\tau l_{\omega_N}}{l}} \left(1 - \tau l_{\omega_N}\omega_g - \pi_g\right) \tag{20}$$

In essence, the interpretation of equation (20) is the following. As before, LHS are the marginal benefits received by household when an additional unit of public input is provided. In other hand, two terms can be distinguished in the RHS. The first one is the marginal cost of the public funds ($MCPF$), which is bigger than one under the assumption of $l_{\omega_N} > 0$. The second term is the tax revenue effect that arises so long as g may affect positively or negatively tax base through economic profits and wage rate; in such a way, the indetermined sign of π_g does not ensure that the marginal production cost of the public input to be reduced through tax collection. By contrast, term $\tau l_{\omega_N}\omega_g$ unambiguously lowers the production marginal cost. Both terms of RHS define the marginal cost of providing a public input (MCP). Whereas in the case of a consumption public good the $MCPF$ and the MCP are equal, the distinction is required when a public input is considered .

When expressions (5) and (8) are inserted into (20), manipulation gives again the production efficiency condition (17). At this point we obtain a result found firstly by Diamond and Mirrless (1971), and later confirmed by Feehan and Matsumoto (2002). As is well-known, we have that production effects of public inputs are equal to its marginal production cost, though distortionary (but optimally set) taxation is used. Since there are no differences in terms of efficiency in production between lump-sum and distorting taxes, condition (17) cannot be used to see what happens with the quantities provided of public input under each tax scenario. Hence, in the following section we use a numerical approach to solve the equations systems defined by first order conditions (14)-(15) and (18)-(19) plus government budget constraint.

4 Numerical simulation

Literature provides different approaches to solve numerically our theoretical framework. One of the main methods is using the gradient in a derivative-based methodology; this is the case of the well-known Newton's procedure.

On the other hand, a non-derivative methodology based on the multisection of the initial set and on the evaluations of the objective function also can be used. We have chosen this one (with a simpler implementation but also theoretically-consistent) because it allows us to proxy the real government's behavior through the iterative process followed to find the optimal solution. The different stages of this method, to be explained shortly, can be interpreted as different requirements have to be faced by policy-makers to design taxes. Moreover, our methodology permits to answer simple questions that politicians ussually ask to economists, such as what is the tax rate needed to finance a determined level of public spending.

Roughly, the methodology we follow is based on Casado, Csendes and García (2000), and consists of a subdivision of the initial decision variables set. Then we select the points of the grid which satisfy the budget restriction of the government with a determined precision in each stage[5]. This process continues until the maximum previously-fixed precision is achieved. Whereas Casado, Csendes and García (2000) do not consider different levels of precision (they use a comparison of values of the objective function as stopping criteria), our numerical approach has been adapted to take the precision which the restriction is satisfied into account as the primary criterium.

A brief formal description of the algorithm we use is next. First, we describe all the elements which participate in the method. Let V be the objective function we want to optimize:

$$\begin{array}{rcl} V : FV \times P \subseteq \mathbb{R}^N \times \mathbb{R}^J & \longrightarrow & \mathbb{R} \\ (f, p) & \longrightarrow & V(f, p), \end{array}$$

where FV is the set of feasible values for the decision variables (f); FV may be one interval or the union of them. P is the set of parameter values (p) and they will be fixed throughout all the process. N is the number of decision variables and J the number of parameters. In the same way, let R be the set of restrictions in our problem:

$$\begin{array}{rcl} R : FV \times P \subseteq \mathbb{R}^N \times \mathbb{R}^J & \longrightarrow & \mathbb{R}^M \\ (f, p) & \longrightarrow & R(f, p) \end{array}$$

where M is the number of restrictions. In addition, given $\epsilon > 0$ and a set $Z \subseteq FV$, we define the set of compatible values, $BR(\epsilon, Z)$, as a subset where

[5] The results we obtain in each stage could be interpreted as the solution of the problem with the precision we have required.

the restrictions are satisfied with a precision ϵ, i.e.:

$$BR(\epsilon, Z) = \{z \in Z \mid R(z, p) < \epsilon\}$$

Roughly, the problem we are interested in solving is:

$$\left\{ \begin{array}{l} \max V(f, p) \\ f \in BR(\epsilon, FV), p \in P \end{array} \right. \tag{21}$$

In order to perform our numerical procedure, we define some instrumental but essential parameters. As the resolution of the problem consists of using different stages where we fix a precision and a number of subdivisions for each interval, we set:

- Precision path: $PG = [PG_1, ..., PG_{ST}]$
- Subdivision path: $NG = [NG_1, ..., NG_{ST}]$

where ST is the number of stages. PG means the precision we require in each stage and NG refers to the number of subdivisions of each interval we use to obtain PG.

Finally, in the stage $k \in \{1, \ldots, ST\}$, we define $\tilde{f} \in BR(PG_k, FV_k)$ as the solution to the problem (21), that is, the point which satisfies the condition:

$$V(\tilde{f}, p) > V(f, p), \forall f \in BR(PG_k, FV_k) \tag{22}$$

For each new stage $k+1$, we form a new initial set based in the points which satisfies the restriction with the precision we require in the previous stage, that is the set FV_{k+1} is equal to $BR(PG_k, FV_k)$.

More details on this numerical procedure can be found in Martinez and Sanchez (2004). Moreover, they compare this methodology with the well-known Newton-Raphson method, obtaining results in favor of our numerical methodology.

In order to implement our numerical simulation, we distinguish two scenarios. As is said before, the first one comes from using lump-sum taxation. The second one uses distortionary taxes to finance productive public spending. Functions and parameters used are defined next.

According to the above theoretical model, we assume the following utility function: $V(\omega, t) = x^a(\bar{L} - l(\omega, t))^{1-a}$, where t refers to tax rate (both lump-sum and per unit tax on labor), $a = 0.4$ and $\bar{L}$ is time endowment of households and equal to 24. Production function we employ is $F(l, g) = l(\omega, t)^{\alpha} g^{1-\alpha}$, where $\alpha = 0.6$. Initial set of values where numerical procedure will begin its search is $[0, 3]$ for t, $[0.1, 8]$ for g in the distortionary tax setting and $[0.1, 37]$ for g in the lump-sum scheme[6]. Precision requirements we impose for searching solutions is the parameters vector $PG = [1, 10^{-2}, 10^{-4}]$. With the aim of achieving this precision, we use the next number of subdivision of each interval $NG = [30, 10, 10]$. Routines used in the simulations are available upon request.

Given these functions and parameters, we solve the lump-sum problem -equations (14) and (15)- and the distortionary scenario -equations (18) and (19). Obsviously, both cases employ government budget constraint.

5 Discussion of the results

Numerical simulation provides results for different levels of precision. Firstly, we focus on the optimal solutions found after three stages with a precision for the grid of 10^{-4}. Table 1 shows details of the optimum under two tax settings: lump-sum and distorting taxation. At least two comments can be made in viewing these computations. The first one is a striking point: utility level is bigger in a second best scenario than under a lump-sum tax setting. So far we can only provide a tentative explanation; it is based on tax revenue effect that the provision of public inputs may produce through the positive impact of g on labor supply. Indeed, the RHS of expression (20) indicates that the social marginal cost of providing g depends not only on the MCPF but also on the tax revenue effect that may reduce the production marginal cost of public input. Note that in the optimal rule for the provision of g with lump-sum taxation -expression (16)-, this tax revenue effect is limited to π_g, whose sign in indetermined, while in the second best tax setting the positive term $\tau l_{\omega_N} \omega_g$ is found as well.

Insert Table 1

A second comment that is worth to note from Table 1 is the bigger level of public input provided under labor taxes respect to the first best framework. This issue is related with the debate about the difference between the first

[6]These intervals are large enough to obtain feasible solutions under each scenario.

order conditions (for the provision of g) and the *level* of public input pointed out in the Introduction. Again the tax revenue effect linked to distortionary taxation have played a role, decreasing social marginal cost of providing the public input.

Regarding labor market, the results are consistent. Representative individual works more under a lump-sum scenario than when distorting taxes are considered. Although Table 1 does not report on net wage, it is easy to note that per unit taxes on labor lower gross wage and then desincetive the labor supply. This fact supports a part of the above argument on the relevance of the tax revenue effect. Indeed, differences in wages and labor supply make explicit the consequences of distortionary taxation in terms of deadweight loss; but what is also clear is that the magnitude of the revenue effect involved through productive public spending will be high enough to overshoot the tax burden excess.

Insert Table 2 and 3

Table 2 and 3 report the results we have obtained in the penultimate stage (the second in our simulation) around the optimum value in a lump-sum scenario and with distorting taxes, respectively. Given an interval, we inform about its initial point (using subindex ini), final point (using subindex end) and precision required in computation for each decision variable (t, g). We report the compatible values found and the value of indirect utility function in those case. When no compatible values are found, the minimum of the government restriction R is compared to the precision we have required PG. This fact shows the trade-off between the precision of the results and the ability of government to achieve an efficient result. Both tables are an illustration of the multisection iterative process followed by the numerical simulation. They show how relaxing the precision requirements by searching optimal values, a bigger number of combinations of t and g satisfy government budget constraint, and they can be considered as solutions of problem (13). However, if these solutions are ranked by the utility they produced, the optimal combinations provided in table 1 are those maximizing social welfare.

From another point of view, Tables 2 and 3 allow us to give an idea about limitations of the policy-makers in the real world. Governments are usually subject to constitutional restrictions that force them to set tax rates inside a determined interval. If the welfare-maximizing t is not included among the legislative possibilities of the policy-makers, government can proxy as close as possible the tax rate to the optimum t, according to the utility derived from each (t, g) pair. In such a way, we offer a battery of combinations of (t, g) related to utility level achieved by each one.

6 Concluding remarks

This paper has dealt with an issue on which literature has not paid much attention: the optimal *level* of public inputs under different tax settings. Previous contributions have focused on the case of consumption public goods or have discussed the optimal *rules* of productive public spending. However, both social welfare implications of taxation and characterization of public inputs as growth-enhacing public instruments drive to consider this issue as relevant for policy-makers.

We have built a simple general equilibrium model where public inputs provide productive services. Two different tax settings have been considered: one with lump-sum taxation and other using per unit tax on labor. Firstly, we have found the optimal conditions for the provision of public inputs under each scenario. At this point, a distinction between the marginal cost of public funds and the marginal cost of providing public inputs is required. The reason for that is the tax revenue effect caused by the productive public spending through increases in tax bases.

Secondly, a new numerical procedure described in Martinez and Sanchez (2004) has been implemented to obtain values for tax rates and public input maximizing social welfare. Methodology is based on an multisection iterative process of the initial set that evaluates the representative agent's utility function (as objetive function), obtaining compatible values of tax rates and public inputs under several precision requirements. More stages we consider, more detailed values we will obtain for t and g.

Results show that utility achieved under a distorting tax setting is bigger than when a first best scenario is considered. This may be caused by the tax revenue effect linked to the productive public spending. Level of public input provided is also higher with labor taxes than with lump-sum taxes. Moreover, a battery of non-optimal results is shown by comparing to the efficient solution and by allowing some restrictions in the government's performance.

This paper can be improved along several directions. Firstly, influence of non-distorting taxes on profits upon the results in favor of the second best solution should be studied in a deeper way. Secondly, results obtained for each stage of the numerical procedure can be used to measure the inefficiency of non-optimal compatible values and how far they are in terms of social welfare from the optimum values. Thirdly, we are interested in studying other theoretical concepts described in the theorical framework (MCPF, MCP, etc) and, in such a way, to determine the optimum value with a more itemized criterium.

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Tables

Table 1: Comparison of the optimum obtained in the different theorical schemes

Theorical scheme	Utility	Public Input	Tax Rates	Labor	Wages
Distorting_1	7.3003	4.9711	0.1351	11.9644	0.4233
Lump sum_2	6.8934	4.5551	0.0391	16.2896	0.3601

(1): R = $6.63*10^{-5}$; (2) R = $8.29*10^{-8}$

Table 2: Lump sum tax setting. Around the optimum.

STAGE = 2 / 3	**INTERVAL = 1 / 23**	
$t_{ini} = 0.0001$	$t_{fin} = 0.1001$	$BW_t = 0.01$
$g_{ini} = 0.1001$	$g_{fin} = 0.7001$	$BW_g = 0.01$
No compatible values found		
Min R = 0.0929284 PG = 0.01		
STAGE = 2 / 3	**INTERVAL = 2 / 23**	
$t_{ini} = 0.0001$	$t_{fin} = 0.1001$	$BW_t = 0.01$
$g_{ini} = 4.5001$	$g_{fin} = 8.3001$	$BW_g = 0.01$
t	g	$V_{\max}$
0.0401	4.5701	6.88969
0.0501	4.9101	6.8225
0.0601	5.2401	6.7593
0.0701	5.5901	6.69609
0.0801	5.9101	6.63729
0.0901	6.2301	6.57775
0.1001	6.5501	6.5222
STAGE = 2 / 3	**INTERVAL = 3 / 23**	
$t_{ini} = 0.1001$	$t_{fin} = 0.2001$	$BW_t = 0.01$
$g_{ini} = 0.1001$	$g_{fin} = 0.7001$	$BW_g = 0.01$
No compatible values found		
Min R = 0.231506 PG = 0.01		
STAGE = 2 / 3	**INTERVAL = 4 / 23**	
$t_{ini} = 0.1001$	$t_{fin} = 0.2001$	$BW_t = 0.01$
$g_{ini} = 4.5001$	$g_{fin} = 11.3001$	$BW_g = 0.01$
t	g	$V_{\max}$
0.1001	6.5501	6.5222
0.1101	6.8701	6.46775
0.1201	7.1901	6.41554
0.1301	7.5101	6.36386
0.1401	7.8201	6.31608
0.1501	8.1301	6.26861
0.1601	8.4401	6.22329
0.1701	8.7501	6.17863
0.1801	9.0601	6.13545
0.1901	9.3701	6.09295
0.2001	9.6701	6.05314
STAGE = 2 / 3	**INTERVAL = 5 / 23**	
$t_{ini} = 0.2001$	$t_{fin} = 0.3001$	$BW_t = 0.01$
$g_{ini} = 0.1001$	$g_{fin} = 0.4001$	$BW_g = 0.01$
No compatible values found		
Min R = 0.3669 PG = 0.01		

Source: Simulation with the conditions described above.

Table 3: Distorting tax setting. Around the optimum.

STAGE = 2 / 3	**INTERVAL = 1 / 30**	
$t_{ini} = 0.0001$	$t_{fin} = 0.1001$	$BW_t = 0.01$
$g_{ini} = 0.1001$	$g_{fin} = 7.1001$	$BW_g = 0.01$
t	g	$V_{\max}$
0.0001	3.1601	7.16566
0.0101	3.3801	7.16904
0.0201	3.6101	7.17654
0.0301	3.8301	7.18215
0.0401	3.9901	7.19618
0.0501	4.1701	7.20777
0.0601	4.3301	7.22184
0.0701	4.4701	7.23714
0.0801	4.5901	7.25315
0.0901	4.7001	7.26608
0.1001	4.8001	7.2785
STAGE = 2 / 3	**INTERVAL = 2 / 30**	
$t_{ini} = 0.1001$	$t_{fin} = 0.2001$	$BW_t = 0.01$
$g_{ini} = 0.1001$	$g_{fin} = 7.5001$	$BW_g = 0.01$
t	g	$V_{\max}$
0.1001	4.8201	7.27903
0.1101	4.8601	7.28965
0.1201	4.9101	7.29694
0.1301	4.9401	7.30036
0.1401	4.9901	7.29968
0.1501	4.9801	7.29276
0.1601	4.9101	7.27622
0.1701	4.8701	7.25222
0.1801	4.7601	7.21989
0.1901	4.6301	7.16241
0.2001	4.4701	7.09824

Source: Simulation with the conditions described above.

Table 4: (Cont.) Distorting tax setting. Around the optimum.

STAGE = 2 / 3	**INTERVAL = 3 / 30**	
$t_{ini} = 0.2001$	$t_{fin} = 0.3001$	$BW_t = 0.01$
$g_{ini} = 0.1001$	$g_{fin} = 7.5001$	$BW_g = 0.01$
t	g	$V_{\max}$
0.2001	4.4501	7.09097
0.2101	4.2301	6.99536
0.2201	3.9901	6.88204
0.2301	3.7101	6.72404
0.2401	3.3101	6.48676
0.2501	2.8501	6.18171
0.2601	2.4301	5.85104
0.2701	1.9001	5.34797
0.2801	1.3001	4.64585
0.2901	0.7201	3.68484
0.3001	0.3701	2.80972

Source: Simulation with the conditions described above.