Optimal Taxation with Entry Barriers.

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Abstract:

This paper addresses the issue of optimal income taxation in an economy with entry barriers to firms and labor using an infinite horizon general equilibrium approach. The first benchmark model is one with monopolistic competition amongst firms producing a continuum of intermediate input goods, which finds that (i) the optimal labor income tax rate is lower as compared to a competitive market analogue; (ii) the optimal steady state capital income tax rate is nonzero. The second modified model introduces heterogeneity of agent type and entry barriers to private labor in public sector firms, and finds that the optimal sector specific labor income tax rates in this setting clearly suggest a labor income tax trade off between the public and private sectors. For strong (weak) entry barriers the model prescribes a relatively higher (lower) tax on public sector income. The optimal steady state capital income tax rate is nonzero, and its magnitude depends on the elasticity of demand of the publicly produced good.

Keywords: Optimal Taxation, Monopolistic Competition, Entry Barriers, Private Agent, Public Agent.

JEL Classification Codes: *D43, E62, H21, H30.*

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1. Introduction:

Until very recently, the optimal taxation literature seemed more or less silent about the departure from the simplifying assumption of economy-wide competitive markets. To my knowledge, the first attempt to formally address the issue of optimal fiscal policy under imperfect competition in private markets appeared with the recent paper by Schmitt-Grohe & Uribe (2004a), where they study optimal fiscal (and monetary) policy under imperfect competition in a stochastic, flexible-price production economy without capital. The literature on optimal taxation is, therefore, in most parts based on the simplifying assumption that the private sector of the economy is characterized by perfect competition in all markets. Standard *Ramsey* taxation models established in literature that deal specifically with optimal income taxation considers environments without imperfections in private markets. In practice, however, this assumption is too restrictive, and does not always seem to be a realistic description of the incentive structure underlying policy.

The substantive findings of *Ramsey* taxation literature, which are often regarded as fiscal policy prescriptions for governments, are therefore drawn from models with the underlying assumption of economy-wide competitive markets. These findings are subject to verification if one introduces private market imperfection in some form; since the optimal policy then not only must be responsive to the efficiency considerations, but also must attempt to cure the distortions created by private market imperfections. For the current paper, what stimulates this argument against the simplifying assumption of competitive markets is that particular production sectors often practice monopoly rights in pricing since there are barriers to entry for new firms in that particular industry. Such entry barriers may exist for several reasons. Of the many, this paper will consider the following three possible broad forms, which result in imperfection in product and/or factor markets, acknowledging that there may be many others.

Firstly, innovations may raise concerns related to protection of technology which requires patenting. The innovator in such cases will practice monopoly rights in pricing until the economy adopts or copies the technology and starts producing competitively. This may also be the case in the labor market with innovation of specific skills. Secondly, in the event the economy does not possess a particular technology and is unable to innovate locally, it may require to import, which brings in a foreign producer who is indispensable until the innovation or adoption of technology happens locally. The practice of monopoly rights for the foreign producer in this case is again obvious since the producer will not take the market price as given. Lastly, if there is a market failure and consequently a government intervention in production of a particular good, there may be entry barriers to labor services from the private sector. This phenomenon is not new, since most public sector firms have formal insider-outsider frictions in factor employment, and public recruitment of private agents is often subject to institutional and non-institutional barriers. Monopoly rights in labor market in this paper is considered in a more general sense which can be attributed to entry barrier to private labor in public sector firms, or in other words, the right of coalitions of insiders to block the use of efficient technologies and best practice working arrangements simply because they have a monopoly in supplying labor in certain sectors (see for instance, Herrendorf & Teixeira, (2004)).

The reason why this paper considers public sector production a probable rationale behind entry barriers to labor is because governments often enjoy a natural monopoly over provision of many publicly provided goods and services, such as property rights, law and order, and contract enforcement. Many of these services require confidentiality, job-specific training and in most cases the perceived monopoly power of insiders unionizes them who work as coalitions. These *public* agents' coalitions typically restrict *private* agent participation in these sectors on grounds of, among possibly many others, confidentiality. Besides, a variety of public policies have the direct or indirect effect of limiting entry into competitive markets. For instance, regulations may indirectly create barriers to entry by creating cost differentials between existing and new firms by mandating safety, training, environmental, or building code standards on new entrant but not on firms already in the market. Similar entry barriers may also hold for a particular factor required in production for public sector firms.

As mentioned earlier, literature concerning optimal income taxation in particular in competitive settings has established a few substantive results which are often prescribed for government's choice of fiscal policy. The famous contribution by Chari & Kehoe (1999) presents a comprehensive and technical review of these celebrated results. Erosa & Gervais (2001) provides a brief coverage of these models and results in both infinite horizon and overlapping generations set ups, which I rate as a rather summarized yet complete non-technical version of Chari & Kehoe (1999). One of these celebrated results is the prescription of zero optimal steady state tax on capital income, which was seminally proposed by Chamley (1986). This result is judicious since a positive tax on the return from today's savings effectively makes consumption next period more expensive relative to consumption in the current period. As mentioned by Judd (1999), a current period non-zero tax on capital income implies explosive distortions on future periods since capital tax compounds over time, creating non-uniform distortions over time. In an infinitely-lived agent's model, therefore, a positive tax on capital income in the steady state implies that the implicit tax rate of consumption in future has an unbounded increasing trend.

Subsequent investigations of this startling result have found that it is robust for most neoclassical models which are characterized by competitive markets as long as the government's commitment power is perfect (see for instance, Chari & Kehoe (1999), Jones *et. al* (1993 & 1997), among others). In a relatively more recent paper, Golosov *et. al* (2003) models taxation in an environment where agents' skills are private information and shows that if source of distortion is not only confined to taxation, a positive tax on capital income may be sustainable as a pareto efficient outcome. More generally, under a wide variety of circumstances, an optimal tax system maintains aggregate production efficiency, i.e. an optimal tax system maintains the equality between the marginal rates of transformations across production sectors. But if the source of distortion is not only taxation, and if it distorts the relation between the marginal rates of transformations across sectors and thereby induces aggregate production inefficiency, a non-zero tax on capital income may be sustainable and can be used as a corrective device. This is because under such circumstances, the optimal policy must not only be responsive to maintaining aggregate production efficiency but also to cure the distorted margins. This argument extends the usefulness of the present paper, which investigates, among others, if this celebrated result holds if one or more markets are characterized by imperfect competition. The assumption that the government's commitment power is perfect is maintained throughout. In the presence of monopolistic competition where some market power is exercised, economic agents bear the burden of a distortion each period which induces a deadweight loss in allocations of period equilibrium, unless agents are offered some form of compensation. Such distortions can be due to excessively high or low elasticity of substitution between differentiated goods, market for which are typically imperfectly competitive. This distortion often affects factor returns, and the government must use some tax or subsidy device to offset the effects of this distortion. If such distortion affects capital returns, the government *may* find it optimal to introduce a capital tax or subsidy device to offset the effect.

This paper first proposes a benchmark model of optimal taxation in a simple two sector production economy with entry barrier to firms in a particular production sector. I name this simple model the benchmark due to its usefulness and extension into a more comprehensive model (the modified model) that introduces entry barrier to firms and factors, which is presented in the later part of the paper. In other words, the benchmark model is designed to provide simple insights into the optimal taxation problem when the assumption of economy-wide competitive markets is relaxed. The key findings of this model is that (1) with entry barriers to firms in intermediate goods sector, labor services in that sector enjoys a relatively lower tax rate as compared to a competitive market analogue; (2) the capital income from that sector is taxed at a nonzero rate in the steady state. Next, the paper extends the benchmark model by introducing agent type heterogeneity and public production of intermediate input goods, and characterizes entry barrier to firms in public sector production and entry barriers to private labor in public sector. The interesting finding of this extension is that the government faces a labor income tax trade off between the private and public sector labor income tax instruments, although simple linear tax rules under the current settings provide analytically rather indistinct results. The capital income tax is nonzero in the limit. In addition, stronger entry barriers for private labor in public sector forces the government to introduce higher public sector labor income tax and a relatively low private sector labor income tax.

If there are entry barriers to both firms and labor participation (the modified model, e.g.), and the particular sector where such barriers are practised is indispensable, the optimal taxation problem is likely to derive complex analytical results. This is largely due to higher order non-linearity in agents' equilibrium reaction functions, which is turn is a consequence of firms having distinct demand functions for agent type specific labor. These equilibrium reaction functions are incorporated in the optimal taxation problem of the government, in order to ensure that while maximizing welfare subject to its budget constraint, the government also considers equilibrium reactions of economic agents and firms. The non-linearity however adds to the advantage of finding a labor income tax trade off between sectors. A key finding from this extension is that if it is institutionally or non-institutionally highly (less) restricted to work in the public sector being a private agent, the private sector labor income tax must be lower (higher) as compared to the public sector labor income tax. In any case, simple linear tax rates fail to uncover for which sector labor income is taxed relatively heavily. Another interesting finding, as stems directly for the strategic interaction between the two types of agents in the modified model, is that although the private sector labor income tax depends on its counterpart, the public sector labor income tax is independent of its counterpart. The intuitions of these results will follow their derivation in different subsections of the paper.

While general equilibrium models that consider such forms of imperfection in goods and factor markets have been established in literature to address cross country growth differences or real business cycle effects, the fiscal policy choice for the government in similar settings is left mostly unexplored. There has been considerable amount of contribution addressing the issue of taxation with migration possibility of labor, as may be found in Wilson (1992) and Mirrlees (1982), which is attributable to cross border labor mobility and taxation rather than sector specific entry barriers to firms and/or labor and taxation. One of the key findings of this stream of research is that high marginal taxes are justifiable with higher propensity to migrate, such that opening borders to migration leads to a higher marginal tax rate. However, simple linear taxation of factor income fails to uncover this stimulating result (Wilson, 1992). One of the reasons why, presumably, one may find the current paper interesting, is because to my knowledge, except for Schmitt-Grohe & Uribe (2004a & b), there are no specific attempts of such kind established in literature, while the issue addressed is of considerable academic and empirical interest. The literature on public economic theory and optimal taxation has covered the choice optimal fiscal policy issue under variants of macroeconomic and growth models with competitive markets as a maintained hypothesis. While these pioneering attempts are strong benchmarks and departure point of the current stream of research, from a critical point of view there is a likelihood that one considers these to be increasingly stylized. The current paper relaxes one crucial and strong assumption of these benchmark models and finds some interesting and useful insights of the optimal choice of income tax rates.

The paper is organized as follows. Section 2 presents a benchmark model with monopolistic competition and addresses the issue of optimal income tax rates under this setting. Entry barrier is modelled simply as restricting new firms entry into a particular sector producing intermediate input goods, market for which is characterized by monopolistic competition. Section 3 presents a modified version of the benchmark model that introduces heterogeneity of agents (public and private agents), public sector production of intermediate goods and in addition to monopolistic competition amongst public sector firms, an entry barrier to private labor participation in public sector firms. It addresses the optimal taxation problem of the government under the modified setting. Section 4 concludes.

2. The Benchmark Model:

In this section, I will present a simple benchmark general equilibrium model with imperfectly competitive intermediate goods market as a starting point. This simple model can be extended and/or applied to a number of different set ups that address the optimal taxation problem in an otherwise similarly structured economy. The model is standard in literature that deal macroeconomics of imperfect competition, and the set up can otherwise resemble the real business cycle model typically used in Benassy (2002). The model presents presumably the simplest form of market imperfection in a general equilibrium over infinite horizon and thus allows one to realize the potential deviation from standard competitive-market-assumption based *Ramsey* tax rules. The purpose of presenting this simple model as the benchmark case is to see whether and how optimal tax rules differ when distortions from market imperfection affect the productive efficiency of the economy thereby affecting allocations in the macroeconomic equilibrium. This model can be conveniently extended to address issues related to taxation of income from public sector, and this task is undertaken in the modified model presented in section 3[1].

[1] From an expositional point of view, the paper is constructed in a way such that the benchmark model presented in section 2 and the modified model presented in section 3 are completely independent, although most symbols are repeated for consistency.

2.1 The Environment:

Time is discrete and runs forever. There is no uncertainty. The environment considered is a dynamic general equilibrium with a continua of measure one of identical, infinitely lived households. There are two production sectors in the economy indexed by *s*, with *s* = *y*, *z*, denoting sectors which produce final manufacturing goods *y* and intermediate input goods *z*. In the remainder of the paper I will hold the final manufacturing good *y* as the numeraire. There is a continua of measure one of identical firms in sector *y* that own a technology with which a perishable final manufacturing good, *y*, can be produced combining *j* intermediate goods z_j with $j \in [0,1]$. The final manufacturing good can be used for private consumption (*c*), government consumption (*g*) and to augment capital stock for investment (*x*). Capital depreciates at a constant rate δ with $\delta \in (0,1)$, and its law of motion is given by $k_{t+1} = (1-\delta)k_t + x_t$. The intermediate input goods sector *z* has *j* intermediate input producing firms who own a technology with which the continuum of intermediate input goods $j \in [0,1]$ can be produced combining capital k_j and labor services l_j.

The representative household supplies labor services and capital to *j* firms in sector *z*. Since all households are identical, they have identical preferences over consumption of final good and labor supply. At each period *t*, the representative household derives utility from consumption (c_t) and disutility from labor services (l_t). Preferences for the representative household working in firm *j* of sector *z* are given by:

$$\sum_{t=0}^{\infty} \beta^t [\ln c_{jt} + \{1 - \mathrm{V}(l_{jt})\}] \qquad (1)$$

where $\mathrm{V} : \mathrm{R}_+ \rightarrow \mathrm{R}$ is a convex function and $\beta \in (0,1)$ is the subjective discount rate. c_{jt} is the private consumption of final good *y* where index *j* denotes a particular firm in the intermediate goods production sector, *z*, where the household supplies its labor, l_{jt}. The assumption that the utility function is additively separable in consumption and labor supply (or leisure) is standard in relevant literature. The assumption that the function **V(.)** is convex (and not strictly convex) is required to avoid unnecessary complications of having cross derivatives and second derivatives of utility function with respect to labor services[2]. In other words, the assumption is made in purpose of having the utility linear in labor services, which may be justified by the lottery argument of Hansen (1985). All households are endowed with $k_0 > 0$ units of capital at $t = 0$ and one unit of time at each instant. The representative household is also endowed with the property rights of the

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[2] This assumption restricts the utility function with $\mathbf{V}_{ll}(t) = \mathbf{V}_{lc}(t) = \mathbf{V}_{cl}(t) = 0$.

representative firm in sector *y*, and hence receives all profits that may exist in equilibrium. Since there is competitive market for the final goods, equilibrium profits will eventually be zero, and hence will be ignored in the household's budget constraint.

The property rights of *j* firms in intermediate input goods sector *z* are held by an external agent who does not belong to the model economy. This is sensible if one assumes that the economy under consideration do not own the technology required to produce intermediate input goods, and thus requires external firms to operate and take back any proceeds that may exist in equilibrium. Given the main purpose of this paper, this assumption is fairly innocuous and does not inhibit the general equilibrium analysis of the economy in any way. The reason why this assumption is held will become clearer in section 2.4 where I introduce the monopolistic competition amongst *j* firms operating in the intermediate input goods sector. With monopolistic competition in sector *z* and as long as $j^* \in [0,1)$ there will be nonzero profits in equilibrium. Imperfection in input goods market is therefore more compatible with the assumption that the technology used to produce the particular good, for which such market imperfection exists, is not locally owned. Apart from this conjecture, the assumption avoids the unnecessary complications of including nonzero profits in the representative household's budget constraint. The assumption, however, can easily be relaxed, and the fact that imperfection in goods market may be due to other reasons, as may be found in the literature concerning Industrial Organizations, is humbly acknowledged.

2.2 Government:

The government consumes exogenous g_t of the final good each period and has, at its disposal, taxation of factor income as the sole instrument to finance the predetermined revenue target g_t. The government generates its revenue with flat rate taxes on all incomes from labor and capital. The government taxes labor income from this sector at a homogeneous rate, independent of which particular firm *j* the representative worker is working in. The same assumption holds for capital income taxation. The tax rates on per unit labor income and capital income are τ_t and θ_t, respectively. The government's period *t* budget constraint is given by:

$$g_t \leq \tau_t \int_0^1 w_{jt} l_{jt} dj + \theta_t \int_0^1 r_{jt} k_{jt} dj \qquad (2)$$

In order to avoid potential time consistency problem of optimal taxation, the assumption that the government has access to an effective commitment technology with which it can sustain all initially announced tax plans, is maintained throughout the paper. Hence all optimal tax plans are time consistent, and no-commitment outcomes, as may be found in Persson *et. al.* (1987), Benhabib & Rustichini (1997) and Phelan & Stacchetti (2001) in similar settings, are suppressed. The government is benevolent, i.e. it maximizes welfare of the economy. This paper abstracts from introducing government bonds, and thus assumes that the government runs a balanced budget each period. This is because government savings is not one of the focuses of this particular research, and introducing bonds in the model will not change the key results of the paper (see Ljungqvist & Sargent (2000) for details). The model, however, is flexible and government savings can conveniently be incorporated at the mere cost of algebra only.

2.3 Final/manufacturing Goods Sector:

The final good, y_t, the numeraire, is purchased by households for private consumption (c_t) and investment (x_t), and purchased by government for government consumption (g_t). The final good can be produced combining a continuum of intermediate goods z_{jt} with $j \in [0,1]$ using the following technology:

$$y_t \leq A_y \left(\int_0^1 z_{jt}^{\sigma} dj \right)^{\frac{1}{\sigma}} \qquad (3.1)$$

with $\sigma < 1$ and A_y is the time invariant sector specific TFP parameter[3].

With p_{jt} denoting the relative price of the intermediate input *j*, the representative firm takes the resource constraint *(3.1)* with equality and competitively maximizes profits. Assume that the sole supplier of the intermediate input *j* is the representative firm *j* in the intermediate input goods producing sector, and households are prohibited to purchase these intermediate goods and rent it to firms in sector *y*. The representative firm in the final goods producing sector faces the following sequence of static problems:

$$\max_{y_t, z_{jt}} \quad A_y \left(\int_0^1 z_{jt}^{\sigma} dj \right)^{\frac{1}{\sigma}} - \int_0^1 p_{jt} z_{jt} dj \qquad (3.2)$$

The necessary condition with respect to a change in z_{jt} yields the input price as a function of the input demand:

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[3] With this production technology, the term $\frac{1}{1-\sigma}$ is the elasticity of substitution between any two intermediate input goods, and the technology exhibits Constant Returns to Scale for all values of σ.

$$p_{jt} = (A_y)^{\sigma} (y_t)^{(1-\sigma)} z_{jt}^{(\sigma-1)} \qquad (3.3)$$

Inverting it yields the equilibrium demand for intermediate input as:

$$z_{jt} = (A_y)^{\frac{\sigma}{(1-\sigma)}} y_t (p_{jt})^{-\left(\frac{1}{1-\sigma}\right)} \qquad (3.4)$$

Note that *(3.4)* has standard features of a demand correspondence. The related price elasticity of demand is $\frac{1}{\sigma-1}$ which is strictly negative since $\sigma < 1$ by assumption.

2.4 Intermediate Goods Sector:

Firms producing intermediate goods are indexed by *j* and combine labor (l_{jt}) and capital (k_{jt}) to produce a continuum of intermediate goods z_{jt}. The technology for firm *j* is given by:

$$z_{jt} \leq A_z k_{jt}{}^{\eta} l_{jt}{}^{1-\eta} \qquad (4.1)$$

Where A_z is the time invariant sector specific TFP parameter, $\eta \in (0,1)$ is the capital share parameter and (k_{jt}, l_{jt}) are units of capital and labor employed in firm *j*, respectively.

Firm *j* sets the price p_{jt} for intermediate good z_{jt} in a framework of monopolistic competition. The institutional settings are such that firm *j* in this sector is allowed to sell its output z_{jt} only to firms in the final goods sector (and not to the households). As for wages in this sector, they are also firm specific. Assume all workers in a firm *j* in this sector forms a union and choose their wage w_{jt} so as to maximize the utility of the representative worker in the firm. Since the intermediate input is only supplied to the final/manufacturing goods sector firms, the representative firm *j* takes into consideration the equilibrium demand for intermediate input *j* which is determined in the profit maximization problem of the representative firm in sector *y*. Hence the representative firm *j* in this sector faces the following sequence of static problems:

$$\max_{z_{jt}, k_{jt}, l_{jt}} \quad p_{jt} z_{jt} - r_{jt} k_{jt} - w_{jt} l_{jt} \qquad (4.2)$$

s.t. $\quad z_{jt} = A_z k_{jt}{}^{\eta} l_{jt}{}^{1-\eta}$

$$p_{jt} = (A_y)^{\sigma} (y_t)^{(1-\sigma)} z_{jt}^{(\sigma-1)}$$

Incorporating both constraints in *(4.2)*, the profit maximization problem of the representative firm *j* in this sector can be rewritten as:

$$\max_{k_{jt},l_{jt}} \quad A_y^{\sigma}(y_t)^{(1-\sigma)} A_z^{\sigma}(k_{jt})^{\eta\sigma} l_{jt}{}^{\sigma(1-\eta)} - r_{jt}k_{jt} - w_{jt}l_{jt} \tag{4.3}$$

Necessary conditions for an optimum for changes in factors give the distribution of incomes:

$$r_{jt}k_{jt} = \sigma\eta p_{jt}z_{jt} \tag{4.4}$$

$$w_{jt}l_{jt} = \sigma(1-\eta)p_{jt}z_{jt} \tag{4.5}$$

Where $p_{jt}z_{jt} = A_y^{\sigma}(y_t)^{(1-\sigma)}(z_{jt})^{\sigma}$

Interesting to note here that the equilibrium demand for intermediate input good z_{jt} crucially depends on the parameter σ. The parameter σ represents the degree of market power each intermediate input goods firm possesses, and hence will stand for the distortions created by the monopolistic competition amongst the *j* intermediate input goods firms. The lower the value of σ, the greater is the market power of individual firms in sector *z*. Alternatively, as $\sigma \to 1$, market power diminishes, and the market for intermediate input goods moves towards a perfectly competitive structure.

The values of p_{jt}, z_{jt}, l_{jt} and k_{jt} are solution to the system comprising four equations:

$$z_{jt} = A_z k_{jt}{}^{\eta} l_{jt}{}^{1-\eta} \tag{5.1}$$

$$z_{jt} = (A_y)^{\frac{\sigma}{(1-\sigma)}} y_t (p_{jt})^{-\left(\frac{1}{1-\sigma}\right)} \tag{5.2}$$

$$r_{jt}k_{jt} = \sigma\eta p_{jt}z_{jt} \tag{5.3}$$

$$w_{jt}l_{jt} = \sigma(1-\eta)p_{jt}z_{jt} \tag{5.4}$$

(5.1) and *(5.2)* give an expression for l_{jt} as follows:

$$(l_{jt})^{[1-\sigma(1-\eta)]} = \sigma(1-\eta)A_y^{\sigma}(y_t)^{(1-\sigma)} A_z^{\sigma}(k_{jt})^{\sigma\eta}(w_{jt})^{-1} \tag{5.5}$$

Using *(5.3)* and *(5.1)*, an expression for k_{jt} is as follows:

$$k_{jt} = \{\sigma\eta A_y^{\sigma}(y_t)^{(1-\sigma)} A_z^{\sigma} l_{jt}^{\sigma(1-\eta)}(r_{jt})^{-1}\}^{\frac{1}{(1-\sigma\eta)}} \quad (5.6)$$

Substituting *(5.6)* in *(5.5)* yields the equilibrium demand for labor in firm *j* in this sector:

$$l_{jt} = \Omega_{jt}(w_{jt})^{\frac{(1-\sigma\eta)}{(\sigma-1)}} \quad (5.7)$$

Where $\Omega_{jt} \equiv [\sigma(1-\eta)^{(1-\sigma\eta)}\eta^{\sigma\eta}(A_y A_z)^{\sigma}(y_t)^{(1-\sigma)}(r_{jt})^{-\sigma\eta}]^{\frac{1}{(1-\sigma)}}$

Note that although wages are firm specific, rental rates of capital are not. Hence $r_{jt} = r_t$ for all *j*. Because of the model's symmetry, therefore, Ω_{jt} does not depend on *j*. Hence it is convenient to define $\Omega_{jt} \equiv \Omega_t$.

2.5 Household's problem:

The program of a representative household working in a particular firm *j* for sector *z* is to choose private consumption, labor services and a period ahead capital stocks in order to maximize its utility defined by *(1)* subject to sequence of its budget constraints and the demand function for the type of labor it supplies in sector *z*, defined by *(5.7)*. The program can be written as:

$$\max \quad \sum_{t=0}^{\infty} \beta^t [\ln c_{jt} + \{1 - \mathrm{V}(l_{jt})\}]$$

s.t.

$$c_{jt} + x_{jt} \le (1-\tau_t) w_{jt} l_{jt} + (1-\theta_t) r_{jt} k_{jt}; \quad (6.1)$$

$$l_{jt} = \Omega_t (w_{jt})^{\frac{(1-\sigma\eta)}{(\sigma-1)}}; \quad (6.2)$$

$$k_0 > 0 \quad (given) \quad (6.3)$$

Since demands for labor are symmetrical for all firms in sector *z*, the programs are same for all households despite the different firms in sector *z* in which they work. All households thus make exactly the same decisions. So without loss of generality, the index *j* pertaining to the firm in sector *z* with which a household is affiliated can be omitted. Also, since capital in sector *z* is not firm specific, aggregate investment follows:

$$x_t = k_{t+1} - (1-\delta) k_t \quad (6.4)$$

Defining $R_t \equiv [r_t(1-\theta_t)+(1-\delta)]$, and using *(6.2)* and *(6.4)* in *(6.1),* the time t budget constraint for the representative household can be written as:

$$c_t + k_{t+1} \le (1-\tau_t)\left(\frac{1}{\Omega_t}\right)^{\frac{(\sigma-1)}{(1-\sigma\eta)}} l_t^{\frac{\sigma(1-\eta)}{1-\sigma\eta}} + R_t k_t \qquad (6.5)$$

The representative household's problem is, therefore, to choose c_t, l_t and k_{t+1} in order to maximize the j invariant version of *(1)* subject to *(6.5)*. With $\beta^t \lambda_t^h$ as the multiplier associated with the time t budget constraint *(6.5),* the necessary conditions for an optimum include the budget constraint *(6.5)* itself and the followings:

$$c_t: \quad \lambda_t^h = (c_t)^{-1} \qquad (6.5a)$$

$$l_t: \quad \mathbf{V}_l(t) = \lambda_t^h (1-\tau_t)\frac{\sigma(1-\eta)}{(1-\sigma\eta)} w_t \qquad (6.5b)$$

$$k_{t+1}: \quad \frac{\lambda_t^h}{\lambda_{t+1}^h} = \beta R_{t+1} \qquad (6.5c)$$

and the Transversality condition that puts a restriction on the terminal value of the household's capital stock in terms of utility:

$$\mathbf{lim}_{t\to\infty}[\lambda_t^h k_{t+1}] = 0 \qquad (6.5d)$$

Consolidating the necessary conditions yields:

$$\mathbf{V}_l(t) = (c_t)^{-1}(1-\tau_t)\frac{\sigma(1-\eta)}{(1-\sigma\eta)} w_t \qquad (6.6a)$$

$$\frac{c_{t+1}}{c_t} = \beta R_{t+1} \qquad (6.6b)$$

$$\mathbf{lim}_{t\to\infty} \frac{k_{t+1}}{c_t} = 0 \qquad (6.6c)$$

These optimality conditions are intuitive and simple to interpret. Condition *(6.6a)* equates the marginal rate of substitution between labor and consumption to the distortion adjusted after tax relative price of one unit of labor in terms of consumption. The term $\frac{\sigma(1-\eta)}{(1-\sigma\eta)}$ in *(6.6a)* can be interpreted as the inverse of the distortion created by the monopolistic competition in the intermediate goods producing sector. Recall that $\sigma < 1$ and lower σ indicates greater market power for the firm *j* in sector *z* creating higher distortion $\frac{(1-\sigma\eta)}{\sigma(1-\eta)}$ in equilibrium for the representative household. In this setting $\sigma \to 1$ indicates very low market power, which is tantamount to saying that the market for intermediate goods is asymptotically competitive, and implies $\frac{(1-\sigma\eta)}{\sigma(1-\eta)} \to 1$ such that *(6.6a)* yields the competitive equilibrium condition with distorting taxes on labor income. Hence the representative household will maximize utility at the point where its marginal rate of substitution between labor and consumption equals its net earning of per unit labor employed in that particular sector after adjusting for the distortion created by monopolistic competition.

Condition *(6.6b)* is the standard Euler equation for deterministic infinite horizon optimization problem and here illustrates the dynamic path of private consumption in this economy. The transversality condition *(6.6c)* implies that the discounted lifetime utility is maximal when the terminal value of the capital stock in terms of private consumption is zero. In other words, it states that for an optimal consumption allocation the present discounted value of the household's capital stock must be zero as time goes to infinity.

2.6 Macroeconomic Equilibrium:

With distorting taxes, any competitive equilibrium (from a set of possibly many indexed by different tax policies) is a second-best outcome, and hence not pareto optimal. In this model economy, the equilibrium concept cannot be one of a competitive equilibrium, and with added distortions from the monopolistic competition in the intermediate input goods sector, the equilibrium is likely to be one step less efficient than a second-best outcome. I will restrict attention to equilibria with the following properties: (i) they are symmetric with respect to the intermediate input goods; (ii) they are recursive, in the sense that in each period all decision makers condition their actions on the state variables only.

In order to define the equilibrium concept, I will first propose a set of definitions, using symbols without time subscripts to denote the one-sided infinite sequence for the corresponding variable.

Definition 1.1: A private allocation is a sequence $\{c, l, x, k, z, y\}$ that satisfies the resource constraints given by (3.1), (4.1) and (6.4). ■

Definition 1.2: A government policy is a 2-tuple of sequences $\{\tau, \theta\}$. ■

Definition 1.3: A government allocation is a sequence $\{g\}$. ■

Definition 1.4: A price system is a 3-tuple of non-negative bounded sequences $\{p, w, r\}$. ■

The following equilibrium concept is proposed:

Definition 1.5 (Macroeconomic Equilibrium):

A *Macroeconomic Equilibrium* is a private allocation, a government allocation, a price system and a government policy such that

(a) Given the price system, the private allocation $\{y, z\}$ solves the problem of the representative firm in sector y.

(b) Given the price system and derived demand function for z, the private allocation $\{k, l\}$ solves the problem of firm j in sector z.

(c) Given the price system, government policy and derived labor demand function for l_z, the private allocation $\{c, l, k, x\}$ solves the problem of the representative household.

(d) All markets clear in the long run. ■

There are many macroeconomic equilibria indexed by different tax policies of the government. The multiplicity of the macroeconomic equilibria motivates the optimal taxation problem for the government.

Definition 1.6 (Optimal Taxation Problem):

Given the time 0 initial endowments of capital stock and the government allocation (which is the preset revenue target), the *Optimal Taxation Problem* for the government is to choose a macroeconomic equilibrium that maximizes expression (1). ■

For a given welfare criterion as in *(1)*, which the government uses to evaluate different allocations, the *optimal taxation problem* is to pick the fiscal policy (or one of them if there are many) that generates the macroeconomic equilibrium allocation giving the highest value of the welfare criterion. Combining the necessary conditions for an optimum derived from the solutions to the representative household's and firms' problems, the following system which, together with the transversality condition, characterizes the macroeconomic equilibrium dynamics for the model economy under consideration:

$$y_t \geq c_t + x_t + g_t \tag{7.1a}$$

$$y_t \leq A_y \left(\int_0^1 z_{jt}^{\sigma} dj \right)^{\frac{1}{\sigma}} \tag{7.1b}$$

$$z_{jt} \leq A_z k_{jt}^{\eta} l_{jt}^{1-\eta} \tag{7.1c}$$

$$k_t = \int_0^1 k_{jt} dj \tag{7.1d}$$

$$x_t = k_{t+1} - (1-\delta) k_t \tag{7.1e}$$

$$\int_0^1 l_{jt} dj \leq 1 \tag{7.1f}$$

$$p_{jt} = (A_y)^{\sigma} (y_t)^{(1-\sigma)} z_{jt}^{(\sigma-1)} \tag{7.1g}$$

$$r_{jt} k_{jt} = \sigma \eta p_{jt} z_{jt} \tag{7.1h}$$

$$w_{jt} l_{jt} = \sigma (1-\eta) p_{jt} z_{jt} \tag{7.1i}$$

$$\mathbf{V}_l(t) = (c_t)^{-1} (1-\tau_t) \frac{\sigma(1-\eta)}{(1-\sigma\eta)} w_t \tag{7.1j}$$

$$\frac{c_{t+1}}{c_t} = \beta [(1-\theta_{t+1}) r_{t+1} + (1-\delta)] \tag{7.1k}$$

Expression *(7.1a)* illustrates how the final manufacturing good is exhausted for private consumption, government consumption and investment each period. *(7.1b)* and *(7.1c)* are the two technologies in operation that produces the final manufacturing good and intermediate input goods. *(7.1d)* and *(7.1f)* are the market clearing conditions for raw capital and labor. *(7.1e)* states the investment dynamics. *(7.1g)* presents the equilibrium demand for intermediate input good, and *(7.1h)* and *(7.1i)* present income distribution for the representative household. The rest two expressions are macroeconomic equilibrium reactions of the representative household for any arbitrarily chosen tax rates. In order for any tax rule to be implementable in equilibrium, it must satisfy these two restrictions, which in optimal taxation literature are analogous to implementability constraints.

The macroeconomic equilibrium dynamics is characterized by the solution to the symmetric version of system *(7.1)* comprising eleven expressions in eleven unknowns which are $(c_t, k_t, x_t, y_t, l_t, p_t, z_t, w_t, r_t, \tau_t, \theta_t)$. Symmetry in the macroeconomic equilibrium is drawn with a fairly innocuous assumption that all intermediate input goods are same, such that the equilibrium system *(7.1)* is *j* invariant. This assumption and restriction towards symmetric equilibrium aids the theoretical tractability of the model and therefore makes the closed form solutions relatively more convenient to achieve.

2.7 The Optimal Taxation Problem:

Following definition 1.6, and following Chamley's (1986) approach, the optimal taxation problem for the government assumes that net returns to the factors are constrained (and assumed to be greater than some arbitrary value). Suppose rather than choosing tax rates, the government chooses after-tax rental rates of capital, $\tilde{r}_t$, and after-tax wage rate, $\tilde{w}_t$, such that $\tilde{r}_t = (1-\theta_t)r_t$ and $\tilde{w}_t = (1-\tau_t)w_t$.

Since the government is benevolent, it maximizes the household's utility. In choosing the optimal after tax returns, this optimization problem involves not just the government's own budget constraint, but also a set of other constraints. In a symmetric equilibrium where all intermediate input goods are same, it is simple to show, using the linear homogeneity property of *(4.1)* that Euler's theorem imply:

$$\sigma A_y A_z k_t^{\eta} l_t^{1-\eta} = r_t k_t + w_t l_t \qquad (8.1)$$

Hence the symmetric equilibrium version of the government budget constraint, assuming that it balances each period, can be restated as:

$$g_t = \sigma A_y A_z k_t^{\eta} l_t^{1-\eta} - \tilde{r}_t k_t - \tilde{w}_t l_t \qquad (8.2)$$

Expression *(8.2)* is the modified budget constraint that incorporates the necessary conditions from the firm's optimization problem. The government's choice of after-tax returns is also constrained by the aggregate resource constraint and macroeconomic equilibrium reactions of the representative household. In a Primal approach analogue of this problem, due to Jones *et. al.* (1997), Chari & Kehoe (1999) and Ljungqvist & Sargent (2000), these equilibrium reactions are similar to the implementability constraints, which restricts the optimally chosen tax rules to be implementable in equilibrium.

The aggregate resource constraint of the economy, under the assumption that all intermediate inputs goods are same can be stated as:

$$c_t + k_{t+1} + g_t = A_y A_z k_t^{\eta} l_t^{1-\eta} + (1-\delta)k_t \qquad (8.3)$$

Expression *(8.3)* is therefore the economy's aggregate technology constraint, and is derived substituting for the technology that produces homogeneous intermediate goods z_t. The symmetry condition supports this simple substitution since the intermediate input good is not a final good, and the final manufacturing good can only be produced using a single factor which is the intermediate input good. This simplification, however, would not be appropriate if one had assumed that the production technology of final manufacturing good requires household's labor services (or capital stocks) as inputs in addition to intermediate input goods. This will be highlighted in the next modified version of the model with entry barriers in labor employment.

The macroeconomic equilibrium reactions of the representative household, or alternatively the set of implementability constraints, are the necessary conditions *(6.6a)* and *(6.6b)* from the household's optimization problem, which are repeated here for convenience:

$$\mathrm{V}_l(t) = (c_t)^{-1} \frac{\sigma(1-\eta)}{(1-\sigma\eta)} \tilde{w}_t \qquad (8.4)$$

$$c_{t+1} = c_t \beta[\tilde{r}_{t+1} + (1-\delta)] \qquad (8.5)$$

The optimal taxation problem for the government, therefore involves maximizing household's utility subject to a set of constraints *(8.2)*, *(8.3)*, *(8.4)* and *(8.5)*. Assume that the government's consumption expenditure, g_t, is small enough such that the problem's constraint set is convex. The Lagrangian of the problem is:

$$\begin{aligned} L^G = \; & \sum_{t=0}^{\infty} \beta^t \{[\ln c_t + \{1 - \mathrm{V}(l_t)\}] \\ & + \psi_t [\sigma A_y A_z k_t^{\eta} l_t^{1-\eta} - \tilde{r}_t k_t - \tilde{w}_t l_t - g_t] \\ & + \phi_t [A_y A_z k_t^{\eta} l_t^{1-\eta} + (1-\delta)k_t - c_t - g_t - k_{t+1}] \\ & + \mu_t [\mathrm{V}_l(t) - (c_t)^{-1} \frac{\sigma(1-\eta)}{(1-\sigma\eta)} \tilde{w}_t] \\ & + \xi_t [c_{t+1} - c_t \beta(\tilde{r}_{t+1} + 1 - \delta)]\} \end{aligned} \qquad (9)$$

Where $\beta^t \psi_t$, $\beta^t \phi_t$, $\beta^t \mu_t$ and $\beta^t \xi_t$ are the Lagrange multipliers associated with constraints *(8.2)*, *(8.3)*, *(8.4)* and *(8.5)*, respectively. In computing the optimal tax rates, I will restrict attention to asymptotic steady state only. While this may be arguably a limitation for analyzing capital income tax rates, it is in no way inhibiting for analyzing optimal tax rates on labor income.

2.7.1 Optimal Labor Income Tax:

Maximizing *(9)* with respect to labor supply yields the following necessary condition in order for an optimal labor income tax rate to be optimal and implementable in macroeconomic equilibrium:

$$l_t: \qquad \mathbf{V}_l(t) = \psi_t \tau_t [\sigma(1-\eta) A_y A_z k_t^{\eta} l_t^{-\eta}] + \phi_t \frac{\sigma(1-\eta) A_y A_z k_t^{\eta} l_t^{-\eta}}{\sigma} \qquad (9.1)$$

Condition *(9.1)* is fairly intuitive. A marginal increase in labor services in period *t* increases the quantity of available final goods, via an increase in the production of intermediate input goods, by the amount $\frac{[\sigma(1-\eta) A_y A_z k_t^{\eta} l_t^{-\eta}]}{\sigma}$, which has social marginal value of ϕ_t. In addition, there is an increase in tax revenues for the government which equals $\tau_t [\sigma(1-\eta) A_y A_z k_t^{\eta} l_t^{-\eta}]$ and has social marginal value ψ_t. This enables the government to reduce other tax burden by an amount equal to $\psi_t \tau_t [\sigma(1-\eta) A_y A_z k_t^{\eta} l_t^{-\eta}]$. In equilibrium, the sum of this two effects must equal the marginal disutility of labor services each period. Notice that a marginal increase in labor services increases the amount of available final goods, via an increase in the production of intermediate input goods not exactly by an amount equal to the marginal revenue product of labor, and the amount is distorted in the margin by the parameter σ. Hence the optimal labor income tax in this case must be designed in a way to cure this distorted margin.

Condition *(9.1)* yields the optimal labor income tax rule as:

$$\tau_t^* = \frac{1}{\psi_t} \left[\frac{\mathbf{V}_l(t)}{\sigma(1-\eta) A_y A_z k_t^{\eta} l_t^{-\eta}} - \frac{\phi_t}{\sigma} \right] \qquad (9.2)$$

Since the government runs a balanced budget each period, optimal tax rule is finite as long as σ is nonzero. In order to compare this optimal tax rule with others that may exist in similar settings, consider the case where $\sigma \to 1$. Assume there exists a similar economy, with the exception that intermediate input producing firms have no market power at all, i.e. an economy with $\sigma \to 1$.

Proposition 1: The optimal labor income tax rule *(9.2)* is lower than competitive equilibrium *Ramsey* optimal labor income tax rule, as long as the demand for the intermediate input good is elastic.

Proof: Consider *(9.2)* with $\sigma \to 1$, and define the optimal tax rule as $\tilde{\tau}_t$. Under the current setting, $\tilde{\tau}_t$ can be considered as the otherwise perfect competition analogue of optimal labor income tax rate. Now examine the difference between these two optimal tax rules, which is:

$$\tau_t^* - \tilde{\tau}_t = \frac{\phi_t(\sigma - 1)}{\psi_t \sigma} \qquad (9.3)$$

which is a strictly negative quantity as long as $0 < \sigma < 1$, and both the government budget constraint and resource constraint bind. ■

Proof of proposition 1 is based on the condition that demand for the intermediate input good is highly elastic. The key result implied by proposition 1 therefore is that labor services enjoy a relatively lower income tax if employed in a monopolistically competitive firm as compared to a perfectly competitive firm. Entry barriers to new firms in the industry thus forces the government to reduce its tax rate on labor income in order to provide incentives for labor services, or as a way to compensate for the distortion created by market power. The intuition behind a lower labor income tax rule becomes clearer when one considers equilibrium reaction of representative household. Unlike the typical competitive equilibrium reaction, condition *(8.4)* incorporates a distortion term. In choosing an optimal tax rate, the government must compensate this distortion by setting the labor income tax rate lower. However this does not necessarily hold if the intermediate input good is demanded inelastically. I will defer the intuition regarding the elasticity of the intermediate input good towards later sections of the paper.

2.7.2 Optimal Capital Income Tax:

The reason why only steady state of optimal tax rule is considered exclusively becomes clearer when one considers the properties of optimal capital income tax rule. Due to investment dynamics, a one period distortion created by a nonzero capital income tax rate grows exponentially over time. Due to this trivial and chaotic consequences of a nonzero capital income tax rate, literature on optimal taxation in competitive markets prescribes setting capital income taxes equal to zero in the long run (see for instance, Chamley (1986), Judd (1985 & 1999), Jones *et. al.* (1993 & 1997), Chari & Kehoe (1999), among others). This celebrated and insightful result is often challenged by varying the assumption about government's ability to commit to future tax rates (see for instance, Benhabib & Rustichini (1997), and Phelan & Stacchetti (2001)). What this paper shows that even with a perfect commitment technology, this phenomenal result may not hold if one relaxes the assumption of perfectly competitive markets.

Considering *(9)*, the necessary condition for a change in one period ahead capital stock, k_{t+1}, can be written as:

$$k_{t+1}: \quad \phi_t = \beta\left[\psi_{t+1}\theta_{t+1}(\eta\sigma A_y A_z k_{t+1}^{\eta-1} l_{t+1}^{1-\eta}) + \phi_{t+1}\left(\frac{\eta\sigma A_y A_z k_{t+1}^{\eta-1} l_{t+1}^{1-\eta}}{\sigma} + 1 - \delta\right)\right] \qquad (9.5)$$

The steady state version of the Euler equation, condition *(8.5)*, from household's maximization problem is:

$$1 = \beta[\tilde{r} + 1 - \delta] \qquad (9.6)$$

Now consider the steady state version of *(9.5)*, in terms of steady state prices,

$$\phi = \beta[\psi(r - \tilde{r}) + \phi(\sigma^{-1} r + 1 - \delta)] \qquad (9.7)$$

Proposition 2: Given the government's optimal taxation problem *(9)*, the steady state optimal capital income tax is nonzero.

Proof: Suppose government expenditure stay constant after some period *T*, such that the optimal taxation problem *(9)* converges to a steady state, i.e. all endogenous variables remain constant. Consider steady state version *(9.7)* and substitute *(9.6)* in it. This gives:

$$\theta^* = \frac{\phi\left[1 - \frac{1}{\sigma}\right]}{[\phi + \psi]} \qquad (9.8)$$

And clearly $\theta^* \neq 0$ since $\sigma < 1$ and both the government budget constraint and resource constraint bind. ■

Hence for proposition 2, the important point in this context is to see how the celebrated competitive market *Ramsey* taxation result of zero steady state capital income tax collapses under the current setting. What drives this startling result is once again the market power parameter σ. The government in this setting designs the optimal tax rules not just to maintain production efficiency, but also to cure for the distortions created by the monopolistic competition. In doing so, it finds it optimal to tax (or subsidize, depending on the magnitude of σ) capital income in the long run. Intuitively, one can find this result justifiable considering the necessary condition *(9.5)*. A marginal increment of capital stock in period *t* in the intermediate input good sector increases the quantity of available final goods at period *t+1*, via increased production (and lower prices) of intermediate input goods, by the amount $(\sigma^{-1}r_{t+1} + 1 - \delta)$, which has social marginal value ϕ_{t+1}. The marginal increment in final goods is distorted by the amount σ. This enables the government to reduce other taxes by the amount $(r_{t+1} - \tilde{r}_{t+1})$, but in doing so it must compensate for the adverse effects caused by the distortion of monopolistic competition. Note that for any non-negative σ, indicating higher elasticity of substitution between two intermediate input goods (and elastic demand for the intermediate input good), it is optimal for the government to subsidize capital income. Hence in reducing the other tax burden by the amount $(r_{t+1} - \tilde{r}_{t+1})$, the government must also compensate capital owners for σ affected return by allowing a subsidy for capital income. For inelastic substitution possibility between two intermediate goods, the government finds it optimal to tax capital income. Once again I will defer the intuition behind it until subsection 3.7.2. The competitive market analogue of this result (consider $\sigma \to 1$ in *(9.8)*) such that the market for intermediate goods is asymptotically competitive, steady state capital income tax converges to zero, i.e. the model establishes the celebrated Chamley-Judd result of zero capital income taxation in steady state.

3. The Modified Model:

In this section, a modified version of the preceding model that incorporates, in addition to entry barriers to new firms, entry barriers to labor services in a particular sector, is presented to address the issue of optimal choice of tax rates. As it has already been mentioned, this model can be a useful benchmark to address optimal taxation of public sector income in presence of entry barriers to labor in public sector. This simple model can also be interpreted as a model that incorporates public-private participation and optimal taxation with heterogeneous agents.

The issue of sellers' taxation when public sector joins production side by side with private sector was covered to a considerable extent by Schmitz Jr. (2001). Existing literature on optimal taxation has a handful of important contributions that deal with income taxation with heterogeneous agents, as may be found in Barreto & Alm (2003) and Judd (1985). Of these two, the former may be treated as more relevant to the kind of task undertaken in this section of the paper, since the heterogeneity of agents in Barreto & Alm (2003) is defined as public and private agent, and they present an endogenous growth model to investigate, among others, tax preferences of public and private agents. The main focus of that particular paper was however, taxation and growth effects in the presence of corruption. It seems, therefore, that there is much room for cross-fertilization between the optimal income tax literature and the literature focusing on contemporaneous public-private participation in production, which is exactly where this modified model is intended to contribute.

3.1 The Environment:

Time *t* is discrete, runs forever, and there is no uncertainty. The two production sectors indexed by *s*, with *s* = *y*, *z*, produce the final good, *y*, which can be consumed, invested or used for government consumption, and a continuum of intermediate input goods *z*, which is used in connection with labor services to produce the final manufacturing good *y*. This is one crucial assumption where the model is slightly modified from the preceding one, since the production of the final good now requires labor services along with the intermediate input goods. I will explain the reason why this modification is undertaken later. The final goods sector *y* is the private sector in terms of ownership, and the intermediate input goods sector *z* is the public sector, meaning that it is owned by the government. A note of clarification about the public sector production is worth mentioning. In this environment, one should not confuse the publicly produced good (the intermediate input good) with a publicly provided good, where the latter may have specific properties (like non-rival and non-excludable, or congestion, for instance) of a public good and may enter the agents' utility function. The intermediate input good in this environment is publicly produced but has a market price and possess no properties of a public good, i.e. although it is publicly produced, it has all features of a private good in the market. There is a finite measure of agents representing households, who can be private agents or publicly employed agents (hereafter, public agents). All private agents are identical, and all public agents are identical. Hereafter, I will use subscript *i*, with *i = 1, 2*, to denote private and public agents, respectively. Specifically, there is a unit square of private agents, or agent type 1, and a unit square of public agents, or agent type 2.

Agents derive utility from consumption and disutility from labor services. Preferences of agent *i* over sequences of consumption and labor services are represented by the time separable utility function over infinite horizon:

$$\sum_{t=0}^{\infty} \beta^t [\ln c_{jit} + \{1 - \mathrm{V}^i(l_{yit}, l_{zjit})\}] \qquad (10)$$

where $i = 1, 2$, denotes type of agent, $V^i : R_+^n \rightarrow R$ is a convex function with $n = 1, 2$, and $\beta \in (0,1)$ is the subjective discount rate which is agent type independent. c_{jit} is the consumption of final good y by agent type i working in firm j with $j \in [0,1]$ in the intermediate goods production sector z, and l_{sit} is the agent i's labor service to production sector s, with $s = y, z$ denoting the final goods and intermediate input goods sector, respectively. Assume that the private agent can work and save, but the public agent is only allowed to work. The private agent is endowed with $k_0 > 0$ units of installed capital at period 0. Moreover both types of agents are endowed with one unit of time at each instant. Installed capital depreciates at constant rate δ with $\delta \in (0,1)$, and its law of motion is given by $k_{t+1} = (1-\delta)k_t + x_t$, where x_t denotes private agent's investment at period t.

3.2 Production:

Entry barrier for new firms in the intermediate input goods sector is similar to that used in the benchmark model. All firms in sector z are public sector firms, and therefore property rights of these firms are owned by the government. There are j intermediate input goods firms in sector z (the public sector) with $j \in [0,1]$ denoting a continuum of intermediate input goods produced by these firms. Firm j in sector z combines labor and capital for production, sets the price p_{jt} of its produced intermediate input good in the framework of a monopolistic competition, and returns any nonzero profits that may exist in equilibrium to the government each period.

Entry barrier in labor market is modelled in two ways. First, the public agent is not allowed to work in the private sector. This is an entry prohibition, which eliminates l_{y2t} from the public agent's utility function (and private sector's technology). It does not take much to realize why this simplification is made, since the assumption is sensible from a real world point of view. The second form of entry barrier is introduced in the public sector, where the public sector (or public agents) set a parameter $\omega \in [0,1)$ to restrict entry for private labor in the public sector. A similar form of entry barrier is presented in Herrendorf & Teixeira (2004)'s exogenous growth model where insiders restrict the entry of outsiders. Real world examples of such entry restrictions for private labor services in the public sector are plentiful, starting from simple cognitive and competitive examinations to various steps of certification, licensing etc.

The technology for public sector firm j in sector z is represented by:

$$z_{jt} \leq A_z k_{jt}{}^{\eta} l_{zjt}{}^{1-\eta} \qquad (11.1)$$

$$l_{zjt} = (1-\omega) l_{zj1t} + b_{jt} l_{zj2t} \qquad (11.2)$$

$$l_{zj1t} \geq 0, \qquad l_{zj2t} > 0 \qquad (11.3)$$

Components of expression *(11.1)* have already been introduced in the benchmark model. Note that the restrictions $l_{zj1t} \geq 0$ and $l_{zj2t} > 0$ presented by *(11.3)* are jointly instrumental in interpreting the underlying assumptions proposed to model entry barriers in the labor market. The first restriction leaves the possibility for private agent to stop providing labor services in this sector, which might happen for very large values of ω. This is why a slight modification in the technology of the final good sector is proposed in this environment. The second restriction ensures that public agents work in the public sector.

Expression *(11.2)* represents the combination of labor services which is used in production of z_{jt}. $b_{jt} \in [1-\omega,1]$ is a choice variable that affects the productivity of the public agent in the public sector firm *j*. On the other hand, $\omega \in [0,1)$ is the parameter that affects the productivity of the private agent. Assume, for instance that $\omega = 0$ and $b_{jt} = 1$. This is the case where private labor and public labor services are perfect substitutes. But for $\omega = 0$ and $b_{jt} < 1$, private labor is more productive than the public labor. In other words, condition $\omega = 0$ would imply that public agents have no monopoly power. Now consider the case where $\omega > 0$ and $b_{jt} = \omega$. This again implies that private and public labor services are perfect substitutes, but in addition implies that the public agents have some monopoly power. With $\omega > 0$, if $b_{jt} > 1-\omega$, then private and public labor services are imperfect substitutes and the public agent is more productive. Hence any case involving $\omega > 0$ would imply that public agents have monopoly power, and consequently there exists some entry barriers for private labor to participate in producing z_{jt}. Following Herrendorf and Teixeira (2004), $\omega > 0$ is interpreted as summarizing the legal and institutional restrictions on private labor's entry into the *j*-th intermediate input goods firm that affect its productivity in public sector. The institutional arrangements are such that the public agent working in firm *j* in sector *z* has the right to choose b_{jt}. The assumption that $b_{jt} \in [1-\omega,1]$ implies that the most inefficient productivity level the public agent is allowed to choose is $1-\omega$.

Interesting to note that with the restriction *(11.2)*, the technology *(11.1)* is still strictly concave but does not follow standard Inada conditions in all of its inputs. This is one of the main reasons to hold restriction *(11.3)*, which states that for high values of the parameter ω implying very low productivity, the private agent might decide to stop working in the public sector. One can now justify why private labor services is included as a factor of production in the final goods sector. The final manufacturing good, *y*, is once again the numeraire. The private sector that produces final manufacturing good y combines a continuum of intermediate input goods z_{jt} and labor services from private agent (l_{y1t}) with the following technology:

$$y_t \leq A_y \left[\left(\int_0^1 z_{jt}^{\sigma} dj \right)^{\frac{1}{\sigma}} \right]^{\nu} l_{y1t}^{1-\nu} \qquad (12.1)$$

$$l_{y1t} > 0 \qquad (12.2)$$

with $\sigma < 1$ and A_y is the time invariant sector specific TFP parameter. The parameter $\sigma < 1$ otherwise indicates the degree of market power exercised by intermediate input goods firms, and lower value of the parameter indicates greater market power, and vice versa. The parameter $\nu \in (0,1)$ is the share parameter of intermediate input goods. As before, market for the final good is assumed to be perfectly competitive. The restriction *(12.2)* ensures that private agent supplies labor services in this sector. This is crucial since the technology *(12.1)* satisfies standard Inada conditions in all inputs, and the only source of labor for this sector is from the private agent.

3.3 Firms' problems and solutions:

There is a continua of measure one of identical private sector firms in sector y. The representative firm in sector y faces the following sequence of static problems:

$$\max_{y_t, z_{jt}, l_{y1t}} \; A_y \left[\left(\int_0^1 z_{jt}^{\sigma} dj \right)^{\frac{1}{\sigma}} \right]^{\nu} l_{y1t}^{1-\nu} - \int_0^1 p_{jt} z_{jt} dj - w_{yt} l_{y1t} \qquad (13.1)$$

where w_{yt} is the sector specific wage rate. The necessary condition with respect to a change in z_{jt} yields the input price as a function of the intermediate input j and labor services:

$$p_{jt} = \nu.(A_y)^{\frac{\sigma}{\nu}} (y_t)^{\left(1-\frac{\sigma}{\nu}\right)} (l_{y1t})^{\frac{\sigma(1-\nu)}{\nu}} (z_{jt})^{\sigma-1} \qquad (13.2)$$

The necessary condition with respect to a change in l_{y1t} yields:

$$w_{yt} = (1-\nu) A_y \left[\left(\int_0^1 z_{jt}^{\sigma} dj \right)^{\frac{1}{\sigma}} \right]^{\nu} l_{y1t}^{-\nu} \qquad (13.3)$$

The equilibrium demand for intermediate input good *j* can be derived inverting *(13.2)*, which is as follows:

$$z_{jt} = \nu^{\frac{1}{1-\sigma}} (A_y)^{\frac{\sigma}{\nu(1-\sigma)}} (y_t)^{\frac{\nu-\sigma}{\nu(1-\sigma)}} (l_{y1t})^{\frac{\sigma(1-\nu)}{\nu(1-\sigma)}} (p_{jt})^{-\left(\frac{1}{1-\sigma}\right)} \qquad (13.4)$$

Equilibrium demand for intermediate good *j* presented by *(13.4)* is inversely related to its equilibrium price, which is one of the desirable properties of demand functions for normal goods. The corresponding price elasticity of demand is $\frac{1}{\sigma-1}$, which is strictly negative since $\sigma < 1$ by assumption.

Firms in the public sector producing a continuum of *j* intermediate input goods z_{jt}, set the price of intermediate input good in the framework of a monopolistic competition. Each public sector firm *j* faces the following sequence of static problems:

$$\max_{z_{jt}, k_{jt}, l_{zj1t}, l_{zj2t}} \quad p_{jt} z_{jt} - r_{jt} k_{jt} - w_{zj1t} l_{zj1t} - w_{zj2t} l_{zj2t} \qquad (14.1)$$

s.t. $\quad z_{jt} = A_z k_{jt}^{\eta} l_{zjt}^{1-\eta}$

$$p_{jt} = \nu.(A_y)^{\frac{\sigma}{\nu}} (y_t)^{\left(1-\frac{\sigma}{\nu}\right)} (l_{y1t})^{\frac{\sigma(1-\nu)}{\nu}} (z_{jt})^{\sigma-1}$$

$$l_{zjt} = (1-\omega) l_{zj1t} + b_{jt} l_{zj2t}$$

$$l_{zj1t} \geq 0, \qquad l_{zj2t} > 0$$

Where r_{jt} is the rental rate of capital employed in public sector firm *j*, w_{zj1t} is the wage paid to per unit private labor supplied in public sector firm *j*, and w_{zj2t} is the wage paid to per unit public labor supplied in public sector firm *j*. Substituting the constraints the problem can be transformed into an unconstrained maximization problem, stated as follows:

$$\max_{k_{jt}, l_{zj1t}, l_{zj2t}} \nu.(A_y)^{\frac{\sigma}{\nu}} (y_t)^{\left(1-\frac{\sigma}{\nu}\right)} (l_{y1t})^{\frac{\sigma(1-\nu)}{\nu}} A_z^{\sigma} k_{jt}^{\eta\sigma} [(1-\omega) l_{zj1t} + b_{jt} l_{zj2t}]^{\sigma(1-\eta)} - r_{jt} k_{jt} - w_{zj1t} l_{zj1t} - w_{zj2t} l_{zj2t} \qquad (14.2)$$

The necessary conditions for changes in the three factors, $k_{jt}, l_{zj1t}, l_{zj2t}$, can be summarized as:

$$r_{jt} k_{jt} = \sigma\eta p_{jt} z_{jt} \qquad (14.3)$$

$$w_{zj1t} l_{zj1t} = \sigma(1-\eta)(1-\omega) p_{jt} z_{jt} \frac{l_{zj1t}}{l_{zjt}} \qquad (14.4)$$

$$w_{zj2t} l_{zj2t} = \sigma(1-\eta) b_{jt} p_{jt} z_{jt} \frac{l_{zj2t}}{l_{zjt}} \qquad (14.5)$$

These necessary conditions explain the distribution of income earned by the three factors employed in public sector. Note that because of monopolistic competition in intermediate input goods market and the entry barriers to private labor in the public sector, both private and public agent, in addition to budget constraints, must take into account the public sector demand for their labor services when maximizing their utility. The demand for both types of labor services in public sector is derived solving the system comprising *(11.1)*, *(11.2)*, *(13.4)*, *(14.3)*, *(14.4)* and *(14.5)*, and can be stated as follows:

$$l_{zj1t} = (1-\eta)(1-\omega)\Omega_{jt} (l_{zjt})^{\frac{\sigma-1}{1-\sigma\eta}} (l_{y1t})^{\frac{\sigma(1-\nu)}{\nu(1-\sigma\eta)}} \frac{l_{zj1t}}{w_{zj1t}} \qquad (14.6)$$

$$l_{zj2t} = (1-\eta) b_{jt} \Omega_{jt} (l_{zjt})^{\frac{\sigma-1}{1-\sigma\eta}} (l_{y1t})^{\frac{\sigma(1-\nu)}{\nu(1-\sigma\eta)}} \frac{l_{zj2t}}{w_{zj2t}} \qquad (14.7)$$

Where $\Omega_{jt} = [\sigma\nu(y_t)^{\left(1-\frac{\sigma}{\nu}\right)} (A_y)^{\frac{\sigma}{\nu}} (A_z)^{\sigma} \eta^{\sigma\eta} (r_{jt})^{-\sigma\eta}]^{\frac{1}{1-\sigma\eta}}$

A few remarks deserve special attention in interpreting these two labor demand functions. In solving the system, it is only possible to derive closed form solutions for the wage rates, and not of the particular labor demand. Hence the labor demand functions *(14.6)* and *(14.7)* are derived from the wage equations, which is why l_{zjit} appear in both sides. Note also that equilibrium demand for private labor and public labor both depend on overall public sector labor demand l_{zjt} and private sector labor demand l_{y1t}. Although labor in public sector is firm specific, the capital stocks are not specified so. Hence rental rate to capital is independent of *j*, which simplifies $\Omega_{jt} = \Omega_t$.

3.4 Government:

The government runs a balanced budget each period and finances its exogenous expenditure g_t through flat rate taxes on factor incomes. The government's commitment power is perfect which restricts it to sustain all initially announced tax plans. All initially announced optimal tax rules are therefore, time consistent. The tax rate on income from labor employed in sector *s* is τ_{st} with *s* =

y, z, and tax rate on income from capital is θ_t. There is no explicit restriction that the two sector specific labor income tax rates are same. They may potentially be different for private sector income and public sector income[4].

In addition to the tax revenue, the government, being the owner of the public sector firms in sector z also receives any nonzero profits, π_{zjt}, that may exist in equilibrium due to the monopolistic competition. Assume for simplicity that each period aggregate profits are small enough and insufficient for financing the exogenous stream of government consumption expenditure. These profits are fully redistributed to the agents in the form of lump sum transfers, TR_t^i, such that for all time t, $\sum_{i=1}^{2} TR_t^i = \int_0^1 \pi_{zjt} dj$ holds. In the unlikely event of negative equilibrium profits one can assume that the government uses a lump sum tax for both agents to recover the loss. This paper does not highlight on the method of lump sum transfers or taxes since they are treated irrelevant to the main purpose. One can assume that none of these acts affect the utility and corresponding equilibrium decisions of the agents. For instance, these transfers may come with the provision of public goods that do not enter the utility functions of either type of agents.

The government's period t budget constraint is given by:

$$g_t + TR_t^1 + TR_t^2 = \tau_{yt} w_{yt} l_{y1t} + \tau_{zt} \int_0^1 w_{zj1t} l_{zj1t} dj + \tau_{zt} \int_0^1 w_{zj2t} l_{zj2t} dj + \theta_t \int_0^1 r_{jt} k_{jt} dj + \int_0^1 \pi_{zjt} dj \tag{15.1}$$

The government is benevolent, i.e. it maximizes a social welfare function defined as a positively weighted average of individual utilities. The government attaches weights $\alpha_i \geq 0$ to agent type i's utility with $\sum_{i=1}^{2} \alpha_i = 1$. Once again there is no specific restriction on the equality of these weights for private and public agent, so $\alpha_1 \neq \alpha_2$ is maintained in general.

3.5 Agents' problems and solutions:

The environment gives rise to a simple sequential game between the two types of agents. The sequence of events is therefore important in designing the agents' maximization problems. At time *0*, the government announces tax policies. In addition to initial capital stock endowment for private agent and working time endowment for both, all agents (and the government) are endowed

[4] There are no restrictions on the magnitudes and signs of these tax rates, in order to allow for possible subsidies and rather unlikely event of confiscatory taxation.

with perfect foresight. Both type of agents, therefore respond with equilibrium decision of consumption, investment (for private agent only), productivity level (for public agent only) and labor supply. Firm *j* in the public sector decides production of z_j and its price considering the equilibrium demand for intermediate input good by representative firm in private sector *y*. The public agent is assumed to go first by choosing productivity b_j and labor supply l_{zj2} taking as given the equilibrium investment and labor supply decisions of the private agent. The representative firm in private sector producing *y* competitively maximizes profit and requires (or allows) labor service from the private agent only. Because price and output decisions of the public sector firm *j* depends on private agent's labor supply to the private sector, the public agent, in her maximization problem, takes this decision as given. The private agent has perfect knowledge of the entry barrier parameter ω, and hence can correctly surmise the equilibrium decisions of the public agent. Once the public agent has decided, the private agent chooses labor supply and investment decisions taking public agent's decisions of productivity b_j and labor supply l_{zj2} as given[5]. Since demands for labor are symmetrical for all public sector firms as may be found in *(14.6)* and *(14.7)*, the programs are same for all identical private agents and identical public agents. All private agents and public agents thus make exactly the type-wise same set of decisions. Moreover, although wages are firm and sector specific, rental rates of capital are not. So the index *j* pertaining to the firm in public sector with which an agent is affiliated can be omitted, without loss of generality.

3.5.1 Private agent's problem:

The representative private agent chooses consumption levels, labor supply (to both sectors) and a period ahead capital stocks to maximize her preferences over infinite horizon subject to her budget constraint, public sector's private labor demand function *(14.6)* and public sector's aggregate labor demand constraint *(11.2)*, treating the equilibrium decision of productivity and labor supply by the public agent as exogenous (or correctly surmised, precisely). All expressions are *j* invariant since all private agents are identical. The problem can be written as the following program:

$$\max_{c_{1t}, l_{y1t}, l_{z1t}, k_{t+1}} \quad \sum_{t=0}^{\infty} \beta^t [\ln c_{1t} + \{1 - \mathrm{V}^1(l_{y1t}, l_{z1t})\}]$$

s.t.

$$c_{1t} + x_t \le (1-\tau_{yt}) w_{yt} l_{y1t} + (1-\tau_{zt}) w_{z1t} l_{z1t} + (1-\theta_t) r_t k_t + TR^1 \qquad (16.1)$$

---

[5] In an alternative setting, one can illustrate the environment giving rise to a dynamic game instead of a simple sequential game between the agents. This is possible if one allows the public agents to choose productivity level a period ahead of all decisions. This introduces dynamics in the productivity variable for the public agent. This paper, however, does not focus on growth effects, and hence treats this potential extension as relatively unimportant. As for reference, a similar dynamic game is modelled in Herrendorf & Teixeira (2004).

$$l_{z1t} = (1-\eta)(1-\omega)\Omega_t (l_{zt})^{\frac{\sigma-1}{1-\sigma\eta}} (l_{y1t})^{\frac{\sigma(1-\nu)}{\nu(1-\sigma\eta)}} \frac{l_{z1t}}{w_{z1t}} \tag{16.2}$$

$$k_{t+1} = (1-\delta)k_t + x_t \tag{16.3}$$

$$l_{zt} = [(1-\omega)l_{z1t} + b_t l_{z2t}] \tag{16.4}$$

$$b_t, l_{z2t} \quad and \quad k_0 > 0 \ \ given \tag{16.5}$$

Consolidating *(16)* and defining $R_t \equiv [r_t(1-\theta_t)+(1-\delta)]$, the private agent's maximization problem can be restated as:

$$\max_{c_{1t}, l_{y1t}, l_{z1t}, k_{t+1}} \sum_{t=0}^{\infty} \beta^t [\ln c_{1t} + \{1 - \text{V}^1(l_{y1t}, l_{z1t})\}]$$

s.t.

$$c_{1t} + k_{t+1} \le (1-\tau_{yt})w_{yt} l_{y1t} + (1-\tau_{zt})(1-\eta)(1-\omega)\Omega_t (l_{zt})^{\frac{\sigma-1}{1-\sigma\eta}} (l_{y1t})^{\frac{\sigma(1-\nu)}{\nu(1-\sigma\eta)}} l_{z1t} + R_t k_t + TR^1 \tag{17.1}$$

where $l_{zt} = [(1-\omega)l_{z1t} + b_t l_{z2t}]$

For $\beta^t \lambda_{1t}$ as the Lagrange multiplier associated with the period *t* version of *(17.1)*, the necessary conditions for a maximum are the budget constraint *(17.1)*, the Transversality condition that puts a restriction on the terminal value of capital stock in terms of consumption, and the followings:

$$c_{1t}: \quad \lambda_{1t} = (c_{1t})^{-1} \tag{17.2}$$

$$l_{y1t}: \quad \text{V}^1_{ly1}(t) = (c_{1t})^{-1}\left[(1-\tau_{yt})w_{yt} + (1-\tau_{zt})w_{z1t}\frac{\sigma(1-\nu)}{\nu(1-\sigma\eta)}(l_{y1t})^{-1}\right] \tag{17.3}$$

$$l_{z1t}: \quad \text{V}^1_{lz1}(t) = (c_{1t})^{-1}\left[(1-\tau_{zt})w_{z1t}\left\{1+\frac{\sigma-1}{1-\sigma\eta}(1-\omega)\frac{l_{z1t}}{l_{zt}}\right\}\right] \tag{17.4}$$

$$k_{t+1}: \quad \frac{c_{1t+1}}{c_{1t}} = \beta R_{t+1} \tag{17.5}$$

Conditions *(17.3)*, *(17.4)* and *(17.5)* are equilibrium reactions of private agent for any chosen tax rates, and hence in constructing the optimal taxation problem the government treats these three as implementability constraints. As long as any chosen tax plan satisfies these three conditions, the tax plan is implementable in an equilibrium for the private agent.

3.5.2 Public agent's problem:

The representative public agent chooses consumption levels, labor supply (to public sector only) and productivity level to maximize her preferences over infinite horizon subject to her budget constraint, public sector's public labor demand function *(14.7)* and public sector's aggregate labor demand constraint *(11.2)*, taking as given the equilibrium decision of the private agent. All expressions are *j* invariant since all public agents are identical. The problem can be written as the following program:

$$\max_{c_{2t},l_{z2t},b_t} \sum_{t=0}^{\infty} \beta^t [\ln c_{2t} + \{1 - \mathrm{V}^2(l_{z2t})\}]$$

s.t.

$$c_{2t} \leq (1-\tau_{zt}) w_{z2t} l_{z2t} + TR^2 \qquad (18.1)$$

$$l_{z2t} = (1-\eta) b_t \Omega_t (l_{zt})^{\frac{\sigma-1}{1-\sigma\eta}} (l_{y1t})^{\frac{\sigma(1-\nu)}{\nu(1-\sigma\eta)}} \frac{l_{z2t}}{w_{z2t}} \qquad (18.2)$$

$$l_{zt} = [(1-\omega) l_{z1t} + b_t l_{z2t}] \qquad (18.3)$$

$$l_{z1t} \quad \textit{and} \quad l_{y1t} \quad \textit{given} \qquad (18.4)$$

Consolidating *(18)*, the public agent's maximization problem can be restated as:

$$\max_{c_{2t},l_{z2t},b_t} \sum_{t=0}^{\infty} \beta^t [\ln c_{2t} + \{1 - \mathrm{V}^2(l_{z2t})\}]$$

s.t.

$$c_{2t} \leq (1-\tau_{zt})(1-\eta) b_t \Omega_t (l_{zt})^{\frac{\sigma-1}{1-\sigma\eta}} (l_{y1t})^{\frac{\sigma(1-\nu)}{\nu(1-\sigma\eta)}} l_{z2t} + TR^2 \qquad (19.1)$$

where $l_{zt} = [(1-\omega) l_{z1t} + b_t l_{z2t}]$

For $\beta^t \lambda_{2t}$ as the Lagrange multiplier associated with the period *t* version of *(19.1)*, the necessary conditions for a maximum are the budget constraint *(19.1)* and the followings:

$$c_{2t}: \qquad \lambda_{2t} = (c_{2t})^{-1} \qquad (19.2)$$

$$l_{z2t}: \qquad \mathrm{V}^2_{lz2}(t) = (c_{2t})^{-1} \left[(1-\tau_{zt}) w_{z2t} \left\{ 1 + \frac{\sigma-1}{1-\sigma\eta} b_t \frac{l_{z2t}}{l_{zt}} \right\} \right] \qquad (19.3)$$

$$b_t: \qquad (1-\tau_{zt})\frac{w_{z2t}l_{z2t}}{b_t}\left[1+\frac{\sigma-1}{1-\sigma\eta}b_t\frac{l_{z2t}}{l_{zt}}\right]=0 \qquad (19.4)$$

In a similar way to that of the private agent, conditions *(19.3)* and *(19.4)* are implementability constraints for the government's optimal taxation problem which represents the equilibrium reactions of public agent for any chosen tax plan.

3.6 Equilibrium:

I will restrict my attention for the modified model economy's equilibria to those which are symmetric and recursive. This assumes that all intermediate input goods are same and hence the index *j* can be dropped without loss of generality. Since both private agent and public agent consume from the same source, aggregate consumption follows $c_{1t}+c_{2t}=c_t$. All investment decisions are private agents' decisions.

There may be multiple equilibria due to (i) the non-linearity (in corresponding labor supply decision) of necessary conditions *(17.3)*, *(17.4)* and *(19.3)* which represent the equilibrium reaction of private and public agents due to small changes in labor supply decision; or (ii) indexation by different tax policies chosen by the government; or (iii) both. The equilibrium definition will follow the set of necessary definitions, where a symbol without time subscript denotes the one sided infinite sequence of the corresponding variable.

Definition 2.1: An allocation is a sequence $\{c,l_{y1},l_{z1},l_{z2},x,k,z,y\}$ that satisfies the resource constraints given by (11.1), (11.2), (11.3), (12.1) and (12.2). ■

Definition 2.2: A government policy is a 3-tuple of sequences $\{\tau_y,\tau_z,\theta\}$.

■

Definition 2.3: A government allocation is a sequence $\{g\}$. ■

Definition 2.4: A price system is a 5-tuple of non-negative bounded sequences $\{p,w_y,w_{z1},w_{z2},r\}$. ■

Definition 2.5: A productivity policy for the public agent is a bounded sequence $\{b\}$.

■

Note that the labor services do not necessarily follow adding up constraint since both types of agents are endowed with same working time, and public agent's labor is sector specific. Moreover, since there is an entry barrier to private labor in public sector, private agent's labor supply to public sector may be zero depending on the magnitude of the entry barrier parameter (which is treated exogenous throughout). Note also that in definition 2.3 the government allocation does not contain the lump sum transfers. This is because the aggregate lump sum transfer each period exactly matches the integrated nonzero profit (if any) received by the government, by assumption. This implies that if in a particular period *t* the government receives zero profits its aggregate lump sum transfer is zero. The restriction $\sum_{i=1}^{2} TR_t^i = \int_0^1 \pi_{zjt} dj$ makes the presence of lump sum transfers as elements in the set of government allocation redundant, which allows one to completely ignore both profits and lump sum transfers in the government period *t* budget constraint[6].

The following equilibrium concept is proposed:

Definition 2.6 (Equilibrium):

An *Equilibrium* is

-- an allocation,

-- a government allocation,

-- a price system,

-- a government policy, and

-- a productivity policy for the public agent, such that

(a) Given the price system, the allocation $\{z, y, l_{y1}\}$ solves the problem of the representative private sector firm producing *y*.

(b) Given the price system and derived demand function for *z*, the allocation $\{z, k, l_{z1}, l_{z2}\}$ solves the problem of firm *j* in sector *z*.

(c) Given the price system, government policy, productivity policy of public agent and derived labor demand function for l_{z1}, the allocation $\{c_1, l_{y1}, l_{z1}, k\}$ solves the problem of the representative private agent.

[6] This will be maintained hereafter, i.e. the government's budget constraint hereafter will be presented removing the lump sum transfers and the profits.

(d) Given the price system, government policy and derived labor demand function for l_{z2}, the allocation $\{c_2, l_{z2}\}$ and productivity policy $\{b\}$ solve the problem of the representative public agent.

(e) All markets clear in the long run. ■

The equilibrium dynamics is characterized by the following system:

$$y_t \geq c_{1t} + c_{2t} + x_t + g_t \tag{20.1a}$$

$$y_t \leq A_y \left[\left(\int_0^1 z_{jt}^{\sigma} dj \right)^{\frac{1}{\sigma}} \right]^{\nu} l_{y1t}^{1-\nu} \tag{20.1b}$$

$$z_{jt} \leq A_z k_{jt}^{\ \eta} l_{zjt}^{\ 1-\eta} \tag{20.1c}$$

$$l_{zjt} = (1-\omega) l_{zj1t} + b_{jt} l_{zj2t} \tag{20.1d}$$

$$x_t = k_{t+1} - (1-\delta) k_t \tag{20.1e}$$

$$k_t = \int_0^1 k_{jt} dj \tag{20.1f}$$

$$l_{y1t} + \int_0^1 l_{zj1t} dj \leq 1 \tag{20.1g}$$

$$\int_0^1 l_{zj2t} dj \leq 1 \tag{20.1h}$$

$$z_{jt} = \nu^{\frac{1}{1-\sigma}} (A_y)^{\frac{\sigma}{\nu(1-\sigma)}} (y_t)^{\frac{\nu-\sigma}{\nu(1-\sigma)}} (l_{y1t})^{\frac{\sigma(1-\nu)}{\nu(1-\sigma)}} (p_{jt})^{-\left(\frac{1}{1-\sigma}\right)} \tag{20.1i}$$

$$w_{yt} = (1-\nu) A_y \left[\left(\int_0^1 z_{jt}^{\sigma} dj \right)^{\frac{1}{\sigma}} \right]^{\nu} l_{y1t}^{-\nu} \tag{20.1j}$$

$$r_{jt} k_{jt} = \sigma \eta p_{jt} z_{jt} \tag{20.1k}$$

$$w_{zj1t} l_{zj1t} = \sigma(1-\eta)(1-\omega) p_{jt} z_{jt} \frac{l_{zj1t}}{l_{zjt}} \tag{20.1l}$$

$$w_{zj2t} l_{zj2t} = \sigma(1-\eta) b_{jt} p_{jt} z_{jt} \frac{l_{zj2t}}{l_{zjt}} \tag{20.1m}$$

$$\mathbf{V}_{ly1}^{1}(t) = (c_{1t})^{-1} \left[(1-\tau_{yt}) w_{yt} + (1-\tau_{zt}) w_{z1t} \frac{\sigma(1-\nu)}{\nu(1-\sigma\eta)} (l_{y1t})^{-1} \right] \tag{20.1n}$$

$$\mathbf{V}^1_{lz1}(t) = (c_{1t})^{-1}\left[(1-\tau_{zt})w_{z1t}\left\{1+\frac{\sigma-1}{1-\sigma\eta}(1-\omega)\frac{l_{z1t}}{l_{zt}}\right\}\right] \qquad (20.1o)$$

$$\frac{c_{t+1}}{c_t} = \beta[(1-\theta_{t+1})r_{t+1} + (1-\delta)] \qquad (20.1p)$$

$$\mathbf{V}^2_{lz2}(t) = (c_{2t})^{-1}\left[(1-\tau_{zt})w_{z2t}\left\{1+\frac{\sigma-1}{1-\sigma\eta}b_t\frac{l_{z2t}}{l_{zt}}\right\}\right] \qquad (20.1q)$$

$$(1-\tau_{zt})\frac{w_{z2t}l_{z2t}}{b_t}\left[1+\frac{\sigma-1}{1-\sigma\eta}b_t\frac{l_{z2t}}{l_{zt}}\right] = 0 \qquad (20.1r)$$

Expression *(20.1a)* explains how the final manufacturing good in exhausted. Expressions *(20.1b-d)* state the technologies of the economy, and *(20.1e)* is the law of motion for capital. Expressions *(20.1f-h)* represent the factor market clearing conditions. Equilibrium price of (and inverse demand for) intermediate good is represented by *(20.1i)*. The next four conditions are income distribution of different factors. The remaining expressions are equilibrium reactions of two types of agents. The equilibrium dynamics is characterized by the solution to the symmetric version of system *(20.1)* comprising eighteen expressions in unknowns which are $(c_{1t}, c_{2t}, k_t, x_t, y_t, l_{y1t}, l_{z1t}, l_{z2t}, b_t, p_t, z_t, w_{yt}, w_{z1t}, w_{z2t}, r_t, \tau_{yt}, \tau_{zt}, \theta_t)$.

3.7 Optimal Taxation Problem:

The formulation of the optimal taxation problem for the government follows Chamley's (1986) approach and assumes that net returns to the factors are constrained (and assumed to be greater than some arbitrary value). Rather than choosing tax rates, the government chooses after-tax rental rates of capital, $\tilde{r}_t$, and after-tax wage rates, $\tilde{w}_{yt}$ and $\tilde{w}_{zit}$, such that $\tilde{r}_t = (1-\theta_t)r_t$, $\tilde{w}_{yt} = (1-\tau_{yt})w_{y1t}$ and $\tilde{w}_{zit} = (1-\tau_{zt})w_{zit}$.

The government's budget constraint with symmetry and after incorporating the necessary conditions from the firms' maximization problem is:

$$g_t = (1-\nu)A_y\left[\left(\int_0^1 z_{jt}^{\sigma}dj\right)^{\frac{1}{\sigma}}\right]^{\nu}(l_{y1t})^{1-\nu} + \sigma\eta p_t z_t + \sigma(1-\eta)(1-\omega)p_t z_t\frac{l_{z1t}}{l_{zt}}$$
$$+\sigma(1-\eta)b_t p_t z_t\frac{l_{z2t}}{l_{zt}} - \tilde{w}_{yt}l_{y1t} - \tilde{w}_{z1t}l_{z1t} - \tilde{w}_{z2t}l_{z2t} - \tilde{r}_t k_t \qquad (21.1)$$

where $l_{zt} = (1-\omega)l_{z1t} + b_t l_{z2t}$.

Since the private agent supplies labor services to the private sector, and labor is required in connection to intermediate inputs in producing the final good, this economy is characterized by the following two resource constraints:

$$c_{1t} + c_{2t} + k_{t+1} + g_t = A_y \left[\left(\int_0^1 z_{jt}^{\sigma} dj \right)^{\frac{1}{\sigma}} \right]^{\nu} l_{y1t}^{1-\nu} + (1-\delta)k_t \qquad (21.2)$$

$$z_{jt} = A_z k_{jt}^{\eta} l_{zjt}^{1-\eta} \qquad (21.3)$$

where $l_{zt} = (1-\omega)l_{z1t} + b_t l_{z2t}$.

The government's choice of optimal tax rates is also constrained by individual agent's budget constraints *(16.1)* and *(18.1)*, and their equilibrium reaction functions *(17.3)*, *(17.4)* and *(17.5)* for the private agent and *(19.3)* and *(19.4)* for the public agent. As stated earlier in section 3.4, the benevolent government maximizes a social welfare function defined as a positively weighted average of individual utilities with the weight $\alpha_i \geq 0$ to agent type *i*'s utility.

The optimal taxation problem for the government is therefore to maximize a social welfare function defined as the weighted average of individual utilities with weight $\alpha_i \geq 0$, subject to its own budget constraint *(21.1)* that incorporates necessary conditions from firms' maximization problems, two resource constraints *(21.2)* and *(21.3)*, type specific agent's budget constraints *(16.1)* and *(18.1)*, and type specific agents' equilibrium reaction functions *(17.3)*, *(17.4)* and *(17.5)* for the private agent and *(19.3)* and *(19.4)* for the public agent. Let $\beta^t \psi_t$, $\beta^t \phi_{yt}$, $\beta^t \phi_{zt}$, $\beta^t \varepsilon_{1t}$, $\beta^t \varepsilon_{2t}$, $\beta^t \mu_{1yt}$, $\beta^t \mu_{1zt}$, $\beta^t \xi_{1t}$, $\beta^t \mu_{2zt}$ and $\beta^t \xi_{2t}$ be the ten period *t* Lagrange multipliers associated with the ten constraints *(21.1)*, *(21.2)*, *(21.3)*, *(16.1)*, *(18.1)*, *(17.3)*, *(17.4)*, *(17.5)*, *(19.3)* and *(19.4)*, respectively, of this optimal taxation problem[7]. Also, the Lagrangian function

[7] I acknowledge that the problem seems cluttered by notations, but I maintain symmetry of the multiplier notations according to sectors, type of constraint and agent types using subscripts in order to aid smooth reading. The corresponding Lagrangian function of this problem is similar in formation to *(9)*, and hence it is deemed unimportant to write down the function. One can easily identify the multipliers and provide interpretations with the help of the subscripts used. The multiplier corresponding to government budget constraint, ψ_t has only time subscript, and hence is easily identifiable. The multipliers associated with the two resource constraints have sector and time subscripts, denoted by ϕ_{st}. The associated multipliers to the two budget constraints are agent type specific, and hence have subscripts *i* and *t*, denoted by ε_{it}. The multipliers μ_{ist} correspond to the constraint representing agent *i*'s equilibrium reaction for changes in labor services in sector *s*. The multipliers ξ_{it} correspond to the constraint representing agent *i*'s equilibrium reaction for changes in non-labor control variables, i.e., a period ahead capital stock for agent type 1 and productivity policy for agent type 2.

associated with this problem is denoted as $\Im$, such that the associated maximum value Lagrangian is $\Im^*$.

3.7.1 Optimal Labor Income Taxes:

Differentiating the Lagrangian function of the government's optimal taxation problem defined in section 3.7 with respect to l_{y1t} yields the following necessary condition for the sector *y* specific labor income tax rate (the labor income tax rate for the private sector) to be optimal:

$$\alpha_1 \mathrm{V}_{ly1}^1(t) = \psi_t[(1-\nu)w_{yt} - \tilde{w}_{yt}] + \phi_{yt} w_{yt} + \varepsilon_{1t}\tilde{w}_{yt} + \mu_{1yt}\left[(c_{1t})^{-1}\left\{\tilde{w}_{z1t}\frac{\sigma(1-\nu)}{\nu(1-\sigma\eta)}(l_{y1t})^{-2}\right\}\right] \quad (22.1)$$

which in turns can be used to derive the optimal private sector labor income tax rule:

$$\tau_{yt}^* = 1 - \frac{\alpha_1 \mathrm{V}_{ly1}^1(t)}{w_{yt}(\varepsilon_{1t} - \psi_t)} + \frac{[\psi_t(1-\nu) + \phi_{yt}]}{(\varepsilon_{1t} - \psi_t)} + \frac{\mu_{1yt}[(c_{1t})^{-1}\tilde{w}_{z1t}\frac{\sigma(1-\nu)}{\nu(1-\sigma\eta)}(l_{y1t})^{-2}]}{w_{yt}(\varepsilon_{1t} - \psi_t)} \quad (22.2)$$

It is certain that *(22.2)* is analytically less useful unless some points are clarified. The optimal private sector labor income tax rule is finite as long as $\varepsilon_{1t} \neq \psi_t$, i.e. for $\Im^*$ denoted as the maximum value Lagrangian of the government's optimal taxation problem, *(22.2)* is finite as long as marginal sensitivity of $\Im^*$ with respect to changes in government's budget constraint and private agent's budget constraint differ. Since only private agent supplies labor in private sector, the optimal tax rate on labor income only affects the private agent. The private agent's equilibrium reaction for any changes in government's exogenous decision would be much larger than the government's own reaction, such that $[\varepsilon_{1t} - \psi_t] > 0$.

From the comparative static properties of the derived optimal private sector labor income tax rule, interesting to note that it is a function of (among others) the optimal public sector labor income tax rate, and the two are inversely related. This implies a labor income tax trade off for the government between the private and the public sector. The underlying reason for this trade off becomes clearer if one considers that the optimal private sector labor income tax rate is also a negative function of private labor supply in private sector. Since entry to public sector is not prohibited for private agents but there exists a barrier to entry, private agents will decide on labor supply diversification based on tax incentives. The higher she supplies labor to her own sector, the lower the wage, and the lower the wage, the lower the tax rate. This lower tax rate on private sector income must be matched with a relatively higher tax on public sector income. If the private agent decides to supply less labor in her own sector (possibly due to low degree of entry barrier in public sector), she earns a higher wage in private sector but pays a higher tax. This is matched by a relatively low tax on public sector income.

Differentiating $\Im$ with respect to l_{z1t} yields the following necessary condition for the sector z specific labor income tax rate (the labor income tax rate for the public sector) to be optimal[8]:

$$\alpha_1 \mathrm{V}_{lz1}^1(t) = \psi_t(w_{z1t} - \tilde{w}_{z1t}) + \phi_{zt}\frac{w_{z1t}}{p_t\sigma} + \varepsilon_{1t}\tilde{w}_{z1t} + \mu_{1zt}\tilde{w}_{z1t}\left[(c_{1t})^{-1}\frac{(\sigma-1)}{(1-\sigma\eta)}\left\{\frac{-b_t l_{z2t}(1-\omega)}{(l_{zt})^2}\right\}\right]$$
$$+\mu_{2zt}\tilde{w}_{z2t}\left[(c_{2t})^{-1}\frac{(\sigma-1)}{(1-\sigma\eta)}\left\{\frac{b_t l_{z2t}(1-\omega)}{(l_{zt})^2}\right\}\right] + \xi_{2t}\tilde{w}_{z2t}\left[(c_{2t})^{-1}\frac{(\sigma-1)}{(1-\sigma\eta)}\left\{(1-\omega)\left(\frac{l_{z2t}}{l_{zt}}\right)^2\right\}\right] \qquad (22.3)$$

This in turns yields the optimal public sector labor income tax rate:

$$\tau_{zt}^* = 1 - \frac{\alpha_1 \mathrm{V}_{lz1}^1(t) - w_{z1t}[\psi_t + (p_t\sigma)^{-1}\phi_{zt}]}{w_{z1t}\mathrm{M}_t - w_{z2t}\mathrm{N}_t} \qquad (22.4)$$

where

$$\mathrm{M}_t \equiv \left[(\varepsilon_{1t} - \psi_t) + \mu_{1zt}(c_{1t})^{-1}\frac{1-\sigma}{1-\sigma\eta}\left\{\frac{(1-\omega)b_t l_{z2t}}{l_{zt}}\right\}\right]$$

$$\mathrm{N}_t \equiv \left[(c_{2t})^{-1}(1-\omega)\frac{1-\sigma}{1-\sigma\eta}\left\{\mu_{2zt}\frac{b_t l_{z2t}}{(l_{zt})^2} + \xi_{2t}\left(\frac{l_{z2t}}{l_{zt}}\right)^2\right\}\right]$$

Unlike the optimal private sector labor income tax rule, the optimal public sector labor income tax rule does not depend on its private sector counterpart. This tax rule is analytically less clear to explain, and the complexity is largely due to the two terms M_t and N_t which involve nonlinear correspondence to l_{zt} and l_{z2t}. However, because of their interdependence suggested by *(22.2)*, it

[8] The optimal public sector labor income tax rule can also be derived Differentiating $\Im$ with respect to l_{z2t}, since all tax rates are agent type independent, i.e. the government targets labor income from two different sectors for two different tax rates, and does not target agent type.

is rather inconclusive which sector's labor income is relatively highly taxed. But it is certain from the intuition that it depends on the degree of entry barrier to private labor in public sector. The substantive finding regarding optimal labor income tax, as this paper advocates, is that a higher private sector tax must be accompanied by a relatively lower public sector tax, and vice versa. The degree of this trade off depends indirectly on the entry barrier parameter. Both *(22.2)* and *(22.4)* reconfirm this intuitive and extremely useful result.

3.7.2 Optimal Capital Income Tax:

Differentiating $\Im$ with respect to k_{t+1} yields the following necessary condition for the capital income tax rate to be optimal:

$$\phi_{yt} + \varepsilon_{1t} = \beta[\psi_{t+1}(r_{t+1} - \tilde{r}_{t+1}) + \phi_{yt+1}(1-\delta) + \phi_{zt+1}(\sigma)^{-1} r_{t+1} + \varepsilon_{1t+1}(\tilde{r}_{t+1} + 1 - \delta)] \qquad (23.1)$$

The steady state version of *(23.1)* can be presented as:

$$\phi_y + \varepsilon_1 = \beta[\psi(r - \tilde{r}) + \phi_y(1-\delta) + \phi_z(\sigma)^{-1} r + \varepsilon_1(\tilde{r} + 1 - \delta)] \qquad (23.2)$$

The steady state version of the Euler equation *(17.5)* from private agent's maximization problem is:

$$1 - \beta[\tilde{r} + 1 - \delta] = 0 \qquad (23.3)$$

Substituting *(23.3)* in *(23.2)*, it is straightforward to derive the optimal steady state capital income tax rate, which is:

$$\theta^* = \frac{\phi_y \sigma - \phi_z}{\sigma(\phi_y + \psi)} \qquad (23.4)$$

Expression *(23.4)* is finite as long as $\phi_y + \psi \neq 0$ which holds since $\phi_y > 0$ (both resource constraints bind since all markets clear in the long run), and ψ is non-negative. With the final manufacturing good as the numeraire, $p\phi_y$ is a measure of the steady state social marginal value of available intermediate input good for sector *y*, which is equal to ϕ_z. Interesting to note that the optimal steady state capital income tax is nonzero, if the steady state relative market price for the intermediate input good does not exactly compensate for the distortion created by monopolistic competition, i.e. unless $(\sigma)^{-1}p = 1$, and once again whether or not it is optimal for the government to tax capital income or subsidize it depends on the parameter σ, which determines the degree of substitutability of two intermediate goods. With monopolistic competitive pricing, therefore, the government must introduce a nonzero capital tax device to cure for the uncompensated distortions created at the margin. Since preferences are strictly monotone, the relative price of the intermediate input goods is strictly positive. In addition, if σ is positive which indicates that substitutability between two intermediate input goods is highly elastic (and demand for intermediate input good is highly elastic), then it is optimal for the government to subsidize capital income. On the other hand, if substitutability between two intermediate input goods is inelastic (and so is demand for intermediate input good), the model suggests the government should tax capital income.

Why would it be optimal for the government to subsidize capital income (or offer lower tax on labor income in the benchmark model) from public sector if demand for intermediate input good is highly elastic? The intuition behind this result stems from Solow's (1998) Federico Caffe lectures on Monopolistic Competition and Macroeconomic Theory. In the presence of monopolistic competition in the intermediate input goods market, a demand shock typically has multiplier like effect. If the intermediate input good's demand is highly elastic in addition, a small increase in its price will reduce its demand more than proportionately, and holding labor supply constant this in turn will reduce the production of final manufacturing good. Since investment is made from the single homogeneous final good, installed capital stock next period will decrease, which will make public sector firms increase unit cost of capital in offer. But with a relatively low quantity of capital, production of intermediate input goods will fall, which makes the public sector firm increase its price further. This is the multiplier effect of a demand shock under monopolistic competition. The private agent being the sole investor will bear the adverse partial equilibrium effect of a demand shock. The only way the government can compensate for this effect is to introduce a capital income subsidy. Note also that with such features of market price determination, the constant distortion created by the monopolistic competition does not change over time although the steady state price is much different from what it has been in any transition period. Hence given all other endogenous variables of the model, it is of measure zero that in the steady state the relative price exactly offsets the distorted margins, which in turn implies that capital income tax in the steady state is unambiguously nonzero.

4. Concluding Remarks:

In order to examine the optimal choice of income tax rates in a model which is characterized by imperfect competition in one or more markets due to entry barriers to firms and labor services, this paper has followed the dynamic general equilibrium approach with infinite horizon and first proposed a benchmark model of optimal taxation with entry barriers to firms in the production of intermediate input goods that generates monopolistic competition amongst existing firms in the market for intermediate input goods. The associated labor income and capital income tax rates are analytically derived and their properties are discussed. Next, a similar approach is adopted with a modified model to address the issue of optimal choice of income tax rates in the presence of entry barriers to labor in addition to entry barriers to firms. This model introduced agent type heterogeneity and public sector participation in production. The analytically derived labor and capital income taxes associated with this model are found to be relatively more complex, but an attempt has been made to analyze some insights related to their steady state properties.

The benchmark model established that the optimal steady state capital income tax, which is typically found to be zero in competitive market analogues, is nonzero, and may be a tax or a subsidy depending on the degree of the distortion created by monopolistic competition, or alternatively, and more intuitively depending on the magnitude of the elasticity of substitution between two intermediate input goods. It is optimal for the government to subsidize capital income if demand for intermediate input good is elastic, and to tax capital income if demand is inelastic. The underlying intuition behind this result can be drawn from Solow's (1998) analysis of monopolistic competition in Macroeconomic Theory, which states that due to myopic quantity expectation of a monopolistic producer, a demand shock typically has multiplier like effect. If demand for the monopolistically produced good is elastic in addition, this induces more than proportionate change in the amount of installed capital resulting in further change in the price of that particular good. Although the economy seems to be vulnerable to such shocks, the consequences are undesirable and unsustainable for equilibrium. The only device at the government's disposal to offset this effect is to subsidize capital income. Similar result and intuition hold for the modified model where entry barrier to labor services is modelled. This finding can be compatible with Golosov *et. al* (2003) which shows that a nonzero capital income tax is otherwise pareto efficient if the source of inefficiency is not confined to taxation only.

The optimal labor income tax rate in the benchmark model suggests that it is much lower than its competitive market analogue if the demand for the monopolistically produced good is elastic, implying in general that greater market imperfection for an elastically demanded good induces lower optimal marginal income tax rates. This is largely due to the government's attempt to neutralize the distortions created by monopolistic competition which affect factor returns. The derived optimal labor income tax rule suggests that for greater market power exercised, the difference between a competitive market (or *Ramsey*) tax rule and the derived tax rule increases, implying that the higher the degree of monopoly power the lower the corresponding labor income tax, given that the demand for the intermediate input good is elastic. Unless there are changes in the structural parameters of the model, the optimal labor income tax rule is roughly constant over the long run, which is standard in literature concerned with *Ramsey* taxation (see for instance, Chari *et. al* (1994), Chari & Kehoe (1999)).

In the modified model, agent type heterogeneity and public production were introduced in order to derive optimal income tax rates in the presence of entry barriers to private labor in public sector production. Results from this model suggest that the optimal taxation problem in the presence of sector specific wages and entry barriers is much more complex than usual, and the derived optimal labor income tax rules are analytically indistinct. However, some useful insights about the tax mix are clearly identifiable and fairly intuitive from the comparative static properties of the solutions. The private sector optimal labor income tax is found to be negatively related to its counterpart public sector labor income tax, which suggests that in equilibrium the government faces a constraint of trade off between the two labor income tax rates. This result is drawn without any exogenous restrictions on the equality of the two tax rates. The weaker the entry barrier, the higher is the private labor participation in public sector, and the lower the private labor participation in private sector. This drives the private sector wage higher resulting in a higher private sector labor income tax rate. Due to the trade off, the government must set contemporaneous public sector labor income tax rate much lower. The expression for optimal public sector labor income tax rate is rather indistinct and analytically less useful. But unlike the private sector tax, the public sector labor income tax is independent of its counterpart. It is also inconclusive that which tax rate is higher in equilibrium, but results suggest that their relative magnitude depend on, among others, the entry barrier parameter. The stronger the entry barrier, the less the private agent supplies her labor to public sector firm, and the higher the wage she earns paying a relatively higher tax. Due to the trade off, and since private labor's working time increases in private sector reducing her private sector wage, the government must set a relatively lower tax on private sector labor income. Hence an optimal policy choice for the government may be to set weak entry restrictions and a tax mix comprising high private sector labor income tax and low public sector labor income tax, or a strong entry restriction and a tax mix comprising high public sector labor income tax and low private sector labor income tax.

The reason why this paper prefers to use the Chamley's (1986) approach to model optimal taxation problem leaving the *primal* approach (due to Jones *et. al* (1997), Chari & Kehoe (1999), among others) is that the latter is more useful in dealing with dynamic tax issues such as capital income taxes, and it is the Chamley's (1986) approach which directly identifies period by period expressions of labor income tax rules. This is not a limitation of the paper, since the *primal* approach should necessarily derive the same set of analytical results, with perhaps a somewhat different set of insights. A potential extension as may be suggested by many is to introduce a dynamic game between the agents allowing the public agent to choose its productivity level a period ahead. My own idea about this manipulation, or potential extension, is that it is more useful in settings where the focus is on balanced growth effects of flat rate taxes, or political economy consequences of dynamic taxation. In modelling public sector production of intermediate input goods, the paper assumes that the technology and associated TFP remains unchanged. This is not a maintained hypothesis in Schmitz (2001), who introduces inefficiency in public production of investment goods by multiplying the private sector's TFP parameter by a time invariant parameter which can take nonnegative fractional values. While this specification is simple in exposition and aids the desired results for Schmitz (2001), in the present environment it is rather useless. In other words, in modelling public sector production of intermediate input goods, this paper does not explicitly assume that the public sector produces with an inefficient TFP.

The macroeconomic implications of monopolistic competition have been thorough rigorous investigations, and I acknowledge that there exists, as always, two slightly stylized schools of thoughts. Robert Solow's (1998) Federico Caffe lectures provide a brief yet complete review of the existing literature relevant to this debate. Some of the issues raised by the otherwise alternative school of thought, as may be found in Blanchard & Kiyotaki (1987), provide useful insights on short and medium run equilibrium dynamics. While those *may* be useful for the present analysis when optimal taxation is considered in transition, their long run implications are rather unimportant. The present analysis of long run optimal tax structures is unlikely to consider these arguments as serious shortcomings. The main findings of this paper are analytically robust in similar settings, and provide reassuring and clear intuitions which can be attributed to empirically observed direct taxation policies of governments.

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