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Allowances: Are Really Better than Tax Credits?

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ABSTRACT

In this paper we develop an analytical framework devoted to compare family size related tax credits and allowances. As a main result, we obtain that, in a social welfare context *à la* Atkinson and Bourguignon (1987), if tax credits and allowances are compared in an individual context, it could be assured that a progressive income tax with family size related tax allowances is never welfare superior to an equal-yield income tax involving tax credits.

Theoretical results are checked on empirical grounds for the Spanish personal income tax (IRPF) in force in 1999. We develop a micro-simulation exercise, using the Panel of IRPF Returns, which belongs to the Institute of Fiscal Studies. Two results can be derived from this exercise. Firstly, that, both adopting an individual or household approach, allowances and tax credits are welfare superior to an equal yield proportional tax. And secondly, that, although a tax credit system reduces global income inequality more than what allowances does, also from an individual or household approach, it is not possible to determine superiority in welfare terms of a system over the other.

KEY WORDS: Personal Income Tax, Inequality, Social Welfare, Family Size, Tax Credit, Allowances

J.E.L. CLASSIFICATION: D31, D63, H23, H24,

I. Introduction

The most recent reform of Spanish Personal Income Tax (IRPF) incorporates the substitution of a system of family size related tax credits by a tax allowances system regarded as better. This reform was analysed in welfare terms in Badenes, López-Laborda and Onrubia (1998) from an empirical approach. In that paper, a comparison among tax credits and allowances was undertaken by using Atkinson and Bourguignon (1987) methodology, and assuming both tax structures were yield neutral. The number of children was not the only non-income attribute incorporated, since we added marital status and the number of income earners in the tax unit. No welfare superiority for any structure compared could be assessed.

In this paper we start adopting a theoretical approach, in a social welfare context *à la* Atkinson and Bourguignon (1987). As a main result, we obtain that, if tax credits and allowances are compared in an individual context -assuming yield neutrality and taxes with the same tax schedule- it could be assured that a progressive income tax with family size related tax allowances is never welfare superior to an income tax involving tax credits. This statement and the validity for a family context are developed in section II.

Recently, Lambert and Yitzhaki (1995, 1997) have proposed a utilitarian criterion embodying both horizontal and vertical equity commands which determines the equitable income tax. They show that a family size related tax credit cannot be considered equitable, whereas an allowance can, if a particular social evaluation function defined over income and needs is adopted. In section III, the social evaluation function proposed by Lambert and Yitzhaki (1997) is confronted with the one proposed by Atkinson and Bourguignon (1987), and we conclude that tax credits can be considered equitable by evaluating welfare with a function *à la* Atkinson and Bourguignon (1987).

In section IV, theoretical results are checked on empirical grounds for the Spanish IRPF in force in 1999. We have developed a micro-simulation exercise, using the Panel of IRPF Returns, which belongs to the Institute of Fiscal Studies (Ministry of Finances). The paper finishes with some concluding remarks.

II. Tax credits versus allowances in a social welfare context *à la* Atkinson and Bourguignon (1987)

Atkinson and Bourguignon (1987) have proposed a very attractive method for a welfare analysis of the tax treatment conceded to different types of taxpayers, according to non-income attributes (for example, family size): this is the sequential generalised Lorenz dominance criterion. This criterion allows the analysis to be made in income terms (without needing equivalence scales) so, maintaining essential elements of average utility of income approach.

The starting point consists of reaching certain agreement about differential treatment that income units must be given by personal income tax. As stated by Atkinson, Bourguignon and Chiappori (1987:347): "there is a degree of agreement but not complete agreement about the relative treatment of different family types".

As Lambert (1993, 1994) explains, the available non income information is used to subdivide the population into groups $i=1,\dots,n$ with different levels of need, which rank from the neediest downwards. The idea is that for each given x , some income units are more deserving of additional resources than others.

These differences in needs are recognised by the social decision-maker which assigns a different utility of income-function $U^i(x)$ to income units in

each group. Each $U^i(x)$ is increasing and concave, i.e., the decision-maker is inequality averse when focusing on income distribution within any group.

The social welfare function evaluates average utility-of-income across the whole population¹:

$$W_2 = \sum p_i \cdot W_i \quad [1]$$

where p_i is the proportion of income units belonging to group i , and $W_i \in W_1$ is average utility-of-income within group i ,

$$W_1 = \frac{1}{N} \sum U(x), \quad U' > 0, \quad U'' < 0, \quad \forall x \geq 0 \quad [2]$$

where N is the population.

Evaluation in welfare terms of a progressive income tax which applies over non income attributes requires, as a necessary condition:

Theorem 1 (Atkinson and Bourguignon, 1987)

If overall inequality is unambiguously reduced by an income tax involving differences in tax treatment, then it is a necessary condition for welfare superiority over an equal-yield proportional tax for all $W \in W_2$.

Lambert has proved that conditions for overall inequality reduction are not trivial at all, and that separate progressive taxation -with between-groups

¹ Recently, Ok and Lambert (1999) have demonstrated that it is no necessary to use the utilitarian method for the aggregation of the welfare levels of the groups in order to obtain the overall welfare of the population. The methodology of Atkinson an Bourguignon (1987) is applicable to all increasing and need-based social welfare functions, i.e., functions which record an increase in overall welfare when a cardinal welfare transfer is made from less needy to needier.

redistribution to the needy and within-group redistribution to the poor- do not necessarily involve overall inequality reduction.

Restricting the vector of utility functions $\langle U^1(x), \dots, U^n(x) \rangle$ to describe an attitude to needs on the part of the social decision-maker, we can find necessary and sufficient conditions for an unambiguous welfare recommendation.

These properties of the vector of social utility functions are the following: for each $i:1,2,\dots,n-1$, $[dU^i/dx] - [dU^{i+1}/dx]$ is both positive and decreasing in income x . This means that at every level of income, the social decision-maker attributes a higher marginal social utility of income to some type of income units than to others, and also that the systematic difference in marginal social utility at each income level decreases with income.

Assuming these hypothesis, the following theorem applies:

Theorem 2 (Atkinson and Bourguignon, 1987)

It is necessary and sufficient for welfare improvement -for all $W \in W_2$ - that there is generalised Lorenz dominance of income tax over the equal-yield proportional tax, for the sub-populations consisting of the j most needy groups, for each $j=1, \dots, n$.

Necessary condition for the means is the following:

$$\sum_{i=1}^j p_i (\mu_{x-IR}^i - \mu_{x-IP}^i) > 0, \quad \forall j \quad [3]$$

Where IR is the progressive income tax, and IP is the proportional tax.

Theorem 2 shows the sequential generalised Lorenz dominance criterion.

Let us show, in this context, the main two options for family size treatment at Personal Income Tax: tax credits and allowances. We adopt two hypothesis in this paper: first, there is an only tax schedule with separated taxation in case of marriage; second, overall yield is the same in both evaluated alternatives.

Substitution of tax credits by allowances is a single cross reform, in which taxpayers at the bottom of income distribution are losers, taxpayers at the top are winners, and taxpayers not affected by family circumstances remain unchanged. This assessment can be easily explained.

Let us identify family size with number of children, h_s , that taxpayer s have in charge. We can define individual tax savings generated by tax credits or allowances as follows:

$$D_s = t(x_s) - [t(x_s) - d(h_s)] = d(h_s) \quad [4]$$

$$R_s = t(x_s) - t[x_s - r(h_s)] = r(h_s) \cdot t'[x_s - r(h_s)] \quad [5]$$

Where $t(x)$ is a progressive tax schedule, $t'(\cdot)$ is the marginal tax rate, and $r(\cdot)$ and $d(\cdot)$ are, respectively, the allowance and tax credit for each child. Making equal the alternative tax savings, so that $D_s=R_s$, we obtain the following indifference relation: $t'(\cdot) = (d/r)$. Since we assume yield neutrality, there is an only value of r for each given d that assures that,

$$\sum_{s=1}^n D_s = \sum_{s=1}^n R_s \quad [6]$$

So, the following relations derive straightforward: For those individuals whose marginal tax rate is smaller than d/r , tax saving using tax credits will be greater than using allowances, that is $D_s>R_s$, while for the opposite situation fulfils

$D_s < R_s$. Since marginal tax rate is increasing with income, it may be concluded that the comparison posed is a single cross and regressive reform.

When interest is focused on individual income distribution, the following theorem applies:

Theorem 3: (Dardanoni and Lambert, 1988)

In a yield-neutral reform, if tax $t^2(x)$ crosses tax $t^1(x)$ once from below, there is Lorenz dominance of post- $t^2(x)$ income over post- $t^1(x)$ income.

According with this theorem, there will be Lorenz dominance of after tax income including tax credits over after tax income with allowances. So, using tax allowances, necessary condition set in theorem 1 will never fulfil, and this tax structure will never be welfare superior to an structure involving tax credits for a welfare function $W \in W_2$. Since necessary condition for welfare enhancement does not fulfil, there is no point in checking generalised Lorenz dominance for each one of the sub-populations as described in theorem 2: such a dominance will not be possible at least for the whole population, $j=n$. These results allow us to propose the following theorem:

Theorem 4:

A progressive income tax with family size related tax allowances is never welfare superior to an equal yield income tax involving tax credits, for any welfare function $W \in W_2$.

This theorem can not be interpreted as assuring welfare superiority of tax credits over allowances. Necessary condition set in theorem 1 will fulfil, but nothing can be said about the fulfilling of the necessary and sufficient condition set in theorem 2.

So far, we assume that comparison is made among individuals. If we focused on distribution of family income, the results presented in theorems 3 and 4 could not be maintained. It could be possible that substitution of tax credits by allowances would damage those families placed at the top of the income distribution, which are composed by individual taxpayers placed at the bottom. In that case, dominance of after tax income with allowances over tax credits could be expected, and necessary condition would fulfil.

Comparison between tax credits or allowances in the context of linear income tax arouse no interest, both in familial or individual approach. Tax savings are identical since they are obtained at the same marginal tax rate.

Next, we present some simulations which illustrate theoretical results. Tax credits and allowances are evaluated taking successively individual and family as unit of analysis. Family circumstances are measured by number of children in charge. Tax schedule used for simulation exercise is presented in Table 1.

TABLE 1

Net tax base up to (m.u.)	Gross liability (m.u.)	Remaining net tax base up to (m.u.)	Rate applicable (%)
50	0	100	20.00
150	20	100	30.00
250	50	250	40.00
500	150	onwards	50.00

Tax allowance for each child, r , equals 21.425 monetary units (m.u), and the tax credit which allows the same overall revenue is $d= 10$ m.u. We consider three individuals $s=(1,2,3)$ whose taxable income x_s are $x_1=200$, $x_2=500$ and $x_3=600$ respectively. Family size is $h_1=0$, $h_2=1$, and $h_3=2$.

We begin with an individual approach. Table 2 shows that there is generalised Lorenz dominance of tax credits over allowances. Then, necessary condition required in theorem 1 does not fulfil, and the result of theorem 4

applies: under yield neutrality, substitution of tax credits by allowances does not lead to welfare improvement for the considered social welfare functions.

TABLE 2
Exercise 1. Individual approach

s	X	T(x)	D	R	TD(x)	TR(x)	X-TD	X-TR	Ac. X-TD	Ac. X-TR
1	200.00	35.00	0.00	0.00	35.00	35.00	165.00	165.00	165.00	165.00
2	500.00	150.00	10.00	8.57	140.00	141.43	360.00	358.57	525.00	523.57
3	600.00	200.00	20.00	21.43	180.00	178.57	420.00	421.43	945.00	945.00
Σ_s	1,300.00	385.00	30.00	30.00	355.00	355.00	945.00	945.00		

T(x): Gross liability
 TD(x): Tax liability in case of tax credit depending on children
 TR(x): Tax liability in case of allowance depending on children
 D: Tax credit depending on children, so that $D=d \cdot h$
 R: Tax saving when allowance applies
 X-TD: Post-tax income with children tax credits
 X-TR: Post-tax income with children allowance
 Ac. X-TD: Accumulated post-tax income with children tax credit
 Ac. X-TR: Accumulated post-tax income with children allowance

The following simulations show how this result could not be maintained when the unit of analysis is not an individual but a family. All possible combinations of marriage among the three individuals have been considered. Composition of families $f=(1,2)$ is supposed as follows: in exercise 2 the first family is composed only by individual $s=3$, and the second one is a marriage of $s=1$ with $s=2$. In exercise 3, the first home is a single individual, $s=2$, and $s=1$ plus $s=3$ compose the second family. Exercise 4 assumes a single home $s=1$, and a family with $s=2$ and $s=3$.

In case of marriage, tax schedule applies separately, even though overall family tax revenue is the amount of tax revenues of each member of the family.

TABLE 3
Exercise 2. Household approach

<i>f</i>	X	T(x)	D	R	TD(x)	TR(x)	X-TD	X-TR	Ac. X-TD	Ac. X-TR
1	600.00	200.00	20.00	21.43	180.00	178.57	420.00	421.43	420.00	421.43
2	700.00	185.00	10.00	8.57	175.00	176.43	525.00	523.57	945.00	945.00
Σf	1,300.00	385.00	30.00	30.00	355.00	355.00	945.00	945.00		

TABLE 4
Exercise 3. Household approach

<i>f</i>	X	T(x)	D	R	TD(x)	TR(x)	X-TD	X-TR	Ac. X-TD	Ac. X-TR
1	500.00	150.00	10.00	8.57	140.00	141.43	360.00	358.57	360.00	358.57
2	800.00	235.00	20.00	21.43	215.00	213.57	585.00	586.43	945.00	945.00
Σf	1,300.00	385.00	30.00	30.00	355.00	355.00	945.00	945.00		

TABLE 5
Exercise 4. Household approach

<i>F</i>	X	T(x)	D	R	TD(x)	TR(x)	X-TD	X-TR	Ac. X-TD	Ac. X-TR
1	200.00	35.00	0.00	0.00	35.00	35.00	165.00	165.00	165.00	165.00
2	1,100.00	350.00	30.00	30.00	320.00	320.00	780.00	780.00	945.00	945.00
Σf	1,300.00	385.00	30.00	30.00	355.00	355.00	945.00	945.00		

Exercises 2, 3 and 4 corroborate what Moyes and Shorrocks (1998) state: “there seems little prospect of improving greatly on the laborious practice of examining the redistributive impact of the present tax structure, and the impact of proposed reforms, on a case-by-case basis”. Given certain individual income distribution, and adopting a family approach, redistributive analysis of a progressive income tax depends on the grouping among families. So, theoretical proposals presented above loses generality, being necessary a particular evaluation for each possible combination.

In exercise 2, a substitution of tax credits by allowances allows a reduction in overall income inequality. Explanation is very simple: by grouping in family $f=2$ the poorest two individuals, tax saving due to children allowance with separated taxation is less than saving obtained with tax credits ($R_2=8.57 < D_2=10$). On the contrary, the individual with the greatest income (which is now the poorest family, $f=1$) obtains a tax saving through tax allowance greater than with tax credit ($R_1=21.43 > D_1=20$). The substitution of tax credits by allowances causes in this case a change in familial after tax-income that diminishes overall inequality (necessary condition required in theorem 1).

In exercise 3, the grouping in family $f=2$ lead to the same result obtained with individual approach. In this case, reranking of income does not prevent that tax savings derived from application of allowances are increasing with family income, so that is impossible to achieve a greater diminishing of overall inequality than using tax credits.

Finally, exercise 4 shows how is possible to find –with family approach– income distributions which generate identical distributional results. Grouping the greatest incomes in family $f=2$ where tax credits and allowances are solely enjoyed, makes both systems indifferent, since tax savings are the same.

III. Tax Credits and Equity

Lambert and Yitzhaki (1997) undertake a different analysis for comparison by using allowances instead of tax credits. Following Lambert and Yitzhaki (1995, 1997), the just money income tax $T(x,h)$, i.e., the tax which embodies the horizontal and vertical equity commands, is the one defined by the following equation:

$$U_h(x - T(x,h)) = U_h(x) - \tau(U_h(x)), \quad \forall x, \forall h \quad [7]$$

where $\tau(\cdot)$ is the tax function on social utility which meets the vertical equity command.

Lambert and Yitzhaki (1997) show that under quite mild restrictions on the social evaluation function $U(x,h)$ and form of the utility tax $\tau(\cdot)$, a family size related tax credit cannot be considered equitable. Nevertheless, tax allowances are equitable, if it is assumed that income net of the family size related exemption, $r(h)$, can be taken as a measure of social utility:

$$U_h(x) = x - r(h) \quad [8]$$

This social welfare function has not been specified as proposed by Atkinson and Bourguignon (1987), because the properties of the vector of social utility functions do not fulfil:

$$\frac{dU_h(x)}{dx} - \frac{dU_{h-1}(x)}{dx} = 0 \quad [9]$$

$$\frac{d^2U_h(x)}{dx^2} - \frac{d^2U_{h-1}(x)}{dx^2} = 0 \quad [10]$$

Nevertheless, a function as presented below, in which social utility is measured as the difference between income and an increasing with size but decreasing with income deduction, fulfils Atkinson and Bourguignon (1987) properties:

$$U_h(x) = x - r(x, h) \quad [11]$$

with $r_h > 0$, $r_x < 0$, $r_{xx} > 0$, $r_{xh} < 0$. From which it follows that:

$$\frac{dU_h(x)}{dx} - \frac{dU_{h-1}(x)}{dx} > 0 \quad [12]$$

$$\frac{d^2U_h(x)}{dx^2} - \frac{d^2U_{h-1}(x)}{dx^2} < 0 \quad [13]$$

As a consequence, decreasing with income deductions could be justified in equity and, if theorem 2 holds, in welfare terms. Now then, it must be take into account that such a decreasing deductions could be equivalent to a fixed tax credit when taxation is progressive, if deductions have a decreasing rate equivalent to the marginal tax rates increasing pace. So, we could also rely on an equity justification for tax credits.

IV. Empirical approach: treatment for dependents in the Spanish Personal Income Tax

In this section we carry out an empirical contrast of the theoretical results presented in section II. We choose the personal income tax today in force in Spain for developing the simulations.

Every exercise has been made using a sample of 5.969 returns from the Panel of taxpayers of the Spanish Institute of Fiscal Studies. Three different

types of taxpayers can be identified in the 1994 sample²: a) single persons, b) married couples taxed jointly and c) married couples taxed separately. When household approach is adopted, the returns corresponding to the third group are reduced exactly to a half, so the total size of the sample is 5.163.

The amount of the allowances for the first and second child in the Spanish Personal Income Tax is 200,000 pesetas, and 300,000 for the third and followings. We want to compare allowances and tax credits in a yield neutral context, and for the design of a tax credit system, we maintain the proportion of $2/3$, so the tax credit for the first two children will be of 52,925 pesetas, and 79,388 for the rest.

Let us suppose that the social decision maker distinguishes four groups among income units (individuals or families), according to the number of children in charge, which rank from the neediest downwards. Each one of these groups is taxed in a different way depending on the need:

i=1: income units with three or more children in charge.

i=2: income units with two children in charge.

i=3: income units with one child in charge.

i=4: income units without children.

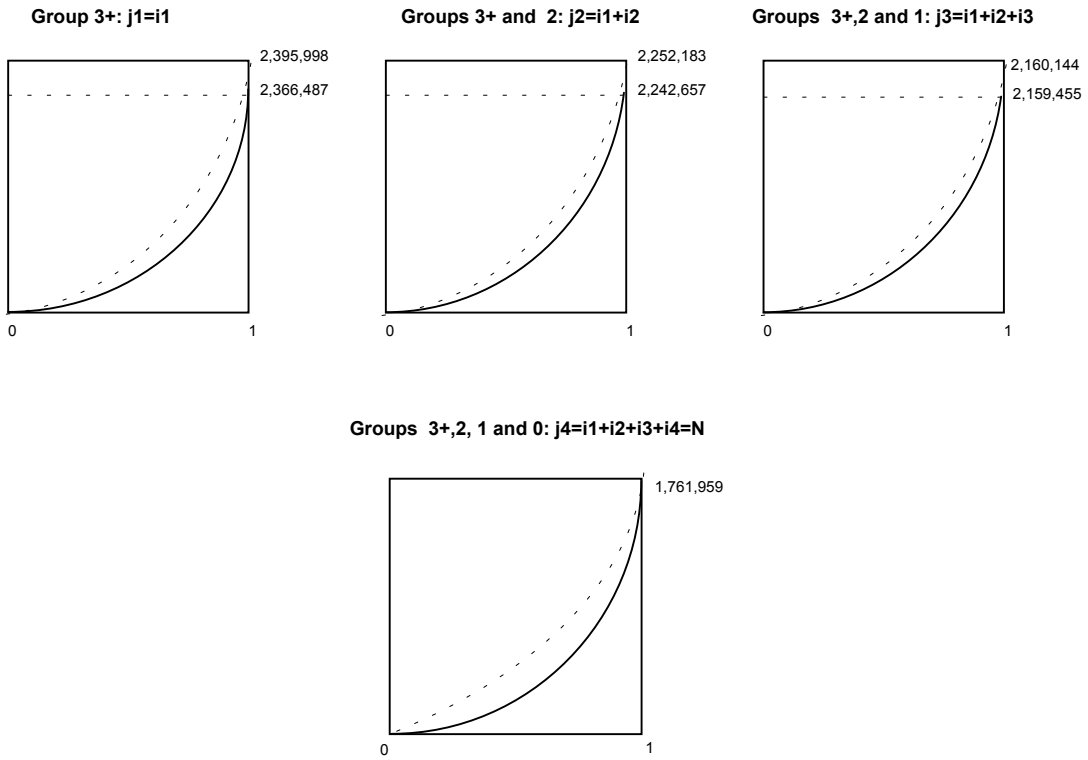
We assume that, at every level of income, the social decision-maker considers the first group the most deserving of additional resources, and also that the systematic difference in marginal social utility at each income level decreases with income.

In this context, the necessary and sufficient condition for a system to be welfare superior to other, is the existence of sequential generalised Lorenz

² 1994 was the most recent disposable year. The tax structure applicable for 1999 was developed from 1994 data.

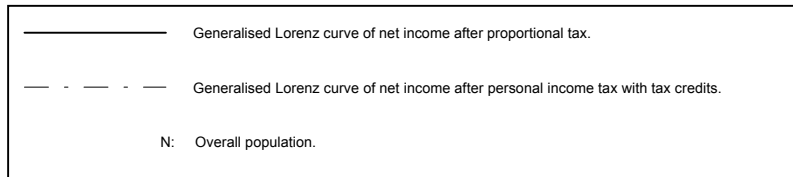
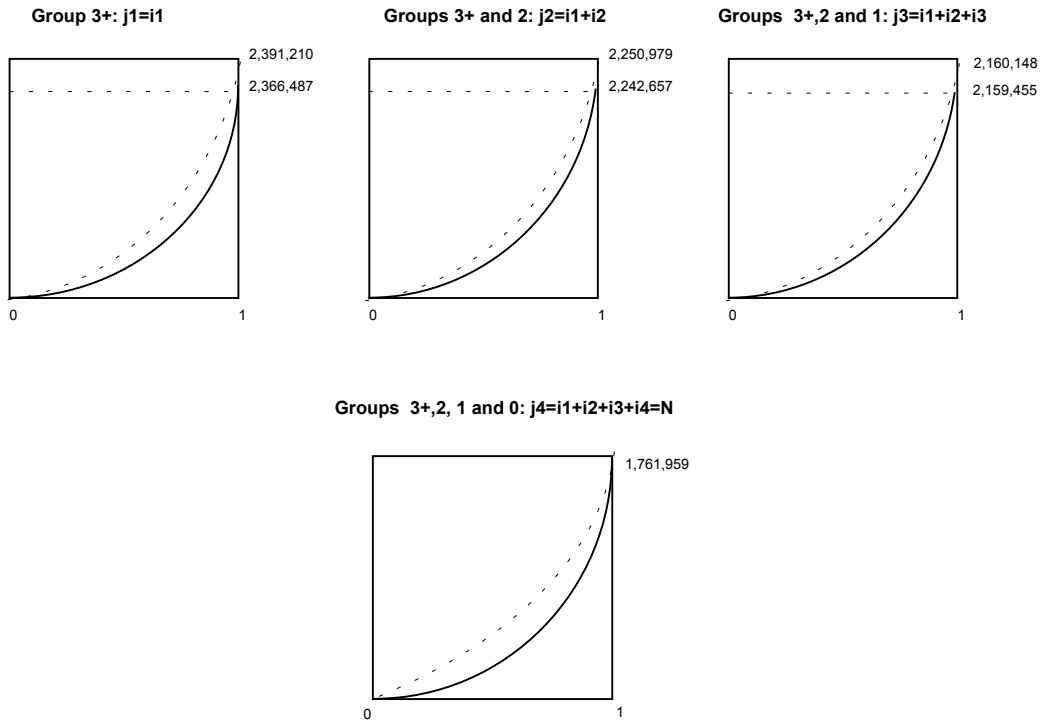
dominance. The curves that allow us for checking the welfare superiority are showed next. Figures IV.1.1 to IV.1.3 are referred to individual approach, and figures IV.2.1 to IV.2.3, to the familiar one.

Figures IV.1.1. Allowances versus proportional tax. Individual approach.

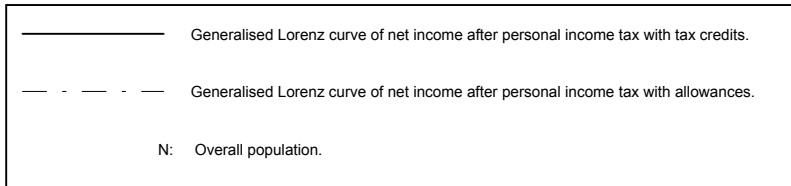
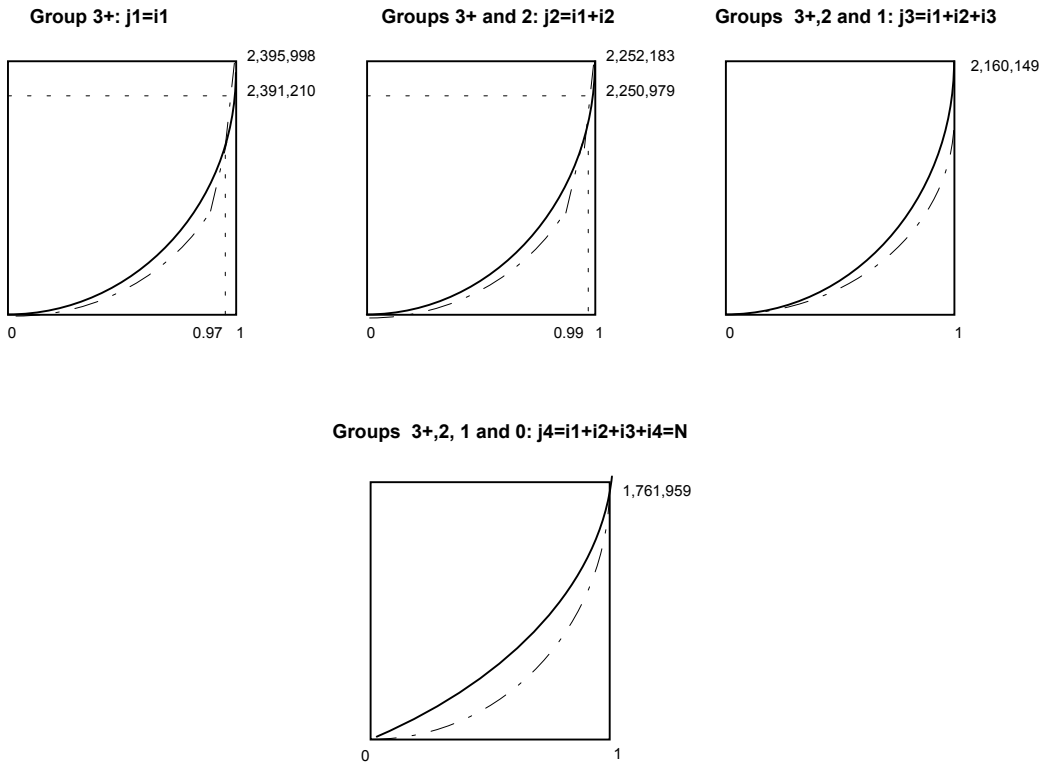


	Generalised Lorenz curve of net income after proportional tax.
	Generalised Lorenz curve of net income after personal income tax with allowances.
N:	Overall population.

Figures IV.1.2. Tax credits versus proportional tax. Individual approach.

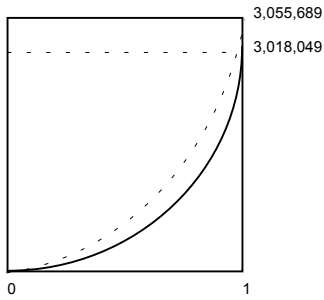


Figures IV.1.3. Tax credits versus allowances. Individual approach.

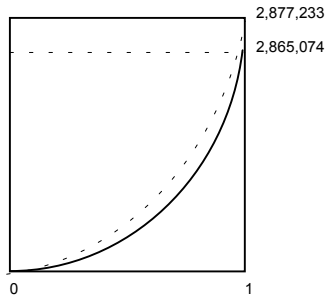


Figures IV.2.1. Allowances versus proportional tax. Household approach.

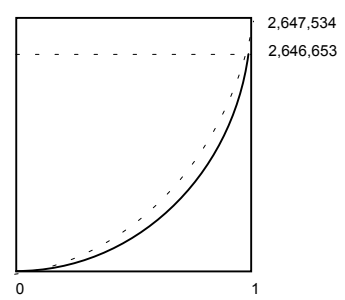
Group 3+: $j_1=i_1$



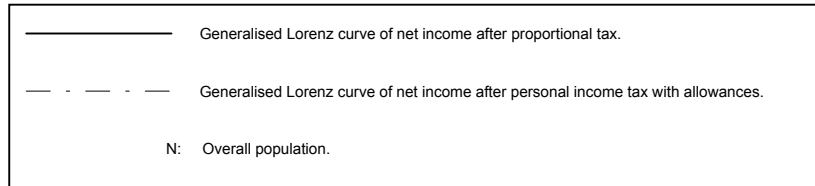
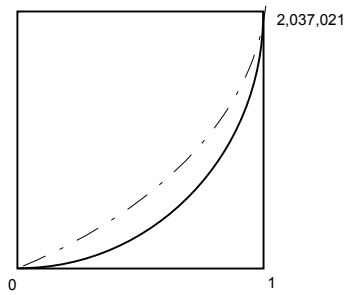
Groups 3+ and 2: $j_2=i_1+i_2$



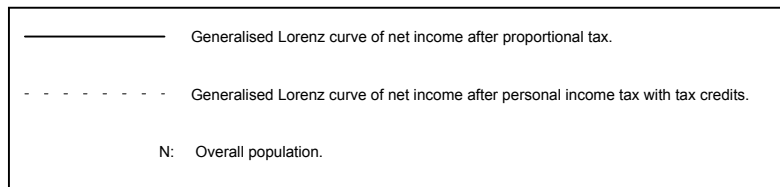
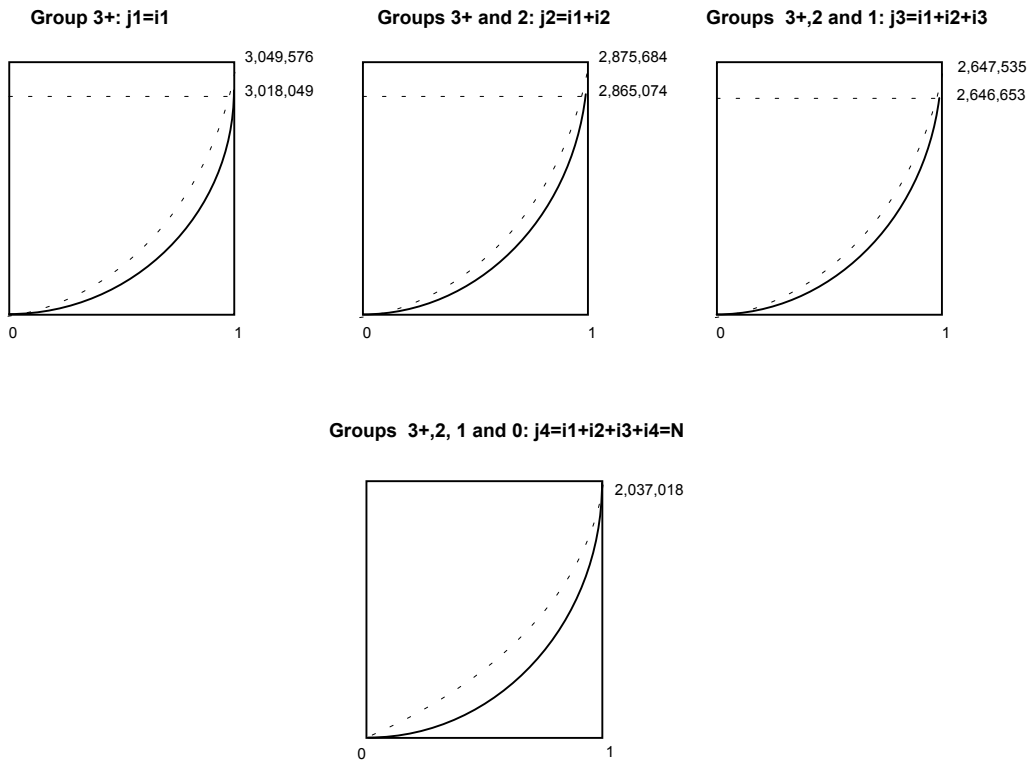
Groups 3+,2 and 1: $j_3=i_1+i_2+i_3$



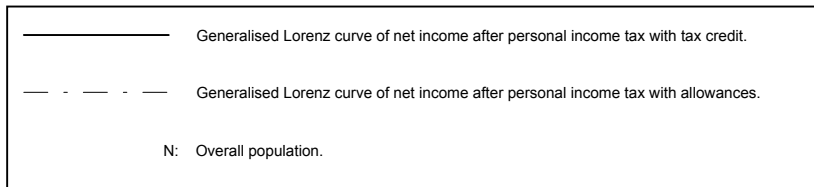
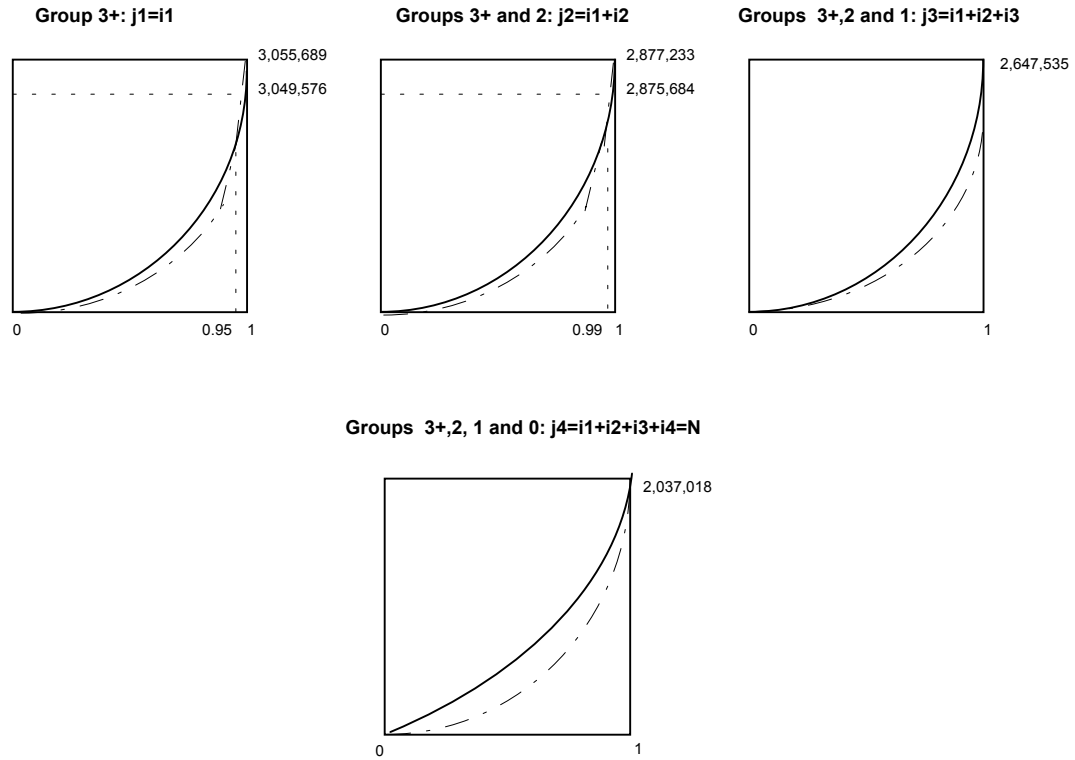
Groups 3+,2, 1 and 0: $j_4=i_1+i_2+i_3+i_4=N$



Figures IV.2.2. Tax credits versus proportional tax. Household approach.



Figures IV.2.3. Tax credits versus allowances. Household approach.



There are some remarkable results derived from the figures presented above. First, **both adopting an individual or household approach, allowances and tax credits are welfare superior to an equal yield proportional tax**, for every social welfare function $W \in W_2$. Figures IV.1.1, IV.1.2, IV.2.1 and IV.2.2, show that the allowances or tax credits design dominates over the proportional tax (where each group is not treated differently) for every sub-population.

In figures IV.1.3 and IV.2.3, we can see that **a tax credit system reduces global income inequality more than what allowances does, so, necessary condition for allowances to be welfare superior to tax credits never fulfils** for every social welfare function $W \in W_2$. Table IV.1 shows the value of the Gini index that corroborates this statement.

Table IV.1
Gini index of net income after tax credits or allowances

	INDIVIDUAL APPROACH	HOUSEHOLD APPROACH
Income tax with tax credits	0,3931	0,4117
Income tax with allowances	0,3942	0,4126

Since figures IV.1.3 and IV.2.3 show crosses in generalised Lorenz curves for the two first sub-populations (income units with three or more children and with two or more children) on the last centiles, necessary and sufficient condition does not fulfil. So, **it is impossible to determine superiority in welfare terms of an allowances or tax credit system for a design of an income tax that recognises household charges.**

V. Concluding remarks

In this paper we have developed an analytical framework devoted to compare family size related tax credits and allowances. As a main result, we

have obtained that, in a social welfare context *à la* Atkinson and Bourguignon (1987), if tax credits and allowances are compared in an individual context, it could be assured that a progressive income tax with family size related tax allowances is never welfare superior to an income tax involving tax credits.

Two results can be derived from the micro-simulation exercises. Firstly, that, both adopting an individual or household approach, allowances and tax credits are welfare superior to an equal yield proportional tax. And secondly, that, although a tax credit system reduces global income inequality more than what allowances does, also from an individual or household approach, it is not possible to determine superiority in welfare terms of a system over the other.

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