Optimal Redistribution when the Educational Background Matters

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Abstract

Higher education plays an important role in determining lifetime earnings. In turn, the decision to become educated depends on innate ability and family characteristics like wealth and educational background. We abstract from family income differences to concentrate on the effects of fiscal policies on educational decisions when the cultural background matters. In a dynamic framework, where generations are linked by the educational background, we derive optimality conditions for a linear income tax and a lump-sum subsidy to education. Factors determining their sign and magnitude include concerns for redistribution, efficiency and the educational externality on future generations.

Keywords: Optimal linear income tax, Subsidies, Higher education, Educational background

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1 Introduction

It is generally accepted that there are three main inputs to the production of human capital: initial endowments like innate ability, time spent by the individual in human capital investment, and other inputs that are purchased by individuals or provided by their families or the government (see Behrman *et al.* [96]). Among the last, it is worth mentioning the role of both family wealth and cultural background (for instance, the level of education of the parents).

Human capital formation has an undeniable effect on market productivity. For this reason, educational policies are widely recognized to be one of the most important policy tools in our societies. This is probably the reason why in most industrialized countries there are substantial public subsidies, financed by general taxation, for higher education.

Consequently, it is important to determine which of the determinants of human capital can be most efficiently controlled. Empirical work concerned with this issue (for instance, Behrman and Taubman [76]) suggests that the degree of education is mostly determined by ability and by family characteristics. Ability is innate and, thus, given. Family wealth and educational background remain as the main variables that could be influenced by public policy.

In this chapter, we abstract from initial income differences to concentrate on the role of the educational background. We consider a government that cares for both efficiency and redistribution. In order to compensate individuals with less initial endowments, of both ability and educational background, the government can choose to use an income tax system. However, the taxation of earnings has disincentive effects on both labor supply and education. If less individuals decide to attend university today, the number of unfavorably endowed individuals, in terms of lower educational background, will be larger tomorrow.

Alternatively, the government can choose to subsidize education in order to promote it and increase present and future income. However, as noted by Hamada [74], Hare and Ulph [79] and Creedy [95] among others, the recipients of these subsidies turn out to be the individuals with higher lifetime earnings. The subsidy to education may then be considered as a regressive policy.

The aim of this chapter is to identify the optimal combination of income taxes and subsidies to education that brings together all these considerations. In order to do so, we consider a dynamic framework in which individuals belonging to two different educational backgrounds supply labor and demand higher education. Young individuals differ in both their ability to benefit from education and their educational background (i.e., the level of education of their parents). These two variables affect the costs and, thus, the decision to undertake education. Following Montgomery [92] productivity, measured by the wage an individual can earn, is the result of formal and home education. When the government tries to redistribute income, it affects the educational decisions made by individuals and, hence, the proportions of educated and uneducated parents in the following generation. It also affects labor supply and thus tax revenue today.

Other papers have considered the effects of family wealth inequality on human capital investment. For instance, in Barham *et al.* [95], financing for education is obtained from within the family and the unequal distribution of income results in unequal opportunities to acquire higher education. In their model, although parents are not altruistic, they have a better knowledge of their children's abilities to succeed in education and are hence willing to lend them the funds they need to pursue their studies. Other models consider altruistic parents (Borjas [92], Torvik [93]). Parents leave bequests or invest in the education of their children because they derive a higher utility the higher the welfare of their children. The conclusions are similar to that in Barham *et al.* [95]: differences in income imply differences in opportunity.

However, the inequality of opportunity to acquire education may not only be due to family income differences. In fact, the most serious obstacles to attending university in many countries turn out to rely on the cultural background of the potential student (i.e., the level of education of her family). Indeed, according to OECD[98], "despite continuing increases in the proportion of the population completing secondary and tertiary education, the educational chances of individuals remain heavily influenced by their own family background". Children of highly educated parents are between two and three times as likely to gain tertiary education than those of poorly educated parents.

The family educational background of future generations can also be affected by public policy. Although individuals cannot influence their inherited family background, they are responsible for the backgrounds they pass on to future generations. Thus, the decision to undertake higher education by an individual will affect his earnings, but also the likelihood that his children attend higher education. In other words, individual's decisions impose an externality on their children.

The chapter is organized as follows. In section 2, we describe the model and we provide a characterization of the labor supply and educational choices of individuals. We also analyze the dynamics of the model and compute the first-best solution. In section 3, we characterize the behavior of the government. First, we study the effects of fiscal policy on individual decisions. Induced distortions on the decision to attend higher education will modify the proportions of educated and uneducated individuals in the economy. Finally, we analyze the conditions for optimality of the labor income tax rate and the subsidy to education. Section 4 concludes.

2 The model

2.1 Individual's labor supply and educational choice

In this section we analyze the behavior of individuals belonging to a given generation. Individuals differ both in their ability to benefit from education and in their educational background. Ability to benefit from education, denoted by a, is stochastically determined at birth. For simplicity, we consider that a is uniformly distributed between 0 and 1. An individual's educational background is represented by the education of his parents, e_{-1}. We assume that the level of education chosen by an individual, e, can take one of two values: either 0, if the individual does not attend university, or 1, if she does.

Individuals live for three periods. In the first period, individuals work for a low wage $\bar{w}$ and may or may not attend higher education. Studying entails a financial cost that depends on their ability to benefit from education and on the education of their parents. We assume this cost to be $\gamma_{e_{-1}} C(a)$, where the parameter $\gamma_{e_{-1}}$represents the effect of the parents' education on their children's educational costs. We posit $\gamma_0 > \gamma_1$ to reflect the fact that education is more costly for the children of uneducated parents. $C(.)$ is a decreasing and convex function of ability (i.e., $C' < 0, C'' > 0$).

In the second period, due to either the experience gained at work or the acquired education, individuals earn a higher wage $w^e_{e_{-1}} > \bar{w}$. As throughout the chapter, the subindex accounts for the education level of the parents, e_{-1}, and the superindex accounts for the education level of the individual, e. In general, due to the effect of inherited human capital, children of educated parents earn higher wages, whether they invest on education or not: $w^0_1 > w^0_0$ and $w^1_1 > w^1_0$. Moreover, we assume that the benefit in terms of wages of investing in education is at least as large for children of educated parents as it is for children of uneducated parents: $w^1_1 - w^0_1 \geq w^1_0 - w^0_0$.

During their working lives, individuals can save in order to consume in the third period, when they are retired. Younger individuals can also disave (e.g., borrow against future earnings in period 1). The capital market is assumed to be perfect, which precludes individuals from being liquidity constrained. For simplicity, we also assume that the population is constant and that the interest rate is zero.

Once the decision to study or not has been taken, productivity and thus wages in the second period are determined. Then, individuals decide how much to work. Let $h(l)$ be the disutility of labor measured in units of consumption at each period, with l being the number of hours spent at work each period. We assume that the disutility of labor is increasing and convex $(h' > 0, h'' > 0)$. Following Diamond (1998), the preferences for consumption and leisure are represented by a quasi-linear utility function (linear in consumption). Individual lifetime utilities are thus given by:

$$u^0_0 = \bar{w}\,\bar{l} - h(\bar{l}) + w^0_0\,l^0_0 - h(l^0_0) \quad (1)$$

$$u^0_1 = \bar{w}\,\bar{l} - h(\bar{l}) + w^0_1\,l^0_1 - h(l^0_1) \quad (2)$$

$$u^1_0 = \bar{w}\,\bar{l} - h(\bar{l}) + w^1_0 l^1_0 - h(l^1_0) - \gamma_0 C(a) \quad (3)$$

$$u^1_1 = \bar{w}\,\bar{l} - h(\bar{l}) + w^1_1 l^1_1 - h(l^1_1) - \gamma_1 C(a) \quad (4)$$

Individuals choose, in the first and second periods, the labor supply that maximizes their lifetime utility ($u^e_{e_{-1}}$). At the optimum, they supply the amount of labor that satisfies in each case:

$$\bar{w} = h'\left(\bar{l}\right), w^e_{e_{-1}} = h'\left(l^e_{e_{-1}}\right)$$

Each individual chooses whether to become educated or not by comparing lifetime utility with and without education. The educational decision determines, for each type e_{-1}, a threshold value of ability above which individuals will attend university. We will denote by $\widehat{a}_{e_{-1}}$ the ability level for which individuals of type e_{-1} are indifferent between undertaking higher education or not.

Given the education of parents, e_{-1}, the threshold ability level is hence determined by equating lifetime utility of an uneducated and an educated individual. Given that first period wage and labor supply are the same for both types of individuals, the resulting condition is

$$\gamma_0 C(\widehat{a}_0) = \left(w^1_0 l^1_0 - h\left(l^1_0\right)\right) - \left(w^0_0 l^0_0 - h\left(l^0_0\right)\right) \tag{5}$$

$$\gamma_1 C(\widehat{a}_1) = \left(w^1_1 l^1_1 - h\left(l^1_1\right)\right) - \left(w^0_1 l^0_1 - h\left(l^0_1\right)\right) \tag{6}$$

Thus, at the threshold ability level $\widehat{a}_{e_{-1}}$, the cost of education equals the gain in terms of net lifetime earnings.

All individuals with parents with education e_{-1} whose ability is larger than $\widehat{a}_{e_{-1}}$ (i.e., education costs lower than $\gamma_{e_{-1}} C(\widehat{a}_{e_{-1}})$) will attend university. Individuals of ability $a_{e_{-1}} < \widehat{a}_{e_{-1}}$ will not.

Education is more costly for the children of uneducated parents (since $\gamma_0 > \gamma_1$). Further, given our assumption that $w^1_1 - w^0_1 \geq w^1_0 - w^0_0$, $w^1_1 l^1_1 - h\left(l^1_1\right) - (w^0_1 l^0_1 - h(l^0_1)) \geq w^1_0 l^1_0 - h\left(l^1_0\right) - (w^0_0 l^0_0 - h(l^0_0))$.[1]

As a consequence, $\widehat{a}_0 > \widehat{a}_1$: the threshold ability level is higher if parents are uneducated. Hence, less children of uneducated parents will attend university and the ones that do will in average be more able than the children of educated parents.

[1] From the envelope theorem we get $\frac{\partial}{\partial w}(wl - h(l)) = l > 0$. On the other hand, $\frac{\partial l}{\partial w} = \frac{1}{h''} > 0$. Then, the effect of a higher wage on earnings net of the disutility of labor is positive and increasing.

2.2 Dynamics of the model

The evolution over time of the proportions of educated and uneducated people in this economy can be described by a Markov chain with the following transition matrix:

$$P = \begin{pmatrix} \widehat{a}_0 & 1-\widehat{a}_0 \\ \widehat{a}_1 & 1-\widehat{a}_1 \end{pmatrix}$$

where $(1-\widehat{a}_0)$ is the probability of attending university for a child whose parents have not and $(1-\widehat{a}_1)$ is the probability of attending university when the parents have. Recall that a is uniformly distributed between 0 and 1. Hence, $\widehat{a}_0$ and $\widehat{a}_1$ are the probabilities of remaining uneducated when having an unfavorable and favorable family background, respectively.

At the steady state, it must be the case that:

$$(\pi_0, \pi_1) = (\pi_0, \pi_1)\ P \tag{7}$$

The proportion of educated and uneducated people in the economy replicates itself once the steady state has been reached. We can easily obtain the vector of limiting or steady state probabilities by substituting the matrix P into (7) and using the fact that $\pi_0+\pi_1=1$:

$$\pi_0 = \frac{\widehat{a}_1}{1-\widehat{a}_0+\widehat{a}_1} \tag{8}$$

$$\pi_1 = \frac{1-\widehat{a}_0}{1-\widehat{a}_0+\widehat{a}_1} \tag{9}$$

Hence,

$$\frac{d\pi_0}{d\widehat{a}_0} = \frac{\pi_0}{1-\widehat{a}_0+\widehat{a}_1} = -\frac{d\pi_1}{d\widehat{a}_0} \tag{10}$$

$$\frac{d\pi_0}{d\widehat{a}_1} = \frac{\pi_1}{1-\widehat{a}_0+\widehat{a}_1} = -\frac{d\pi_1}{d\widehat{a}_1} \tag{11}$$

With a constant population, an increase in the probability of remaining uneducated for children of either family background increases the number of uneducated individuals in the steady state and reduces the number of educated individuals in exactly the same amount.

2.3 The first best

It seems convenient to characterize the first best solution that provides the maximum value of social welfare. We assume, at this stage, that the government can control all the instruments to achieve full redistribution.

In this ideal situation, the government would determine both the level of education and the labor supply of each individual in order to maximize the utility generated in the overall economy. Then, it would equally distribute this global utility among the citizens. For given levels of education in this economy, it would choose the labor supply levels that maximize consumption net of labor disutility at the steady state. The government hence maximizes*:*

$$\pi_0 Eu_0 + \pi_1 Eu_1$$

where:

$$Eu_0 = \widehat{a}_0 \ u_0^0 + \int_{\widehat{a}_0}^{1} u_0^1 \ da \tag{12}$$

$$Eu_1 = \widehat{a}_1 \ u_1^0 + \int_{\widehat{a}_1}^{1} u_1^1 \ da \tag{13}$$

stand for the expected net contributed income of children of uneducated parents ($e_{-1} = 0$) and educated parents ($e_{-1} = 1$), respectively. The conditions for optimality regarding labor supply are:

$$\bar{w} = h'\left(\bar{l}\right) \text{ and } w_{e_{-1}}^e = h'\left(l_{e_{-1}}^e\right). \tag{14}$$

The first-best levels of labor supply coincide with the levels individuals would choose in a decentralized economy, without government intervention.

The government also determines the optimal threshold ability levels, $\widehat{a}_0$ and $\widehat{a}_1$, that maximize the average net consumption. The first best cut-off ability levels will be denoted by $\widehat{a}_0^{FB}$ and $\widehat{a}_1^{FB}$. After some rearranging, the optimality conditions regarding $\widehat{a}_0$ and $\widehat{a}_1$ when the optimum is interior are respectively:[2]

$$\frac{Eu_0 - Eu_1}{1 - \widehat{a}_0 + \widehat{a}_1} + (w_0^0 l_0^0 - h\left(l_0^0\right) - (w_0^1 l_0^1 - h\left(l_0^1\right)) + \gamma_0 C(\widehat{a}_0)) = 0 \tag{15}$$

$$\frac{Eu_0 - Eu_1}{1 - \widehat{a}_0 + \widehat{a}_1} + (w_1^0 l_1^0 - h\left(l_1^0\right) - (w_1^1 l_1^1 - h\left(l_1^1\right)) + \gamma_1 C(\widehat{a}_1)) = 0 \tag{16}$$

[2]Second-order conditions for maximum can also be shown to hold.

Since $(w_1^1 l_1^1 - h(l_1^1)) - (w_1^0 l_1^0 - h(l_1^0)) \geq (w_0^1 l_0^1 - h(l_0^1)) - (w_0^0 l_0^0 - h(l_0^0))$, $\gamma_0 > \gamma_1$ and $C(.)$ is decreasing, it follows that $\widehat{a}_0^{FB} > \widehat{a}_1^{FB}$.

In this model, the government controls all the instruments required to accomplish full redistribution. Thus, by minimizing the costs associated to the optimal level of investment in education it manages to redistribute more income. This is the reason why, at the first best, less children of uneducated than of educated parents will enroll in higher education: education is more costly for the former than for the latter for a given ability level. Although this clearly leads to inequality of opportunities at the optimum, note that equalization of the threshold ability levels is a goal that has not been explicitly included in the objective function of the planner. Therefore, not surprisingly, it does not result.

On the other hand, it is worth noticing that the first best threshold values of *a do not coincide* with those resulting from the decentralized individual decisions in the absence of government intervention or laissez-faire (see (5), (6) and (15), (16)).

Since, at the laissez-faire, the expected lifetime utility is larger for the children of educated parents, who earn higher net incomes whether they study or not ($Eu_1 > Eu_0$) individuals who make their educational decision in the absence of government intervention end up consuming too little education. In other words, individuals underinvest in education at the laissez-faire solution. Note that when educational background does not matter, the first-best level of threshold ability and the laissez-faire one coincide. The underinvestment in education when educational background matters is due to the fact that individuals fail to take into account the effect of their educational decision on their children's behavior and, thus, on average income in the steady state. In the absence of full redistribution, individuals will try to maximize their net lifetime earnings[3], thus neglecting the effect of their decisions on future generation's expected income. There is hence a role for public policy.

[3] Once an identical level of utility is ensured to all individuals, it is in their own interest to maximize average utility.

3 The government

The government may influence the levels of individual incomes and, thus, the level of social welfare in the economy by means of its fiscal policy. We assume that the government's available policy tools are a linear income tax and a lump-sum subsidy for education[4]. The linear income tax system is characterized by a marginal tax rate τ and a demogrant G. While the income tax applies to all individuals, the lump-sum subsidy S is targeted to individuals who undertake higher education. Both the labor supply and the educational choices of the individual will be affected by these policy variables.

Once taxes and subsidies are introduced, individual utilities become:

$$
\begin{aligned}
v_0^0 &= (1-\tau)\left(\bar{w}\bar{l} + w_0^0 l_0^0\right) - h(\bar{l}) - h(l_0^0) + 2G && (17)\\
v_1^0 &= (1-\tau)\left(\bar{w}\bar{l} + w_1^0 l_1^0\right) - h(\bar{l}) - h(l_1^0) + 2G && (18)\\
v_0^1 &= (1-\tau)\left(\bar{w}\bar{l} + w_0^1 l_0^1\right) - h(\bar{l}) - h(l_0^1) - \gamma_0 C(a) + S + 2G && (19)\\
v_1^1 &= (1-\tau)\left(\bar{w}\bar{l} + w_1^1 l_1^1\right) - h(\bar{l}) - h(l_1^1) - \gamma_1 C(a) + S + 2G && (20)
\end{aligned}
$$

The condition that determines the optimal levels of labor supply is:

$$(1-\tau)\, w = h'(l) \qquad \forall w, l. \tag{21}$$

It is worth noticing that the only policy tool that affects labor supply is the tax rate: $l = l\left((1-\tau)\, w\right)$. Because of the quasi-linear utility function the labor supply is not subject to income effects and, thus, is not affected by S and G.

Regarding the educational choice, equations (5), (6) become now:

$$
\begin{aligned}
\gamma_0 C(\widehat{a}_0) &= (1-\tau)(w_0^1 l_0^1 - w_0^0 l_0^0) - \left(h\left(l_0^1\right) - h\left(l_0^0\right)\right) + S && (22)\\
\gamma_1 C(\widehat{a}_1) &= (1-\tau)(w_1^1 l_1^1 - w_1^0 l_1^0) - \left(h\left(l_1^1\right) - h\left(l_1^0\right)\right) + S && (23)
\end{aligned}
$$

and the threshold ability levels will depend on the tax rate and the lump-sum subsidy for education.

[4] It can be argued that a linear tax system does not make much sense when we have discrete values for labour income. It would then be equivalent to differentiated lump-sum taxes. However, the assumptions about the wage structure rule out this possibility.

3.1 Comparative statics

As noted previously, the only policy tool that affects the labor supply decision is the tax rate. Given the characteristics of the disutility of labor function, $h(.)$, it is straightforward to show that an increase in the tax rate induces a decrease in labor supply.

How do τ and S affect the decision to undertake higher education? By checking the effect of these parameters on the threshold ability value, we will be able to determine the induced effect on the number of students of each group, and on the composition of the population at the steady state.

Differentiation of (22), (23) when $dS = 0$, yields:

$$\begin{aligned} \gamma_0 C'(\widehat{a}_0)\, d\widehat{a}_0 &= (w_0^0 l_0^0 - w_0^1 l_0^1)\, d\tau \\ \gamma_1 C'(\widehat{a}_1)\, d\widehat{a}_1 &= (w_1^0 l_1^0 - w_1^1 l_1^1)\, d\tau \end{aligned}$$

Then,

$$\frac{d\widehat{a}_0}{d\tau} = \frac{w_0^0 l_0^0 - w_0^1 l_0^1}{\gamma_0 C'(\widehat{a}_0)} > 0 \tag{24}$$

$$\frac{d\widehat{a}_1}{d\tau} = \frac{w_1^0 l_1^0 - w_1^1 l_1^1}{\gamma_1 C'(\widehat{a}_1)} > 0 \tag{25}$$

A marginal increase in the tax rate involves a disincentive to undertake education. Individuals who become educated in the first period will pay higher taxes in the second. On the contrary, the lump-sum subsidies to education will provide incentives to study. By total differentiation of (22), (23) when $d\tau = 0$, we obtain:

$$\frac{d\widehat{a}_0}{dS} = \frac{1}{\gamma_0 C'(\widehat{a}_0)} < 0 \tag{26}$$

$$\frac{d\widehat{a}_1}{dS} = \frac{1}{\gamma_1 C'(\widehat{a}_1)} < 0 \tag{27}$$

Note that there exists a relationship between the effects of taxes and subsidies on the threshold ability levels:

$$\frac{d\widehat{a}_0}{d\tau} = (w_0^0 l_0^0 - w_0^1 l_0^1)\frac{d\widehat{a}_0}{dS}$$

and

$$\frac{d\widehat{a}_1}{d\tau} = (w_1^0 l_1^0 - w_1^1 l_1^1)\frac{d\widehat{a}_1}{dS}$$

When we increase the tax rate, individuals have less incentives to become educated, whereas increasing the subsidy promotes education. The terms of proportionality in the previous expressions, $(w^1_{e_{-1}} l^1_{e_{-1}} - w^0_{e_{-1}} l^0_{e_{-1}})$, determine the quantity by which the subsidy should be increased, when the tax rises, in order to keep $\widehat{a}_{e_{-1}}$ constant. It is worth noticing that, since educated children with educated parents receive higher wages than educated children with uneducated parents, the rule of proportionality differs by types e_{-1}. Therefore, an increase in the subsidy in response to an increase in the tax that seeks to leave $\widehat{a}_0$ unchanged will not be enough to leave $\widehat{a}_1$ unaffected. The latter will nevertheless increase in response to the tax: less children of educated parents become educated. This is so because in order to compensate individuals with higher wages for the increase in the tax rate we would need higher subsidies.

It would then be interesting to determine how public policy affects the gap between $\widehat{a}_0$ and $\widehat{a}_1$. If we had $\widehat{a}_0 = \widehat{a}_1$, individuals would have the same probability of becoming educated independently of their cultural background. Equality of opportunity may be defined, in the present context, as university attendance being independent of parental education. The greater the difference $(\widehat{a}_0 - \widehat{a}_1)$, the greater the inequality of opportunity. It can be shown that:

$$\frac{d\left(\widehat{a}_0 - \widehat{a}_1\right)}{dS} \begin{array}{c} > 0 \\ < 0 \end{array} \Leftrightarrow \frac{\frac{C^{'}(\widehat{a}_1)}{C(\widehat{a}_1)}}{\frac{C^{'}(\widehat{a}_0)}{C(\widehat{a}_0)}} \begin{array}{c} < \\ > \end{array} \frac{(1-\tau)(w^1_0 l^1_0 - w^0_0 l^0_0) - \left(h\left(l^1_0\right) - h\left(l^0_0\right)\right) + S}{(1-\tau)(w^1_1 l^1_1 - w^0_1 l^0_1) - \left(h\left(l^1_1\right) - h\left(l^0_1\right)\right) + S}$$

Since labor earnings, net of the disutility of labor, are higher for individuals with higher net wages, the right hand side of the second inequality can be shown to be smaller than one. Its left hand side is larger than one if $C^{'}(.)/C(.)$ is nonincreasing in a.[5] In this case, the gap is reduced by the marginal increase in the subsidy to education.

Even under this last assumption, the effect of the tax rate on equality of opportunity is ambiguous. Note that the relationship between the effects of taxes and subsidies on the

[5] Note that $\frac{d}{da}\left(\frac{C^{'}(a)}{C(a)}\right) = \frac{C^{'}(a)}{C(a)}\left(\frac{C^{''}(a)}{C^{'}(a)} - \frac{C^{'}(a)}{C(a)}\right)$. It then suffices for $C^{'}(.)/C(.)$ to be decreasing in a that the degree of convexity of the cost function is not too large.

gap is given by:

$$\left(\frac{d\widehat{a}_0}{d\tau}-\frac{d\widehat{a}_1}{d\tau}\right) = (w_0^0 l_0^0 - w_0^1 l_0^1)\left(\frac{d\widehat{a}_0}{dS}-\frac{d\widehat{a}_1}{dS}\right) + ((w_1^1 l_1^1 - w_1^0 l_1^0) - (w_0^1 l_0^1 - w_0^0 l_0^0))\frac{d\widehat{a}_1}{dS}.$$

When the gap is reduced by the increase in the subsidy, it tends to be increased by the increase in the tax. However, if the difference in the benefit from education between educated children of educated and uneducated parents is high enough, it could be the case that the combined increases in S and τ tend to reduce the gap (i.e., reduce the inequality of opportunities). This is due to the fact that subsidy increases compensate type-0 individuals more than type-1 individuals for the rise in the tax. Hence, type-1 individuals are relatively more distorted by the tax.

Finally, the effect of the policy parameters on the proportion of educated and uneducated individuals at the steady state is given by:

$$\frac{d\pi_0}{d\tau} = \frac{\pi_0\frac{d\widehat{a}_0}{d\tau}+\pi_1\frac{d\widehat{a}_1}{d\tau}}{1-\widehat{a}_0+\widehat{a}_1},$$

$$\frac{d\pi_0}{dS} = \frac{\pi_0\frac{d\widehat{a}_0}{dS}+\pi_1\frac{d\widehat{a}_1}{dS}}{1-\widehat{a}_0+\widehat{a}_1}.$$

3.2 Optimal taxes and subsidies

The objective of the government is to maximize expected utility. We assume that the government applies a concave transformation $U(.)$ to individual utilities v (which now include the fiscal parameters). The higher the government's aversion to inequality, the more concave the welfare function will be. $U(v_{e_{-1}}^e)$ is hence the social valuation of this individual's utility. The government's problem is the following:

$$\max_{\tau,G,S} \quad \pi_0\left(\widehat{a}_0 U(v_0^0)+\int_{\widehat{a}_0}^{1} U(v_0^1)da\right)+\pi_1\left(\widehat{a}_1 U(v_1^0)+\int_{\widehat{a}_1}^{1} U(v_1^1)da\right)$$

subject to (λ) :

$$\tau\left(\bar{w}\ \bar{l}+\pi_0\widehat{a}_0 w_0^0 l_0^0+\pi_1\widehat{a}_1 w_1^0 l_1^0+\pi_0(1-\widehat{a}_0)w_0^1 l_0^1+\pi_1(1-\widehat{a}_1)w_1^1 l_1^1\right)=2G+\pi_1 S$$

where λ is the Lagrange multiplier associated to the government's budget constraint. The expression in brackets in the budget constraint represents the tax base. We will denote it by B. If, in addition, we denote the expected social (lifetime) utility of a child with uneducated and educated parents, respectively, by:

$$EU(v_0) = \widehat{a}_0 U(v_0^0) + \int_{\widehat{a}_0}^{1} U(v_0^1) da, \tag{28}$$

$$EU(v_1) = \widehat{a}_1 U(v_1^0) + \int_{\widehat{a}_1}^{1} U(v_1^1) da. \tag{29}$$

The resulting Lagrangian is:

$$L = \pi_0 EU(v_0) + \pi_1 EU(v_1) + \lambda (\tau B - 2G - \pi_1 S)$$

and the first-order conditions with respect to the three decision variables are:

$FOC(G)$:

$$\pi_0 \widehat{a}_0 U'(v_0^0) + \pi_0 \int_{\widehat{a}_0}^{1} U'(v_0^1) da + \pi_1 \widehat{a}_1 U'(v_1^0) + \pi_1 \int_{\widehat{a}_1}^{1} U'(v_1^1) da \equiv EU' = \lambda \tag{30}$$

$FOC(\tau)$:

$$\frac{\partial \pi_0}{\partial \tau}(EU(v_0) - EU(v_1)) + \pi_0 \frac{\partial EU(v_0)}{\partial \tau} + \pi_1 \frac{\partial EU(v_1)}{\partial \tau} = -\lambda(B + \tau \frac{\partial B}{\partial \tau} - S \frac{\partial \pi_1}{\partial \tau}) \tag{31}$$

$FOC(S)$:

$$\frac{\partial \pi_0}{\partial S}(EU(v_0) - EU(v_1)) + \pi_0 \frac{\partial EU(v_0)}{\partial S} + \pi_1 \frac{\partial EU(v_1)}{\partial S} = -\lambda(\tau \frac{\partial B}{\partial S} - S \frac{\partial \pi_1}{\partial S} - \pi_1) \tag{32}$$

EU' in (30) stands for expected social marginal utility and can be rewritten as follows:

$$EU' = \pi_0 \widehat{a}_0 U'(v_0^0) + \pi_1 \widehat{a}_1 U'(v_1^0) + \pi_0 (1 - \widehat{a}_0) EU'(v_0^1) + \pi_1 (1 - \widehat{a}_1) EU'(v_1^1),$$

where

$$EU'(v_0^1) = \frac{\int_{\widehat{a}_0}^{1} U'(v_0^1)}{(1 - \widehat{a}_0)} \text{ and } EU'(v_1^1) = \frac{\int_{\widehat{a}_1}^{1} U'(v_1^1)}{(1 - \widehat{a}_1)}$$

are the expected social marginal utility of educated children with uneducated and educated parents, respectively. We can substitute λ by EU' into equations (31) and (32).

3.2.1 Optimal income tax rate

The term

$$\frac{\pi_0 \frac{\partial EU(v_0)}{\partial \tau} + \pi_1 \frac{\partial EU(v_1)}{\partial \tau}}{EU'} + B$$

in (31) can, after some manipulation, be written as

$$\begin{aligned} & \pi_0 \widehat{a}_0 \left(1 - \frac{U'(v_0^0)}{EU'}\right) w_0^0 l_0^0 + \pi_1 \widehat{a}_1 \left(1 - \frac{U'(v_1^0)}{EU'}\right) w_1^0 l_1^0 + \\ & \pi_0 \left(1 - \widehat{a}_0\right) \left(1 - \frac{EU'(v_0^1)}{EU'}\right) w_0^1 l_0^1 + \pi_1 \left(1 - \widehat{a}_1\right) \left(1 - \frac{EU'(v_1^1)}{EU'}\right) w_1^1 l_1^1 \\ = & -cov\left(\frac{EU'(v_{e-1}^e)}{EU'}, w_{e-1}^e l_{e-1}^e\right) \end{aligned}$$

The optimality condition for the tax rate can then be stated as:

$$\frac{\frac{\partial \pi_0}{\partial \tau}(EU(v_0) - EU(v_1))}{EU'} - cov\left(\frac{EU'(v_{e-1}^e)}{EU'}, w_{e-1}^e l_{e-1}^e\right) + \tau \frac{\partial B}{\partial \tau} - S\frac{\partial \pi_1}{\partial \tau} = 0 \qquad (33)$$

The covariance in (33) can in turn be decomposed in:

$$\begin{aligned} & \pi_0 \left(\hat{a}_0 \left(\tfrac{U'(v_0^0)}{EU'} - \tfrac{EU'(v_0^e)}{EU'}\right) w_0^0 l_0^0 + (1 - \hat{a}_0) \left(\tfrac{EU'(v_0^1)}{EU'} - \tfrac{EU'(v_0^e)}{EU'}\right) w_0^1 l_0^1\right) \\ & + \pi_1 \left(\hat{a}_1 \left(\tfrac{U'(v_1^0)}{EU'} - \tfrac{EU'(v_1^e)}{EU'}\right) w_1^0 l_1^0 + (1 - \hat{a}_1) \left(\tfrac{EU'(v_1^1)}{EU'} - \tfrac{EU'(v_1^e)}{EU'}\right) w_1^1 l_1^1\right) \\ & + \pi_0 \left(\tfrac{EU'(v_0^e)}{EU'} - 1\right) E\left(w_0^e l_0^e\right) + \pi_1 \left(\tfrac{EU'(v_1^e)}{EU'} - 1\right) E\left(w_1^e l_1^e\right) \end{aligned}$$

Therefore

$$\begin{aligned} cov\left(\tfrac{EU'(v_{e-1}^e)}{EU'}, w_{e-1}^e l_{e-1}^e\right) &= \pi_0 cov\left(\tfrac{EU'(v_0^e)}{EU'}, w_0^e l_0^e\right) + \pi_1 cov\left(\tfrac{EU'(v_1^e)}{EU'}, w_1^e l_1^e\right) \\ & + cov\left(\tfrac{EU'(v_{e-1})}{EU'}, E(w_{e-1} l_{e-1})\right) \end{aligned} \qquad (34)$$

The first covariance in (34) refers to the children of uneducated parents. From (17), (19) and (22) the children of uneducated parents who earn higher wages have higher net incomes and, since marginal utility is decreasing in income, $cov\left(EU'(v_0^e)/EU', w_0^e l_0^e\right) < 0$. The same argument applies, from (18), (20) and (23) to the children of educated parents and, therefore,

$cov\left(EU'(v_1^e)/EU', w_1^e l_1^e\right) < 0.$

On the other hand, the last covariance term in (34) is the covariance between the average wage earnings of each type of individual (children of educated and uneducated parents respectively) and their expected marginal utilities. We know that children of educated parents earn more than children of uneducated parents, whether they study or not. Besides, they support lower educational costs. Then expected earnings and utility are higher for children of educated parents and $cov\left(EU'(v_{e_{-1}})/EU', E(w_{e_{-1}} l_{e_{-1}})\right) < 0$.

The sign of the covariance term in (33) is thus negative.

The term $\partial B/\partial\tau$ represents the change in the tax base at the steady state due to the change in the population composition and to changes in labor supply induced by the tax. Both effects can be shown to be negative: when the tax rate increases, the tax base diminishes both due to the decrease in labor supply and to the induced changes in the composition of the population: τ has a positive effect on π_0 and therefore a negative effect on π_1. Since individuals in π_1 (i.e., educated individuals) earn higher wages, the total effect on tax revenue is negative.

Following the tradition of the optimal linear taxation literature, we can write:

$$\tau^* = \frac{\dfrac{\frac{\partial \pi_0}{\partial \tau}\left(EU(v_1) - EU(v_0)\right)}{EU'} + cov\left(\dfrac{EU'(v_{e_{-1}}^e)}{EU'}, w_{e_{-1}}^e l_{e_{-1}}^e\right) + S\dfrac{\partial \pi_1}{\partial \tau}}{\dfrac{\partial B}{\partial \tau}} \tag{35}$$

which provides us with the optimal tax rate for a given subsidy S. This kind of expression, which is traditionally known as "à la Sheshinsky", is somehow misleading since τ affects as well the right hand side of the expression. However, it also allows to isolate the four main elements that underlie the sign and magnitude of the tax rate.

In the denominator of (35) we find, as usual, an efficiency term. It includes the disincentive effects of income taxation on both labor supply and education. Through its disincentive to education, the tax influences the composition of the population.

Regarding the numerator of (35), some terms play in favor of a larger marginal tax rate and others against. The covariance term is associated with the willingness to redistribute income from richer to poorer individuals. It has a negative sign which, together with the negative sign of the denominator, implies that the marginal tax rate is larger the more we care for income redistribution.

On the other hand, the first term in the numerator of (35) represents the difference of expected utility between children of educated and uneducated parents, weighted by the effect of the tax on the proportions of educated and uneducated individuals in the steady state. It thus represents the effect of the education externality on future generations. The larger this externality effect, the smaller will be the marginal tax rate compatible with a given disincentive to education.

Finally, the last term in the numerator reflects the fact that, by affecting the proportions of educated individuals, the government changes the proportions of individuals receiving the subsidy S. If τ increases, the number of educated individuals, π_1, falls and subsidy payments fall accordingly.

3.2.2 Optimal subsidy to education

As we did with the optimality condition for the tax rate, we rearrange the optimality condition for the subsidy:

$$\begin{aligned} \frac{\frac{\partial \pi_0}{\partial S}(EU(v_0) - EU(v_1))}{EU'} + \pi_0 (1 - \widehat{a}_0) \left(\frac{EU'(v_0^1)}{EU'} - 1 \right) & \\ + \pi_1 (1 - \widehat{a}_1) \left(\frac{EU'(v_1^1)}{EU'} - 1 \right) + \tau \left(\frac{\partial B}{\partial S} \right) - S \frac{\partial \pi_1}{\partial S} &= 0 \end{aligned} \tag{36}$$

We can then express the optimal level of S à la Sheshinsky, as a function of a given τ:

$$S^* = \frac{\frac{\frac{\partial \pi_0}{\partial S}(EU(v_0) - EU(v_1))}{EU'} + cov\left(\frac{EU'(v_{e-1}^e)}{EU'}, \frac{\partial v_{e-1}^e}{\partial S} \right) + \tau \frac{\partial B}{\partial S}}{\frac{\partial \pi_1}{\partial S}} \tag{37}$$

The covariance term can now be written as

$$\pi_0 \left((1 - \hat{a}_0) \left(\tfrac{EU'(v_0^1)}{EU'} - \tfrac{EU'(v_0^e)}{EU'} \right) \right) + \pi_1 \left((1 - \hat{a}_1) \left(\tfrac{EU'(v_1^1)}{EU'} - \tfrac{EU'(v_1^e)}{EU'} \right) \right)$$
$$+ \pi_0 \left(\tfrac{EU'(v_0^e)}{EU'} - 1 \right) E\left(\frac{\partial v_0^e}{\partial S} \right) + \pi_1 \left(\tfrac{EU'(v_1^e)}{EU'} - 1 \right) E\left(\frac{\partial v_1^e}{\partial S} \right)$$

The two first elements are clearly negative since the marginal expected utility of income is lower for those who get educated within each group. Since $\frac{\partial v_0^e}{\partial S} = \frac{\partial v_1^e}{\partial S}$, the last two terms dissapear. Therefore, the covariance term in (37) is, once again, negative.

Note that, due to the specification of the utility function in this model, labor supply has no income effect and is hence unresponsive to changes in S. For this reason, the term $\partial B/\partial S$ in (37) reflects only the effect of the subsidy on the composition of the population. The denominator is then positive: the number of educated individuals increases as a result of an increase in the subsidy.

All the elements that played in favor of a larger tax rate, such as the concern for redistribution, play now against a large lump-sum subsidy to education, the subsidy being a regressive measure. Similarly, the internalization of the externality and the efficiency term, that would induce a lower tax rate, imply now a larger lump-sum subsidy to education, since individuals are then more willing to both work and undertake education.

Since redistribution, efficiency and educational externality are simultaneously considered by the government, the optimal policy will be given by (35) and (37): the mix of taxes and subsidies that best conciliates these conflicting objectives.

4 Concluding remarks

We have analyzed optimal fiscal policies when the lifetime income of an individual depends on his educational level and, in turn, his decision to become educated is determined by both ability to benefit from education and educational background (i.e., the level of education within the family). To concentrate on the role of the educational background, we have abstracted from the effect that family income could have on the human capital investment decision, already treated in a number of papers.

We have considered a dynamic framework in which individuals from two different educational backgrounds decide whether or not to undertake higher education. We have assumed that individuals face different educational costs according to their innate ability and to the educational level of their parents. The individual decision to attend higher education today has a positive externality effect on future generations' expected income.

Failure to account for this externality leads to underinvestment in education.

Subsidies to education can serve the purpose of internalizing this externality. However, since expected lifetime incomes of educated individuals are higher than that of uneducated ones, the subsidy to education is a regressive policy. To fight inequality, we may use taxation. However, an income tax induces, in this framework, distortions to both the labor supply and the educational choices. In particular, the income tax affects negatively the decision to undertake higher education and, through the induced negative effect on future generations, tends to reduce total income in the long run. The optimal policy is then a mix of both instruments, the one that best conciliates the conflicting objectives of the government: redistribution, efficiency and the educational externality on future generations.

Let us conclude with a few words on equality of opportunity. Note that, in our model, equality of opportunity has not been considered as an explicit objective of the government. Education is more costly for the children of uneducated parents. A policy that would aim at equalizing opportunities might result in a lower proportion of educated individuals in the steady state. Global income would then be reduced. Our conjecture is therefore that such an objective might be too costly in terms of efficiency. The relationship between equality of opportunity and redistribution deserves, however, further research.

References

[1] Barham, V., Boadway, R., Marchand, M. and P. Pestieau, 1995, Education and the poverty trap, European Economic Review 39, 1257-1275.

[2] Behrman, J.R., Rosenzweig, M.R. and Taubman, P., 1996, College choice and wages: estimates using data on female twins,. Review of Economics and Statistics 78 (4), 672-85.

[3] Behrman, J.R. and Taubman, P., 1976, Intergenerational transmission of income and wealth, American Economic Review 66(2), 436-40.

[4] Borjas, G. J., 1992, Ethnic capital and intergenerational mobility, Quaterly Journal of Economics 107(1), 123-150.

[5] Creedy, J., 1995, The economics of higher education: An analysis of taxes versus fees (Edward Elgar Publishing Limited, England).

[6] Diamond, P.A., 1998, Optimal income taxation: An example with a U-shaped pattern of optimal marginal tax rates, The American Economic Review 88(1), 83-95.

[7] Hamada, K., 1974, Income taxation and educational subsidy, Journal of Public Economics 3, 145-158.

[8] Hare, P. and D.T. Ulph, 1979, On education and distribution, Journal of Political Economy 87, S193-S212.

[9] OECD, 1998, Education at a glance.

[10] Torvik, R., 1993, Talent, growth and income distribution, Scandinavian Journal of Economics 95(4), 581-596.