

# **SPECTRAL METHODS IN BUSINESS CYCLE ACCOUNTING\***

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November 2005

## **ABSTRACT**

We apply the Business Cycle Accounting Method developed in Chari, Kehoe and McGrattan (2005) to the U.S. Great Depression and to postwar U.S. data. We develop a spectral decomposition based on the population properties of the stochastic process generated by the model. We decompose the variance of output into the variance induced by each orthogonalized innovation to the wedges. We show that investment wedges play a minor role in the Great Depression and a modest role in postwar data.

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\* This paper is based on a keynote address given by V. V. Chari at the XX Jornadas de Economía Conference in Montevideo, Uruguay. The authors thank the National Science Foundation for support. The views expressed herein are those of the authors and not necessarily those of the Federal Reserve Bank of Minneapolis or the Federal Reserve System.

In Chari, Kehoe and McGrattan (2005), we propose and demonstrate the use of a simple method, called *business cycle accounting* for guiding researchers in developing quantitative models of economic fluctuations. Our method has two components: an equivalence result and an accounting procedure.

In our earlier work, we show an equivalence result, that a large class of models, including models with various frictions, are equivalent to a prototype growth model with time-varying wedges which resemble time-varying productivity, government consumption, labor taxes, and capital income taxes. We label the time-varying wedges *efficiency wedges*, *labor wedges*, *investment wedges* and *government consumption wedges*.

In our earlier work, we focus primarily on an episodic method of applying our accounting procedure. This procedure uses data together with the equilibrium conditions of a prototype growth model to measure the wedges, then feeds the values of these wedges back into the growth model, one at a time and in combinations, to assess what fraction of the output movements can be attributed to each wedge separately and in combinations.

In our earlier work, we apply our episodic procedure to two actual U.S. business cycle episodes: the Great Depression, and the 1982 recession. We show that the investment wedge cannot account for either the downturn or the slow recovery during the Great Depression and plays only a minor role in the 1982 recession. We also show that a similar procedure applied to postwar U.S. data yields similar results: the investment wedge plays a minor role in U. S. business cycles.

In this paper, we investigate a complementary spectral decomposition based on the population properties of the model's stochastic process. The results with this spectral decomposition match those of the initial decomposition: the investment wedge plays a minor role in the prewar period and a modest role in the postwar period.

Our findings suggest that models with credit market frictions operating through investment channels, such as those in Bernanke and Gertler (1989) are not promising avenues for studying the Great Depression and that these frictions play a limited role in postwar recessions. Our findings also suggest that sticky wage mechanisms with monetary shocks, as in

Bordo, Erceg, and Evans (2000) or models with fluctuating monopoly power, as in Cole and Ohanian (forthcoming), are more promising avenues.

This paper confirms our earlier substantive contribution that existing models of credit market frictions, such as those of Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997) can account for only a small fraction of the fluctuations in the Great Depression and that sticky wage mechanisms with monetary shocks, as in Bordo, Erceg, and Evans (2000) or models with fluctuating monopoly power, as in Cole and Ohanian (forthcoming), are more promising avenues.

## 1. Prototype Growth Model

The prototype economy that we use in our accounting procedure is a growth model with four stochastic variables: the *efficiency wedge*  $A_t$ , the *labor wedge*  $1 - \tau_{lt}$ , the *investment wedge*  $1/(1 + \tau_{it})$ , and the *government consumption wedge*  $g_t$ . In the model, consumers maximize expected utility over per capita consumption  $c_t$  and per capita labor  $l_t$ ,

$$E_0 \sum_{t=0}^{\infty} \beta^t U(c_t, l_t) N_t$$

subject to the budget constraint

$$c_t + (1 + \tau_{xt})x_t = (1 - \tau_{it})w_t l_t + r_t k_t + T_t,$$

and the capital accumulation law

$$(1 + \lambda)k_{t+1} = (1 - \delta)k_t + x_t \tag{1}$$

where  $k_t$  denotes the per capita capital stock,  $w_t$  the wage rate,  $r_t$  the rental rate on capital,  $\beta$  the discount factor,  $\delta$  the depreciation rate of capital,  $N_t$  the period  $t$  population equal to  $(1 + \lambda)^t$ , and  $T_t$  lump-sum taxes.

The firms' production function is  $F(k_t, (1 + \gamma)^t l_t)$ , where  $(1 + \gamma)^t$  is labor-augmenting technical progress that is assumed to grow at a constant rate. Firms maximize  $A_t F(k_t, (1 + \gamma)^t l_t) - r_t k_t - w_t l_t$ . The equilibrium is summarized by the resource constraint,

$$c_t + x_t + g_t = y_t, \quad (2)$$

where  $y_t$  and  $g_t$  denote per capita aggregate output and per capita government consumption, together with

$$y_t = A_t F(k_t, (1 + \gamma)^t l_t) \quad (3)$$

$$-\frac{U_{l_t}}{U_{c_t}} = (1 - \tau_{l_t}) A_t (1 + \gamma)^t F_{l_t} \quad (4)$$

$$U_{c_t} (1 + \tau_{x_t}) = \beta E_t U_{c_{t+1}} [A_{t+1} F_{k_{t+1}} + (1 - \delta)(1 + \tau_{x_{t+1}})] \quad (5)$$

where, here and throughout, we use notation like  $U_{c_t}$ ,  $U_{l_t}$ ,  $F_{l_t}$  and  $F_{k_t}$  to denote the derivatives of the utility function and the production function with respect to their arguments. We assume that  $g_t$  fluctuates around a trend of  $(1 + \gamma)^t$ .

Notice that in this benchmark prototype economy the efficiency wedge resembles the productivity parameter and the labor wedge and the investment wedge resemble tax rates on labor income and investment income.

## 2. Estimating the Stochastic Process for the Wedges

We choose parameters of preferences and technology in standard ways, as in the quantitative business cycle literature, and then use the equilibrium conditions of our prototype economy to estimate the parameters of a stochastic process for the wedges.

In terms of the data, we proceed as follows. Throughout we use annual data. Given data on investment  $x_t$  and an initial choice of capital stock  $k_0$ , we construct a series for the capital stock using the capital accumulation equation  $k_{t+1} = (1 - \delta)k_t + x_t$ . We also adjust output and its components to remove sales taxes and military compensation and to add the service flow for consumer durables. (In a technical appendix, available on request, we describe our data sources and computational methods in detail.)

We estimate the stochastic process for the wedges as follows. We assume that the production function has the form  $F(k, l) = k^\alpha l^{1-\alpha}$  and the utility function has the form  $U(c, l) = \log c + \psi \log(\bar{l} - l)$ . We choose the capital share  $\alpha = .35$ , the depreciation rate  $\delta = .046$ , the discount factor  $\beta = .97$ , the time allocation parameter  $\psi = 2.24$ , and the endowment of time  $\bar{l}$  equal to 5,000 hours per year.

Next consider equations (2)–(5), which summarize the equilibrium of the prototype economy. We substitute for consumption  $c_t$  in (4) and (5) using the resource constraint (2) and then log-linearize (3)–(5) to obtain three linear equations. We specify a vector AR1 process for the (demeaned) four wedges  $s_t = (\log A_t, \tau_{lt}, \tau_{xt}, \log g_t)$  of the form

$$s_{t+1} = P_0 + P s_t + Q \eta_{t+1} \quad (6)$$

where  $\eta_t$  is standard normal and i.i.d. and  $Q$  is lower triangular. We then have seven linear equations, three from the equilibrium and four from (6). We can then solve this system of equations for linear decision rules for output  $y_t$ , investment  $x_t$  and labor  $l_t$ .

We then use the maximum likelihood procedure described in McGrattan (1996) to estimate the parameters  $P_0$ ,  $P$ , and  $Q$  of the vector AR1 process for the wedges using data on output, investment, labor, and government consumption. We choose initial conditions for the wedges so that in the starting year, the economies are on a balanced growth path, at their observed initial values for consumption, investment, government consumption, capital stock, and employment. We estimate separate sets of parameters for our Great Depression and our postwar analyses. The parameters for our Great Depression analysis are estimated using data for 1901–1940 while those used in our postwar analysis are estimated using data for 1955–2000. (For a more detailed discussion of our estimation, see the technical appendix available upon request.)

In our Great Depression analysis, we impose the restriction that the covariance between the innovations to government consumption and to the other wedges is zero. We impose this restriction to avoid having the large movements in government consumption associated with World War I dominate the estimation of the stochastic process. Table 1 gives the

parameter values for  $P$  and  $Q$  and the associated standard errors for our two periods. It is worth noting that for the postwar data, we found several parameter configurations with likelihoods only slightly lower than the parameter estimates that we use.

### 3. Spectral Decomposition of Variance

In our earlier work, we develop a decomposition of the movements in the data based on the realizations measured using the model. In this paper, we develop a decomposition based on the population properties of the stochastic process generated by the model. In our spectral method, we begin by orthogonalizing the innovations to the wedges. At each frequency, we then decompose the variance of output into the variance induced by each orthogonalized innovation.

The spectral method is complementary to the episodic method in our earlier work. The spectral method has the advantage that is based on the population properties of the model. As such, it captures not just the behavior of a single episode that actually occurred, but it also captures behavior in other episodes that could have occurred but did not. The disadvantage of this method is that it requires us to orthogonalize the innovations to the wedges. Our difficulty in interpreting these orthogonalized innovations makes drawing sharp lessons about underlying models harder using this procedure than using our episodic method.

We orthogonalize the innovations to the wedges as follows. We choose one of 12 possible orderings of the wedges. Consider first the following ordering: the efficiency wedge first, followed in sequence by the labor, investment, and government consumption wedges. Given this ordering, we rewrite (6) as

$$s_{t+1} = Ps_t + Q\tilde{\epsilon}_{t+1}$$

where  $Q$  is the lower triangular matrix that solves  $QQ' = V$  and the covariance matrix of  $\tilde{\epsilon}_t$  is the identity matrix. With this ordering, the innovation to the efficiency wedge affects all the other wedges contemporaneously, while the innovation to the labor wedge affects the labor, investment, and government consumption wedges only and so on.

We can write our equilibrium in state-space form as follows. Let  $X_t = (\log k_t, s_t)$  denote the state in period  $t$ . The state evolves according to

$$X_{t+1} = AX_t + D\varepsilon_{t+1}. \quad (7)$$

The first row of (7) is the transition law for the capital stock, and the associated value of  $\varepsilon_t$  is identically zero. The rest of the system describes the vector autoregressive process for the four wedges. The matrix  $D$  is given by

$$D = \begin{bmatrix} 0 & 0 \\ 0 & Q \end{bmatrix}.$$

Let  $Y_t = (\log y_t, \log l_t, \log x_t, \log g_t)'$  denote the vector of output, labor, investment, and government consumption. Using the linear decision rules, we can rewrite this vector as

$$Y_t = CX_t. \quad (8)$$

Using standard methods (as, for example, Sargent 1987), we see that the spectral matrix of  $Y$  is given by

$$S(\omega) = C(e^{i\omega}I - A)^{-1}DD'(Ie^{-i\omega} - A')^{-1}C' \quad (9)$$

where  $\omega$  measures frequency. Let  $S_{ij}(\omega)$  be the element in the  $i$ th row and  $j$ th column of this matrix. Each such element can be decomposed into four pieces that sum up to one at each frequency  $\omega$  as follows. Define the spectral matrix associated with each innovation  $k$ ,  $k = 1, \dots, 4$ ,

$$S^k(\omega) = C(e^{i\omega}I - A)^{-1}De_{kk}D'(Ie^{-i\omega} - A')^{-1}C'$$

where  $e_{kk}$  is a matrix with a one in the  $kk$  element and zeros elsewhere, and let  $S_{ij}^k(\omega)$  denote the  $ij$  element of  $S^k(\omega)$ . Since output is the first variable in  $Y_t$ , our decomposition of the variance of output is given by

$$\left[ \frac{S_{11}^1(\omega)}{S_{11}(\omega)}, \frac{S_{11}^2(\omega)}{S_{11}(\omega)}, \frac{S_{11}^3(\omega)}{S_{11}(\omega)}, \frac{S_{11}^4(\omega)}{S_{11}(\omega)} \right]$$

The term  $S_{11}^k(\omega)/S_{11}(\omega)$  is interpreted as the fraction of variance of output at frequency  $\omega$  attributable to the innovation in wedge  $k$ .

So far we have illustrated our procedure using a specific ordering of the wedges. For each of the 12 possible orderings, the same procedure applies.

### *Findings*

For each wedge, we compute the average contribution to the spectrum over the 12 possible orderings. In Figure 1, we plot this average for the efficiency, labor, and investment wedges for the period from 1901 to 1940. We see that at business cycle frequencies (between two and six years) the combined contribution of the efficiency and labor wedges is over 80% and the contribution of the investment wedge is less than 15%. This result reinforces our basic finding that investment wedges played at best a minor role in the prewar era. In Figure 2 we plot the analog of Figure 1 for the period from 1955 to 2000. Here the combined contribution of the efficiency and labor wedges is roughly 60% while investment wedges contribute a little over 30%. This result suggests that investment wedges played a somewhat more important role in the postwar era.

## **4. Conclusion**

This study, like our earlier work, is aimed at applied theorists who are building detailed models of economic fluctuations. Once such theorists have chosen the primitive sources of shocks, they need to choose the mechanisms through which such shocks lead to fluctuations. Our accounting procedure, by focusing on the wedges, can be used to suggest promising mechanisms and rule out less promising ones.

We have found that investment wedges play, at best, a minor role in the Great Depression and a modest role in postwar business cycles. The findings imply that existing models of credit market frictions, such as those of Bernanke and Gertler (1989) and Carlstrom and Fuerst (1997), can account for only a small fraction of the fluctuations in the Great Depression or more typical U.S. downturns. These findings are our substantive contribution.



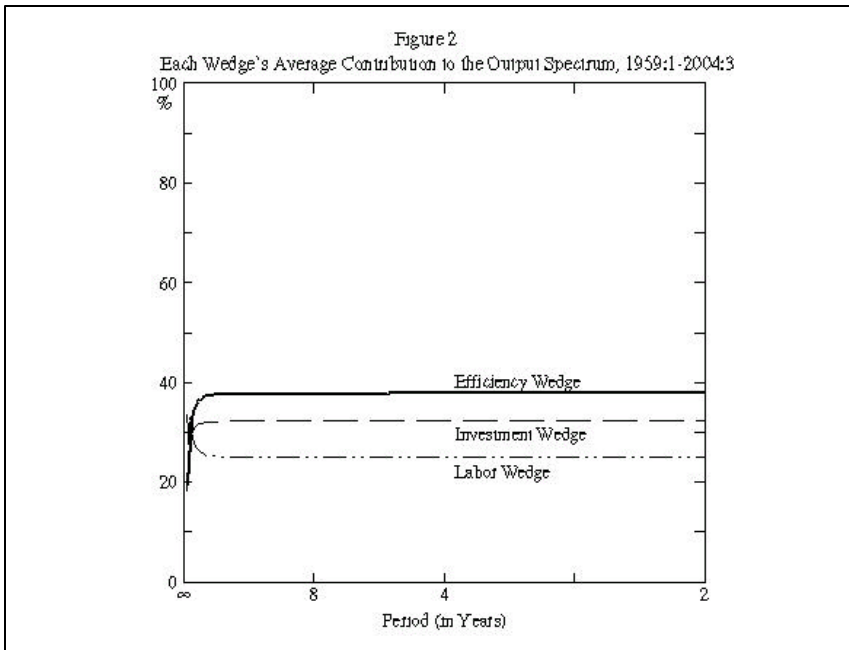
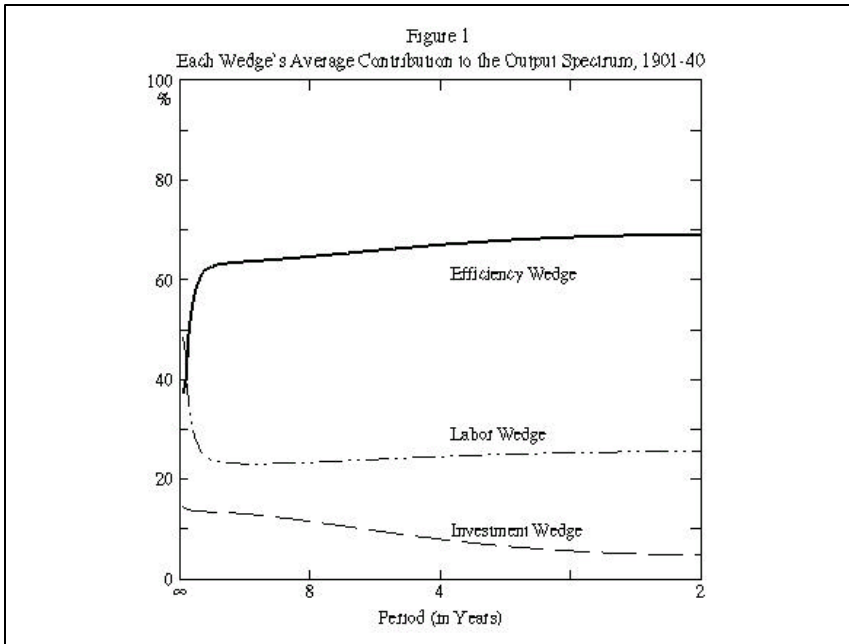
These findings do not imply, of course, that frictions in financial markets are irrelevant for business cycle fluctuations. Indeed, we have shown in Chari, Kehoe and McGrattan (2005) that a detailed economy with input-financing frictions is equivalent to a prototype economy with efficiency wedges. In this sense, while existing models of credit market frictions are not promising, new models in which financial frictions show up as efficiency and labor wedges are.

It would be interesting to apply our methods to other countries, including Uruguay, to see if our substantive findings hold up in other cases.

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## ANEXO



**TABLE 1**  
**PARAMETERS OF VECTOR AR(1) STOCHASTIC PROCESS IN TWO HISTORICAL EPISODES<sup>a</sup>**  
 Estimated Using Maximum Likelihood with U.S. Data on Output, Labor, Investment, and Government Consumption

<b>Annual Data, 1901 – 40</b>			<b>Coefficient matrix on shocks (Q)</b>	
Coefficient matrix on lagged states (P)				
.804	-.150		.0516	0
(.544, .1.05)	(-.0304, .145)	(-.561, .502)	(.0381, .0622)	0
-.0924	1.05	.538	-.0154	.0459
(-.362, .221)	(.870, 1.10)	(-.0466, .96 0)	(-.0309, .00544)	(.0231, .05468)
-.0262	-.0304	.170	-.00725	-.00471
(-.468, .205)	(-.226, .152)	(-.310, .390)	(-.0394, .0167)	(-.0193, .0123)
0	0	.747	0	0
		(.413, .805)		.221
				(.145, .277)
Means of states = [.544 (.506, .595), -.186 (-.262, -.080), .278 (.216, .355), -2.78 (-2.94, -2.53)]				
<b>Quarterly Data, 1959:1 – 2004:3<sup>b</sup></b>			<b>Coefficient matrix on shocks (Q)</b>	
Coefficient matrix on lagged states (P)				
.764	.0455	.434	-.0486	0
(.0315, .0451)	(.397, .411)	(-.0882, -.0433)	(.00874, .0128)	0
-.0236	.995	.0397	-.00245	.00554
(-.0392, -.0141)	(.949, .995)	(.028, .0475)	(-.00487, -.00245)	(.00108, .00443)
-.0958	.0240	1.17	0.00457	.000607
(-.0899, -.0670)	(.0212, .0366)	(1.12, 1.15)	(.00219, .00419)	(-.000598, .00161)
-.0254	.0398	0.000948	-.00418	.0136
(-.0384, -.00233)	(.0439, .0699)	(-.00333, .0185)	(-.00870, -.00301)	(.00472, .0132)
		(.973, .991)		.697e-7
				(7.39e-9, 1.87e-5)
Means of basic data: See Chari, Kehoe, and McGrattan (2005). Means of states = [-.0375 (-.0472, -.0223), .304 (.294, .316), .356 (.326, .359), -1.570 (-1.60, -1.56)]				

<sup>a</sup> To ensure stationarity, we added a penalty term to the likelihood function proportional to  $\max(|\lambda_{\max}| - .995, 0)^2$ , where  $\lambda_{\max}$  is the maximal eigenvalue of  $P$ . Numbers in parentheses are 90 percent confidence intervals for a bootstrapped distribution with 500 replications.

<sup>b</sup> The (1, 1) element of  $P$  is set residually after imposing the condition that one eigenvalue is equal to 0.995. This was done to achieve better performance in hill climbing when computing confidence intervals.