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Pierre M. Picard, Takatoshi Tabuchi

ABSTRACT: This paper considers the spatial structure of a city subject to final demand and vertical linkages. Individuals consume differentiated goods (or services) and firms purchase differentiated inputs (or services) in product (or service) markets where forms compete under monopolistic competition. Workers rent their residential lots in an urban land market and contribute to the production of differentiated goods and inputs. We show that firms and workers co-agglomerate and endogenously form a city. We characterize and discuss the spatial distribution of firms and consumers in such cities on one- and two-dimensional spaces (linear city and planar city). We show that final demand and vertical linkages raise the urban density and reduce the city spread. We finally show that a city is too much dispersed compared to the social optimum.

JEL Codes: C62, F12, R12

Keywords: Co-agglomeration, continuous distribution, new economic geography, forward and backward linkages, housing space.

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1 Introduction

Many international economists and economic geographers highlight the importance of market externalities in the location of economic activities between and within countries. They explain how the endogenous agglomeration of economic activities in specific locales can be the result of specific linkages. While forward linkages (or supply linkages) entice consumers to locate closer to producers in order to benefit from a larger diversity of less expensive products, backward linkages (or demand linkages) entice the producers to locate closer to their final consumers or client firms in order to save on transport costs (Krugman 1991, Krugman and Venables 1995). Such a literature contrasts with the urban economic literature that traditionally discusses the endogenous formation of cities in the light of non-market externalities. In particular, cities are seen as business information centers or social interaction centers that build on the agglomeration forces stemming from human interactions, face-to-face communications or exogenous technological spillovers (Beckmann, 1976; O'Hara, 1977; Ogawa and Fujita, 1980, Fujita and Ogawa, 1982; Lucas and Rossi-Hansberg, 2001; Mossay and Picard, 2009). In this context, agents have incentives to locate close to each others because proximity to other agents increases the efficiency of communications and interactions.

Non-market face-to-face interaction is certainly an important factor of the formation of many central business districts, but it is probably not the sole driver of the economic agglomeration process within most city centers. For instance, Tabuchi and Fujita (1997) study the face-to-face interactions that are likely to take place within headquarters and central functions of Japanese firms. However, by the time of their study, most of those activities were concentrated in Tokyo and Osaka. As a result, face-to-face interactions between headquarters are most probably not the main agglomeration driver in all other Japanese cites that counted each more than a million residents. By contrast, there is evidence of vertical linkages (or input-output linkages) in cities. Indeed, most cities consume a significant share of their own production. For example, in 2000, a prefecture like Tokyo sold some 33% and 30% of its production to its own residents and firms whereas it sold only 7% and 29% to residents and firms outside the prefecture. In line with this idea, we observe that large cities generate a significant share of the market potential for their own firms. Mathae and Shwachman (2009) compute the market potentials of EU regions (NUTS 2) as the transportation-cost-weighted market capacities of each region and break down those market potentials according to the share generated by the region itself, its neighboring regions, and the rest of EU regions. Those authors show that the economic activities of regions hosting a large city significantly contributes their own market potential. Drawing on this study, the economic activities of the regions of Inner London and Brussels-Capital contribute up to 78% and 75% of the own market potential that their firms can access.[1] Knowing that the metropolitan areas of London and Brussels are geographically larger than the regions of Inner London and Brussels-Captial, one can safely conclude that London and Brussels significantly contributes their own market potentials. The same conclusion is likely to apply for the other major EU cities because they are most often the unique significant metropolitan areas of their regions.

This paper investigates the spatial structure of a city subject to backward and vertical linkages. We consider endogenous urban location of individuals and firms who consume the set of differentiated goods or services that they produce. We present a model where residents have hyperbolic preferences for residential space and quadratic preferences for differentiated varieties of goods or services and where residents work in the firms that produce and sell those varieties under monopolistic competition. Firms hire their workforce around their production sites while they ship and sell products or offer their services directly to consumers. This set-up allows us to study how the urban structure is shaped by forward and backward linkages: firms move to regions where intermediate goods or

[1] We gratefully thank Dr Mattae and the Central Bank of Luxembourg for the supply of disagregated data.

production factors are supplied with low prices and firms are attracted by regions where their intermediate or final goods is highly demanded.

Due to the linkages, firms prefer to locate and hire labor in locales closer to consumers and other firms that also purchase their intermediate output but compete with them. As a result, firms and workers tend to co-agglomerate and form a city center endogenously. We characterize and discuss the shape of the residential distribution in such cities on a one- and two-dimension geographical spaces (linear and planar cities). We show that residential distribution is continuous, symmetric and single-peaked. To our knowledge, this is the first formal model of urban spatial structure based on a microeconomic foundation of forward and backward linkages in the framework of new economic geography. We furthermore disentangle the impact of supply linkages from final demand linkages and show that both effects foster the agglomeration and raise land rents within cities.

The paper relates to the literature as follows. Since the pioneering work of von Thünen (1826), the study of urban structure has often focused on the assumption of central business districts, where firms locate on a spaceless point as in Alonso's (1964) residential location theory and on a set of such points as in Fujita and Krugman's (1995) new economic geography. In those models, the spatial distribution inside a city center(s) remains an unsolved issue. On the other hand, the formation of city and the distribution of urban activities within a city have been the focus of a small set of contributions initiated by Beckmann (1976) and followed by Ogawa and Fujita (1980), Tauchen and Witte (1984), Tabuchi (1986) and Mossay and Picard (2009). These studies have paid attention to the impact of social interactions and technological externalities on urban structure mainly through face-to-face communications that enhance the productivity of pairs of firms or workers. Whereas the production mechanisms of face-to-face communications modeled in those studies have produced convenient analytical properties, they remain black boxes because they do not take into account the actual nature of economic interactions within a city. In a similar vein, Lucas and Rossi-Hansberg (2002) model the positive pecuniary externalities between the firms as exogenous spillovers that decay with distance. To our knowledge, none of those contributions have opened the black box of the externalities accruing between the firms. By contrast, this paper presents some microeconomic foundation of the pecuniary externalities that generate backward and forward linkages developed by Krugman (1991), Ottaviano et al. (2002) and Krugman and Venables (1995). In such microeconomic foundations, the prices of goods and services are determined in general equilibrium under monopolistically competitive markets.

The new economic geography literature offers only a small set of theoretical studies on firms' spatial distribution over a continuous space. The scarcity of such a research mainly results from the analytical difficulties involved in the study of location incentives on a spatial continuum. In particular, Fujita et al. (1999) and Mossay (2003) have reduced the study to uniform distributions of firms (flat earth) while Picard and Tabuchi (2010) have enlarged it to a larger class of spatial distributions. Because of the absence of preference for residential space, those authors find that continuous equilibrium distributions are most often intrinsically unstable. This suggests that economic activities must agglomerate in spaceless points, which are then called cities. By contrast, this paper introduces preferences for residential space and shows existence of a unique equilibrium distribution of firms and workers, that is continuous, symmetric and single-peaked.

The remainder of the paper is organized as follows. Section 2 presents the urban model with final demand linkages. Section 3 characterizes spatial equilibrium of the model, while Sections 4 and 5 apply this discussion to the one- and two-dimension space set-ups. Section 6 extends the previous set-up to vertical linkages and to exporting cities. Section 7 compares the spatial equilibrium to the socially optimal configuration of a city. Section 8 concludes. Proofs are relegated to the Appendix.

2 The Model

We assume two sets of perfectly mobile individuals and differentiated product varieties whose residence and production are distributed on a geographical compact space $\mathcal{X} \subseteq \Re^X$, $X = 1, 2$. On the one hand, the mass of individuals residing at location x is defined by the density function $\lambda(x)$, where $x \in \mathcal{X}$ is the individual's coordinate. Without loss of generality, we assume that the total mass of individuals is unity: $\int_{\mathcal{X}} \lambda(x)\mathrm{d}x = 1$. On the other hand, each variety is produced at a single location with coordinate $y \in \mathcal{X}$ while the mass of varieties produced at location y is defined by the distribution function $\mu(y)$. The total mass of varieties is given by $M = \int_{\mathcal{X}} \mu(y)\mathrm{d}y$. Product varieties can be interpreted as services. In the case of a two-dimensional geographical space $\mathcal{X}$, x and y are the vectors of coordinates (x_1, x_2) and (y_1, y_2) whereas $\lambda(x)\mathrm{d}x$ and $\mu(y)\mathrm{d}y$ are equivalent to $\lambda(x_1, x_2)\mathrm{d}x_1\mathrm{d}x_2$ and $\mu(y_1, y_2)\mathrm{d}y_1\mathrm{d}y_2$.

2.1 Consumers' preferences and demands

Individuals consume the differentiated product varieties, residential space and a numéraire good. The preferences of an individual located at location x are given by the following utility function:

$$U\left[q_0, q\left(\cdot, x\right), s(x)\right] = C\left[q\left(\cdot, x\right)\right] - \frac{\theta}{2}\frac{1}{s(x)} + q_0$$

where q_0 is the consumption for numéraire, $s(x)$ is the consumption for space, $q\left(\cdot, x\right)$ is the consumption profile for differentiated product varieties that are offered at location x, and $C\left[q\left(\cdot, x\right)\right]$ is a composite good function aggregating those product varieties. The parameter θ reflects the preference for residential space, a larger θ implying a stronger preference for space. The preference for space reflects a decreasing marginal utility from use of residential space.[2] The preference for residential space acts as a dispersion force in

[2]The present hyperbolic preference for space and the Beckmann's (1976) logarithmic preference for space are two instances of the same class of preferences $(s^{1-\rho} - 1)/(1-\rho)$ where $\rho = 2$ and $\rho \to 1$

this model.[3]

As in Ottaviano et al. (2002) the composite good is expressed as

$$C\left[q\left(\cdot,x\right)\right]=\alpha\int_{\mathcal{X}}q(y,x)\mu(y)\mathrm{d}y\ -\frac{\beta}{2}\int_{\mathcal{X}}\left[q(y,x)\right]^{2}\mu(y)\mathrm{d}y-\frac{\gamma}{2}\left[\int_{\mathcal{X}}q(y,x)\mu(y)\mathrm{d}y\right]^{2}$$

This function is made of individual consumption $q\left(y,x\right)$ of a variety produced at location y and consumed at location x. In this expression, $\alpha>0$, $\beta>0$ and $\gamma>0$ are parameters reflecting the preference for the goods or services. Ceteris paribus, a higher α implies a more intense preference towards the varieties compared to the numéraire, a higher β means a more bias toward a dispersed consumption of varieties (i.e. the love for variety), and a higher γ implies a higher degree of substitutability between varieties.

The budget constraint of an individual located at x is equal to

$$\int_{\mathcal{X}}p(y,x)q(y,x)\mu(y)\mathrm{d}y+R(x)s(x)+q_{0}\leq w(x)+\overline{q}_{0} \tag{1}$$

where $p(y,x)$ is the price of a variety produced at location y sold at location x, $R(x)$ is the land rent, $w(x)$ is the individual's income when she resides at location x, and $\overline{q}_0$ is her initial endowment of numéraire. We assume that the endowment $\overline{q}_0$ is large enough so that consumers have positive demands for the numéraire in equilibrium. We also assume that product varieties are exchanged for any configurations of firms and consumers. We now present the production side of the economy.

respectively, which yield iso-elastic demands for residential space with price elasticity respectively equal to 1/2 and 1. So, the present hyperbolic preference represents an intermediate setting between Beckmann's demand and the inelastic demand for residential space that is regularly used in standard urban economics (e.g. Fujita and Ogawa 1980). As will be shown below, the hyperbolic sub-utility for space yields more convenient analytical properties.

[3] In new economic geography, the dispersion force is usually obtained by the assumption of productive land that creates a localized demand for workers in the farming sector. This analytically convenient assumption is often criticized on the ground that farming products must be undifferentiated and transported at zero cost (see e.g. Fujita et al. 1999, Picard and Zeng 2005).

2.2 Production

Each firm produces a single variety of goods or services, sets the prices of its goods or services and delivers them to the consumer locations incurring a transport cost. Each firm faces a monopolistic competition in the following sense. First, it faces price competition with other firms that sell imperfect substitutes. Second, because the mass of each firm is negligible in the whole market, it cannot determine its price strategically. Finally, there exist many potential firms that enter until profits fall to zero.

As in most modern cities, each firm uses some intermediate inputs that are produced in the city. Like Krugman and Venables (1995), we make things simple by assuming that firms use the same set of differentiated goods or services as those purchased by final consumers and that their benefits from using those goods or services take the same form as the consumers' preferences. To be more precise, we now assume that production requires no variable inputs (without loss of generality as in Ottaviano et al. 2002) but that it requires three different fixed inputs: labor, physical capital equipment and intermediate goods or services. We assume that, a firm producing at location y must hire an inelastic unit of labor and pay a wage $w(y)$. We assume that a too high commuting cost to entice workers to commute. Hence, each worker resides at the same place as its firm.[4] As a result, when the labor market clears, the mass of varieties is equal to the mass of workers: $M = 1$ and $\mu(y) = \lambda(y)$.

In addition, to build up and use its production equipment, the firm must acquire the K units of physical capital which costs K units of numéraire. Alternatively, the firm can buy $q^i(\cdot, y)$ units of intermediate goods at a price $p(\cdot, y)$ to reduce its cost of physical capital or operation. Physical capital and intermediate goods are therefore input substitutes. One interpretation is that a part of the physical capital can be replicated by

[4]The study of commuting patterns is out of the scope of the current paper. See Ogawa and Fujita (1980) and Fujita and Ogawa (1982), among others, for spatial structure with commuting.

a set of intermediate inputs at a lower cost. More specifically, the use of a set of $q^i(\cdot, y)$ intermediate inputs reduces the requirement for physical capital to $K - C[q^i(\cdot, y)]$ units of numéraire where $C[q^i(\cdot, y)]$ takes the same form as the composite good in the consumers' preferences. Such an assumption replicates Krugman and Venables' (1995) assumption on firms' production functions to Ottaviano et al. 's (2002) quadratic utility model. Note that firms use no space.

Given this set-up, the firm located at y maximizes a profit $\Pi(y) = e(y) - f(y)$ that embeds the operating profit

$$e(y) \equiv \int_{\mathcal{X}} [p(y,x) - \tau(y,x)] \left[q^c(y,x)\lambda(x) + q^i(y,x)\mu(x)\right] \mathrm{d}x \tag{2}$$

and the fixed cost

$$f(y) \equiv K - C[q^i(\cdot, y)] + \int_{\mathcal{X}} p(z,y)q^i(z,y)\lambda(z)\mathrm{d}z + w(y) \tag{3}$$

where $q^c(y,x)$ is the demand of a good produced at location y by consumers at location x and $\tau(y,x)$ is the unit transport cost from locations y to x. The firm thus makes two choices: one about its prices $p(y, \cdot)$ and one about its own demand $q^i(\cdot, y)$ of intermediate inputs to other firms. Because the former decision affects operating profits and the latter fixed costs, the two decisions can be disentangled into operating profit maximization and cost minimization. We first analyze the latter decision.

We first discuss the short-run equilibrium where land, product and labor markets clear. We then discuss the long run spatial equilibrium where firms and workers relocate within the city.

3 Short run equilibrium

In this section we consider the individuals' demands for products or services and for residential space, the firms' demands for intermediate inputs and finally the market prices and profits. We finally determine the individual's location incentives within the city.

3.1 Product and land demands

Following the tradition of urban economic models à la Alonso (1964), assume that land is owned by absentee landlords who do not participate in production and consumption. The behavior of landlords dictates the consumption behaviors of residents. Indeed, because landlords extract the maximum land rent, the bid rent at location x is defined as

$$\Psi(x) = \max_{s(x), q(\cdot,x), q_o(x)} \frac{w(x) + \overline{q}_0 - q_0 - \int_{\mathcal{X}} p(y,x) q(y,x) \mu(y) \mathrm{d}y}{s(x)}$$

from (1).Since no resident wishes to relocate in the city in equilibrium, the bid rent is subject to the constraint that the resident's utility $U(q_0, q(\cdot,x), x)$ is no less than the equilibrium utility U^*. The bid rent is in fact the ratio between the net expenses on residential space (numerator) and the residential space consumption (denominator). Maximization of the bid rent $\Psi(x)$ with respect to q_0 is to solve $U^* = U(q_0, q(\cdot,x), x)$ for q_0 and plug this solution into $\Psi(x)$, which yields

$$\Psi(x) = \max_{s(x), q(\cdot,x)} \frac{w(x) + \overline{q}_0 - \frac{\theta}{2}\frac{1}{s(x)} - U^* + C[q(\cdot,x)] - \int_{\mathcal{X}} p(y,x) q(y,x) \mu(y) \mathrm{d}y}{s(x)}$$

Maximizing $\Psi(x)$ with respect to $q(\cdot,x)$yields the individual consumption of residential space and varieties of goods and services as follows. We first solve for $q(\cdot,x)$ to obtain the individual's demand:

$$q^c(y,x) = \frac{\alpha}{\beta+\gamma} - \frac{1}{\beta} p(y,x) + \frac{\gamma}{\beta(\beta+\gamma)} P(x)$$

where the superscript c stands for final consumers and $P(x) = \int_{\mathcal{X}} p(y,x)\mu(y)\mathrm{d}y$ is the price index for consumers at location x (see Picard and Tabuchi, 2010). The consumer surplus that an individual located at x obtains from consuming the differentiated goods is given by

$$\begin{aligned} S[p(\cdot,x)] = {} & \frac{\alpha^2}{2(\beta+\gamma)} - \frac{\alpha}{\beta+\gamma} \int_{\mathcal{X}} p(y,x)\mu(y)\mathrm{d}y \\ & - \frac{\gamma}{2\beta(\beta+\gamma)} \left[\int_{\mathcal{X}} p(y,x)\mu(y)\mathrm{d}y\right]^2 + \frac{1}{2\beta}\int_{\mathcal{X}} [p(y,x)]^2 \mu(y)\mathrm{d}y \end{aligned}$$

which depends on the price profile $p(\cdot, x)$. Plugging those equilibrium values in the utility function yields

$$V(x) = w(x) + S[p(\cdot, x)] - \theta \frac{1}{s(x)} + \overline{q}_0 \tag{4}$$

and the bid rent

$$\Psi(x) = \max_{s(x)} \frac{w(x) + \overline{q}_0 - \frac{\theta}{2}\frac{1}{s(x)} - U^* + S[p(\cdot, x)]}{s(x)}$$

Then, we solve for $s(x)$ to get the demand for residential space: $s(x) = \theta/[w(x) + \overline{q}_0 - U^* + S[p(\cdot, x)]]$. Finally, the equilibrium rent is equal to the bid rent: $R(x) = \Psi(x)$. Hence, the land rent is determined as

$$R(x) = \frac{\theta}{2}\frac{1}{s(x)^2} \tag{5}$$

We now turn to the firms' demand for intermediate inputs and to their prices decisions.

3.2 Intermediate input demands and equilibrium prices

The firm's cost minimization has the same form as the consumer's utility maximization. It therefore yields a firm's consumption of

$$q^i(y, x) = \frac{\alpha}{\beta + \gamma} - \frac{1}{\beta}p(y, x) + \frac{\gamma}{\beta(\beta + \gamma)}P(x)$$

units of intermediate inputs, which is the same as the consumer' consumption $q^c(z, y)$. The minimized fixed cost (3) is then given by

$$K - S[p(\cdot, y)] + w(y) \tag{6}$$

That is, the minimized fixed cost is equal to the cost of physical capital K minus the cost savings $S[p(\cdot, y)]$ plus the wages to workers $w(y)$.

We can now establish the prices that firms set for their consumers and client firms. The demand addressed to each firm located at y is equal to $q^t(y, x) \equiv q^c(y, x) + q^i(y, x) =$

$2q^c(y,x)$. The firm located at y finds the price profile $p(y,\cdot)$ that maximizes it operating profit $e(y)$ in (2). The first-order condition for profit maximization yields the optimal price of variety produced at location y and sold to a consumer residing at location x:

$$p^*(y,x) = \overline{p}(x) + \frac{1}{2}\tau(y,x) \quad \text{where} \quad \overline{p}(x) = \frac{2\alpha\beta + \gamma\int_{\mathcal{X}}\tau(y,x)\lambda(y)\mathrm{d}y}{2(2\beta+\gamma)} \tag{7}$$

At the equilibrium the demand is equal to $q^{t*}(y,x) = 2[p^*(y,x) - \tau(y,x)]/\beta$ so that the operating profit is given by

$$e^*(y) = \frac{2}{\beta}\int_{\mathcal{X}}[p^*(y,x) - \tau(y,x)]^2\lambda(x)\,\mathrm{d}x$$

In the long run, entry occurs until firms earn zero profit. This means that $\Pi^*(y) = 0$ in any location y where a firm can set up its production activity. Using (6),the wage paid by a firm at location y should then be equal to

$$w^*(y) = e^*(y) - K + S[p^*(\cdot,y)] \tag{8}$$

The presence of vertical linkages impacts on the worker's wage in two ways. First, it increases the operating profit$e^*(y)$ because her production is sold not only to consumers but also to other firms. Second, it increases her firm's profit through the capital savings that are induced by the use of intermediate inputs. Those capital savings take the same form as her own surplus from consumption.

As mentioned above, we impose that product varieties can be exchanged for any pair of locations and for any distribution of firms and consumers. This means that the firms' prices net of transport costs on a variety produced in x and sold in y are always positive, i.e.,

$$p^*(y,x) - \tau(y,x) > 0 \qquad \forall y,x$$

Let $\mathcal{B}\subset\mathcal{X}$ be the support of the city and let the maximal distance between any two points of $\mathcal{B}$ be $2b \equiv \max_{x,y\in\mathcal{B}} T(x-y)$. Then, the possibility of exchanging varieties from any location requires that

$$\tau\ (2b)^2 < \frac{2\alpha\beta}{2\beta+\gamma} \tag{9}$$

Note that the feasible exchange condition (9) involves endogenous variable b that must be replaced by the equilibrium value. This is determined in later sections.

3.3 Location incentives

In this model, workers reside at their firm's location. Because of fixed labor requirements, firms and workers have the same spatial distribution. As a result the spatial distribution of firms is driven by the location of workers $\lambda(x)$. The workers' incentives to reside in some locations are given by their utility differentials. We here collect the above results to establish their utility level when product markets, land market and labor markets clear.

Because each unit of geographical space hosts $\lambda(x)$ firms that each hires a unit mass of individuals, each individual uses $s(x) = 1/\lambda(x)$ units of space. As a results, we can write the consumer-worker's indirect utility function (4) as

$$V(x) = w^*(x) + S\left[p^*(\cdot, x)\right] - \theta\lambda(x) + \overline{q}_0$$

which includes its surplus from consumption, its wage and the residential disutility that more dense locales imposes on him through higher land rents. Using the equilibrium wage (8), we finally get

$$V(x) = e^*(x) + 2S\left[p^*(\cdot, x)\right] - \theta\lambda(x) - K + \overline{q}_0 \tag{10}$$

The presence of vertical linkages increases the operating profits $e^*(y)$ because the production is sold to both consumers and firms and it doubles the consumer surplus $S\left[p^*(\cdot, x)\right]$ because intermediate inputs allow capital cost savings that are exactly equal to the consumer surplus (as in Krugman and Venables,1995).

We finally break down the transport cost from locations x to y as

$$\tau(x, y) \equiv \tau T(x - y)$$

where τ is the *amplitude* of transportation costs and T captures the *shape* of transport costs. Collecting the above results, the consumer-worker's indirect utility can be rewritten as the following function of $\lambda(\cdot)$ and x

$$V(x) = W_0 - W_1 f_1(x) + W_2 f_2(x) - W_3 f_3(x) - W_4 [f_1(x)]^2 - W_5 \lambda(x) \tag{11}$$

where f_1, f_2 and f_3 are three "accessibility measures" defined as

$$\begin{aligned} f_1(x) &\equiv \int_{\mathcal{X}} T(x-z)\,\lambda(z)\,\mathrm{d}z \\ f_2(x) &\equiv \int_{\mathcal{X}} [T(x-z)]^2\,\lambda(z)\,\mathrm{d}z \\ f_3(x) &\equiv \int_{\mathcal{X}\times\mathcal{X}} T(x-y)\,T(y-z)\,\lambda(y)\,\lambda(z)\,\mathrm{d}y\mathrm{d}z \end{aligned}$$

and where W_0 is a constant and

$$\begin{aligned} W_1 &= \frac{2\tau\alpha(3\beta+2\gamma)}{(2\beta+\gamma)^2} \qquad W_2 = \frac{3\tau^2}{4\beta} \qquad W_3 = \frac{\tau^2\gamma}{\beta(2\beta+\gamma)} \\ W_4 &= \frac{\tau^2\gamma^2}{4\beta(2\beta+\gamma)^2} \qquad W_5 = \theta \end{aligned}$$

All constants W_j's are all positive and 'generically' different from zero in the sense that $W_j > 0$ for any non-zero measure of parameters $(\alpha, \beta, \gamma, \tau, \theta)$ (see Picard and Tabuchi, 2007 and 2010). Note that expression (11) applies for any dimension of the geographical space, $\mathcal{X}$.

We are now equipped to analyze the long-run spatial equilibrium.

4 Spatial equilibrium

In a spatial equilibrium, workers have no incentives to relocate and therefore get the same utility level everywhere. Formally, a spatial equilibrium is such that $\lambda(x) > 0$ if $V(x) = \overline{V}$ and $\lambda(x) = 0$ if $V(x) < \overline{V}$. To our knowledge, the spatial equilibrium condition $V(x) = \overline{V}$ has no explicit solution for the spatial distribution $\lambda(x)$ for a general class of transport cost functions. Yet, one important class of spatial distribution is readily spotted for the following quadratic transport costs:

$$T(x-y) \equiv \|x-y\|^2 = \begin{cases} (x-y)^2 & \text{if} \quad X = 1 \\ (x_1-y_1)^2 + (x_2-y_2)^2 & \text{if} \quad X = 2 \end{cases}$$

where x denotes the coordinates of a consumer, y the coordinates of a firm, and $\|\cdot\|$ the Euclidean distance. Such transport cost functions are commonly used in Hotelling models and its various applications (see Anderson et al. 1992). Economides (1986) has discussed the analytical difficulties and absence of pure strategy equilibrium under non-quadratic transport cost functions. Quadratic transport cost functions imply that travel/shipping costs increase more than proportionally with distance. This is likely to be the case in a city where larger distance implies changes of modes of transportation and higher travel/ship cost. For instance, activities requiring close distance can be done by foot, those requiring longer distance needs to combine foot and metro or buses whereas much longer distance require additional change of metro or buses.

Under quadratic costs, the accessibility measures become

$$
\begin{aligned}
f_1(x) &= \int_{\mathcal{B}} \|x-z\|^2 \lambda(z)\,\mathrm{d}z \\
f_2(x) &= \int_{\mathcal{B}} \|x-z\|^4 \lambda(z)\,\mathrm{d}z \\
f_3(x) &= \int_{\mathcal{B}\times\mathcal{B}} \|x-y\|^2 \|y-z\|^2 \lambda(y)\,\lambda(z)\,\mathrm{d}y\mathrm{d}z
\end{aligned}
$$

which are polynomials of x of order 2, 4 and 2, respectively. Similarly, the expression $[f_1(x)]^2$ is a polynomial of x of order 4. As a result, because the spatial equilibrium imposes $V(x) = \overline{V}\ \forall x \in \mathcal{B}$, the spatial equilibrium distribution of workers $\lambda(x)$ must also be a polynomial of order 4.

Lemma 1 *Under quadratic transport costs, the equilibrium distribution of workers $\lambda(x)$ is a polynomial of order* 4.

In the case of a unidimensional geographical space ($X = 1$), the equilibrium distribution of workers is equal to

$$\lambda(x) = \sum_{k=0}^{4} a_k x^k \tag{12}$$

where $a_k \in \Re$. In the case of a bidimensional geographical space ($X = 2$), it is equal to

$$\lambda(x_1, x_2) = \sum_{k=0}^{4} \sum_{\ell=0}^{k} a_{k\ell} x_1^{\ell} x_2^{k-\ell} \tag{13}$$

where $a_{k\ell} \in \Re$.

As a corollary, the support $\mathcal{B}$ of the city must be a bounded and convex set. Indeed, the fact that $\lambda(x)$ is a polynomial implies that $\{x : \lambda(x) > 0\}$ is a convex open set and the same fact combined with $\int_{\mathcal{B}} \lambda(x)\mathrm{d}x = 1$ implies that $\{x : \lambda(x) > 0\}$ is a bounded set.

5 Linear city

We now study the case of a linear city that spreads on the space $\mathcal{X} = \Re$. We normalize the city width to unity so that $\lambda(x)$ measures the density of workers residing at location $x \in \Re$. In other words, any rectangular space $\mathrm{d}x \times 1$ of the city includes $\lambda(x)\mathrm{d}x$ workers. Let the support of the city be $\mathcal{B} = (-b, b)$ where $\pm b$ are the city borders, so that workers are distributed about the location $x = 0$. We assume that the (farming) opportunity cost of land is nil. So, the land rent is nil at the city border, which implies that $\lambda(b)$ is equal to 0.

The spatial equilibrium is defined by a spatial distribution function $\lambda^*(x)$, a city border b^* and a utility level $\overline{V}^*$ that solve the three equalities $V(x) = \overline{V}^*$, $\lambda^*(b^*) = 0$ and $\int_{-b}^{b} \lambda^*(x)\mathrm{d}x = 1$. The first equality implies that

$$V'(x) = 0, \quad V''(x) = 0, \quad V'''(x) = 0, \quad V''''(x) = 0 \quad \forall x \in \mathcal{B}$$

which can be applied at $x = 0$ to infer the coefficients (a_1, a_2, a_3, a_4) that define the polynomial $\lambda(x)$ in (12). The equilibrium is therefore obtained in the following way. First, it can be readily shown that the equalities $V'(x) = 0$ and $V'''(x) = 0$ imply that the coefficients a_1 and a_3 are both equal to zero, which confirms that the spatial distribution is symmetric about $x = 0$. Second, simultaneously solving $V''''(x) = 0$, $\lambda(b) = 0$ and

$\int_{-b}^{b} \lambda(x)\mathrm{d}x = 1$ we can get the workers' spatial distribution:

$$\lambda^*(x) = \phi \left[\left(x^2 - c_1\right)^2 - (c_2)^2 \right] \tag{14}$$

where

$$c_1 \equiv \frac{-a_2}{2a_4} = \frac{24b^5\phi + 15}{40b^3\phi} \quad \text{and} \quad c_2 \equiv \sqrt{\frac{a_2^2}{4a_4\phi} - \frac{a_0}{\phi}} = \frac{15 - 16b^5\phi}{40b^3\phi}$$

and

$$\phi \equiv \frac{\tau^2}{\theta} \frac{6\beta^2 + 6\beta\gamma + \gamma^2}{2\beta\left(2\beta + \gamma\right)^2} \tag{15}$$

Finally, plugging (14) into $V''(x) = 0$, we get $g\left(b\right) = 0$ where

$$\begin{aligned} g\left(b\right) \equiv\ & 1575\beta^2\left(2\beta + \gamma\right)^4\theta^2 - 4200\alpha\beta^2\left(2\beta + \gamma\right)^2\left(3\beta + 2\gamma\right)\theta\tau b^3 \\ & + 420\beta\left(2\beta + \gamma\right)^2\left(36\beta^2 + 32\beta\gamma + \gamma^2\right)\theta\tau^2 b^5 \\ & - 128\left(6\beta^2 + 6\beta\gamma + \gamma^2\right)\left(9\beta^2 + 7\beta\gamma + \gamma^2\right)\tau^4 b^{10} \end{aligned} \tag{16}$$

Then, the equilibrium city border b^* is determined by the solution of $g\left(b^*\right) = 0$. Although (16) is 10th-order polynomial, we show in Appendix 1(a) that it has a unique positive root.

We need to check the condition under which exchanges between any city locations are feasible at the equilibrium. Using $g\left(b\right) = 0$ and (9), we have

$$\tau < \frac{7\alpha^5\beta^3\left[\sqrt{7}(84\beta^2 + 108\beta\gamma + 35\gamma^2) + \sqrt{C_1}\right]^2}{88200\left(2\beta + \gamma\right)^2\theta^2} \tag{17}$$

where $C_1 \equiv 51120\beta^4 + 130080\beta^3\gamma + 124632\beta^2\gamma^2 + 53336\beta\gamma^3 + 8607\gamma^4$. Therefore, exchanges are likely to be feasible between any locations when transport costs are small and when consumers have intense preferences towards the varieties and weak preferences for residential space. We assume condition (17) in the sequel of this section.

We can now discuss the urban structure properties. On the one hand, note that, because $\lambda^*(b^*) = 0$, the spatial distribution $\lambda^*(x)$ cannot be increasing at the city border $x = b^*$. Then, because $\lambda^{*\prime}(x) = 4\phi x\left(x^2 - c_1\right)$,

$$\lambda^{*\prime}(b^*) \leq 0 \iff b^{*2} \leq c_1 \iff 16\phi b^5 \leq 15 \tag{18}$$

so that the city border b^* cannot exceed $\sqrt[5]{15/16\phi}$. On the other hand, it can readily checked that the spatial distribution $\lambda(x)$ describes a single-peaked function on the interval $[-\sqrt{c_1}, \sqrt{c_1}]$, which contains the support of the city $(-b^*, b^*)$. In addition, Appendix 2 shows that the spatial distribution is concave.

Based upon the foregoing, we establish the following.

Proposition 1 *The workers' distribution $\lambda^*(x)$ in a linear city is unique, single-peaked, concave and symmetric about$x = 0$.*

Intuitively, this means that *there can exist a city (with one bump centered on $x = 0$) that is necessarily contained in the interval* $[-b^*, b^*]$. This also means that *there exists no city with more than one bump if varieties of products or services are accessible from everywhere in the city.*

Next, we the equilibrium land rent (5) is rewritten as

$$R^*(x) = \frac{\theta}{2}\lambda^*(x)^2$$

Because the workers' distribution given by (14) is single-peaked and symmetric about $x = 0$ from Proposition 1, the land rent is also single-peaked and symmetric about the city center. Furthermore, we have $R^*(\pm b) = R^{*\prime}(\pm b) = 0$ and

$$\begin{aligned} R^{*\prime\prime}(0) &= -\frac{3\theta}{200b^4}\left(4\phi b^5 + 15\right)\left(8\phi b^5 + 5\right) < 0 \\ R^{*\prime\prime}(\pm b) &= \frac{\theta}{100b^4}\left(16\phi b^5 - 15\right)^2 > 0 \end{aligned}$$

implying that the land rent is concave near the city center and convex near the city edges. That is, the equilibrium land rent is bell-shaped, whereas the equilibrium workers' distribution is concave. The concave part of the rent function is also obtained in the completely integrated configuration in Ogawa and Fujita (1980, Figure 3). The concavity near the city center results from the fact that the access to all other firms and consumers are convex in x. The convex part of the rent function is consistent with that in standard textbooks of urban economics a la Alonso (1964) and is explained by the substitution between access and space for land.

We now turn to the comparative statics of the equilibrium urban structure.

Comparative statics The residential density at the city center is given by $\lambda(0) = a_0 = (4\phi b^5 + 15)/(20b)$. Differentiating it with respect to b, we get

$$\frac{\partial \lambda(0)}{\partial b} = \frac{16\phi b^5 - 15}{20b^2} < 0$$

where the inequality is due to (18). Hence, the city spread $2b$ is inversely related to the residential density at the city center.

Using the implicit function theorem and employing the result $\partial g/\partial b < 0$ obtained in Appendix 1(a), we can derive the following comparative statics for the city border b. It can be readily verified from expression (16) that $\partial g/\partial \alpha < 0$. It can also be verified that $\partial g/\partial \theta > 0$ given $g(b) = 0$. This can be computed by solving $g(b) = 0$ for parameter α and plugging it into $\partial g/\partial \theta$ as:

$$\left.\frac{\partial g}{\partial \theta}\right|_{g(b)=0} = 1575\beta^2 (2\beta + \gamma)^4 \theta + 128 \left(6\beta^2 + 6\beta\gamma + \gamma^2\right) \left(9\beta^2 + 7\beta\gamma + \gamma^2\right) \tau^4 b^{10}/\theta > 0$$

Likewise, we can show $\partial g/\partial \gamma > 0$. In sum, we get

$$\frac{\mathrm{d}b^*}{\mathrm{d}\alpha} = -\frac{\partial g/\partial \alpha}{\partial g/\partial b} < 0 \qquad \frac{\mathrm{d}b^*}{\mathrm{d}\gamma} = -\frac{\partial g/\partial \gamma}{\partial g/\partial b} > 0 \qquad \frac{\mathrm{d}b^*}{\mathrm{d}\theta} = -\frac{\partial g/\partial \theta}{\partial g/\partial b} > 0$$

That is, more intense preferences towards the varieties reduce the city spread and makes the city center more dense. Also stronger product substitution between varieties imply a larger city spread and a lower density near the city center. This is because individuals value more consuming of the whole set of varieties and entice all firms to locate closer to them. Finally, stronger preferences for residential space enlarge the city spread and decrease residential density at the center. This is a so-called suburbanization experienced in many big cities. The comparative statics with respect to the three parameters agree with the findings in new economic geography:

Proposition 2 *The population density near the city center falls and the city borders expand for less intense preference towards varieties (α smaller), stronger product substitutability (γ larger), and stronger preference for residential space (θ larger).*

6 Planar city

We now study the case of a city that spreads on the bidimensional space $\mathcal{X} = \Re^2$. Without loss of generality, we suppose that workers are distributed about the location $x = 0$ and we let $\lambda(x)$ measure the density of workers residing at location $x \in \Re^2$. We here show that the circular city is a spatial equilibrium. Towards this aim we replace the Cartesian coordinates (x_1, x_2) by the polar coordinates (r, φ) where r is the distance of point x to the origin $(0, 0)$ and where φ is the respective polar angle with the horizontal axis Ox_1. An infinitesimal unit of space $\mathrm{d}x_1\mathrm{d}x_2$ must be converted to $r\mathrm{d}\varphi\mathrm{d}r$ under polar coordinates. In a circular city, the individuals' density function is expressed as $\lambda(r, \varphi) \equiv \lambda(r)$. The support of a circular city is $\mathcal{B} = [0, b] \times [-\pi, \pi]$ where $2b$ is equal to the city diameter.

The first accessibility measure can successively be written as

$$\begin{aligned}
f_1(r, \varphi) &= \int_{\mathcal{B}} \left[(s\cos\phi - r\cos\varphi)^2 + (s\sin\phi - r\sin\varphi)^2\right] \lambda(s)\ s\mathrm{d}\phi\mathrm{d}s \\
&= \int_{\mathcal{B}} \left[s + r - 2sr\cos(\varphi - \phi)\right] \lambda(s)\ s\mathrm{d}\phi\mathrm{d}s \\
&= 2\pi \int_0^b \left(r^2 + s^2\right) \lambda(s)\ s\mathrm{d}s
\end{aligned}$$

where $(s, \phi) \in \mathcal{B}$ are polar coordinates for the integration variables. By the same argument, the second and third accessibility measures can be computed as

$$\begin{aligned}
f_2(r, \varphi) &= 2\pi \int_0^b \left(r^4 + 4r^2s^2 + s^4\right) \lambda(s)\ s\mathrm{d}s \\
f_3(r, \varphi) &= 4\pi^2 \int_0^b \int_0^b \left[\left(r^2 + s^2\right)\left(r^2 + t^2\right)\right] \lambda(s)\lambda(t)\ st\mathrm{d}s\mathrm{d}t
\end{aligned}$$

s and $t \in [0, b]$ are distance-to-origin coordinates of the integration variables. Obviously, those accessibility measures depend only on r. Because the spatial distribution $\lambda(r)$ of the circular city also depend only on r, the indirect utility $V(r,\varphi)$ depends only on r. The circular city is therefore consistent with a spatial equilibrium.

The urban structure of the circular city is derived in the same way as in the linear city. Note that, by Lemma1, spatial distributions are 4th-order polynomials of the Cartesian coordinates (x_1, x_2). In addition, to satisfy circular symmetry, this should be a polynomial function of $r^2 = x_1^2 + x_2^2$; that is, it should have the following form: $a_4(x_1^2 + x_2^2)^2 + a_2(x_1^2 + x_2^2) + a_0$. So, the class of spatial distributions that satisfy that is 4th-order polynomials and has circular symmetry is given by $\lambda(r) = a_4 r^4 + a_2 r^2 + a_0$. As a result, the equilibrium spatial distributions $\lambda(r)$ of a circular must also have a zero slope at its peak ($\lambda'(0) = 0$). This property results from the combination of circular symmetry and quadratic transport costs (which drives Lemma1).

We assume again that the (farming) opportunity cost of land is nil so that the land rent is nil at the city border: $\lambda(b) = 0$.

The spatial equilibrium is then defined by a spatial distribution function $\lambda^*(r)$, a city border b^* and a utility level $\overline{V}^*$ that solve the three equalities $V(r) = \overline{V}^*$, $\lambda^*(b^*) = 0$ and $\int_{-\pi}^{\pi}\int_{-b}^{b} \lambda^*(r) r \mathrm{d}\varphi \mathrm{d}r = 1$ where the last integral is equal to $2\pi \int_{-b}^{b} \lambda^*(r) r \mathrm{d}r$. As before, the first equality can be used to give the following necessary conditions

$$V'(r) = 0, \quad V''(r) = 0, \quad V'''(r) = 0, \quad V''''(r) = 0 \ \ \forall r \in [0, b]$$

which can be applied at $r = 0$ to find the coefficients $(a_{0,} a_2, a_4)$. The equilibrium is obtained in the same way as in the case of a linear city, except that the conditions $V'(r) = 0$ and $V'''(r) = 0$ are already satisfied because of the circular symmetry. On the one hand, simultaneously solving $V''''(x) = 0$, $\lambda(b^*) = 0$ and $2\pi \int_{-b}^{b} \lambda^*(r) r \mathrm{d}r = 1$, we get the workers' spatial distribution λ^* given by expression (14) where the parameters are now given by

$$a_0 = \frac{\pi\phi b^6 + 6}{3\pi b^2}, \quad a_2 = -\frac{4\pi\phi b^6 + 6}{3\pi b^4} \quad \text{and} \quad a_4 = \phi \tag{19}$$

and where ϕ is defined exactly as in (15). On the other hand, plugging (19) into $V''(x) = 0$, the city border is given by the equality $g(b) = 0$ where

$$
\begin{aligned}
g(b) = {} & 1152\beta^2(2\beta+\gamma)^4\theta^2 - 1152\pi\alpha\beta^2(2\beta+\gamma)^2(3\beta+2\gamma)\theta\tau b^4 \\
& + 96\pi\beta(2\beta+\gamma)^2\left(48\beta^2+40\gamma\beta+5\gamma^2\right)\theta\tau^2 b^6 \\
& - 4\pi^2\left(6\beta^2+6\gamma\beta+\gamma^2\right)\left(24\beta^2+16\gamma\beta+\gamma^2\right)\tau^4 b^{12}
\end{aligned}
\tag{20}
$$

which is a polynomial equation of order 12. This solution is very similar to the one obtained for the linear city. The existence and uniqueness of b are given by the same argument and shown in Appendix 1(b). So, we can make a proposition similar to Proposition 1.

Proposition 3 *There exists a two-dimension equilibrium distribution $\lambda^*(x)$ with circular symmetry and with a single peak with zero slope.*

It can be verified that the comparative static properties are the same as in the case of a linear city. Because the urban structures of linear and planar cities have very similar properties, the present analysis can be seen as a validation for the generality of studies that focuses on linear cities.

Up to now, we have examined the impacts of final demand linkages on the urban structure. We now extend the model to vertical linkages in order to fully consider forward and backward linkages.

7 Final demand versus vertical linkages

The previous model combines final demand and vertical linkages. Final demand linkages result from the fact that each firm prefer to save transport costs by locating closer to its consumers, whereas vertical linkages result from the fact that each firm prefers to locate closer the firms that purchase its good as intermediate input. In this section, we disentangle the two effects and discuss their roles on the formation of urban structure.

Towards this aim, we alter the preference for the differentiated products or services and assume that the composite good function is now given by

$$C_m\left[q\left(\cdot,x\right)\right]=\alpha\int_{\mathcal{X}}q(y,x)\mu(y)\mathrm{d}y\;-\frac{\beta}{2m}\int_{\mathcal{X}}\left[q(y,x)\right]^2\mu(y)\mathrm{d}y-\frac{\gamma}{2m}\left[\int_{\mathcal{X}}q(y,x)\mu(y)\mathrm{d}y\right]^2 \tag{21}$$

where m is a demand multiplier for the consumer demand. Indeed, one can readily show that the consumer demand is then equal to

$$q_m^c(y,x)=m\left[\frac{\alpha}{\beta+\gamma}-\frac{1}{\beta}p(y,x)+\frac{\gamma}{\beta\left(\beta+\gamma\right)}P(x)\right]=mq^c(y,x)$$

while the consumer surplus becomes $S_m\left[p\left(\cdot,x\right)\right]=mS\left[p\left(\cdot,x\right)\right]$. That is, the consumer demand and surplus are proportional to m.

Similarly, we assume that the capital requirement is given by $K-C_k\left[q\left(\cdot,x\right)\right]$ where we define $C_k\left[q\left(\cdot,x\right)\right]$ as in (21) and where k is the demand multiplier for the intermediate input demands. The demand for intermediate inputs is then equal to $q_k^i(y,x)=kq^i(y,x)$ and the capital saving becomes $S_k\left[p\left(\cdot,x\right)\right]=kS\left[p\left(\cdot,x\right)\right]$. The demand for intermediate inputs and the savings in capital expenditures rise in the same proportion as k.

Under the above assumptions, each firm faces a demand that is equal to $q^t(y,x)=q_m^c(y,x)+q_k^i(y,x)=\left(m+k\right)q^c(y,x)$. Since the parameters m and k have a multiplicative effect on demand, it has the same multiplicative effect on the operating profits. As a result, each firm sets its prices to $p^*\left(y,x\right)=\overline{p}\left(x\right)+\frac{1}{2}\tau\left(y,x\right)$ that are defined in expression (7) and are invariant to the demand multipliers m and k. At the equilibrium, the operating profit is equal to

$$\frac{m+k}{\beta}\int_{\mathcal{X}}\left[p^*\left(y,x\right)-\tau\left(y,x\right)\right]^2\lambda\left(x\right)\mathrm{d}x=\left(m+k\right)\frac{e^*(y)}{2}$$

The location incentives are given by the following indirect utility:

$$V(x)=\left(m+k\right)\left\{S\left[p^*\left(\cdot,x\right)\right]+\frac{e^*(x)}{2}\right\}-\theta\lambda(x)-K+\overline{q}_0$$

When $m=k=1$, this expression reduces to expression (10). Because location incentives are given by utility differentials, the constant $-K+\overline{q}_0$ has no impact on location. Dividing each term by $m + k$, one readily infer that a rise of $m + k$ has the same impact as a fall of θ. Since preferences for residential space generate a dispersion force, the rise of $m + k$ leads to less dispersion. We summarize this result in the following proposition.

Proposition 4 *Stronger demands for final products or services and/or for intermediate products or services leads to stronger final demand linkages and/or stronger vertical linkages, which both increase the density of firms near the city center and reduce the city spread.*

This analysis reflects the individual's trade-off between her demand for residential space, her consumption and her income. Both effects on consumption and production tend to increase firms' and workers' concentration in the vicinity of the city center. When m increases, individuals have larger demands for products or services. This entices firms to save transport costs by locating closer to consumers and to pull their workers closer to the city center. When k increases, firms are able to make larger savings in capital requirements and therefore have larger demand for intermediate products or services. Larger demands for intermediate inputs entice firms to locate closer to each other and save on transport costs. So, firms can raise their operating profits through larger capital savings and through a better proximity to their clients. Finally, because labor markets clear, firms' profits are shifted to workers whose higher incomes are used to pay the higher residential rents near the city center. Note that, under the present specification, final demand linkages and vertical linkages have the same effects on firms' locations.

We now study the socially optimal distribution of firms and workers. For the sake of simplicity, we set again $k = m = 1$.

8 Social optimum

In this section we study the optimal distribution of individuals and firms. We assume that a utilitarian planner allocates residential land to individuals and allows firms to set their product prices. The planner therefore sets the spatial distribution of residents $\lambda(\cdot)$ and the city support $\mathcal{B}^o$ that maximizes the sum of utilities

$$\Omega \equiv \int_{\mathcal{B}^o} V^o(x)\lambda(x)\,\mathrm{d}x$$

subject to the population constraint $\int_{\mathcal{B}^o} \lambda(x)\mathrm{d}x = 1$ where $V^o(x)$ is the utility of the individual residing at location x when firms and workers are distributed according to $\lambda(x)$ and when product prices are set by firms. For the sake of exposition we focus on the linear city that spreads on the support $\mathcal{B}^o \equiv (-b^o, b^o)$.

We have seen before that, for any given distribution of individuals and firms, consumers' product demands and firms' intermediate input demands are independent of the individuals' use of residential space.[5] Therefore, it can readily be shown that demand for products is unaffected by the fact that residential land is now allocated by a planner. Using the same product prices as before, the individual utility can be written as

$$V^o(x) \equiv 2S\left[p(\cdot,x)\right] + e(x) + \overline{q}_0 - K - \frac{\theta}{2}\frac{1}{s(x)}$$

which includes again the individual's consumer surplus $S\left[p(\cdot,x)\right]$, her income stemming for the operating profits $e(x)$ minus capital expenditure $K - S\left[p(\cdot,x)\right]$, and her preference for residential space $\theta/2s(x)$. Using the indirect utility with a land market $V(x)$ as given by (4), we get

$$V^o(x) = V(x) + \frac{\theta}{2}\frac{1}{s(x)} = V(x) + \frac{\theta}{2}\lambda(x) \tag{22}$$

[5]That is, product prices $p(x,y)$ do not depend on $s(x)$ or θ. Changes in the residential market are absorbed by the individuals' consumptions of the numéraire good and the firms' requirement of physical capital $K - S$.

The last term results for land market inefficiencies. Intuitively, for a same spatial distribution $\lambda(x)$, individuals raise their utility by $\frac{\theta}{2}\frac{1}{s(x)} = \frac{\theta}{2}\lambda(x)$ if they do not pay (to absentee landlords) for their use of residential space. The planner indeed receives the whole surplus from the use of residential space whereas a competitive land market allows individuals to receive only the difference between this surplus and the land rents. This effect is the greatest for individuals residing at the city center and nil for the individual residing at the city edge ($\lambda(0) > \lambda(b) = 0$). The planner is therefore willing to increase the residential density about the city center so that this effect represents an agglomeration force.

The planner's incentives to allocate residents can be understood as it follows. For the sake of accuracy, let us express the utility as $V(x, \lambda)$ where λ is the spatial distribution function. Let η be the Lagrange multiplier of the population constraint $\int_{\mathcal{X}} \lambda(x)\mathrm{d}x = 1$. Because of the unit mass population, this multiplier expresses the marginal increase in average utility that would be induced by a marginal immigration of individuals within the city. Then, pointwise maximization of the planners' objective Ω with respect to the density function $\lambda(z)$ yields the following first-order condition:

$$W(x) \equiv V^o(x, \lambda) - \eta + \int_{\mathcal{B}^o} \frac{\partial V^o(z, \lambda)}{\partial \lambda(x)} \lambda(z)\,\mathrm{d}z = 0$$

So, the relocation of (an infinitesimal mass of) individuals into location z has two direct and indirect effects. On the one hand, it increases total welfare if the location z yields a higher utility than in other locations; or more precisely, if individuals at location z have a higher utility than η, the marginal increase in average utility induced by immigration. On the other hand, the relocation of individuals into location z induces a relocation of firms that can improve the consumers' average access to products and services and thus that can increase the average utility. Under the Ottaviano et al.'s (2002) preferences, firms trade off between the access to their consumers and the intensity of competition with other firms. Whereas the former effect entices them to co-agglomerate, the latter effect entices them to disperse. Under such a relocation of firms, the access gains for some consumers many larger than the access losses to others. The planner can use lump sump transfers of numeraire to restore equity amongst individuals. In Appendix 3, the latter contribution can be computed from (11) as it follows:

$$\int_{\mathcal{B}^o} \frac{\partial V^o(z,\lambda)}{\partial \lambda(x)} \lambda(z)\,\mathrm{d}z = V^o(x,\lambda) - W_0 - 2W_4 f_3(x,\lambda) - (W_3 - W_4)\left[f_1(x,\lambda)\right]^2$$

where

$$W_3 - W_4 = \tau^2 \gamma \frac{8\beta + 3\gamma}{4\beta\,(2\beta+\gamma)^2} > 0$$

Note that the firms' relocation to location z increases the average utility in proportion to the local utility $V^o(z,\lambda)$ at z. The relationship between the increase of the average utility and local utility results from the symmetry of transport costs and the linearity in λ of the access measures. In other words, because transport cost are symmetric, a better access to products and services offered at location z by consumers located at x is equal to the access to products and services offered at x by consumers located at z. As a result, the firms' relocation to z implies an increase in the average access f_1 given by $\int_{\mathcal{B}^o} \frac{\partial f_1(x,\lambda)}{\partial \lambda(z)} \lambda(x)\,\mathrm{d}x$, which is equal to $\int_{\mathcal{B}^o} T(z-x)\lambda(x)\,\mathrm{d}x$, or equivalently, $f_1(z,\lambda)$. The same argument applies for f_2. The terms in the access measure f_1^2 and f_3 in the above expression stem from the nonlinearities in λ of f_1^2 and f_3.

Using (22), the first-order condition for optimum can be rewritten as

$$W(x) = 2V(x,\lambda) + \theta\lambda(x) - 2W_4 f_3(x,\lambda) - (W_3 - W_4)\left[f_1(x,\lambda)\right]^2 - \text{constant} = 0 \quad (23)$$

which expresses an impact of a marginal increase in the density function $\lambda(z)$ at location z on the planners' objective Ω.

We now characterize the optimal spatial distribution in more details. Similar to Lemma 1 for the equilibrium distribution, we can readily show that the optimal distribution $\lambda(x)$ is also a polynomial function of order 4. This is because (11) and (23) are similar in that both are linear combinations of $f_1(x)$, $f_2(x)$, $f_3(x)$, $[f_1(x)]^2$ and $\lambda(x)$. Furthermore, from Lemma 2 $\lambda^o(b) = 0$, where b^o is the optimal border, Solving the integral equation (23) using the same method, we obtain the optimal distribution given by

$$\lambda^o(x) = \phi^o \left[\left(x^2 - c_1^o\right)^2 - \left(c_2^o\right)^2 \right] \tag{24}$$

where

$$c_1^o \equiv \frac{24\left(b^o\right)^5 \phi^o + 15}{40\left(b^o\right)^3 \phi^o} \qquad \text{and} \qquad c_2^o \equiv \frac{15 - 16\left(b^o\right)^5 \phi^o}{40\left(b^o\right)^3 \phi^o}$$

and

$$\phi^o \equiv \frac{\tau^2}{\theta} \frac{24\beta^2 + 16\beta\gamma + \gamma^2}{8\beta\left(2\beta + \gamma\right)^2}$$

One can check that the optimal distribution $\lambda^o(x)$ is also single-peaked and concave. Because the optimal distribution $\lambda^o(x)$ given by (24) is qualitatively similar to the equilibrium one $\lambda^*(x)$ (14), the same comparative statics can be applied.

Next, the first-order condition (23) encompasses both inefficiencies in the land market and in the firms' location decisions. The main question is whether the spatial equilibrium implies too much dispersion or too much concentration. In order to answer to this question, plug the equilibrium distribution $\lambda^*(x)$ into (23):

$$\theta\lambda^*(x) - 2W_4 f_3\left(x, \lambda^*\right) - \left(W_3 - W_4\right)\left[f_1\left(x, \lambda^*\right)\right]^2 - \text{constant} \tag{25}$$

Because $\lambda^*(x)$ is symmetric about $x = 0$ and decreasing in $x \in [0, b^*]$, $f_1\left(x, \lambda^*\right)$ and $f_3\left(x, \lambda^*\right)$ can be shown to be symmetric about $x = 0$ and increasing in $x \in [0, b^*]$. Hence, (25), which is an impact of a marginal increase in $\lambda\left(x\right)$ at x on Ω, is a decreasing function of x. This implies that the planner has more incentives to increase the density $\lambda(x)$ at locations closer to the city center $x = 0$ than at the city edge $x = b^*$. Finally, because the population is constant, the expression (25)must be positive at $x = 0$ and negative at $x = b^*$ so that the planner increases the population density at the city center and decreases at the city edge. We have therefore proved the following proposition:

Proposition 5 *The city is too much dispersed at the equilibrium.*

Excessive dispersion results from two effects. First, the land market does not allow residents to extract the full surplus from their residential lots. Second, firms tends disperse too much in equilibrium in order to reduce competition. In addition, as in new economic geography, there exist the pecuniary externalities that are positive accruing from final demand linkages. Because firms do not fully take the externalities into account, they tend to be dispersed as compared to the social optimum. This finding coincides with the literature on face-to-face communications (Ogawa and Fujita, 1980; Tauchen and Witte, 1984; and Tabuchi, 1986).

9 Conclusion

To our knowledge this paper presents the first formal discussion of the endogenous urban structure of a city that is subject to endogenous backward and forward linkages. Workers consume the goods or services they produce while they must rent their residential lots in an urban land market. Firms produce under increasing returns to scale and sell their differentiated goods in markets where demand and supply balance. As in most modern cities, firms are subject to vertical linkages, which we model in the spirit of Krugman and Venables (1995). The production and market structures are exactly the same as those found in standard new economic geography models (Krugman, 1991; Ottaviano et al., 2002).

We showed that firms and workers co-agglomerate about a city center symmetrically with a single peak. We examined the shape of the residential distribution in such cities on a one- and two-dimension geographical space (linear city and planar city). We extended the model to disentangle the effect of final demand and vertical linkages and verified that those two effects are complement. Finally, we showed that the city is too dispersed from a social point of view.

The present model therefore confirms a series of properties that are known in the existing urban literature or expected from the new economic geography literature. The present paper also shows that the combination of specific transport costs and preferences for space offers a good level of analytical tractability. This parallels the tractability properties of address models of spatial competition (Anderson et al. 1992),where the transport cost is a quadratic function of distance, but where consumers are nevertheless immobile. As those properties permit to determine spatial equilibrium conditions in one- or two-dimensional spaces without the recourse to numerical exercises, they also offer some research perspectives for extensions in various directions such as first-best allocations, labor market specificities, system of cities and possibly endogenous commuting. In particular, multiple city centers are likely to arise when firms incur a too high shipping cost to distant places or when workers are able to commute to firms. At the present stage, those extensions are left for further research.

Appendix 1

(a) Linear city

We prove the uniqueness of the city border b. First, note that at the city border b, it must be that $\lambda'(b) \leq 0$. Indeed, if $\lambda'(b) > 0$, this implies that $\lambda(b - \varepsilon) < 0$ for any sufficiently small positive $\varepsilon \in [0, b]$, a contradiction. Hence, by (18), the city border b must be lower than $\sqrt[5]{15/16\phi}$. Second, we show that $\partial g(b^*)/\partial b < 0$ at $b = b^*$ and that $b^* < \sqrt[5]{15/16\phi}$. Indeed, differentiating $g(b)$ with respect to b and substituting the solution in α of $g(b) = 0$ the result yields

$$\frac{\partial g(b^*)}{\partial b} = -\frac{C_2}{b^*}\left(b^{*5} - \frac{15}{16\phi}\right)\left[b^{*5} - \frac{15}{16\phi}\left(1 + \frac{\gamma\,(4\beta+\gamma)}{2\left(9\beta^2+\gamma^2+7\beta\gamma\right)}\right)\right]$$

where C_2 is a positive constant. This expression is negative for all $b^* < \sqrt[5]{15/16\phi}$.

(b) Planar city

The uniqueness of the city border b in the planar city is similarly shown as in (a). We know that $\lambda'(b) \leq 0$. The counterpart of (18) is given by

$$\lambda'(b) < 0 \iff b < \sqrt[6]{\frac{3}{\pi\phi}} \tag{26}$$

We then show that $\mathrm{d}g(b)/\mathrm{d}b < 0$ whenever $g(b) = 0$ and $b < \sqrt[6]{3/\pi\phi}$. Plugging the solution in α of $g(b) = 0$ into $\mathrm{d}g(b)/\mathrm{d}b$ yields

$$\frac{\mathrm{d}g(b)}{\mathrm{d}b} = -C_3\left(b^6 - \frac{3}{\pi\phi}\right)\left[b^6 - \frac{3}{\pi\phi}\left(1 + \frac{8\beta\gamma + 3\gamma^2}{24\beta^2 + 16\beta\gamma + \gamma^2}\right)\right]$$

where C_3 is a positive constant. This expression in negative for all $b < \sqrt[6]{3/\pi\phi}$.

Appendix 2

It is convenient to substitute the variables (b, τ) by (b, r) where

$$r = 2\frac{2\beta + \gamma}{\alpha\beta}b^2\tau \tag{27}$$

By the feasible exchange condition (9) we have that $r < 1$. Therefore, we define $h(b, r) \equiv g(b)$ where we replace τ by (27). So,

$$\begin{aligned} h(b, r) = {} & 1575\beta^2(2\beta + \gamma)^4\theta^2 - \frac{105}{2}\left[12(10 - 3r)\beta^2 + 4(35 - 8r)\beta\gamma + 5(8 - r)\gamma^2\right]\alpha^2\beta^3\theta r\ b \\ & - \frac{2\left(6\beta^2 + 6\beta\gamma + \gamma^2\right)\left(9\beta^2 + 7\beta\gamma + \gamma^2\right)\alpha^4\beta^4r^4\ b^2}{(2\beta + \gamma)^4} \end{aligned}$$

which is positive ($h(0, r) > 0$) and strictly concave (quadratic) in b ($\partial^2h/\partial b^2 < 0$). For each given r, $h(b, r)$ therefore accepts one and only one positive root $b = b^+(r)$, where

$$b^+(r) = \frac{15(2\beta + \gamma)^4\left(C_4(r) + \sqrt{7C_5(r)}\right)}{8\left(6\beta^2 + 6\beta\gamma + \gamma^2\right)\left(9\beta^2 + 7\beta\gamma + \gamma^2\right)\alpha^2\beta r^3} > 0 \qquad r \in (0, 1) \tag{28}$$

and

$$\begin{aligned} C_4(r) &\equiv -84(10 - 3r)\beta^2 - 28(35 - 8r)\beta\gamma - 35(8 - r)\gamma^2 < 0 \\ C_5(r) &\equiv 720\left(140 - 84r + 15r^2\right)\beta^4 + 480\left(490 - 259r + 40r^2\right)\beta^3\gamma \\ & \quad + 8\left(25550 - 11410r + 1439r^2\right)\beta^2\gamma^2 \\ & \quad + 8\left(9800 - 3465r + 332r^2\right)\beta\gamma^3 + \left(11200 - 2800r + 207r^2\right)\gamma^4 \\ & > 0 \end{aligned}$$

It can be easily verified that $(C_4(r))^2 < 7C_5(r)$.

Substituting (**??**) and (28) into $\lambda'(b)$, we have

$$\lambda''(b)|_{\tau=\frac{\alpha\beta r}{2(2\beta+\gamma)b^2},\ b=b^+} = C_6\left(C_7 + 3\sqrt{7C_5}\right)$$

where $C_6 > 0$ and

$$C_7 \equiv -360\,(7-2r)\,\beta^2 - 28\,(105-23r)\,\beta\gamma - (840-101r)\,\gamma^2 < 0$$

It can be shown that $(C_7)^2 < 3^2 7C_5$, and hence, $\lambda''(b^+) < 0$ for all relevant parameter values.

We know that $\lambda(x)$ is a polynomial of order 4 and is symmetric about a single peak at $x = 0$ from (14). Furthermore, $\lambda'(b) < 0$ and $\lambda''(b) < 0$. Hence, it must be that $\lambda''(x) < 0$ for all $x \in (-b, b)$.

Appendix 3

To get the value of $\int_{\mathcal{X}} \frac{\partial V^o(z,\lambda)}{\partial\lambda(x)}\lambda(z)\,\mathrm{d}z$, where $V^o(z) = V(z) + \frac{\theta}{2}\lambda(z)$, we first compute

$$\begin{aligned}
\frac{\partial}{\partial\lambda(x)} f_1(z,\lambda) &= (x-z)^2 \\
\frac{\partial}{\partial\lambda(x)} f_2(z,\lambda) &= (x-z)^4 \\
\frac{\partial}{\partial\lambda(x)} f_3(z,\lambda) &= \frac{\partial}{\partial\lambda(x)}\left[\int_{\mathcal{X}\times\mathcal{X}} (z-y)^2\,(y-w)^2\,\lambda(y)\,\lambda(w)\,\mathrm{d}y\mathrm{d}w\right] \\
&= \int_{\mathcal{X}} (z-x)^2\,(x-w)^2\,\lambda(w)\,\mathrm{d}w + \int_{\mathcal{X}} (z-y)^2\,(y-x)^2\,\lambda(y)\,\mathrm{d}y
\end{aligned}$$

Then, by integrating on the support $\mathcal{X}$, we get

$$\begin{aligned}
\int_{\mathcal{X}} \frac{\partial}{\partial\lambda(x)} f_1(z,\lambda)\lambda(z)\,\mathrm{d}z &= \int_{\mathcal{X}} (x-z)^2\,\lambda(z)\,\mathrm{d}z = f_1(x,\lambda) \\
\int_{\mathcal{X}} \frac{\partial}{\partial\lambda(x)} f_2(z,\lambda)\lambda(z)\,\mathrm{d}z &= \int_{\mathcal{X}} (x-z)^4\,\lambda(z)\,\mathrm{d}z = f_2(x,\lambda) \\
\int_{\mathcal{X}} \frac{\partial}{\partial\lambda(x)} f_3(z,\lambda)\lambda(z)\,\mathrm{d}z &= \int_{\mathcal{X}\times\mathcal{X}} (z-x)^2\,(x-w)^2\,\lambda(w)\,\lambda(z)\,\mathrm{d}w\mathrm{d}z \\
&\quad + \int_{\mathcal{X}\times\mathcal{X}} (z-y)^2\,(y-x)^2\,\lambda(y)\,\lambda(z)\,\mathrm{d}y\mathrm{d}z \\
&= [f_1(x,\lambda)]^2 + f_3(x,\lambda)
\end{aligned}$$

and

$$\int_{\mathcal{X}} \frac{\partial}{\partial \lambda(x)} [f_1(z,\lambda)]^2 \lambda(z) \, \mathrm{d}z = 2 \int_{\mathcal{X}} f_1(z,\lambda) \frac{\partial}{\partial \lambda(x)} [f_1(z,\lambda)] \lambda(z) \, \mathrm{d}z$$
$$= 2 \int_{\mathcal{X}} f_1(z,\lambda) (x-z)^2 \lambda(z) \, \mathrm{d}z$$
$$= 2 f_3(x,\lambda)$$

Hence,

$$\int_{\mathcal{X}} \frac{\partial V^o(z,\lambda)}{\partial \lambda(x)} \lambda(z) \, \mathrm{d}z = -W_1 f_1(x) + W_2 f_2(x) - (W_3 + 2W_4) f_3(x,\lambda) - W_3 [f_1(x,\lambda)]^2 - \frac{W_5}{2} \lambda(x)$$
$$= V^o(x,\lambda) - W_0 - 2W_4 f_3(x,\lambda) - (W_3 - W_4) [f_1(x,\lambda)]^2$$

Furthermore, we have that

$$V^o(x,\lambda) - \eta + \int_{\mathcal{X}} \frac{\partial V^o(z,\lambda)}{\partial \lambda(x)} \lambda(z) \, \mathrm{d}z = 0$$

Therefore, the first-order condition is given by (23).

Appendix 4

Lemma 2 $\max_b \Omega$ *implies that* $\lambda^o(b^o) = 0$*, generically.*

Proof: Let assume that $\lambda(x)$ is the optimal distribution $\lambda^o(x)$ (we drop the superscript ^{o} for the sake of readability). Let us denote the indirect utility $V^o(x)$ and access measures $f_i(x)$, $i = 1,2,3$, by $V^o(x,b)$ and $f_i(x,b)$ to express their dependence with respect to the city border b. We know that $\lambda(x)$ is symmetric about $x = 0$: $\lambda(x) = \lambda(-x)$, $x \in \mathcal{B}_o$. As a result, all access measures $f_1(x,b)$, $f_2(x,b)$, $f_3(x,b)$ and $[f_1(x,b)]^2$ and therefore $V^o(x,b)$ are also symmetric about $x = 0$.

The first-order condition of the optimum b is given by

$$\frac{\partial \Omega}{\partial b} = 2V^o(b,b)\lambda(b) + \int_{-b}^{b} \frac{\partial}{\partial b} V^o(x,b) \lambda(x) \, \mathrm{d}x = 0$$

We compute

$$\begin{aligned}\frac{\partial}{\partial b}V^{o}(x,b) &= \frac{\partial}{\partial b}V(x,b)\\ &= -W_1\frac{\partial}{\partial b}f_1(x,b)+W_2\frac{\partial}{\partial b}f_2(x,b)-W_3\frac{\partial}{\partial b}f_3(x,b)\\ &\quad -2W_4 f_1(x,b)\frac{\partial}{\partial b}f_1(x,b)\end{aligned}$$

where

$$\begin{aligned}\frac{\partial}{\partial b}f_1(x,b) &= [T(x-b)+T(x+b)]\lambda(b)\\ \frac{\partial}{\partial b}f_2(x,b) &= \left[T(x-b)^2+T(x+b)^2\right]\lambda(b)\\ \frac{\partial}{\partial b}f_3(x,b) &= \frac{\partial}{\partial b}\int_{-b}^{b}\int_{-b}^{b}T(x-y)T(y-z)\lambda(y)\lambda(z)\,\mathrm{d}y\mathrm{d}z\\ &= 2\lambda(b)\int_{-b}^{b}[[T(x-z)+T(x-b)]T(z-b)+[T(x-z)+T(x+b)]T(z+b)]\lambda(z)\,\mathrm{d}z\end{aligned}$$

Thus, we get

$$\int_{-b}^{b}\frac{\partial}{\partial b}V^{o}(x,b)\lambda(x)\,\mathrm{d}x=\lambda(b)F(b)$$

where

$$\begin{aligned}F(b) \equiv{} & -W_1\int_{-b}^{b}[T(x-b)+T(x+b)]\lambda(x)\,\mathrm{d}x+W_2\int_{-b}^{b}\left[T(x-b)^2+T(x+b)^2\right](x)\,\mathrm{d}x\\ & -W_3\int_{-b}^{b}\int_{-b}^{b}[[T(x-z)+T(x-b)]T(z-b)+[T(x-z)+T(x+b)]T(z+b)]\lambda(z)\lambda(x)\,\mathrm{d}z\mathrm{d}x\\ & -2W_4\int_{-b}^{b}f_1(x,b)[T(x-b)+T(x+b)]\lambda(x)\,\mathrm{d}x\end{aligned}$$

As a result,

$$\frac{\partial\Omega}{\partial b}=2\lambda(b)[V^{o}(b,b)+F(b)]$$

Because $V^{o}(b,b)+F(b)$ is *generically* different from zero, it must be that $\lambda(b)=0$.

References

[1] Alonso, W. (1964). *Location and Land Use*, Harvard University Press, Cambridge, MA.

[2] Anderson, S.P., A. de Palma and J.-F. Thisse (1992). *Discrete Choice Theory of Product Differentiation*, MIT Press, Cambridge, MA.

[3] Beckmann, M.J. (1976). Spatial equilibrium in the dispersed city, in Papageorgiou G.J. (ed.), *Mathematical Land Use Theory*, Lexington Books, Lexington.

[4] Economides, N. (1986). Minimal and maximal product differentiation in Hotelling's duopoly, *Economics Letters* 21, 67-71.

[5] Fujita, M. and P. Krugman (1995). When is the economy monocentric? von Thünen and Chamberlin unified, *Regional Science and Urban Economics* 25, 505-28.

[6] Fujita, M., P. Krugman and A.J. Venables (1999). *The Spatial Economy: Cities, Regions and International Trade*, MIT Press, Cambridge, MA.

[7] Fujita, M. and H. Ogawa (1982) Multiple equilibria and structural transition of non-monocentric urban configurations, *Regional Science and Urban Economics* 12, 161-196.

[8] Fujita, M. and T. Tabuchi (1997). Regional growth in post-war Japan, *Regional Science and Urban Economics* 27, 643-670.

[9] Krugman, P. (1991). Increasing returns and economic geography, *Journal of Political Economy* 99, 483-499.

[10] Krugman, P. and A. Venables (1995). Globalization and the inequality of nations, *Quarterly Journal of Economics* 110, 857-880.

[11] Lucas, R., and Rossi-Hansberg, E. (2002). On the internal structure of cities, *Econometrica*, 70(4), 1445-1476.

[12] Mathae, Th. and A. Shwachman (2009). Regional wages and market potential in the enlarged EU: an empirical investigation. Working paper N° 37, Banque Centrale du Luxembourg.

[13] Mossay, P. (2003). Increasing returns and heterogeneity in a spatial economy, *Regional Science and Urban Economics* 33, 419-444.

[14] Mossay, P. and P.M. Picard (2009). On spatial equilibria in a social interaction model, CORE DP 2009/63.

[15] Ogawa, H. and M. Fujita (1980). Equilibrium land use patterns in a nonmonocentric city, *Journal of Regional Science* 20, 455-475.

[16] Ottaviano, G., T. Tabuchi, and J.-F. Thisse (2002). Agglomeration and trade revisited, *International Economic Review* 43, 409-436.

[17] Picard, P.M. and T. Tabuchi (2007). Self-organized agglomerations and transport costs, Economics Discussion Paper Series 0712, University of Manchester.

[18] Picard, P.M. and T. Tabuchi (2010). Self-organized agglomerations and transport costs, *Economic Theory* 42, 565-589.

[19] Picard, P.M. and D.-Z. Zeng (2005). Agricultural sector and industrial agglomeration, *Journal of Development Economics* 77, 75-106.

[20] Tabuchi, T. (1986). Urban agglomeration economies in a linear city, *Regional Science and Urban Economics* 16, 421-436.

[21] Tauchen, H. and A.D. Witte (1984). Socially optimal and equilibrium distributions of office activity: Models with exogenous and endogenous contacts, *Journal of Urban Economics* 15, 66-86.

[22] von Thünen, J.H. (1826) *Der Isolierte Staat in Beziehung auf Landwirtschaft und Nationalökonomie,* Perthes, Hamburg.

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