The disadvantage of winning an election[1]

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Abstract

This paper analyzes the problem that an incumbent faces during the legislature when deciding how to react to policy proposals such as the outcome of referenda or popular initiatives. We argue that this is a potential source of electoral disadvantage for the incumbent because his choices factor in the citizens' evaluation of his performance. We analyze the incentives of the incumbent to implement policies that are unpopular but closer to his preferred ones. We characterize conditions under which this potential disadvantage can be transformed into an electoral advantage by the incumbents. We find that the choices of the incumbent during the legislature will be closest to the policy proposals when the intensity of electoral competition is neither too soft nor too tough. Finally, we use our results to discuss some implications of the use of mechanisms such as referenda and popular assemblies on electoral competition.

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1 Introduction

The incumbency advantage is a well documented phenomenon, according to which an incumbent politician is more likely to be reelected than a challenger candidate. Empirical studies, such as Gelman and King (1990) and Lee (2008), estimate the probability of success of an incumbent when facing reelection in the US House. Their results provide strong evidence in favor of the existence of an incumbency advantage. Other studies have analyzed the specific reasons for this advantage. They typically assume that incumbents have better ways to influence voters' decisions than challengers, and that they can do so through different mechanisms such as redistricting (Levitt and Wolfram 1997, Cox and Katz 2002), seniority (McKelvey and Reizman 1992), informational advantages (Krehbiel and Wright 1983), access to campaign resources (Goodliffe 2001, Jacobson 2001), legislative irresponsibility (Fiorina 1989) or pork barrel politics (Cain, Ferejohn, and Fiorina 1987, Ansolabehere, Snyder, and Stewart 2000).

Ansolabehere and Snyder (2002) measured the incumbency advantage in all state executives and found similar empirical support for the incumbent advantage. However, they argue that this advantage does not have its origin in the strategic choices made by incumbents but in their innate characteristics. In this line, Bevia and Llavador (2009) show that only good quality incumbents may enjoy an advantage and Asworth and de Mesquita (2008) show that on average incumbents' quality and ability are higher than the challengers'. Gowrisankaran, Mitchell, and Moro (2006) find that incumbents face weaker challengers than candidates that face open seats and Stone, Maisel, and Maestas (2004) find that incumbents' personal qualities deter strong challengers from running for office.

The present paper provides an explanation to the phenomenon of incumbency advantage that focuses on a strategic mechanism that can potentially constrain elected politicians. During his term in office, the incumbent must often implement some policies on new or emerging issues. These choices may be costly in terms of chances of reelection if they are unpopular among voters. The incumbent is thus facing implicit restrictions on the policies that he can implement if he wants to remain in office. In this paper, we analyze whether and how much policy motivated incumbents will make policy sacrifices during their time in office in order to enhance their electoral prospects. We show under which conditions this potential source of electoral disadvantage can be transformed into an advantage and help the incumbent be reelected.

Although our model is more general, we have in mind a specific type of issues and policy choices as the origin of this potential incumbency disadvantage: the outcomes of different forms of citizen direct political participation.

The outcomes of processes like referenda, citizens' initiatives or popular assemblies may constrain incumbents because their reaction to them factors into voters' evaluation of the incumbent's performance. This can create a disadvantage compared to the case of an incumbent who does not receive any proposal and also compared to the challenger, whose position does not require him to make any pre-election choice and who, as a result, may have a greater chance of winning unless the incumbent is ready to compromise.

In order to analyze of the incumbent's decision, we build a formal model of electoral competition with two candidates, two issues and three stages. In the first stage of the game, the incumbent faces an exogenously given policy proposal on a certain issue, the *popular issue*, on which he has to make a policy choice. The implementation of his choice on the popular issue takes place during the legislature and before the next electoral campaign. In the second stage of the game, both candidates announce simultaneously their policy platforms on a different issue, the *electoral issue*. The electoral issue is defined in the same way as in most models of electoral competition; the candidates' choices in this issue represent their campaign promises. Finally, in the third stage of the game, voters vote for their most preferred candidate.

The model presents two different types of asymmetries. First, voters evaluate the two candidates in different ways. We assume that voters use all the information they have available at the time of the election in order to decide who to vote for. They evaluate each candidate according to their campaign promises. When evaluating the incumbent, in addition to his campaign promises, they take into account his choice on the popular issue during the legislature. However, the policy outcome on the popular issue does not enter into voters' evaluation of the performance of the challenger.

The second asymmetry refers to the two issues. On the one hand, citizens' evaluate the performance of the incumbent on the popular issue by comparing his policy choice with the proposal they had sent him, that is, the outcome of a previous process of direct democracy. On the other hand, they evaluate candidates on the electoral issue by comparing their own preferred policy with each candidate's political platform. Moreover, they assign different weights to the incumbent's choices in each one of the issues.

The optimal policy choices of the incumbent in both issues reflect the incumbent's trade-off between his own policy preferences and reelection. We find that for all parameter values the incumbent has a strategy that allows him to be reelected. The question is whether this winning strategy is always optimal from the incumbent's point of view. And the answer is no.

There are some instances where the incumbent prefers to forgo reelection because obtaining it is too costly in terms of policy. In those cases, the incumbent implements his ideal policy in the popular issue. For this to happen

three conditions must hold: (1) the incumbent must care enough about policy, that is, the value of holding office must be low enough; (2) there must exist an intense conflict of interest between voters and the incumbent over the popular issue; and (3) competition on the electoral issue must be strong, that is, voters must consider the electoral issue to be very important. The intuition for this result is the following: the incumbent has a disadvantage whenever he does not satisfy voters' demands on the popular issue. And he will enjoy the largest advantage at the electoral competition stage when he fully satisfies the their demands on the popular issue. However, this is a costly strategy for a policy motivated incumbent. If the incumbent is policy motivated he will choose the platform that forces him to compromise as little as possible whilst guaranteeing reelection. But when the conflict of interests with voters in the popular issue is too intense or competition with the challenger is very strong, this strategy becomes too costly and the incumbent prefers to implement his ideal policy on the popular issue and lose reelection.

Otherwise, in equilibrium the incumbent chooses a winning strategy that consists of a combination of policies that depend on the weight that voters assign to his performance on each issue. The larger the weight that voters assign to the electoral issue, the more the incumbent will satisfy voters on the electoral issue. Perhaps more surprisingly, this is not the case for the popular issue. The incumbent fully satisfies the voters' demands only when the weight citizens attach to the popular issue is neither too high nor too low. This is because the incumbent does not compete with the challenger in the popular issue. Hence, when citizens care a lot about it, the incumbent can implement a policy closer to his ideal one in this issue without jeopardizing his reelection. This cannot happen in the electoral issue because there the incumbent has to compete against the challenger.

Our model relates to some extent to the literature on spatial competition with valence initiated by Stokes (1963) and later developed by Ansolabehere and Snyder (2000), Groseclose (2001) and Aragones and Palfrey (2002). In those models, one of the candidates holds an advantage due to exogenous non policy factors, called valence factors, such as charisma, better campaign funds or higher intelligence. The difference between our model and these is that in ours the origin of the advantage (if any) is endogenous. In our case, a good performance of the incumbent in the popular issue provides him with an advantage that has an effect on electoral competition similar to the one that valence factors have in the models just mentioned. This choice becomes a source of disadvantage when the incumbent deviates too much from citizens' policy proposal on the popular issue.

The remainder of paper is organized as follows. In the next section we discuss two real political mechanisms the analysis outlined above can apply

to, namely referenda and participatory democracy. Section 3 describes the formal model. Section 4 presents the results and Section 5 discusses them in the light of the mechanisms mentioned above. The last section offers some concluding remarks. All technical proofs can be found in the appendix.

2 Two sources of incumbency disadvantage

Many models of electoral competition assume that after an election takes place, the incumbent can implement any policy he likes. His policy choice will depend only on his objective function. Then, if the incumbent is mostly policy oriented, he will tend to choose a policy close to his ideal policy. But in real life this is not always the case. When an incumbent is deciding which policies to implement, he has to take into account that some choices might have a large negative effect on his chances of reelection. Citizens will factor these choices into their evaluation of the incumbent's performance. Notice that jeopardizing his reelection is not optimal for the incumbent even when he cares about policy because the policy implemented in case he loses will be worse from his point of view than the one that he could have chosen had he won the election. Then, when incumbents are policy motivated, they may compromise on some dimension because they also want to be reelected.

Two mechanisms that can generate this potential incumbency disadvantage are referenda and participatory democracy. Some empirical studies show that these mechanisms of direct democracy are very effective in satisfying voters' preferences and in increasing voters' well being[1]. The characteristics that both referenda and participatory democracy have in common are: (1) there is an issue that a significant part of the population considers to be very important; (2) the incumbent receives a policy proposal on this issue; (3) the incumbent has to make a decision regarding that issue; (4) there is a significant proportion of voters that may base their voting decision on that issue[2]. Next, we elaborate on how these mechanisms fit in our main argument.

2.1 Referenda and popular initiatives

Referenda may be mandatory or facultative. They are mandatory if the law (usually the constitution) directs authorities to hold referenda on specific matters. This is normally the case for amending constitutions, impeaching

[1] See Frey (1994), Frey and Bohnet (1993) and Frey and Slutzer (2000).

[2] When policy proposals are binding they do not create a strategic problem for the incumbent. In those cases he does not have a choice and must implement the proposed policy. Hence, binding proposals do not display any effect of the type we analyze here.

heads of state, ratifying international treaties, etc. Otherwise, when they are facultative, referenda may be initiated at the will of a public authority or at the will of some organized group of citizens (in this case they are also known as popular initiatives). Referenda may be either binding or non-binding. A non-binding referendum is merely consultative or advisory. It is left to the government or legislature to interpret its results and they may even choose to ignore them. A binding referendum forces the incumbent to implement its policy outcome. Referenda may also require the support of a supermajority of votes cast or, as the one that took place in Portugal in 2007 on a new abortion legislation, they may require a minimum level of turn-out to become binding (Herrera and Mattozzi, 2010).

If a non-binding referendum is called during the time of the legislature, the incumbent will have to react to its outcome. If he chooses not to implement the policy corresponding to the referendum outcome he may be punished by voters. The relevance of this policy choice on the decision of voters on the incumbent's reelection will depend on the proportion of voters to whom this issue is relevant.

A referendum initiated by the incumbent himself might have a weaker effect on voters' reaction than a referendum that originates with a popular initiative. The first type of referendum requires a more complicated strategy from the incumbent because he has to decide whether and when it is optimal to call it. The analysis of these strategies is beyond the scope of this paper[3].

2.2 Participatory democracy

Some proposals received by incumbents have their origin in an organized group of citizens. This is the case in systems of participatory democracy, an extended version of the system of representative democracy in which citizens make policy proposals through popular assemblies. Real cases of participatory democracy can be found in the town meetings of New England, a form of local government practiced since colonial times; in the village governance system of the Indian states of Kerala and West Bengal; and in the participatory budgeting system of many Latin-American cities. It has also been applied to school, university, and public housing budgets. The implications of participatory democracy on the behavior of citizens and politicians and on policy outcomes are analyzed in Aragones and Sánchez-Pagés (2009).

In all these cases, popular assemblies and deliberation emerge as governance mechanisms because citizens are interested on a certain issue, normally

[3]See Xefteris (2008) for an analysis of the incumbent's decision about when to call a referendum.

a local one, and they would like certain policies to be implemented. Because they care enough about these issues, their expected benefits from participating in the process may overcome the costs of coordinating in order to elaborate a policy proposal. In those cases, a policy proposal will emerge from the process and it will be sent to the incumbent. The incumbent has formally complete discretion regarding the policies he can implement on the issue. However, because the support to these policy proposals is significant within the population, the incumbent's chances of being re-elected critically depend on his policy choices on that issue.

3 The model

We assume that electoral competition takes place across two dimensions, denoted by x and y. Each dimension is represented by a unit interval of the real line $[0, 1]$. Dimension x represents the electoral issue and dimension y represents the popular issue. There are two candidates: the incumbent and the challenger. The game proceeds in three stages. The first stage takes place during the legislature: the incumbent receives a policy proposal on the popular issue and has to implement a policy on that issue. Both the policy proposed to the incumbent and the policy implemented by him on the popular issue are common knowledge to all candidates and all voters. The second stage is the electoral campaign: both candidates make policy announcements simultaneously on the electoral issue. Again all policy announcements are common knowledge to all candidates and all voters. It is assumed that the winner implements the announced policy on that issue. In the third stage of the game the election takes place: voters decide whether to reelect the incumbent or vote for the challenger. The winner is selected by majority rule and implements the policy announced on the electoral issue.

3.1 Candidates

The two candidates are denoted by L and R. Candidate L is assumed to be the incumbent. Candidates have single peaked preferences over the electoral issue. Without any loss of generality we assume that the ideal point of candidate L on the electoral issue is represented by $x_L = 0$ and the ideal point of candidate R is represented by $x_R = 1$. We assume that the incumbent has single-peaked preferences over the popular issue that are independent of his preferences on the electoral issue. The incumbent's ideal point on the popular issue is represented by $y_L = 0$. As we will argue below, it is not necessary to specify the preferences of the challenger over the popular issue.

Let us denote by $y(L)$ the policy chosen by the incumbent on the popular issue during the legislature. We assume the incumbent to be a unique decision maker. Thus, the present model applies to scenarios where the incumbent holds executive office, or to legislatures where a party holds a parliamentary majority and whose parliamentary representatives vote as a unified bloc.[4] Elections take place at the end of the legislature. When the electoral campaign starts, this choice $y(L)$ has already been made and it is taken as given. We model elections by means of a standard model of electoral competition on the issue x: the incumbent and the challenger simultaneously announce policy platforms denoted by $x(L)$ and $x(R)$ respectively. We assume full commitment, that is, the winner of the election will implement on the electoral issue the policy he announced during the campaign.

We assume that candidates have preferences over policies but that they are also office-motivated. Candidates' payoffs depend on the policy chosen by the incumbent on the popular issue and the policy announcements of both candidates on the electoral issue according to these utility functions:

$$V_L = -\left|y_L - y\left(L\right)\right| + \pi_L\left(K - \left|x_L - x\left(L\right)\right|\right) - \left(1 - \pi_L\right)\left|x_L - x\left(R\right)\right|,$$

$$V_R = -\left|y_L - y\left(L\right)\right| + \left(1 - \pi_L\right)\left(K - \left|x_R - x\left(R\right)\right|\right) - \pi_L\left(\left|x_R - x\left(L\right)\right|\right),$$

where $\pi_L = \pi_L(y(L), x(L), x(R))$ represents the probability that candidate L wins the election, and $1 - \pi_L$ denotes the probability that candidate R wins the election. The probability with which the incumbent is reelected depends on the policy choices made during the legislature and the policy announcements made during the campaign.

K is a non-negative number that represents the utility of holding office. $K = 0$ implies that candidates do not obtain any extra utility from holding office, they only derive utility from the policy implemented. In this case we would have two candidates that are only policy motivated. The larger the value of K the more candidates value being in office. Thus for larger values of K candidates care more about winning. When the value of K is high enough candidates become purely office motivated.

Note that the incumbent obtains a negative payoff whenever he implements a policy on the popular issue that does not coincide with his ideal point on that issue. Observe also that because we assume that the challenger has no power over policy implementation on the popular issue before or after the election, the policy choice of the incumbent on that issue $y(L)$ has an non reversible impact on his payoffs. We elaborate more on this below.

[4] Otherwise, it can be thought of as reduced form model of a more complex (and realistic) governmental system.

For simplicity, we have assumed that the incumbent cares equally about the two issues. Introducing a parameter in the incumbent's payoff function that represents the relative weight that each issue has on the incumbent overall payoffs would not change the main qualitative results obtained.

3.2 Voters

Voters have single-peaked preferences over the electoral issue x. We assume that their ideal points are uniformly distributed over x, thus the ideal point of the median voter on the electoral issue is $x_m = \frac{1}{2}$. Let the ideal point of society in issue y be denoted by $y_m > 0$. The parameter y_m is considered exogenous in our model. It can be interpreted as the outcome of a referendum or of a process of participatory democracy that took place before the beginning of the game. Notice that since the ideal point of the incumbent on the popular issue is assumed to be $y_L = 0$, the value of y_m measures the magnitude of the conflict of interests between the incumbent and the citizens with respect to the popular issue. Here we assume that this proposal is exogenous. The model could be extended by endogenizing the citizens' proposal. We partially addressed this in our previous work (Aragones and Sanchez-Pages, 2009) for the case when the proposal comes from popular assemblies and only the popular issue is relevant for voters.

When facing the election, voters observe the policies announced by both candidates on the electoral issue, $x(L)$ and $x(R)$, the policy implemented by the incumbent on the popular issue, $y(L)$, and then cast their vote. Voters use all the information available in order to evaluate the two candidates. Since they have different kinds of information about the performance of each candidate, their decision rule must exhibit some sort of asymmetry.

We assume that voter i evaluates the incumbent according to the ffunction

$$U_i(L) = -(1-\mu)\,|y_m - y(L)| - \mu\,|x_i - x(L)|,$$

where μ is a parameter that measures the relative weight that voters assign to the electoral issue with respect to the popular issue, and $0 \leq \mu \leq 1$. Values of μ close to one are to be interpreted as a situation in which society considers the popular issue to be not very important. In these cases, voters' evaluation of the incumbent would not be much affected by the incumbent's policy choice on that issue. Values of μ close to zero mean that the popular issue is regarded as very important from voters' point of view and that their evaluation of the incumbent will be strongly affected by the incumbent's policy choice on the popular issue.

Note that voters evaluate the incumbent on the electoral issue by comparing his electoral platform, $x(L)$, to their own ideal point x_i. However,

they evaluate the incumbent on the popular issue by comparing the policy he implemented, $y(L)$, to the policy proposed initially by citizens y_m. Hence, citizens measure the performance of the incumbent on the popular issue in an homogeneous way. This assumption is justified whenever the policy proposal y_m, represents the outcome of referenda, citizens assemblies or polls, that is, when it represents the ideal policy on the popular issue of a substantial subset of the electorate.

On the other hand, voter i evaluates the challenger according to the following function:

$$U_i(R) = -|x_i - x(R)|.$$

The performance of the challenger on the popular issue cannot be evaluated, since he has not implemented any policy during the present legislature. Thus voters can only evaluate the challenger according to his promises on the electoral issue. The reader may argue that the challenger could also make promises on the popular issue that would be implemented in case he wins the election. We are assuming here that the popular issue refers to issues that emerge only once or to issues on which the incumbent policy choices are impossible or too costly to reverse, like an annual budget, abortion legislation, participation in a war or signing an international treaty.

However the model could be extended by assuming that the performance of the challenger on the popular issue whenever he was in office is represented by a given constant. Notice that the voters' evaluation of the challenger's performance on the popular issue cannot be a strategic variable because he is not in office during the current legislature. The analysis would be a bit more combersome but the qualitative results would not change.

Given voters' evaluations of both candidates, voter i will vote for candidate L if and only if

$$-(1-\mu)|y_m - y(L)| - \mu|x_i - x(L)| \geq -|x_i - x(R)|. \qquad (1)$$

Notice that the lower the value of μ the more weight past choices have on the evaluation of the incumbent. At electoral stage, the performance of the incumbent on the popular issue, i.e. the distance $|y_m - y(L)|$, has an effect on voters evaluations very similar to the effect of valence factors.[5]

Given this asymmetry on voters' evaluation of candidates, the standard median voter analysis does not apply at the electoral competition stage. Voters with ideal points at both extremes of the distribution may decide to vote for the same candidate. In fact, when the distance between the policy implemented $y(L)$ and the policy proposal y_m is large enough, the set of voters

[5] Ansolabehere and Snyder (2000), Groseclose (2001) and Aragones and Palfrey (2002)

that decide to vote for the incumbent becomes non-connected. Expression (1) implies that when μ is low enough a citizen with ideal point $x_i = x(L)$ will vote for candidate R whenever

$$\mu \leq 1 - \frac{|x_i - x(R)|}{|y_m - y(L)|}.$$

The set of voters who prefer to vote for the challenger but whose ideal policy x_i is closer to $x(L)$ enlarges as citizens care more about the popular issue and as the incumbent's choice $y(L)$ departs from the citizens' proposal y_m. This shows that the existence of a policy proposal during the legislature is a potential source of electoral disadvantage for the incumbent.

The present specification encompasses as particular cases some standard models of two-party competition. If $\mu = 1$, that is, if voters care only about the electoral issue, we have a standard model of electoral competition. In this case, for very large values of K candidates are purely opportunistic and the model describes a standard downsian framework. For relatively small values of K, candidates become policy motivated, and our model reproduces Wittman's (1983) model of electoral competition. On the other hand, the case of $\mu = 0$, that is, if voters only care about the popular issue, boils down to a more general version of our previous work on participatory democracy (Aragonès and Sánchez-Pagés, 2009).

The incumbent is reelected if an only if the set of voters that prefer the incumbent to the challenger contains a majority of the population[6]. Since the decisions on the two dimensions of the model are made sequentially, one at each stage, we do not have to deal with the complexities of electoral equilibrium in a multidimensional space. In fact, we can solve it as a one dimensional model within each stage. In the next section we study the equilibrium of this game for all values of the parameters K, μ and y_m.

4 Equilibrium results

4.1 Electoral stage

In order to solve the game described above we look for its subgame perfect equilibrium by using backward induction. We start by analyzing the electoral stage, taking as given the choice of the incumbent on the popular issue.

Citizens partially base their evaluation of the incumbent on his performance in the popular issue. Hence, he does not enter the election on the same grounds as the challenger. His choice on the popular issue will have an

[6] That is, we assume that if there is a tie the incumbent is reelected.

impact on electoral competition, as the following lemma illustrates.

Lemma 1 *If* $x(L) = x(R)$, *then* L *obtains at least* $1 - 2|y(L) - y_m|$ *of the votes and* R *obtains at most* $2|y(L) - y_m|$.

When both candidates choose the same position on the electoral issue, that is when $x(L) = x(R)$, only citizens at a distance of at least $|y(L) - y_m|$ from the policy proposed by both candidates vote for the incumbent. Thus, it is possible for the incumbent to capture the vote of extremists if he performs well enough in the popular issue, that is, when $|y(L) - y_m|$ is small enough. It would be wrong to conclude that the fact that citizens' evaluate the incumbent on the popular issue puts him always in a disadvantage. The incumbent's chances of being reelected will be higher the less his policy choice in the popular $y(L)$ issue departs from the society's most preferred policy y_m. As a matter of fact, there exists a threshold on this distance that is critical in determining whether the incumbent has an electoral advantage or not, as the next proposition shows.

Proposition 1 *If* $|y(L) - y_m| < 1/4$, *then* L *wins in equilibrium. Otherwise,* R *wins in equilibrium.*

The incumbent obtains a decisive advantage when he compromises enough on the popular issue. In that case, it is optimal for the incumbent to use a strategy that guarantees his reelection. If, on the contrary, his policy choice on the popular issue departs considerably from the proposal y_m then he might still have a winning strategy at the electoral stage. But he has to compromise so much on the electoral issue in order to win that he rather prefers to lose.

Let us now fully describe the equilibrium at the electoral competition stage given a choice of $y(L)$. The following two propositions characterize the strategies used by the winner of the election in equilibrium. These strategies define the equilibrium policy outcome of the electoral stage as well. First, we describe the equilibrium outcomes of the electoral stage for the case in which the incumbent is reelected in equilibrium.

Proposition 2 *If* $|y(L) - y_m| \leq \frac{1}{4}$, *then* L*'s equilibrium strategies at the electoral stage are:*

i) $x^*(L) = 0$ *if* $|y(L) - y_m| \leq \frac{1-3\mu}{4(1-\mu)}$

ii) $x^*(L) = \frac{3\mu-1}{4\mu} + \frac{1-\mu}{\mu}|y(L) - y_m|$ *if* $|y(L) - y_m| \geq \frac{1-3\mu}{4(1-\mu)}$

This proposition illustrates the trade-off that the incumbent faces. The more he pleases the electorate on the popular issue, the closer his winning electoral platform will be to his ideal policy. The incumbent can even guarantee his reelection by implementing his ideal point on the electoral issue if he satisfies voters enough on the popular issue, provided that the weight voters put on the electoral issue is small enough, i.e μ is low. In order to achieve this, he will need to compromise more on the popular issue the larger the value of μ (note that $\frac{1-3\mu}{4(1-\mu)}$ decreases with μ).

Otherwise, if he implements a policy on the popular issue that departs significantly from the policy proposal y_m, then the incumbent still wins the election in equilibrium but his electoral platform $x^*(L)$ includes a certain degree of compromise. His electoral platform will lie somewhere between his ideal point and the median voter's ideal point. And it will be closer to the median voter's ideal point the larger the distance between the policy he implemented in the popular issue $y(L)$ and the policy proposal y_m. This equilibrium policy choice will also be closer to y_m the more weight voters put on the electoral issue or, in other words, the tougher the competition at the electoral stage, which is represented by higher values of μ[7]. In the limit, when competition at the electoral stage attains a maximum, i.e. μ goes to 1, the policy announced by the incumbent on the electoral issue coincides with the median voter's ideal point. By the same token, as the popular issue becomes more important, i.e. μ decreases, the policy announced by the incumbent on the electoral issue approaches the incumbent's ideal point.

The following proposition describes the equilibrium outcome of the electoral stage when the incumbent decides to forgo reelection. In that case, the equilibrium policy outcome in the electoral issue coincides with the strategies used by the challenger in the equilibrium of the electoral stage.

Proposition 3 *If* $|y(L) - y_m| > \frac{1}{4}$*, then R's equilibrium strategy at the electoral stage is* $x^*(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L) - y_m|$.

When the incumbent has departed significantly from the citizens' ideal point in the popular issue, the challenger wins with a moderate policy in the resulting equilibrium of the electoral stage. Observe that $x^*(R)$ is decreasing in μ so, as before, the tougher the competition at the electoral stage the closer the policy outcome will be to the median voter's ideal point. And the larger the distance between the policy proposal and the policy implemented on the popular issue the closer the policy outcome on the electoral issue will be from the challenger's ideal point.

[7] Straightfoward calculations show that $\frac{\partial x^*(L)}{\partial \mu} = \frac{1}{\mu^2}\left(\frac{1}{4} - |y(L) - y_m|\right) \geq 0$.

4.2 The popular issue

After solving for the equilibrium strategies of the electoral stage of the game, we move backward in the game in order to find the incumbent's optimal policy at the first stage.

Recall that when the incumbent is choosing which policy to implement on the popular issue he is facing a trade-off. If he implements a policy $y(L)$ that is relatively close to the citizens' proposal, y_m, he will be able to get reelected with an electoral platform relatively close to his ideal policy. The closer his choice $y(L)$ is to his own ideal policy on the popular issue, that is, the more his choice departs from y_m, the more he will have to compromise on the electoral issue if he wants to remain in office. This strategy may be too costly if the incumbent is sufficiently policy motivated. Instead, he can implement his most preferred policy on the popular issue and forgo reelection.

In the next few results, we will characterize this trade-off. First we find the best winning strategies and best losing strategies for the incumbent. Then we show under which conditions the incumbent will prefer to be reelected.

The following Lemma describes the best winning policy choice of the incumbent given the degree of political competition with the challenger.

Proposition 4 *The incumbent's best wining strategies are:*

(i) $y^*(L) = \max\left\{y_m - \frac{1-3\mu}{4(1-\mu)}, 0\right\}$ *and* $x^*(L) = 0$ *if* $\mu \leq \frac{1}{3}$;

(ii) $y^*(L) = y_m$ *and* $x^*(L) = \frac{3\mu-1}{4\mu}$ *if* $\frac{1}{3} \leq \mu \leq \frac{1}{2}$;

(iii) $y^*(L) = \max\left\{y_m - \frac{1}{4}, 0\right\}$ *and* $x^*(L) = \frac{1}{2}$ *if* $\mu \geq \frac{1}{2}$.

The relationship between the best winning electoral platform and μ is very intuitive: As competition on the electoral issue becomes tougher, i.e. μ goes up, the incumbent needs to select a platform closer to the median voter's ideal policy in order to win.

Perhaps more surprising is the non-monotonic effect that the weight that citizens put on the popular issue has on the best winning policy that the incumbent can implement in that issue. When the popular issue is important (low μ) the incumbent is virtually facing no opposition. Citizens care almost only about an issue in which the incumbent can act like a monopolist. Actually, when competition on the electoral issue is rather soft ($\mu \leq \frac{1}{3}$) the incumbent can win by implementing her most preferred policy on the electoral issue. As competition on the electoral issue becomes tougher, the concession on the popular issue needed to achieve this increases.

But as μ becomes larger the incumbent cannot longer win the election by implementing his ideal policy on the electoral issue. He can either please citizens on the popular issue by implementing their proposed policy y_m and in return choose a policy close to her ideal one on the electoral one, or alternatively he can pick the median voter's ideal policy on the electoral dimension and select a policy as close as possible to his own ideal one on the popular issue. For intermediate levels of μ the first option is better because electoral competition is still relatively soft and he can implement a policy relatively close to his ideal policy on the electoral issue. That winning electoral promise will be larger (less favorable for the incumbent), the tougher electoral competition becomes, that is, the larger the value of μ. However, when electoral competition is tough, i.e. $\mu > \frac{1}{2}$, the incumbent prefers the second option and he compromises substantially on the electoral issue, that is, he will implement the median voter's ideal point. Hence, the incumbent will implement the citizens' policy proposal on the popular issue only when electoral competition is neither too soft nor to tough.

Figure 1 depicts the incumbent´s winning strategy in both stages of the game as a function of μ.

[Insert Figure 1 here]

Next we find the incumbent's best losing strategy and the corresponding best response of the challenger.

Lemma 2 *The incumbent best losing strategies are* $y^*(L) = 0$ *and* $x^*(L) = 1/2$ *which in turn implies that* $x^*(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu} y_m$.

If the incumbent decides to forgo reelection, the best strategy that he can follow is to implement his preferred policy choice on the popular issue and force the challenger to become as moderate as possible in the electoral one. In equilibrium, he announces the median voter's ideal policy and the challenger wins the election by announcing a platform that will be closer to the median voter the higher the value of μ.

The last step of the analysis amounts to characterize when the incumbent prefers to win the election given the best winning strategies and the best losing strategies described above. His incentives to remain in office will depend on the level of disalignment with the population (simply measured by y_m), the relative weight that voters assign to the electoral issue, i.e. measured by μ, and the value that the incumbent attaches to office K.

Proposition 5 *In equilibrium the incumbent wins*

(*i*) *When* $y_m \leq \frac{1}{4}$ *for any* $K \geq 0$ *and any* $0 \leq \mu \leq 1$.

(*ii*) *When* $\frac{1}{4} \leq y_m \leq \frac{3}{8}$ *if and only if* $K > \max\left\{\frac{2\mu}{1+\mu}y_m - \frac{1}{4}, 0\right\}$

(*iii*) *When* $y_m \geq \frac{3}{8}$ *if and only if*

$$K > \begin{cases} \max\left\{\frac{2\mu}{1+\mu}y_m + \frac{5\mu-3}{4(1-\mu)}, 0\right\} & \text{if } \mu \leq \frac{1}{2} \\ \frac{2\mu}{1+\mu}y_m - \frac{1}{4} & \text{if } \mu \geq \frac{1}{2} \end{cases}$$

When the preferences of the incumbent on the popular issue are aligned with those of society, i.e. $y_m \leq \frac{1}{4}$, the incumbent prefers to win for all values of K and all values of μ. Otherwise, if the preferences of the incumbent on the popular issue are not aligned with the policy proposal on this issue, the incumbent may decide to forgo the reelection. In this case, he will do so only when he is sufficiently policy motivated (for low enough values of K). The tougher electoral competition is, i.e. the bigger μ, the bigger is the range of values of K that induces the incumbent to forgo the election. Intuitively, the more intense electoral competition is and the more costly is to please voters in the popular issue, the more likely is that the incumbent will prefer to lose. The area under the curves in Figure 2 corresponds to the region of the parameter space where the incumbent is not reelected in equilibrium.

[Insert Figure 2 about here]

Incumbents that are highly policy motivated, i.e. have low values of K, are more likely to suffer a disadvantage from being in office. They may find too costly to make a policy choice that will guarantee their reelection when their preferences are not aligned with society's preferences. The cost of being reelected may also be too high when the degree of competition on the electoral issue is high, i.e μ close to 1. In that case, the incumbent will have to propose a very moderate policy on the electoral issue if he wants to beat the challenger. Otherwise, citizens' proposals can be used by the incumbent to obtain a decisive advantage in political competition and become reelected.

5 Discussion

The success of representative democracy relies on the willingness of incumbents to deliver policies that satisfy the preferences of voters. The incentives that such a system offers to politicians often do not go in this direction. Incumbents that are policy motivated, as opposed to office motivated, tend to

undermine voters' preferences. Mechanisms of direct democracy can create a bridge between candidates and voters (1) by transmitting information about voters' preferences to candidates, and (2) by offering incentives to incumbents to satisfy citizens' policy proposals. Our results show that in most cases these mechanisms tend to create an incumbency advantage. However, incumbents will not be reelected if there exists a substantial disalignment between them and voters' views. At the light of these results, we will now revisit the two mechanisms of citizen participation described in Section 2.

5.1 Referenda

There is empirical evidence showing that voters welfare increases when voters make use of mechanisms of direct democracy. Econometric cross-section studies for Switzerland reveal that policy choices regarding provision of public goods correspond better with the preferences of voters in those cantons where mechanisms of political participation are more extensively developed. Public expenditures are ceteris paribus lower in communities where the taxpayers themselves can decide on such matters.

One could argue that the use of referenda to achieve this is unnecessary because it is unlikely that the views of elected representatives will differ substantively from those of voters; or, in other words, that the preferences of incumbents tend to be rather aligned with those of voters. However, empirical evidence suggests that some degree of disalignment between citizens and incumbents is a common scenario, especially in the case of local public services, as shown by Agreen, Dahlberg and Mork (2006) for a sample of Swedish municipalities. For the case of Switzerland, Frey and Bohnet (1993) report that:

> "In September 1992, the citizens of Switzerland turned down two proposals seeking to increase substantially the salaries and the staff of Swiss members of Parliament. Both issues would have become law without Swiss voters taking the optional referendum, and both issues would clearly have been to the benefit of the elected officials."

In addition, the two referenda called in Switzerland to decide whether the country should join the UN and the EU in 1986 and 1992 respectively yielded a rejection of these proposals despite the strong backing of all major political parties[8]. This is part of a more general trend. Between 1948 and 1990, 39%

[8] The referendum of Switzerland joining the United Nations resulted in a rejection by 76% of the voters. A 50% percent of the population voted against Switzerland becoming part of the European Economic Area.

of the referenda held in Switzerland yielded results that opposed the views of the Parliament. It seems then that referenda in the real world allow voters to destroy the agenda control of politicians and bring implemented policies closer to what they want.

5.2 Participatory democracy

Real life experiences of participatory democracy have mainly materialized in processes of "participatory budgeting" at the city level. This is the case of nearly two hundred Brazilian municipalities where popular assemblies coexist with formal political parties and local elections. The most famous experience of participatory budgeting was initiated in 1989 in the city of Porto Alegre after the Workers' Party, leaded by Inacio Lula da Silva gained power. It then extended to the state level in Rio Grande del Sul in 1998 but was abruptly terminated after the Workers' Party lost the election of 2002.

A few years after the participatory budgeting system was first implemented, critics of the system claimed that it was being used as a partisan instrument by the Workers' Party. As a matter of fact, the party had won all municipal elections since 1989 by wide margins. The present paper argues that incumbents enjoy an advantage if they satisfy voters in pre-election issues. Most studies show that that was the case in Porto alegre, as suggested by the higher levels of income redistribution and the patterns of citizen participation in the process (Aragones and Sanchez-Pages, 2009). However, the system did not yield the reelection of the Workers' Party candidate in the 2002 state election. What was the difference between these two scenarios?

As argued by Goldfrank and Schneider (2006), one key issue was the degree of political competition. At the city level, the Workers' Party held strong support in Porto Alegre, and it is likely that the popular issue was dominant in voters' minds when casting their vote in subsequent elections. However, at the state level, the Workers' party faced a much stronger opposition from the rest of parties. That would correspond in our model with a higher level of μ. Our results would suggest that the scenario in which the incumbent party does not enjoy a net advantage would be much more likely to arise in that case. Goldfrank and Schneider (2006) computed the difference between promised investments and actual investments completed for each municipality under the Workers' Party rule and then estimated a strong negative effect of these dashed expectations on the share of municipal votes of the party in the 2002 election. The departure between actual policy choices and proposals in an issue that could be represented by the popular issue of our model, severely undermined the advantage that he incumbent party might have enjoyed in these municipalities.

6 Concluding remarks

The main contribution of this paper is to show how the incumbency advantage can coexist with an incumbency disadvantage. This is the case when the policy choice of the incumbent in a pre-election issue factors into the citizens' evaluation of his performance. We assumed that the performance of the incumbent on that issue is assessed by the distance between the policy proposed by citizens and the policy that the incumbent finally implemented.

We have assumed that voters use an asymmetric rule in order to evaluate the candidates. The reason is that we have identified two different kinds of asymmetries that we had to take into account: (1) only the incumbent is responsible for the policy implemented on the popular issue, and (2) there is a policy proposal made only on the popular issue. Thus, we have assumed that voters evaluate the incumbent according to his performance on the two issues and the challenger only according to the platform he announces in the electoral issue.

We could relax this assumption by having both candidates being evaluated according to both issues. The evaluation of the challenger with respect to the popular issue would just become an exogenous parameter given that the challenger cannot implement any policy during the legislature. This parameter would represent the performance of the challenger in the popular issues in the past. Our results will remain the same as long as the weight that voters assign to the electoral issue when they evaluate the incumbent is smaller than the one they use to evaluate the challenger. Otherwise the incumbent will suffer from a greater disadvantage but qualitatively our results would still go through.

A novel feature of our approach is that it combines elements of both retrospective voting and prospective voting. Voters use retrospective voting to evaluate the performance of the incumbent with respect to the popular issues. And voters use prospective voting to evaluate the campaign promises that candidates announce during the electoral campaign. In order to use all the information available to them at the time of voting, voters combine these two different kinds of evaluations.

We have characterized conditions under which the disadvantages generated by this mechanism are compensated by the advantages, and the incumbent can still run as favorite in the electoral campaign. In all these cases, the incumbent has to adjust his policy choices in order to accommodate the policy proposals he receives, and the final policy outcome is relatively close to the policy outcome most preferred by society. But this is not always the case. When the policy demands made by society are too costly from the incumbent's point of view, the incumbent may decide to forgo reelection. In

this case the final policy outcome is bad from voters' point of view.

From the results of our analysis we conclude that policy proposals that are not aligned with the policy preferences of the incumbent will tend to be neglected. This will be more likely the stronger electoral competition is. Demands on the popular issue that are aligned with the preferences of the incumbent will be satisfied more likely when the intensity of electoral competition is intermediate. Therefore, it may be optimal for the voters to submit policy demands that do not put too much pressure on the incumbent.

The elaboration and submission of a policy proposal through a referendum, a popular initiative or a citizens' assembly is very costly for voters in terms of time and effort, in addition to the cost of coordinating actions. Thus, it will only be optimal for a group of citizens to engage in such a process when the expected benefits are large enough, that is, when the expected change in the incumbent's actions improves the final payoff of voters enough to compensate the cost of the process.

Notice that our analysis applies to policy proposals that are supported by a large enough proportion of the electorate and that are made on issues relevant to a substantial part of the population. Under these conditions, these proposals constitute a potential threat to the incumbent at the voting stage. In general, policy proposals may be put forward by a an organized group of citizens, the government, a party in the opposition, a lobby, etc. The results of polls and surveys on important issues like abortion, terrorism, immigration or military intervention in foreign countries may also be considered as sources of policy proposals.

From this point of view, the present model could be applied to understand the electoral outcomes in the US, the UK and Spain after the invasion of Iraq in 2003. Popular support for the invasion as a response to the terrorist threat raised by 9-11 was widespread in the US but was fairly weak in the other two countries. However, whereas the incumbent party was reelected in the UK, it was not in Spain. One possible explanation comes from the change in the weight that voters gave to that issue as a result of the Madrid bombings that took place two days before the general election of 2004. The interpretation at the light of our model is that the bombings made the popular issue extremely important, that is, they resulted in a drastic increase in μ, at a time in which electoral platforms were already chosen by the two main parties. As a consequence, the incumbent party suffered a large electoral disadvantage because it had implemented a policy in that issue very far from the ideal policy of the vast majority of the population.

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7 Appendix

Proof of Lemma 1. If $x(L) = x(R)$ then $-(1-\mu)|y_m - y(L)| - \mu|x_i - x(L)| \geq -|x_i - x(R)|$ becomes $|y_m - y(L)| \leq |x_i - x(R)|$. Thus L obtains votes from all i such that are at a distance from $x(R) = x(L)$ of at least $|y(L) - y_m|$. This means that R obtains at most $2|y(L) - y_m|$ votes, therefore L obtains at least $1 - 2|y(L) - y_m|$ votes. Notice that in this case R obtains exactly $2|y(L) - y_m|$ if $|y(L) - y_m| \leq x(L) = x(R) \leq 1 - |y(L) - y_m|$. ■

Proof of Proposition 1. First suppose that $\{y(L), x(L), x(R)\}$ is an equilibrium outcome such that $x(R) = x(L)$. Then R cannot win because by the previous lemma R at most can obtain $2|y(L) - y_m| < 1/2$ votes.

Next suppose that $\{y(L), x(L), x(R)\}$ is an equilibrium outcome such that $x(R) \neq x(L)$ and R wins. Then we must have

$$U_L(y(L), x(L), x(R)) = -y(L) - x(R).$$

Consider that L chooses instead $x'(L) = x(R)$. Then by the previous lemma L obtains at least $1 - 2|y(L) - y_m| > 1/2$ votes. Thus L wins and his utility is

$$U_L(y(L), x(R), x(R)) = -y(L) + K - x(R) > -y(L) - x(R),$$

In this case L prefers to win and has a winning strategy. Thus R cannot win in equilibrium.

Now suppose that $|y(L) - y_m| > 1/2$. If L is winning in equilibrium with $x(L)$ and $x(R)$, then consider $x'(R)$ such that $x'(R) = x(L)$ and notice that in this case R obtains more than half of the total vote. Thus R can win the election in this using this strategy, and it is also optimal for R to do so since he obtains an extra payoff of K and his deviation does not involve any change in the policy implemented. The reason is that if $x(L) \leq 1/2$ then R obtains a vote share equal to $x(L) + \min\{1 - x(L), |y(L) - y_m|\}$ which is a majority. Similarly if $x(L) \geq 1/2$ then R obtains $1 - x(L) + \min\{x(L), |y(L) - y_m|\}$ which is also a majority. Thus L cannot win in equilibrium with $|y(L) - y_m| > 1/2$.

Next suppose that $|y(L) - y_m| \in (\frac{1}{4}, \frac{1}{2})$

If $x(L) \in \left[\frac{1}{2} - |y(L) - y_m|, \frac{1}{2} + |y(L) - y_m|\right]$ then R can defeat it with $x(R) = x(L)$. In this case R obtains a vote share of $2|y(L) - y_m| > 1/2$ which allows R to win. R prefers to do so since by mimicking L he obtains an extra payoff of K and his deviation does not involve any change in the policy implemented..

If $x(L) \leq \frac{1}{2} - |y(L) - y_m|$ then R can defeat L with $x(R) \in (\frac{3-2\mu}{4}, \frac{3}{4})$. To show this, note that the set of supporters of R is the interval $[\frac{x(R)+\mu x(L)}{1+\mu} - \frac{1-\mu}{1+\mu}|y(L) - y_m|, 1]$ whenever $x(R) > (1-\mu)(1 - |y(L) - y_m|) + \mu x(L)$. In addition, this number of voters constitutes a majority if and only if $x(R) < \frac{1+\mu}{2} + (1-\mu)|y(L) - y_m| - \mu x(L)$. This defines an interval of platforms that R can use to defeat L. Given the restrictions on $|y(L) - y_m|$ and the assumption on $x(L)$, this interval is at least as large as the interval $(\frac{3-2\mu}{4}, \frac{3}{4})$. Hence, any platform in this interval guarantees R a victory against $x(L)$. Note again that R prefers to win rather than to let L win because, in addition to obtaining K, the policy outcome is closer to his ideal point.

If $x(L) \geq \frac{1}{2} + |y(L) - y_m|$ then the best winning policy for R is $x(R) = \mu x(L) + (1-\mu)(\frac{1}{2} + |y(L) - y_m|)$. We show this by following the same procedure as above to define the set of R's supporters and then check when it constitutes a majority. Next we need to see whether R actually uses this winning strategy. For this to be the case it need to hold that

$$\begin{aligned} K - 1 + \mu x(L) + (1-\mu)(\frac{1}{2} + |y(L) - y_m|) &> -1 + x(L) \\ &\Leftrightarrow \quad x(L) < \frac{K}{1-\mu} + \frac{1}{2} + |y(L) - y_m|. \end{aligned}$$

Hence, L will not able to win with a $x(L)$ in $(\frac{1}{2} + |y(L) - y_m|, 1]$ if $K > (1-\mu)(\frac{1}{2} - |y(L) - y_m|)$. If $K < (1-\mu)(\frac{1}{2} - |y(L) - y_m|)$ we need to check

whether L prefers to win the election with such rightist policy. The best case scenario for L if he wants to win is when $x(L) = \frac{K}{1-\mu} + \frac{1}{2} + |y(L) - y_m|$. In that case, his payoff is just $-y(L) - \frac{\mu}{1-\mu}K - \frac{1}{2} - |y(L) - y_m|$. The best case scenario for L if in the contrary he decides to lose is to set $x(L) = \frac{1}{2} + |y(L) - y_m|$ given that that forces R to choose the same policy. His payoff is just $-y(L) - \frac{1}{2} - |y(L) - y_m|$, so he actually prefers to lose. ■

Proof of Proposition 2. From the previous proposition we know that in this case L wins in equilibrium. Suppose that $x(L)$ and $x(R)$ is an equilibrium outcome such that L wins and $x(R) < x(L)$. Then we must have $U_L(y(L), x(L), x(R)) = -y(L) + K - x(L)$. Consider that L chooses instead $x'(L) = x(R)$. Then by lemma 1 L obtains at least $1 - 2|y(L) - y(A)| > 1/2$ votes and his utility is $U_L(y(L), x(R), x(R)) = -y(L) + K - x(R)$.

Notice that $U_L(y(L), x(R), x(R)) = -y(L) + K - x(R) > -y(L) + K - x(L) = U_L(y(L), x(L), x(R))$ since we assumed that $x(R) < x(L)$. Thus, $x(L)$ and $x(R)$ such that $x(R) < x(L)$ cannot be part of an equilibrium strategy and we must have $x(L) \leq x(R)$.

Let us first characterize the sets of voters that vote for candidate L given $y(L)$, $x(L)$ and $x(R)$.

The set of voters with $x_i < x(L)$ that vote for L is given by all x_i such that

$$x_i < \frac{x(R) - \mu x(L)}{1 - \mu} - |y(L) - y_m| \equiv \underline{x_i}.$$

Similarly, the set of voters with $x_i > x(R)$ that vote for L is given by all x_i such that

$$x_i > \frac{x(R) - \mu x(L)}{1 - \mu} + |y(L) - y_m| \equiv \overline{x_i}$$

Since by proposition 2 $\frac{x(R) - \mu x(L)}{1-\mu} > x(R)$ then we have that $\overline{x_i} > x(R)$. Notice that if $\underline{x_i} < 0$ then $\overline{x_i} < 1$ for all $|y(L) - y_m| < \frac{1}{2}$.

Finally, the set of voters with $x(L) < x_i < x(R)$ that vote for L is given by all x_i such that

$$x_i < \frac{x(R) + \mu x(L)}{1 + \mu} - \frac{1 - \mu}{1 + \mu}|y(L) - y_m| \equiv \widetilde{x_i}$$

Since by proposition 2 $\frac{x(R) + \mu x(L)}{1+\mu} < x(R)$ then we have that $\widetilde{x_i} < x(R) < \overline{x_i}$. However, the comparison between $\underline{x_i}$ and $\widetilde{x_i}$ is not clear-cut. We have that $\underline{x_i} < \widetilde{x_i} < x(L)$ if and only if

$$x(R) - x(L) < (1 - \mu)|y(L) - y_m|.$$

Thus, two cases can emerge:

Case 1: If $x(R)-x(L)\geq(1-\mu)\left|y\left(L\right)-y_m\right|$ then we have that the votes that L obtains are given by $\widetilde{x_i}+\max\left\{0,1-\overline{x_i}\right\}$.

Case 2: If $x(R)-x(L)<(1-\mu)\left|y\left(L\right)-y_m\right|$ then we have that the votes that L obtains are given by $\max\left\{0,\underline{x_i}\right\}+\max\left\{0,1-\overline{x_i}\right\}$.

Suppose in the first place that $x(L)=0$. Then the number of votes that L receives are

$$\#L=\begin{cases}1-\left|y\left(L\right)-y_m\right|-\frac{x(R)}{1-\mu} & \text{if } x\left(R\right)<(1-\mu)\left|y\left(L\right)-y_m\right| \\ 1-\frac{2\mu}{1-\mu^2}x(R)-\frac{2}{1+\mu}\left|y\left(L\right)-y_m\right| & \text{if } x\left(R\right)\in\left[(1-\mu)\left|y\left(L\right)-y_m\right|,(1-\mu)(1-\left|y\left(L\right)-y\right.\right. \\ \frac{x(R)}{1+\mu}-\frac{1-\mu}{1+\mu}\left|y\left(L\right)-y_m\right| & \text{if } x\left(R\right)>(1-\mu)(1-\left|y\left(L\right)-y_m\right|)\end{cases}$$

that attains a minimum when $x\left(R\right)=(1-\mu)(1-\left|y\left(L\right)-y_m\right|$. The number of votes in that case is greater than $\frac{1}{2}$ if and only if

$$\left|y\left(L\right)-y_m\right|\leq\frac{1-3\mu}{4(1-\mu)}.$$

Note that if this holds, $x(L)=0$ is a winning strategy for L. Otherwise, there exists a platform $x(R)$ that can defeat $x(L)=0$.

Second, suppose that $\frac{1-3\mu}{4(1-\mu)}<\left|y\left(L\right)-y_m\right|>\frac{1}{4}$. Let us first show that any platform $x(L)\in(0,\frac{3\mu-1}{4\mu}+\frac{1-\mu}{\mu}\left|y\left(L\right)-y_m\right|)$ can be defeated by $x(R)=\frac{3-\mu}{4}$. First, note that we are in Case 1 since

$$x(R)-x(L)>(1-\mu)\left|y\left(L\right)-y_m\right|\Leftrightarrow x(L)<\frac{3-\mu}{4}-(1-\mu)\left|y\left(L\right)-y_m\right|$$

and in addition we have by assumption that

$$x(L)<\frac{3\mu-1}{4\mu}+\frac{1-\mu}{\mu}\left|y\left(L\right)-y\left(A\right)\right|<\frac{3-\mu}{4}-(1-\mu)\left|y\left(L\right)-y\left(A\right)\right|$$

where the last inequality follows from simple algebra. One can also show that our assumption on $x(L)$ also implies that $\overline{x_i}>1$ which means that the number of votes obtained by L is just $\widetilde{x_i}$ which in turn is smaller than $\frac{1}{2}$ if and only if

$$x\left(L\right)<\frac{3\mu-1}{4\mu}+\frac{1-\mu}{\mu}\left|y\left(L\right)-y_m\right|,$$

which holds by assumption. Hence, L is defeated if he chooses a platform in that interval. From the remainder, let us now show that $x(L)=\frac{3\mu-1}{4\mu}+\frac{1-\mu}{\mu}\left|y\left(L\right)-y_m\right|$ is a dominant strategy.

Again case we have to consider two cases:

1. Suppose that $x(R) > \frac{3\mu-1}{4\mu} - \frac{1-\mu^2}{\mu}\left|y\left(L\right) - y_m\right|$. In that case, the number of citizens who vote for the incumbent are given by $\min\left\{1, \overline{x_i}\right\} - \widetilde{x_i}$. We need to consider two subcases depending on the value of the extremes of this interval.

   i. If $\overline{x_i} > 1$ then R gets $1 - \widetilde{x_i}$ votes and wins the election if and only if

   $$\widetilde{x_i} < \frac{1}{2} \;\rightarrow\; x\left(R\right) < \frac{3-\mu}{4}$$

   Since $\overline{x_i} > 1$ if and only if $x\left(R\right) > \frac{3-\mu}{4}$ then this case cannot arise.

   ii. If $\overline{x_i} < 1$ then R gets $\overline{x_i} - \widetilde{x_i}$ votes. This number of votes is greater than $\frac{1}{2}$ if and only if $x\left(R\right) > \frac{3-\mu}{4}$. Since $\overline{x_i} < 1$ if and only if $x\left(R\right) < \frac{3-\mu}{4}$ again this case is not possible.

2. Suppose instead that $x(R) < \frac{3\mu-1}{4\mu} - \frac{1-\mu^2}{\mu}\left|y\left(L\right) - y_m\right|$.This means necessarily that $\overline{x_i} < 1$ and that the challenger collects votes in $(\max\{0, \underline{x_i}\}, \overline{x_i})$. We need to consider then two different subcases:

   i. If $\underline{x_i} < 0$ the challenger gets $\overline{x_i}$ votes and wins if and only if $x\left(R\right) \geq \frac{1+\mu}{4}$. But this leads to a contradiction because

   $$\frac{1+\mu}{4} > \frac{3\mu-1}{4\mu} - \frac{1-\mu^2}{\mu}\left|y\left(L\right) - y_m\right| \;\Leftrightarrow\; \frac{1-\mu}{4(1+\mu)} > -\left|y\left(L\right) - y_m\right|.$$

   ii. If $\underline{x_i} > 0$ then R gets $\overline{x_i} - \underline{x_i} = 2\left|y\left(L\right) - y_m\right|$ votes. So here R cannot win either.

Thus R cannot win the election for any $x(R)$ he may choose. Still, observe that $x(R) = \frac{3-\mu}{4}$ is a dominant strategy for her.

Since we have shown that L wins in equilibrium when $\left|y\left(L\right) - y_m\right| \leq \frac{1}{4}$, we have that L's most preferred best response is an equilibrium strategy. ■

Proof of Proposition 3. First, suppose that $\left|y\left(L\right) - y_m\right| > 1/2$. If $x(L) > x(R)$ in equilibrium, consider $x'(R)$ such that $x'(R) = x(L)$ and notice that: 1) in this case R obtains more than $\left|y\left(L\right) - y_m\right|$ votes, that is, more than half of the total; and 2) the equilibrium policy outcome is larger, therefore better off for R'. Thus this is a profitable deviation for R and it implies that $x(L) > x(R)$ cannot hold in equilibrium.

Since we know that in equilibrium $x(L) \leq x(R)$ R's best winning strategy is defined by $\overline{x}_i > 1$ and $\widetilde{x}_i < \frac{1}{2}$.This implies that

$$\overline{x_i} = \frac{x\left(R\right) - \mu x\left(L\right)}{1-\mu} + \left|y\left(L\right) - y_m\right| > 1$$

and

$$\widetilde{x_i} = \frac{x(R) + \mu x(L)}{1+\mu} - \frac{1-\mu}{1+\mu}|y(L) - y_m| < \frac{1}{2}$$

Thus the set of winnings strategies for R is defined by

$$(1-\mu)(1-|y(L)-y_m|)+\mu x(L) < x(R) < \frac{1+\mu}{2}+(1-\mu)|y(L)-y_m|-\mu x(L)$$

and among them R prefers the largest one $x(R) = \frac{1+\mu}{2}+(1-\mu)|y(L)-y_m|-\mu x(L)$.

The best response for L in this case is the largest possible value of $x(L)$. So that R's best response to it corresponds to its smallest possible value. Since in equilibrium we need to have $x(L) \leq x(R)$ then $x(L) \leq \frac{1+\mu}{2} + (1-\mu)|y(L)-y_m| - \mu x(L)$ implies $x(L) \leq \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L)-y_m|$. Thus in equilibrium $x(L) = x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L)-y_m|$.

Now suppose that $1/4 < |y(L)-y_m| < 1/2$. If $x(L) \in \left[0, \frac{1}{2} + |y(L)-y_m|\right]$ then R's best response, as in the previous proposition, is defined by $\overline{x}_i > 1$ and $\widetilde{x}_i < \frac{1}{2}$.

This implies that

$$\overline{x_i} = \frac{x(R) - \mu x(L)}{1-\mu} + |y(L) - y_m| > 1$$

and

$$\widetilde{x_i} = \frac{x(R) + \mu x(L)}{1+\mu} - \frac{1-\mu}{1+\mu}|y(L) - y_m| < \frac{1}{2}$$

Thus the set of winnings strategies for R is defined by $(1-\mu)(1-|y(L)-y_m|)+\mu x(L) < x(R) < \frac{1+\mu}{2} + (1-\mu)|y(L)-y_m| - \mu x(L)$

and among them R prefers $x(R) = \frac{1+\mu}{2} + (1-\mu)|y(L)-y_m| - \mu x(L)$

And the best response for L in this case is the largest possible value of $x(L)$. Since in equilibrium we need to have $x(L) \leq x(R)$ then $x(L) \leq \frac{1+\mu}{2} + (1-\mu)|y(L)-y_m| - \mu x(L)$ implies $x(L) \leq \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L)-y_m|$. Thus for $x(L) \in \left[0, \frac{1}{2} + |y(L)-y_m|\right]$ R's best response is $x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L)-y_m|$.

Given that if $x(L) \in \left[\frac{1}{2} + |y(L)-y_m|, 1\right]$ we have that R's best response is $x(R) \in \left[\frac{1}{2} + |y(L)-y_m|, 1\right]$, and for $x(L) \in \left[0, \frac{1}{2} - |y(L)-y_m|\right)$ we have that R's best response is $x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L)-y_m| < \frac{1}{2} + |y(L)-y_m|$, this implies that L's optimal strategy will not be in $\left[\frac{1}{2} + |y(L)-y_m|, 1\right]$.

Therefore the equilibrium if $\frac{1}{4} < |y(L)-y_m| < \frac{1}{2}$ is given by $x(L) = x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu}|y(L)-y_m|$. ■

Proof of Proposition 4. Let us start with the case when $|y(L) - y_m| \leq \frac{1-3\mu}{4(1-\mu)}$.Notice that it can emerge if and only if $\mu \leq \frac{1}{3}$. In that case the incumbent's payoff is increasing with $|y(L) - y_m|$ so his most preferred value of $y(L)$ in this range corresponds to $y(L) = y_m - \frac{1-3\mu}{4(1-\mu)}$. We already know from previous results that in this case that he will then set $x^*(L) = 0$

When $\frac{1-3\mu}{4(1-\mu)} \leq |y(L) - y_m| \leq \frac{1}{4}$, after plugging the incumbent's equilibrium platforms in the electoral issue, it is possible to rewrite his payoff as

$$V_L = -y_m + K - \frac{3\mu - 1}{4\mu} - \frac{1 - 2\mu}{\mu} |y(L) - y_m|, \tag{2}$$

which is decreasing with $|y(L) - y_m|$ as long as $\mu \leq \frac{1}{2}$ and increasing otherwise. In the former case, L's most preferred value of $y(L)$ corresponds to the minimal value of $|y(L) - y_m|$ in this range, that is, $y(L) = \max\left\{y_m - \frac{1-3\mu}{4(1-\mu)}, y_m\right\}$. Hence, if $\mu \leq \frac{1}{3}$ he will set again $y(L) = y_m - \frac{1-3\mu}{4(1-\mu)}$ (and then $x^*(L) = 0$) whereas if $\frac{1}{3} \leq \mu \leq \frac{1}{2}$ he must set $y(L) = y_m$ which in turn implies that $x^*(L) = \frac{3\mu-1}{4\mu}$.

The third case occurs when $\mu \geq \frac{1}{2}$. Then (2) is increasing with $|y(L) - y_m|$. Thus while staying in this range his most preferred value of $y(L)$ corresponds to the one that maximizes $|y(L) - y_m|$, that is, $y(L) = y_m - \frac{1}{4}$, that from previous results it implies $x^*(L) = \frac{1}{2}$. ■

Proof of Lemma 2. We know from previous results that if the incumbent decides to lose by setting $|y(L) - y_m| > \frac{1}{4}$, the challenger will win the election and set $x(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu} |y(L) - y_m|$. In that case, the incumbent receives the payoff

$$V_L = -y_m - \frac{1}{2} - \frac{2\mu}{1+\mu} |y(L) - y_m|,$$

which is increasing in $|y(L) - y_m|$. Thus while staying in this range, his most preferred value of $y(L)$ corresponds to the one that maximizes $|y(L) - y_m|$, that is, $y^*(L) = 0$, which implies that the challenger's best response in this case is $x^*(R) = \frac{1}{2} + \frac{1-\mu}{1+\mu} y_m$. ■

Proof of Proposition 5. Previous results show that since $y_m < \frac{1}{4}$ implies that $|y(L) - y_m| < \frac{1}{4}$ then L prefers to win in this case.

If $y_m \geq \frac{1}{4}$, if the incumbent decides to lose then he receives a payoff equal to

$$V_L = -\frac{1}{2} - \frac{1-\mu}{1+\mu} y_m.$$

If the incumbent decides to use his best winnings strategy then he receives a payoff equal to

when $\mu \leq \frac{1}{3}$ his payoff boils down to

$$V_L = -y_m + K - \frac{3\mu - 1}{4(1-\mu)} \text{ if } \mu \leq \frac{1}{2} \tag{3}$$

and

$$V_L = -y_m + K - \frac{1}{4} \text{ if } \mu \geq \frac{1}{2} \tag{4}$$

Thus, when $\mu \geq \frac{1}{2}$ he prefers to use his winning strategy as long as $-y_m + K - \frac{1}{4} \geq -\frac{1}{2} - \frac{1-\mu}{1+\mu} y_m$, that is, for

$$K > \frac{2\mu}{1+\mu} y_m - \frac{1}{4}$$

Notice that this value is strictly positive for all values of $\mu \in [0,1]$ as long as $y_m > \frac{3}{8}$. For $\frac{1}{4} \leq y_m \leq \frac{3}{8}$ we will have that the incumbent will decide to use a winning strategy for all values of K whenever $\frac{2\mu}{1+\mu} y_m - \frac{1}{4} > 0$, that is, $\mu > \frac{1}{8y_m - 1}$. Notice that the incumbent decides to win for all K whenever $y_m = \frac{1}{4}$. Furthermore, the incumbent always decides to forgo reelection for some positive values of K whenever $y_m > \frac{3}{8}$.

Similarly, when $\mu \leq \frac{1}{2}$ he prefers to use his winning strategy as long as $-y_m + K - \frac{3\mu - 1}{4(1-\mu)} \geq -\frac{1}{2} - \frac{1-\mu}{1+\mu} y_m$, that is, for

$$K > \frac{5\mu - 3}{4(1-\mu)} + \frac{2\mu}{1+\mu} y_m,$$

Notice that this value is strictly negative for small values of μ (in particular for all $\mu \leq \frac{1}{3}$). For those values the incumbent decides to win the election for all K. The set of values of K for which the incumbent decides to use a winning strategy is smaller for larger values of μ in this area. ■

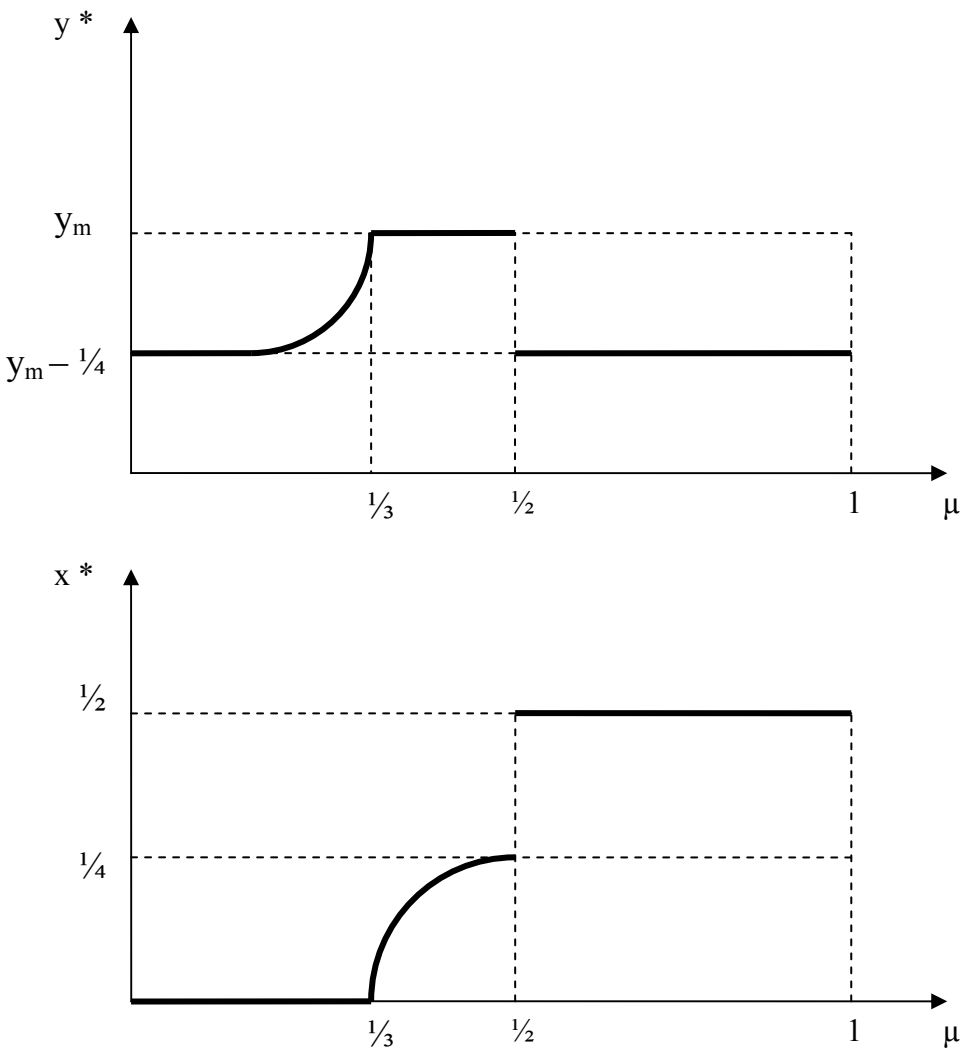

Figure 1: Incumbent's best winning strategies.

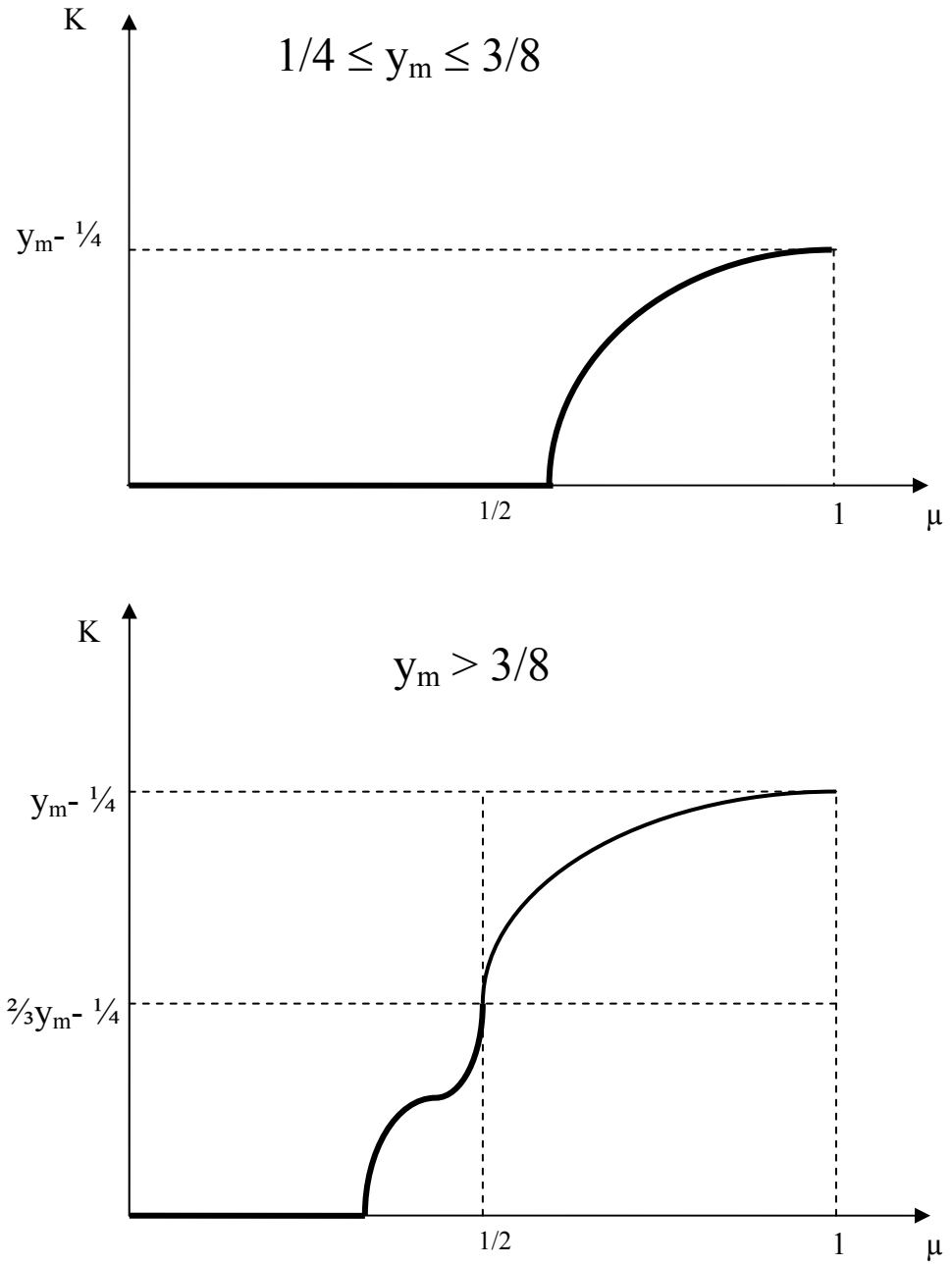

Figure 2 : Minimal values of K for which the incumbent prefers to use a winning strategy in equilibrium.