# CHOOSING TAX SCHEDULES UNDER A VEIL OF IGNORANCE

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## 1. PRELIMINARIES

The issue is to design an appropriate tax scheme for a group of agents with taxable incomes. Our objective is to identify impartial tax schemes when taxable incomes are "morally arbitrary"(Rawls 1971). We take Harsanyi's veil of ignorance approach (Harsanyi 1953, 1955, 1977) to conceptualize an impartial viewpoint. An impartial observer (IO) contemplates becoming, with equal probability, one of the agents in the group, for whom the tax burden is going to be allocated, and obtaining one of the possible taxable incomes under an income distribution with equal probability too. The decision by such an IO is considered as being impartial because he would not represent the interest of a particular person only or a particular income group only. The veil of ignorance necessitates the IO to reflect the interests of all agents equally well and the interests of all income groups equally well too.

The group $N$ consists of $n$ agents and is denoted by $N \equiv \{1,\dots,n\}$. Let $y_i$ be the taxable income of each member $i \in N$. The tax revenue to be collected is fixed at $T > 0$ and the total taxable incomes is not short of collecting this amount $T$, i.e. $\sum_{i\in N} y_i \geq T$. The profile of taxable incomes $(y_1,\dots,y_n)$ and the tax revenue $T$ constitute a tax problem. A tax scheme associates with each problem a profile of tax payments $(t_1,\dots,t_n)$ with $\sum_{i=1}^{n} t_i = T$ and for all $i$, $0 \leq t_i \leq y_i$. Well-known tax schemes are the *head tax* that distributes the tax burden equally subject to no agent paying more more than her income, the *leveling tax* that equalizes post-tax income across agents subject to no agent being subsidized, and the *flat tax* that equalizes tax rates across agents. Formally, under the head tax, $t_i = \min\{-1/\lambda, y_i\}$ for some $\lambda \in \mathbb{R}_-$ with $\sum_{i\in N} \min\{-1/\lambda, y_i\} = T$; under the leveling tax, $t_i = \max\{y_i - 1/\lambda, 0\}$ for some $\lambda \in \mathbb{R}_+$ with $\sum_{i\in N} \max\{y_i - 1/\lambda, 0\} = T$; under the flat tax, $t_i = \lambda y_i$ for some $\lambda \in [0,1]$ with $\sum_{i\in N} \lambda y_i = T$. Note that under the leveling tax, for all $i,j,k$ with $t_i,t_j > 0$ and $t_k = 0$, $y_k \leq y_i - t_i = y_i - t_j$. We

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refer readers to O'Neill (1982), Young (1987, 1988), and Thomson (2003) among others for comprehensive investigations of equitable tax schemes.

Throughout the paper, we will focus on "anonymous" tax schemes and apply them to tax problems that can be obtained from reshuffling $n$-pretax incomes, $y_1, \ldots, y_n$. Thus tax allocations for all profiles $(y_{\pi(i)})_{i \in 1,\ldots,n}$ generated by all permutations $\pi\colon N \to N$ can be written as an $n \times n$ matrix $(t_{ij})_{i,j \in \{1,\ldots,n\}}$ such that for all $i, j \in \{1, \ldots, n\}$, $t_{ij}$ is the tax payment of agent $i$ when his pre-tax income is $y_j$. Due to the feasibility constraint, $(t_{ij})_{i,j \in \{1,\ldots,n\}}$ satisfies $\sum_{i=1}^{n} t_{i\pi(i)} = T$ for all permutations $\pi\colon N \to N$ and due to anonymity, for all $i, j, k$, $t_{ij} = t_{kj}$.

## 2. RESULT

Each agent $i$ has a vNM preferences over wealth lotteries represented by utility index function $u_i(\cdot)$ and a level of pre-tax income $y_i$. The impartial observer has an equal chance of being in the position of agent $i$ with a particular income $y_j$ among $n$ pre-tax incomes. Denote by $(i, y_j, W_{ij})$ the *extended prospect* of being in the position of person $i$ with pre-tax income $y_j$ and post-tax income $W_{ij}$ (hence facing a tax $t_{ij} = y_j - W_{ij}$).

The problem the IO faces is to determine the "fair" tax allocation. Following Harsanyi, we propose that the IO possesses a vNM preferences over lotteries on extended prospects. Let $U(i, y_j, W_{ij})$ be IO's utility index function. When the distribution of post-tax income over the set of person-income pairs $(i, y_j)$ is determined as $(W_{ij})_{i,j=1,\ldots,n}$, the veil of ignorance gives the IO the 'birth lottery'

$$\begin{pmatrix} \frac{1}{n^2} \circ (1, y_1, W_{11}) & \cdots & \frac{1}{n^2} \circ (1, y_n, W_{1n}) \\ \vdots & \vdots & \vdots \\ \frac{1}{n^2} \circ (n, y_1, W_{n1}) & \cdots & \frac{1}{n^2} \circ (n, y_n, W_{nn}) \end{pmatrix},$$

where for all $i, j = 1, \ldots, n$, the extended prospect $(i, y_j, W_{ij})$ is realized with equal probability $1/n^2$ . Now the utility the IO receives from this lottery is, by the expected utility property, equal to

$$\sum_{i,j=1}^{n} \frac{1}{n^2} \cdot U(i, y_j, W_{ij}). \tag{1}$$

As in Harsanyi (1953, 1955, 1977), Rawls (1971), and Dworkin (1981a,b), we view that the impartial nature of the veil of ignorance qualifies the optimal decision by the IO as being a *fair* tax allocation. The decision problem of the IO is to find the distribution of wealth that maximizes the expected utility of (1) subject to the feasibility constraint that the total post-tax wealth equals to $Y - T$ at each realization of type-distributions $(i, y_{\pi(i)})_{i=1,\ldots,n}$ associated with each permutation $\pi : N \to N$, that is, $\sum_{i=1}^{n} W_{i\pi(i)} = Y - T$. For all $i, j \in \{1, \ldots, n\}$, let $t_{ij} \equiv y_j - W_{ij}$. The feasibility constraint can be stated as $\sum_{i=1}^{n} t_{i\pi(i)} = T$ for all permutations $\pi \colon N \to N$. Let $\mathscr{F}(N, y, T)$ be the set of all tax allocations satisfying the feasibility constraint as well as the condition that for all $i, j$, $0 \le t_{ij} \le y_j$.

As in Harsanyi (1953, 1955, 1977), when the IO's position as a particular person $i$ with a particular level of taxable income $y_j$ is fixed, the IO's preference coincides with $i$'s preferences over wealth lotteries.

**The Principle of Acceptance**: For all $i, j \in \{1, \ldots, n\}$, $U(i, y_j, \cdot)$ represents the same vNM preferences on wealth lotteries as $u_i(\cdot)$ represents.

This axiom requires that the IO's vNM preferences do not depend on the pre-tax income level, which means that the IO cares only about the final post-tax wealth, and the level of pre-tax income that has leaded to that final wealth does not matter to the IO.

By the principle of acceptance and the vNM theorem, for all $i$, there exist $a_i > 0$ and $b_i$ such that for all $j$ and all $W$,

$$U(i, y_j, W) = a_i \cdot u_i(W) + b_i \tag{2}$$

Substituting formula (2) into (1), we have

$$\sum_{i,j=1}^{n} \frac{1}{n^2} \cdot U(i, y_j, W_{ij}) = \sum_{i,j=1}^{n} \frac{1}{n^2} \left(a_i \cdot u_i(W_{ij}) + b_i\right) \tag{3}$$

$$= \frac{1}{n^2} \cdot \sum_{i,j=1}^{n} a_i \cdot u_i(W_{ij}) + \frac{1}{n} \cdot \sum_{i=1}^{n} b_i. \tag{4}$$

Thus maximizing (3) is equivalent to maximizing $\sum_{i,j} a_i \cdot u_i(W_{ij})$. Indeterminacy of $a_i$ in Harsanyi (1953, 1977) will not pause any difficulty in identifying fair tax schemes since any value for $a_i$ will generate the same and unique tax scheme preferred by the IO.[1]

As long as the IO considers a tax scheme to determine the tax allocation, tax allocations depend only on taxable incomes, and risk preferences under the veil of ignorance do not play a role in the allocation of tax burden (as noted at the end of Section 1). Thus we may focus on tax allocations satisfying:

**Independence of Risk Preference**: For all $i, j, k \in \{1, \ldots, n\}$, $t_{ij} = t_{kj}$. The

IO's problem of choosing the best tax scheme under the veil of ignorance

[1] See Moreno-Ternero and Roemer (2008) for the determination of $a_i$ through the addition of "objective comparability". See also Karni and Weymark (1998) for a recent development of Harsanyi's impartial observer theorem.

now becomes the problem of choosing a tax profile $(t_1,\dots,t_n)$ to maximize:

$$\max\left\{\sum_{i,j=1}^{n} a_i\cdot u_i(y_j-t_j) : t\in\mathscr{F}(N,y,T)\right\} \tag{5}$$

Now we are ready to state the main result.

**Theorem 1.** *If all agents are risk averse, and at least one of them strictly, then a Harsanyi's impartial observer would select the leveling tax as the most preferred tax scheme for such a group.*

*Proof.* Throughout the proof, we assume for simplicity that individual utility index functions $u_i(\cdot)$ are differentiable. The Lagrangian associated to (4) is given by

$$\mathscr{L}(\cdot) = \sum_{j=1}^{n}\sum_{i=1}^{n} a_i\cdot u_i(y_j-t_j)+\lambda\left(T-\sum_{j=1}^{n} t_j\right)+\sum_{j=1}^{n}\mu_j(y_j-t_j)+\sum_{j=1}^{n}\gamma_j t_j.$$

Provided all agents are risk averse, with one of them strictly, $\sum_{i,j=1}^{n} a_i\cdot u_i(\cdot)$ is strictly concave, and the above program can be solved by the Kuhn-Tucker theorem (e.g., Mas-Colell et al., 1995). We consider two types of solutions to (4) for which the first constraint binds, i.e., solutions $t\in\mathscr{F}(N,y,T)$ such that $\sum_{j=1}^{n} t_j = T$. The first type refers to interior solutions (i.e., those for which $0<t_j<y_j$ for all $j\in N$) and the second one to corner solutions (i.e., those for which either $t_j=0$ or $t_j=y_j$ for some $j\in N$).

Formally,

**Case 1**. $\mu_j=\gamma_j=0$ for all $j\in N$ (the interior solutions).

In this case, we would have to solve the following system of equations:

$$-\sum_{i=1}^{n} a_i\cdot u_i'(y_j-t_j)=\lambda \text{ for all } j\in N.$$

Let $j,k\in N$. Then,

$$\sum_{i=1}^{n} a_i\cdot u_i'(y_j-t_j)=\sum_{i=1}^{n} a_i\cdot u_i'(y_k-t_k).$$

Since $\sum_{i,j} a_i u_i(\cdot)$ is strictly concave, it follows that

$$y_j-t_j=y_k-t_k.$$

In other words, post-tax incomes are equalized across agents.

**Case 2**. $\mu_j\neq 0$ or $\gamma_j\neq 0$ for some $j\in N$ (the corner solutions).

In this case, we would have to solve the following system of equations:

$$-\sum_{i=1}^{n} a_i\cdot u_i'(y_j-t_j)-\lambda-\mu_j+\gamma_j=0 \text{ for all } j\in N.$$

Thus, it is clear that for all those agents $j \in N$ with interior solutions, i.e., $\mu_j = \gamma_j = 0$, then we have $-\sum_{i=1}^{n} a_i \cdot u_i'(y_j - t_j) = \lambda$. Then Case 1 concludes that these agents will have equal post-tax incomes. Let now $j \in N$ be such that $\mu_j > 0$. Then, $t_j = y_j$, which implies that $0 \geq -\sum_{i=1}^{n} a_j \cdot u_j'(0) = \lambda + \mu_j > 0$, a contradiction. Finally, let $j \in N$ be such that $\gamma_j \neq 0$. Then, $t_j = 0$ and, therefore, $-\sum_{i=1}^{n} a_i \cdot u_j'(y_j) = \lambda - \gamma_j$. As $-\sum_{i=1}^{n} a_i \cdot u_i'(y_k - t_k) = \lambda$, for all $k$ such that $t_k > 0$, it follows that $y_j = y_j - t_j \leq y_k - t_k$ for all $j,k$ such that $t_k > 0 = t_j$. □

## REFERENCES


[1] Dworkin, R. (1981a), "What is equality? Part 1: Equality of welfare," Philosophy and Public Affairs, 10, 185-246.

[2] Dworkin, R. (1981b), "What is equality? Part 2: Equality of resources," Philosophy and Public Affairs, 10, 283-345.

[3] Haysanyi, J. C. (1953), "Cardinal utility in welfare economics and in the theory of risk-taking," Journal of Political Economy, 61, 434-435.

[4] Harsanyi, J. C. (1977), Rational behavior and bargaining equilibrium in games and social situations, Cambridge University Press, Cambridge.

[5] Karni, E. and J. Weymark (1998), "An informationally parsimonious impartial observer theorem," Social Choice and Welfare, 15, 321-332.

[6] Moreno-Ternero, J. D., J. E. Roemer (2006), "Impartiality, priority, and solidarity in the theory of justice," Econometrica, 74, 1419-1427.

[7] Moreno-Ternero, J. D., J. E. Roemer (2008), "The veil of ignorance violates priority," Economics and Philosophy, 24, 233-257.

[8] O'Neill, B. (1982), "A problem of rights arbitration from the Talmud," Mathematical Social Sciences 2:345–371.

[9] Rawls, J. (1963), "The sense of justice," Philosophical Review, 72, 281-305.

[10] Rawls, J. (1971), A theory of justice, Cambridge, MA: Harvard University Press.

[11] Thomson, W. (2003), "Axiomatic and game-theoretic analysis of bankruptcy and taxa- tion problems: a survey," Mathematical Social Sciences 45:249-297.

[12] Young, P. (1987), "On dividing an amount according to individual claims or liabilities," Mathematics of Operations Research 12:398-414.

[13] Young, P. (1988), "Distributive justice in taxation," Journal of Economic Theory 44: 321–335.



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