EQUALITY OF OPPORTUNITY: A PROBABILISTIC APPROACH

(Preliminary version)

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Abstract

Modern economic theory of justice recognizes that individual's outcome is a function of effort and circumstances. The measurement of equality of opportunity deals with those inequalities of outcome due to differential circumstances. Standard approaches assume an *a priori* division of individuals into types of circumstances. However, this assumption may be inadequate if some unobserved circumstances like talent or luck are significant. We propose a probabilistic model that considers this uncertainty about the true partitioning of the population. In fact, the standard equality of opportunity model can be seen as a particular case of our model when the probabilities of belonging to a type are fixed (to one or zero). We propose the finite mixture technique as the method to estimate this model.

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Key words: equality of opportunity; circumstances; responsibility; finite mixture models.

1. Introduction

Anthony Martin immigrated from Sicily (Italy) to the United States in 1895, and there he worked like a fireman. Natalie Della Gavarante inmigrated from Liguria (Italy) to the United States in 1897, and there she worked as a part-time abortionist. Both were non-educated inmigrant workers. Any children from this couple would have an obvious initial disadvantage to compete for a position in society. As a matter of fact, standard approaches to the measurement of equality of opportunity would likely classify those (hypothetic) children in the most disadvantage type of society, according to the observed circumstances: parents' education and parents' nationality. However, the only child of this couple would be born with a gift, his voice, and his name would be Frank Sinatra.[1] An unobserved circumstance (talent to sing) would make the difference to compete in society.

Tracing back to Rawls (1971), Sen (1980 and 1985), Dworkin (1981a and 1981b), Arneson (1989) and Cohen (1989), the topic of equality of opportunity (see, Roemer 1993, 1998, 2002 and 2003, Van de Gaer, 1993, Peragine 2002 and 2004, Ruiz-Castillo, 2003, Betts and Roemer 2006, Moreno-Ternero, 2007 and Fleurbaey and Maniquet, 2007) is concerned with the level-the-playing-field principle. Individual's accomplishment is a function of effort (like investment in human capital, number of hours worked and occupational choice) and circumstances (like genes, socioeconomic and cultural background, race and luck). However, individuals are only responsible for their effort as circumstances are beyond the individual's control. Society should, therefore, compensate individuals who suffer from bad circumstances but let the agents exercise their responsibility without trying to distort their outcomes. In other

[1] See http://en.wikipedia.org/wiki/Frank_Sinatra for a complete biography of Frank Sinatra.

words, opportunities must be equalized (*levelling the playing field*) before the competition starts, but after it begins, individuals are on their own.

A first difficulty in this theory is to figure out the line separating effort from circumstances. If we suppose that society has reached a political agreement on the list of circumstances then a second difficulty appears: how to make outcome comparisons between individuals. To do this, people with the same set of circumstances are grouped into types and then, comparisons across types of people with different circumstances are carried out. Types are constructed according to observed circumstances typically parents' education. But, the fact that observed circumstances are not the only factor explaining differences in individual characteristics beyond her/his control makes equality of opportunity difficult to measure. Unobserved circumstances like talent or luck[2] may ruin the classification of people into types according to observed circumstances, bear in mind the example of Frank Sinatra. Therefore, types are not easily identifiable so it can be rather imprudent or risky to classify people *a priori* as members of a type. Provided that there is an informational problem, it would be better from a methodological point of view not to assume an aprioristic classification of individuals (deterministic approach), but rather to attempt to estimate the membership of specific types (probabilistic approach).

As a possible solution, we propose the estimation of a finite mixture model (see, for example, McLachlan and Peel, 2000 and the references therein).[3] This technique allows the individuals in the sample to be classified into different groups and enables us to evaluate the probability of a specific individual belonging to a particular group, even without enough information about his/her membership, or if these groups are not directly observed. That is, if inequality of

[2] See Hurley (2002) and Roemer (2003) for a discussion on the role for luck in the equality of opportunity measurement.

[3] A related technique is latent class models (see, among others, Aitken and Rubin, 1985 and Greene, 2002).

opportunity exists and observable individual circumstances cannot fully determine whether an individual is a member of a type, we can at least evaluate the probability of the individual being a member of such a type using finite mixture models. This task will be undertaken without any *a priori* classification, but instead by utilizing data according to probabilistic criteria.

The proposed model is a generalization of standard approaches. The standard equality-of-opportunity model (see, Roemer 1993, 1998 and 2003, Van de Gaer, 1993, Peragine 2002 and 2004, and Moreno-Ternero, 2007) is deterministic as it assumes fixed membership probabilities (either zero or one). However, the proposed approach is probabilistic and includes the standard model as a particular case, when the estimated probabilities converge for all individuals to either zero or one. We think that any empirical approach should be able to control for this individual heterogeneity without assuming that all the members with a particular set of observed circumstances belong to a particular type.

The paper is organized as follows. Section 2 provides a brief review of the Roemer's pragmatic approach. Section 3 presents a probabilistic model for the measurement of equality of opportunity. The finite mixture model is examined in Section 4, and Section 5 presents the main empirical results. The final section includes some concluding remarks.

2. The deterministic model

This section presents the Roemer's pragmatic approach to equality of opportunity (Roemer 1993, 1998, 2002 and 2003). First, we give some notation and definitions assuming a continuous framework.

Let the members of a population enjoy a certain kind of advantage *u*, for example, income, life expectancy or wage-earning capacity. This advantage is a function of the amount of effort *e* they expend and the amount of resources they consume *r*. Moreover, population is partitioned into a set of types $t=\{1, \ldots, T\}$, where all individuals in a type have the same set of circumstances. Therefore, the achieved level of advantage enjoyed by an individual of type *t* is $u^t(r, e)$.

Suppose that there exists an amount *w* per capita of the resource to allocate among individuals. In order to achieve equality of opportunity, society must choose a policy for allocating *w* among population. Let $\varphi^t : \Re_+ \to \Re_+$ be an allocation rule that indicates the amount of resource that an individual of type *t* receives with respect to the effort she/he exerts. Then, the *T*-tuple $\varphi = (\varphi^1, \ldots, \varphi^T) \in \Gamma$ is the policy of the social planner, where Γ is the set of feasible policies.

Finally, assume that the distribution of effort exerted by individuals of type *t* is G^t_φ, for all $t=1,\ldots, T$ and that $e^t_\varphi(\pi)$ is the level of effort exerted by the individual at the π^{th} quantile of that effort distribution when facing the policy φ.[4] We may hence define the indirect advantage function as follows:

$$v^t_\varphi(\pi) = u(\varphi^t(e^t_\varphi(\pi)), e^t_\varphi(\pi)), \quad (1)$$

[4] Note that the level of effort depends on the whole policy not just the allocation rule for type *t* (see Roemer, 2003, for this generalization).

where $\pi \in [0,1]$.

According to the equality of opportunity Roemer's pragmatic approach two persons of different types have tried equally hard if and only if they are on the same rank of their respective effort distributions.[5] Then, a policy that equalizes opportunities is a policy that equalizes advantage across types, for given quantiles of effort expended. At this point, Roemer proposes to maximize the minimum level of advantage, across all types, of individuals who exert the same degree of effort π^{th}:

$$\varphi_\pi^R = \arg\max_{\varphi\in\Gamma}\left\{\min_{t\in T} v_\varphi^t(\pi)\right\}. \tag{2}$$

Two other proposals are: The average of the policies, $\varphi^R = \int_0^1 \varphi_\pi^R d\pi$, (see Roemer, 2002) and the policy that maximizes the minimum of type-averages of the objective, over types, $\varphi^G = \arg\max_{\varphi\in\Gamma} \min_{t\in T}\left\{\int_0^1 v_\varphi^t(\pi)d\pi\right\}$ (see Van De Gaer, 1993).

Typically, we will have a different policy for each quantile. To adopt a compromise between such hypothetical bundle of policies, Roemer proposes to give the same weight to each policy, hence:

$$\varphi^R = \arg\max_{\varphi\in\Gamma}\left\{\int_0^1 \min_{t\in T} v_\varphi^t(\pi)d\pi\right\}. \tag{3}$$

[5] The use of rank π as an interpersonally comparable measure of effort is precisely justified in Roemer (2003).

Moreover, if we assume that outcomes within a type are monotonically increasing in effort and that all policies treat the members of any type identically, then those at the π^{th} quintile of the effort distribution are exactly those at the π^{th} quintile of the outcome distribution. That is, $F_{\varphi}^{t}(v_{\varphi}^{t}(\pi)) = \pi$, where F_{φ}^{t} is the outcome distribution function in type *t* at policy φ. As a consequence, assuming that F_{φ}^{t} is strictly increasing, the program in (3) can be written as:

$$\varphi^{R} = \arg\max_{\varphi \in \Gamma} \left\{ \int_{0}^{1} \min_{t \in T} F_{\varphi}^{t^{-1}}(\pi) d\pi \right\} \tag{4}$$

In short, to calculate the equal-opportunity policy we need only to know the outcome distribution for each type at a given policy.

Recently, different approaches have been proposed in the literature to take into consideration all the outcomes not just the minimum outcome, for each quantile. Thus, Peragine (2004) proposes to calculate the Generalized Lorenz Curve for each quantile in order to make ordinal comparisons between different distributions of outcome. Moreno-Ternero (2007) provides an alternative cardinal mechanism to construct equality of opportunity policies. In particular, he suggests measuring the inequality between outcomes in the same quantile, across types. The proposed equal-opportunity policy is then the minimization of the inequality average across quantiles. The program for this proposal is the following:

$$\varphi^{M} = \arg\min_{\varphi \in \Gamma} \int_{0}^{1} I(v_{\varphi}^{1}(\pi), \dots, v_{\varphi}^{T}(\pi)) d\pi \tag{5}$$

where $I(\cdot)$ is an inequality index.

However, all these approaches distinguish *a priori* which individuals are in each type. This *a priori* classification of individuals does not reflect the inherent uncertainty of inequality of opportunity. In the following section, we propose a method to overcome this problem.

3. A probabilistic model to measure equality of opportunity

In this section we present a probabilistic model of equality of opportunity. In the next section we propose the finite mixture model as a method to estimate it.

Observed circumstances like parents education are certainly important variables to determine people's group. However, non-observed circumstances like luck are also relevant variables. As a matter of fact, in some contexts they might be the main factor accounting for individual's type. A result of this is that group membership is unknown and each individual with a given (observed) circumstances may have a nonzero probability of belonging to any class.

Let us assume that individuals belong to one of C classes, $c = 1, ..., C$ where class membership is unknown.[6] Then, any individual may probabilistically belong to all those C classes though the unique criteria for computing the probability of class membership has to be individual's observed circumstances. Otherwise, individual effort would be mixed up with individual circumstances. A consequence of this is that all individuals in type t, for $t = 1,\ldots,T$, will have the same probability of class membership as they have the same circumstances. Let $P_t(\delta_c)$ represent the probability of an individual in type t being a member of class c where δ_c is a vector of parameters. In the next section these class probabilities are parameterized as a

[6] We will call the elements of the set $c = \{1,\ldots,C\}$ classes onwards to distinguish them from the set of types $t = \{1,\ldots,T\}$. Note, however, that the real types in our probabilistic model are the elements of the set c.

multinomial logit model depending exclusively on individual circumstances. Let $v_\varphi^t \in \Re$ be a random variable that represents the outcome in type t when facing the policy φ with distribution function $F_\varphi^t(v)$ on $\Re$. Of course, individual observations in type t, for t =1,…,T, are realizations of this random variable. Finally, let $v_\varphi^c \in \Re$ be a random variable that represents the outcome in class c, for c =1,…,C, when facing the policy φ with distribution function $F_\varphi^c(v)$ on $\Re$. In the next section, we assume that $F_\varphi^c(v)$ is a normal distribution function, though other distribution functions can be considered. Then, outcome in type t can be written as a convex function of outcome in class c, for c =1,…,C, where the weights are the probabilities of class membership, that is:

$$v_\varphi^t = \sum_{c=1}^{C} v_\varphi^c \cdot P_t(\delta_c)\,. \tag{6}$$

Note that in the deterministic case, all the probabilities other than one are zero so $v_\varphi^t = v_\varphi^c$ where T = C.

Taking this into consideration, we can rewrite program (3) in the following way:[7]

$$\varphi^{PR} = \arg\max_{\varphi\in\Gamma}\left\{\int_0^1 \min_{t\in T}\left[\sum_{c=1}^{C} v_\varphi^c(\pi)\cdot P_t(\delta_c)\right]d\pi\right\} \tag{7}$$

Now, if we assume that the distribution function of v_φ^c is normal, that is, $F_\varphi^c(v) = \Phi_\varphi^c(v)$, we can also adapt the program in (4) to our probabilistic framework as follows:

[7] The programs in footnote 7 can be also written, *mutatis mutandis*, according to expression (6).

$$\varphi^{PR} = \arg\max_{\varphi \in \Gamma} \left\{ \int_0^1 \min_{t \in T} \left[\sum_{c=1}^{C} \Phi_{\varphi}^{c\;-1}(\pi) \cdot P_t(\delta_c) \right] d\pi \right\} \tag{8}$$

According to this program, we only need to estimate the probabilities of class membership and the distribution function for each class. To obtain these estimates we apply a statistical method called finite mixture model in the next section. Before we do this, we give some intuitions of our probabilistic proposal. Let us write the Moreno-Ternero program (see expression 5) in terms of the *C* classes:

$$\varphi^{PM} = \arg\min_{\varphi \in \Gamma} \int_0^1 I \left[\sum_{c=1}^{C} v_{\varphi}^{c}(\pi) \cdot P_1(\delta_c), \ldots, \sum_{c=1}^{C} v_{\varphi}^{c}(\pi) \cdot P_T(\delta_c) \right] d\pi \tag{9}$$

Assume that there are six types (T = 6) which correspond with two different circumstances: parent's education (university degree, high school degree and school degree or less) and intelligence quotient (above and below average). Now, suppose that we have estimated the corresponding finite mixture model and that the matrix of estimated probabilities of class membership is the following:

Matrix 1

	T1	**T2**	**T3**	**T4**	**T5**	**T6**
C1	**1**	0	0	0	0	0
C2	0	**1**	0	0	0	0
C3	0	0	**1**	0	0	0
C4	0	0	0	**1**	0	0
C5	0	0	0	0	**1**	0
C6	0	0	0	0	0	**1**

where T refers to the type and C refers to the class. Each element in the matrix represents the probability for individuals in the type t to belong to the class c. This matrix of probabilities is the identity matrix what means that our probabilistic model collapses to the deterministic one, so the program in (9) becomes the program in (5). It is easy to see, therefore, that the deterministic model is a particular case of the probabilistic model.

Now, suppose we obtain, after a finite mixture model has been estimated for another population with the same number of types, a very different matrix of probabilities:

Matrix 2

	T1	**T2**	**T3**	**T4**	**T5**	**T6**
C1	0.30	0.30	0.30	0.30	0.30	0.30
C2	0.10	0.10	0.10	0.10	0.10	0.10
C3	0.45	0.45	0.45	0.45	0.45	0.45
C4	0.07	0.07	0.07	0.07	0.07	0.07
C5	0.08	0.08	0.08	0.08	0.08	0.08

In this case, only five classes are found and every type t has the same probability to belong to any class c. Then, we have $\sum_{c=1}^{C} v_{\varphi}^{c} \cdot P_1(\delta_c) = ... = \sum_{c=1}^{C} v_{\varphi}^{c} \cdot P_T(\delta_c)$ and no inequality of opportunity is presented in the population. The interpretation of this result is quite clear: to have a particular set of observed circumstances is irrelevant to belong to a particular class as the columns of the matrix 2 are equal. Therefore, there is not inequality of opportunity due to those observed circumstances.

Finally, in this last example, there are five classes though the observed circumstances cannot explain why. The existence of more than one class, when a matrix like matrix 2 is found, implies that there are some relevant circumstances which produce the appearance of C classes but that are not been observing. For example, suppose that the outcome variable is income and that the mean income for the last class, C5, is close to zero. This class is, then, collecting the observations with incomes equal or close to zero presented in almost every sample of income. If this is the case, we would say under the deterministic model that there are individuals who do not exert any effort in every type. However, under the probabilistic model we would say that there are unobserved circumstances that cause individuals not to exert any effort.

4. The estimation method

Finite mixture models are a statistical method for finding subtypes of related cases (latent classes) from multivariate categorical data (see McLachlan and Peel, 2000). For example, the density function in Figure 1a can be interpreted as a mixture model of two normal density functions with different means and variances (Figure 1b). In our case, they can be used to find different classes, allowing us to find different segments of circumstances and classify individuals into these classes.

The deterministic approach (see Section 2) use predefined types of individuals to estimate equality of opportunity. In our model, it is unnecessary to know beforehand which group produced an observation because both the density functions and the probability of membership of a particular group are estimated *simultaneously*.[8] Individuals are probabilistically separated into several classes, and a density function is estimated for each class. As each observation may have a nonzero probability of belonging to any class, all the observations in the sample are used to estimate all the density functions.[9] Moreover, the proposed methodology allows the sample to be split into classes even when sample-separating

[8] Analysis cluster classifies observations according to *a priori* sample separation information.

[9] In the deterministic procedure, we are *implicitly* restricting the cross-class probabilities to zero and the own probabilities to one. This precludes using observations from other classes to estimate a particular class's probability.

information (in our case circumstances) is not available. In this case, the finite mixture model uses the goodness of fit of each estimated function as additional information to identify classes of individuals.

In finite mixture models, individuals are assumed to belong to one of *C* classes, $c = 1, ..., C$ where class membership is unknown. Let $v_{i\varphi} \in \Re$ denote a random variable that represents the outcome of individual *i* when facing the policy φ with probability density function $f_{v_{i\varphi}}(v) = f_i(v)$ on $\Re$.

The unconditional likelihood for individual *i* is obtained as the weighted sum of her/his *c*-class likelihood functions, where the weights are the probabilities of class membership. That is,

$$f_i(v;\delta) = \sum_{c=1}^{C} f_{\varphi}^{c}(v) \cdot P_i(\delta_c) \quad , \ 0 \le P_i(\delta_c) \le 1, \ \Sigma_c P_i(\delta_c) = 1, \tag{10}$$

where $\delta' = (\delta_{1,}...,\delta_C)$ and $P_i(\delta_c)$ represent the probability of individual *i* being a member of class *c*. The function $f_{\varphi}^{c}(v)$ is the density function of class *c* at the policy φ. In principle, a standard normal density function is assumed, hence it can be written as:

$$f_{\varphi}^{c}(v) = \phi\left(v;\mu_c,\sigma_c\right), \tag{11}$$

where μ_c is the expected value for that class and σ_c is the standard deviation of the corresponding class *c*. Expression (10) shows how the unconditional likelihood for individual *i* becomes finite mixtures.

The class probabilities are parameterized as a multinomial logit model:

$$P_i(\delta_c) = \frac{\exp(\delta_c' q_i)}{\sum_{c=1}^{C} \exp(\delta_c' q_i)} \quad , \quad c = 1,\ldots,C \quad and \quad \delta_C = 0\,, \tag{12}$$

where q_i is a vector of individual-circumstances variables and δ_c is a vector of parameters. For example, if we just consider the parents' education variable as individual circumstances, we will have the following equation: $\delta_c' q_i = \delta_{0c} + \delta_{1c} E1_i + \delta_{2c} E2_i$, where $E1$ represents a high school level of education for the parent and E2 represents a college level of education for the parent, they both are dummies with lower education than high school as the reference variable. In order to estimate equality of opportunity, we have considered the level of education of individuals' parents but we could also consider other circumstances like race, gender or intelligence.

Now, we use the maximum likelihood to the fitting of our mixture model. If we observed the independent outcome data (v_1, …, v_N) where N is the size of the sample, the likelihood function can be expressed as follows:

$$L(v_1,\ldots, v_N \mid \theta) = \prod_{i=1}^{N} f\ (v_i \mid \theta) \tag{13}$$

where $\theta = \{\mu; \sigma; \delta\}$, $\mu' = (\mu_1,\ldots, \mu_C)$ and $\sigma' = (\sigma_1,\ldots, \sigma_C)$.

Then after taking logarithms the overall log-likelihood function resulting from (10) is a continuous function of the vectors of parameters μ, σ and δ, and can be written as:

$$\log L(v_1,\ldots,v_N \mid \theta) = \sum_{i=1}^{N} \log f\ (v_i \mid \theta) = \sum_{i=1}^{N} \log\left\{ \sum_{c=1}^{C} \phi\left(v_i\ ; \mu_c, \sigma_c\right) \cdot P_i(\delta_c) \right\}. \tag{14}$$

The number of types, *C*, is taken as the number of classes that produce convergence in the optimization algorithm. Under the maintained assumptions, maximum likelihood techniques will give asymptotically efficient estimates of all the parameters. Then, once the probabilities of class membership and the distribution function for each class have been estimated, equality of opportunity can be computed in a probabilistic basis by applying programs in (8) or (9).

In short, inequality of opportunity between individuals arise from differences in their probabilities of being members of each class, not because the individuals obtain outcome differently within each class. Thus, outcomes will reflect the uncertainty that we have about the true partitioning of the sample, and we believe that this uncertainty is central to the problem that we are analysing.

In the next section, we apply the proposed methodology to the measurement of equality of opportunity in US during the period 1968-1999.

5. Empirical exercise

TO BE WRITTEN

6. Discussion and conclusions

If unobservable circumstances are significant, observable individual circumstances cannot determine whether an individual is a member of a particular type. However, the traditional equality-of-opportunity approach assumes that people can be classified *a priori* as members of a type. We believe that this assumption can bias results of studies on equality of opportunity.

To solve this problem we propose a probabilistic model where the deterministic approach to equality of opportunity is just a particular case and use finite mixture models to estimate it. Thus, we are able to evaluate the probability for a specific individual of belonging to a particular class.

In principle, we may believe that the larger is the bias achieved by estimating equality of opportunity with the standard deterministic methodology in comparison with the proposed probabilistic procedure, the greater the significance of unobserved circumstances. Then, beyond the scope of this paper, a hypothesis might be tested: to what extent the proposed methodology may allow us to estimate the significance of circumstances hard to be observed like luck (in Lefranc et al., 2006a and 2006b luck is considered, however, it is not measure). This is an open line to be explored in future research.

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Figure 1a. A mixture model

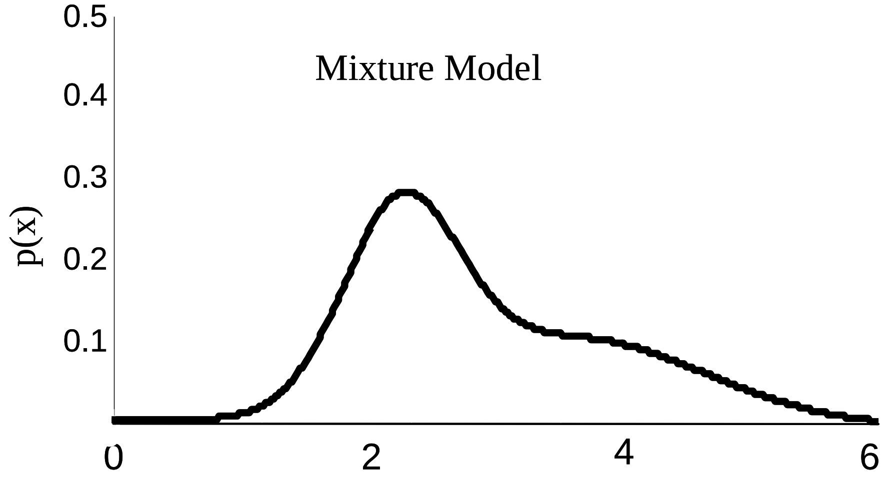

Figure 1b. Components of the mixture model