Experiment showing the motion of a falling object and the influence of air drag



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Abstract

An experimental setup with computer based measurements is introduced. It is possible to observe the object's motion during the fall and see how the limiting speed is reached. Due to the air drag, the motion changes from a constant accelerated motion to a motion with constant velocity. Measurements of the motion of a falling object are done with a graphing calculator and an attached data analyzer. Time-position and time-velocity diagrams of the falling motion are produced to determine the air drag's influence.

Keywords: Computer based measurements, graphing calculator, air drag.

Resumen

Presentamos un equipo experimental basado en mediciones por computadora. Es posible observar el movimiento del objeto durante la caída y ver cómo se alcanza la velocidad límite. Debido a la resistencia del aire, el movimiento cambia de un movimiento acelerado a un movimiento con velocidad constante. Las mediciones del movimiento de un objeto cayendo se realizaron con una calculadora gráfica y un analizador de datos adjunto. Se obtuvieron diagramas de tiempo-posición y tiempo-velocidad del movimiento de caída para determinar la influencia de la resistencia del aire.

Palabras clave: Mediciones basadas en computadora, la calculadora gráfica, resistencia del aire.

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I. INTRODUCTION

In school it is often taught that the acceleration due to gravity is independent of the mass of the falling body. This is only true when the buoyancy force and the air drag can be ignored, which is only the case when the gravitational force is much bigger then the other forces. In real life situations, the students experience the important role of air drag (e.g. falling raindrops or other falling objects, driving a car, etc.). Therefore the air drag should be considered while teaching Newtonain mechanics. Using computer based measurements it possible to conduct experiments showing it's influence on the motion during the fall.

II. THEORY OF THE FREE FALL

Many streams around moving objects are turbulent (e.g. air streams from driving cars, skydivers, flat plate). Then the drag D is dependent on the velocity v quadratically.

$$D = -kv^2, \qquad (1)$$

with *k* as a constant factor [1].

In this case the velocity of a falling object with the boundary condition v(t = 0) = 0 is given through

$$v_{tur}(t) = \sqrt{\frac{m^*g}{k}} \tanh\left(\frac{\sqrt{m^*gk}}{m}t\right), \qquad (2)$$

where m^* is the object's mass reduced by the mass of the displaced fluid, *m* is the object's mass, *g* is the acceleration of gravity and *t* the time.

After some time, a falling object reaches its limiting speed as it is often observed in real life. Then the acceleration is zero, and due to Newton's first law, the object experiences the total force zero. The reason is the increasing drag with increasing velocity. During the motion with constant velocity, the amount of the gravitational force is equal to the drag (when we disregard the buoyancy force).

III. THE MEASUREMENT SYSTEM

The measurement system consists of an ultrasonic distance ranger and a data analyzer which is connected to a graphing calculator¹ as they are provided from common

¹ For this experiment, the ranger EA-2, the data analyzer EA-200 and the calculator ClassPad 300 Plus from Casio are used.

companies for classroom technology. To optimize the experiment a special program should be used to do the measurements and to plot the diagrams [2, 3].

IV. THE EXPERIMENT

To investigate the motion of a falling object, I measured the distance from the starting point to the object during the fall with the described measurement system. A board with the dimensions 60 cm x 32 cm and with a mass of 150 g worked well as the object. It must fall flat with the area perpendicular to the air stream.

The measurements of the distance should be done with a sample rate of 0.02 s. After the data collection, one can immediately plot the time-position and time-velocity diagrams for the falling motion (see Fig. 2). One can compare the motions of the plane board and the board with extra weights attached. To determine the dependency of the air drag on the velocity, the data of mass of the falling object and the corresponding limiting speed must be collected. This should be done for about 5 different masses (150g, 200g,...,350g). From this data, a velocity-mass diagram can be made. The point 0,0 can be added to improve the results for the regression.

A. Setup

The setup is shown in figure 1. The height of the Ranger is about 1.80 m. When one starts the measurement, the distance from the board to the ranger should be about 0.20 m.



FIGURE 1. Setup of the measurement system

B. Use in classroom

This experiment can be done as a demonstration or as a hands-on experiment. To use it as a demonstration a camera should be used to project the calculator's display on a big screen. After observing and discussing the motion of the plane board one can make a couple of measurements for different weights. The students can make or discuss diagrams of the motion or discuss the physics by means of the diagrams.

V. INTERPRETATION OF THE RESULTS

Screenshots of time-position and the corresponding timevelocity diagrams of the falling board are shown in figure 2. Depending on the object's mass, different limiting velocities are observed. The graph of position versus time changes from a parabolic ascent in the beginning to a linear ascent at the end of the motion. This is connected to the velocity graph. It asymptotically approaches a constant value (the limiting speed) when the position graph ascends linearly.



FIGURE 2. Screenshots of the calculator running the measurement program. a) position diagram (thin points) and the corresponding velocity diagram (thick points) of the falling board without attached weights. b) position diagram (thin points) and the corresponding velocity diagram (thick points) of the falling board with attached weights (+150 g).

Knowing the object's mass, it is possible to determine the dependency of the drag on the velocity. The drag is equal to the gravitational force during the motion with constant limiting speed. The gravitational force of the falling object corresponds to the mass, which can be measured with a letter scale. Figure 3 illustrates the dependency of the drag on the velocity. The calculated quadratic regression for this experiment is $y=0.22x^2+0.03x$.



FIGURE 3. The diagram shows the drag (*y*-axis) versus the velocity (*x*-axis) and the calculated quadratic regression as a screenshot.

One can see that the drag depends quadratically on the object's velocity, according to equation 1. This plays an important role with moving cars or trains due to the rising

energy consummation at higher velocities.

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