

# A tutorial for laboratory determination of Planck's constant from the Planck radiation law



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## Abstract

Detailed discussions are presented on the determination of Planck's constant from the Planck radiation law. The laboratory setup consists of a low cost lamp and a photodiode such that a light filter of known wavelength is interposed between the two pieces of apparatus. The lamp filament is the approximate blackbody of which radiation is governed by the Planck's law of radiation. For the purposes of didactics, it is explained that laboratory precautions will enable accurate data. However, if the governing equation is subjected to spurious approximations, errors will be inevitable in the computation. This was the situation before. Now, when the setup is in operation, a phenomenon of resistance-temperature filter effect is observed. Data collected have been subjected to interesting analyses, which can be used to introduce students into numerical experiments. Usually, a pair of intensity values calculated from the observed phenomenon, would be required. Ratio of the intensities is then used in the exact Planck's radiation equations leading to a transcendental exponential equation. Solutions of this equation subsume the precise value of the Planck's constant.

**Keywords:** Planck's constant determination, numerical experiment, phototemperature.

## Resumen

A continuación se presenta un grupo de discusiones detalladas sobre la determinación de la constante de Planck que aparece en la ley de radiación del mismo nombre. En un laboratorio se dispone de una instalación de bajo costo que consta de una lámpara y un fotodiodo, de tal forma que un filtro de luz de longitud de onda conocida es interpuesto entre las dos piezas del aparato. El filamento de la lámpara se comporta como un cuerpo negro cuya radiación está gobernada por la ley de radiación de Planck. Para fines didácticos, se explica que se obtienen datos aproximados como una medida de precaución en el laboratorio. Sin embargo, si la ecuación gobernante está sujeta a aproximaciones espurias, los errores se propagarán inevitablemente en los cálculos. Esta fue una situación anterior. Ahora, cuando se pone en marcha esta nueva configuración, un fenómeno de filtración entre la resistencia y la temperatura es observado. Los datos recolectados han sido sujetos a análisis interesantes, los cuales pueden ser utilizados para introducir a los alumnos en el estudio de los experimentos numéricos. Usualmente, serán requeridos un par de valores de intensidad calculados a partir del fenómeno observado. La razón de las intensidades es entonces usada en las ecuaciones de radiación de Planck llevándonos a una ecuación transcendental exponencial. Las soluciones de esta ecuación nos proveen de un valor preciso para la constante de Planck.

**Palabras clave:** Determinación de la constante de Planck, experimento numérico, fototemperatura.

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## I. INTRODUCTION

In all quantum phenomena, it appears that the Planck's constant,  $h$ , is ever present. A notable experimental determination of,  $h$ , may be attributed to the work of Millikan during the years in the period of 1905 to 1916. With sophisticated apparatus, Millikan's final result was  $(5.57 \pm 0.03) \times 10^{-34}$  Js [1]. This was based on the Einstein interpretation of the photoelectric effect. One might surmise this as the reason for the existence of several form of photoelectric effect experiments in order to determine,  $h$ . In the present days' efforts, there are more sophisticated

sets of apparatus [2], that are based on other principles. It is, however, interesting to notice that the determination of,  $h$ , in all existing experiments is dependent on other fundamental constants [2]. This is also the trend to be observed in many undergraduate laboratory experiments [3, 4]. In this work, too, the Boltzmann's constant,  $k_B$ , is the fundamental constant associating with,  $h$ . We note with enthusiasm, that, there is an absolute interesting laboratory exercise for the determination of  $k_B$  [5].

In all undergraduate laboratory exercises on,  $h$ , the emphases are on precautions in the functions of the apparatus. While this is good training for the students, it is insufficient. Emphases are also necessary on the choices of

methods of analysis of data especially when approximations are to be used. One of the aims of this presentation is an exposition of wrong results owing to misleading approximations of the governing Planck's radiation equation, [4]. Another aim is to further test the phenomenon of resistance-temperature filter effect, RTFE, [6]. The RTFE is due to the physical implications that the filter used in the experimental setup has the ability to filter temperature. This temperature is certainly the phototemperature consequent to the photocurrent detected by the photodiode used.

In Section II, the manners by which the Planck radiation equation is used is explained in more details than that was given in [6]. Section III will give the laboratory setup of the experiment with further explanation of the RTFE. All details of the numerical experiments and analyses are presented in Section IV. Discussions of results are in Section V. Section VI is for conclusions.

## II. THEORY

The Planck radiation law governs the energy distribution in blackbody radiation [7]. In terms of frequency of the radiation, the law can be stated as

$$\rho(\nu, T) d\nu = \frac{8\pi h \nu^3}{c^3} \{ \exp(h\nu/kT) - 1 \}^{-1} d\nu, \quad (2)$$

where  $\rho$ , in units of  $J/m^3/s^{-1}$ , is the radiation energy per unit volume, and per unit frequency interval,  $d\nu$ , at the temperature,  $T$ ,  $c$  is the speed of light,  $k_B$  is the Boltzmann's constant and  $h$  is the Planck's constant. For equation (2) to be applicable it has to be expressed in the standard units of intensity, which are  $W/m^2$ . Multiplying both sides of (2) by  $c/\nu$  gives

$$I(\nu, T) \equiv \rho(\nu, T) \frac{c}{\nu} = \frac{8\pi h \nu^2}{c^2} \{ \exp(h\nu/kT) - 1 \}^{-1}, \quad (3)$$

where  $I$  will be used to denote intensity and  $i$  will denote electric current in what follow. Units of the equation are now  $W/m^2$  as given by equation (3).

For a fixed frequency,  $\nu$ , at different temperatures  $T_j$  and  $T_l$  the ratio of any two intensities  $I_j(T_j)$  and  $I_l(T_l)$  is

$$\frac{I_j(T_j)}{I_l(T_l)} = \frac{\exp(h\nu/kT_l) - 1}{\exp(h\nu/kT_j) - 1}, \quad (4)$$

where  $j \neq l = 1, 2, 3, \dots$

It should be observed that equation (4) is expressible as

$$\exp(h\nu/kT_j) - \frac{I_l}{I_j} \exp(h\nu/kT_l) + \frac{I_l}{I_j} - 1 = 0, \quad (5)$$

so that for measured values of intensities  $I_j$  and  $I_l$ , and known values of  $T_j$ ,  $T_l$  and  $\nu$ , equation (5) is a form of transcendental equation which does not have a closed form of solution for  $h\nu/k$  to be found [9]. It is, however, an interesting exercise to apply anyone numerical procedure such as the Newton's Method or the fixed-point iteration method [8]. The Newton's Method was attempted in this tutorial.

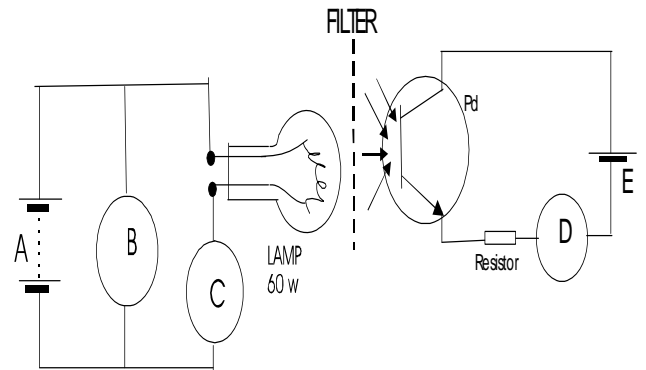
In the visible range of frequencies used, the approximate form of equation (4) based on  $h\nu \gg kT_j$  so that  $\exp(h\nu/kT_j) \gg 1$  is [4]

$$\ln\left(\frac{I_j}{I_l}\right) = \frac{h\nu}{k} \left( \frac{1}{T_l} - \frac{1}{T_j} \right). \quad (6)$$

Results from applications of equations (5) and (6) will be discussed in more detail in the following sections. The laboratory setup involves measurements of series of voltages  $V_j$  with their corresponding electric current,  $i_j$ , of a light bulb filament as shown in Fig. 1. To determining the intensities  $I_j$  and temperature  $T_j$  one requires to use the Stefan-Boltzmann law which can be expressed as [4]

$$W \equiv P/A = \sigma T^4, \quad (7)$$

where  $\sigma$  is the Stefan-Boltzmann constant and  $A$  is the surface area of the filament enclosed in the bulb, and  $P$  is the power emitted.



**FIGURE 1.** Laboratory setup of electric circuit. A and E should be stabilized sources and not ordinary batteries which drain every second. See text for further explanation.

If a power law of the form  $T \propto R^\gamma$  is assumed, and an empirical relation between resistance of the filament,  $R_e$ , and temperature  $T_e$ , through the power dissipated can be expressed as

$$P_e = i_e^2 R_e \equiv A \sigma T_e^4 = \beta R_e^{4\gamma}, \quad (8)$$

in which  $\beta$  and  $\gamma$  are constants, and from equation (8) the empirical relation of  $R_e$  and  $T_e$  is [4]

$$T_e = \left( \frac{R_e}{R_0} \right)^\gamma T_0, \quad (9)$$

$$R_p = R_e \left( \frac{i_p}{i_e} \right)^{2/(4\gamma-1)}. \quad (13)$$

where  $R_0$  denotes the filament resistance at temperature  $T_0$ . The room temperature has been used as  $T_0$  for which  $R_0$  was measured using a MICRONTA digital resistance meter, [6]. Using equation (8) in (9) yields

$$\ln(i_e^2 R_e) = \ln C_1 + 4\gamma \ln R_e, \quad (10)$$

where

$$C_1 = A\sigma \left( \frac{T_0}{R_0^\gamma} \right)^4, \quad (10a)$$

so that a plot of  $\ln(i_e^2 R_e)$  on the vertical axis against  $\ln(R_e)$  on the horizontal axis is expected to give a straight line in order to determine the power,  $\gamma$ , using the least squares fit. One should notice that the previous works reported have been based on the approximation so far explained [4, 9, 10]. Also, the circuit given in Fig. 1 is similar in operation to that was used in [4], implying that similar raw data would be recorded. However, in our present tutorial, we have applied RTFE which is due to filter effect on the emitted power. It is necessary to explain the RTFE at this point.

Figure 1 consists of two circuits: the heat emitting circuit and the detector circuit. Although the pair of circuits operates in complete analogy to thermionic conversion circuits of heat to electricity [11], the later has the photodetector fundamentally of which function is to detect the amount of current that is proportional to the filtered heat. In the filtering process, therefore, there is some content of temperature which is proportional to the detected current. The photopower content  $i_p^2 R_p$ , where  $i_p$  and  $R_p$  denote photocurrent and photoresistance, should have the relation of Stefan-Boltzmann law as expressed by equation (8) for the same power value,  $\gamma$ , already determined. That is, for the detected current,  $i_p$ , the empirical relation with  $R_p$  and  $T_p$  is

$$P_p = i_p^2 R_p \equiv A\sigma T_p^4 = \beta R_p^{4\gamma}, \quad (11)$$

It should be noticed that  $i_{pj}$  are recorded simultaneously with  $V_{ej}$  and  $i_{ej}$  in the raw data. Equation (11) which arises from RTFE enables us to calculate  $R_p$  and  $T_p$ . For example, by taking the ratio of equation (8) and (11) one gets

$$\frac{P_e}{P_p} = \frac{i_e^2 R_e}{i_p^2 R_p} = \left( \frac{R_e}{R_p} \right)^{4\gamma}. \quad (12)$$

Equation (12) leads to

By expressing  $T_p$  in the same form similar to equation (9),  $T_p = (R_p/R_0)T_0$  and then taking the ratio of this expression to equation (9),  $T_p$  is obtained in terms of  $i_p$ ,  $i_e$  and  $T_e$  as

$$T_p = T_e \left( \frac{i_p}{i_e} \right)^{2\gamma/(4\gamma-1)}. \quad (14)$$

It is pertinent to realise that the photointensity of the filament due to RTFE for anyone measured set of  $V_{ej}$ ,  $i_{ej}$  and  $i_{pj}$  is  $I_{pj} = i_{pj}^2 R_{pj}/A$ . Another set of raw data values  $V_{el}$ ,  $i_{el}$  and  $i_{pl}$  gives photointensity  $I_{pl} = i_{pl}^2 R_{pl}/A$  where  $R_{pj}$  and  $R_{pl}$  are calculated using equation (13). With the fixed value of frequency (specified for the filter),  $\nu$ , expressions of the applicable Planck radiation law given by equation (3) can be written for  $I_{pj}$  and  $I_{pl}$ . Ratio of the photointensities taken as in equation (4) would lead to

$$\exp(h\nu/kT_{pj}) - \frac{I_{pl}}{I_{pj}} \exp(h\nu/kT_{pl}) + \frac{I_{pl}}{I_{pj}} - 1 = 0, \quad (15)$$

where  $T_{pj}$  and  $T_{pl}$  are obtainable from equation (14). Theoretically, one deduces that  $h\nu \gg kT_{pj}$  is an improved approximation over  $h\nu \gg kT_{ej}$ , if  $T_{ej} > T_{pj}$ , so that  $\exp(h\nu/kT_{pj}) \gg 1$  is, by implication, more reliable. Now, a form of equation (6) when RTFE is invoked is

$$\ln \left( \frac{I_{pj}}{I_{pl}} \right) = \frac{h\nu}{k} \left( \frac{1}{T_{pl}} - \frac{1}{T_{pj}} \right). \quad (16)$$

It should be observed that in either equation (15) or equation (16), the filament surface area  $A$  is not required [4]. If it is necessary,  $A$  can be determined from  $C_1$  as given by equation (10a).

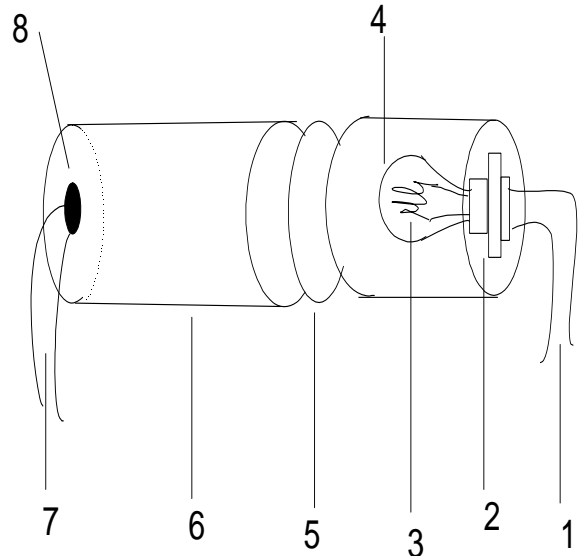
Equation (16) is an improved version of equation (6) as given in ref. [4]. However, the two equations do not give reasonable results as would be seen shortly. Advanced degree students would find it stimulating to attempting solutions of equations (5) and (15) using anyone of several numerics [8]. Even then, there are problems. One problem concerns the large value of  $\exp(h\nu/kT_{ej})$  or  $\exp(h\nu/kT_{pj})$ . Another one is that due to the presence of exponents; either equation has several solutions depending on starting input values when the Newton's Method is applied. In this wise a simple Mathematica statement of FindRoot helps a lot. Overall, analyses show that equation (15) contains an exact value of  $h$ , the Planck's constant.

### III. LABORATORY SETUP OF THE EXPERIMENT

As indicated in Fig. 1. most of the pieces of apparatus are common in many undergraduate laboratories. A is a stabilised variable d.c power source, of range 0.0 V to 350.0 V; B and C are MICRONTA digital multimeters for measurements of voltage and current. D happens to be an analog microammeter as it is an important precaution to avoid saturation currents which may likely occur if measurement of detected currents are in miliamperes. E is a PHYWE stabilised a.c./d.c power source of which the d.c source was operated at a constant value of 11.0 V to biasing the RS 651-995 Photodetector which was chosen because of its large area and flexibility to various optical filter frequencies up to a peak wavelength of 900 nm.

The setup consists of two circuits (see Fig. 1): the heat emitting circuit through a 60W light bulb, and the heat (current) detector circuit essentially by the RS photodetector Pd. The two circuits communicate by one way from the lamp to the photodetector. However, the detected current,  $i_p$ , is measured with a microammeter D and the function of the resistor, 1.0 k $\Omega$ , is to dissipate the detected power. Certainly the circuit (Fig. 1) is simple to operate. By increasing the d.c. voltage, A, in the heat emitting circuit, the filament resistance,  $R_e$ , and consequently its temperature  $T_e$ , increase. The voltage across the filament is measured by B and denoted as  $V_e$  while the electric current through it is  $i_e$  measured by C. Spontaneously, Pd, detects the heat converted to photocurrent,  $i_p$ , measured by D of which value depends on the interposing filter (see Figs. 1 and 2). It is therefore, conformable to reason that the detected current has some

content of resistance  $R_p$ , and consequently temperature  $T_p$  filtered from  $R_e$  and  $T_e$ . This process we refer to as resistance-temperature filter effect, RTFE, resulting into equations (13) and (14).  $V_e$ ,  $i_e$ , and  $i_p$  are the physical quantities of the raw data shown in table 1; other quantities recorded are calculated as indicated.



**FIGURE 2.** Schematic drawing of essential pieces of apparatus: 1. Connecting cords to variable General \purpose (Philip Harris) power supply, 2. Bulb socket, 3. Lamp filament, 4. Lamp bulb, 5. PHYWE light filter, 6. Polyvinyl tube, 7. Connecting cords to biasing voltage (i.e., PHYWE stabilized power source), 8. RS 651-995 Photodetector.

**TABLE I.** Raw experimental data and calculated filament resistance, power, temperature and photocurrent; other quantities are calculated data due to RTFE: photoresistance, photopower, and phototemperature. Parameters used are:  $T_0$  (Room temperature) = 310 K,  $R_0$  (resistance at  $T_0$ ) = 64.5  $\Omega$ , bulb wattage = 60W, tube length,  $L_t$  = 13.0 cm, biasing voltage,  $V_b$  = 11.0 V, filter wavelength,  $\lambda_f$  = 578.0 nm.

$V_e$ (V)	$i_e$ (A)	$R_e = V_e / i_e$ ( $\Omega$ )	$P_e = i_e^2 R_e$ (W)	$T_e$ (K)	$i_p$ ( $\mu$ A)	$R_p$ ( $\Omega$ )	$P_p = i_p^2 R_p$ ( $\mu$ W)	$T_p$ (K)
124.5	0.2065	602.906	25.7093	1735.12	4.0	0.011561	0.18497	0.50534
156.4	0.2313	676.178	36.1753	1890.89	6.0	0.017368	0.62523	0.68561
181.8	0.2492	729.535	45.3046	2001.65	8.0	0.023194	1.48443	0.85162
199.5	0.2612	763.783	52.1094	2071.68	10.0	0.028964	2.89642	1.00591
213.0	0.2700	788.889	57.5100	2122.51	12.0	0.034734	5.00174	1.15264
228.2	0.2795	816.458	63.7819	2177.87	14.0	0.040519	7.94164	1.29370
241.2	0.2879	837.791	69.4415	2220.38	16.0	0.046135	11.8106	1.42591
252.2	0.2944	857.337	74.3066	2259.10	18.0	0.051945	16.8301	1.55848
264.5	0.3000	881.667	79.3500	2306.98	20.0	0.058251	23.3004	1.69824

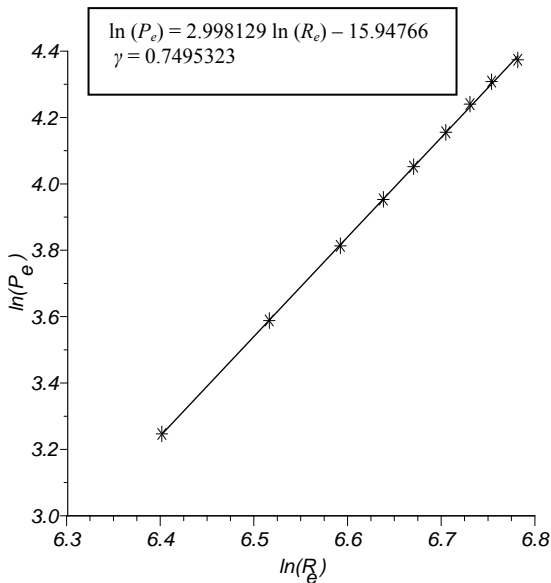
It is found that avoiding darkening the laboratory is both precautionary [4] and applicable to most other experiments in our didactic programme. A blackened-inside polyvinyl tube of diameter 8.0 cm was used inside of which the communication of lamp bulb and photodetector takes place thus avoiding other sources of interfering radiation. Amount of current detected was found to depend on tube length: shorter tube gives more intensity of photocurrent.

Too, the tube fitted exactly into the PHYWE filters used. A simple thermometer was used to measure  $T_0$ , the room temperature, and  $R_0$ , the resistance at  $T_0$ , was measured using MICRONTA Digital Multimeter.  $T_0$  and  $R_0$  are required in equation (9). The data recorded in table 1 are for the PHYWE yellow light filter of wave length,  $\lambda_f$  = 578.0 nm.

Results are also reported here of data collected (see table 3) for PHYWE green and blue light filters at the same  $T_0$ ,  $R_0$  and other parameters recorded in table 1. Actually, the data discussed here constitute just one set of a very large number of sets of data collected and analysed over a period of two years up to now. The effort has so far been to diligently observe any drastic deviation from the RTFE.

#### IV NUMERICAL EXPERIMENTS

In this section we detail our numerical procedures. We begin with the determination of  $\gamma$ -parameter, [4] which is clearly by using equation (10). The resulting straight line (see Fig 3) from the least squares fit yields the value of  $\gamma$  that is subsequently used to calculate the variables given in table I. Specifically, the variables are  $T_e$  from equation (8),  $R_p$  from equation (12),  $T_p$  from equation (13) and  $P_p = i_{pj}^2 R_p/A$ . The significances of equation (10) include determination of surface area of the heat emitting filament through the parameter  $C_l$ .



**FIGURE 3.** Graph of  $\ln P_e$  (emitted power) against  $\ln R_e$  (filament resistance) to determine  $\gamma$ . See table I for calculations of  $P_e$  and  $R_e$ .

We first sought to compute the Planck's constant,  $h$ , using the procedure of reference [4]. By equation (5), the implication of instruction given in, [4], is that  $I_j \equiv i_{pj}$  so that we have

$$\ln\left(\frac{i_{p1}}{i_{pj}}\right) = \frac{h\nu}{k_B}\left(\frac{1}{T_{ej}} - \frac{1}{T_{e1}}\right), \quad (17)$$

where  $\nu = c/\lambda$  with  $c$  meaning the speed of light,  $\lambda$ , the wavelength of filter in use as given with the data of table I,

and  $k_B$  is the Boltzmann constant of which value used here, [12], (IOP, 1999/2000), is  $1.38065812 \times 10^{-23} \text{ JK}^{-1}$ .

In equation (17), it should be noted that  $T_{e1}$  was taken as the reference temperature of the filament so that a plot of  $(1/T_{ej} - 1/T_{e1})$  along the horizontal axis against  $\ln(i_{p1}/i_{pj})$  along the vertical axis should yield a straight line; the slope of this line obtained by the least squares fit should be equivalent to  $h\nu/k_B$ . Using the data given in table 1, the plotted points give straight line. In fact the least squares equation is

$$\ln\left(\frac{i_{p1}}{i_{pj}}\right) = 13031.64\left(\frac{1}{T_{ej}} - \frac{1}{T_{e1}}\right) + 0.264413, \quad (18)$$

where  $j = 2, 3, 4, \dots, 9$ . The graph is not shown here for one obvious reason that  $I_j \neq i_{pj}$ . The correct relation is  $I_e = i_{pj}^2 R_p/A$  where A is the filament surface area.

The gradient value 11585.34 from equation (18) yields  $h = 3.468905 \times 10^{-34} \text{ Js}$ . Other equally carefully collected data yielded values in the range of  $4.9 \times 10^{-34} \text{ Js}$  to  $5.65 \times 10^{-34} \text{ Js}$ .

We next used equation (5) with the incorrect  $I_e - i_p$  relation. Equation (5) would then be

$$\exp\left(\frac{h\nu}{k_B T_{e1}}\right) - \frac{i_{pj}}{i_{p1}} \exp\left(\frac{h\nu}{k_B T_{ej}}\right) + \frac{i_{pj}}{i_{p1}} - 1 = 0. \quad (19)$$

It appears that equation (19) should be soluble with anyone standard numerical methods [8], in which,  $h$ , becomes the unknown variable to be found. However, there are problems. First, the resulting value in the exponent, obtained by evaluating  $\nu/k_B T_{ej}$  is too large for any hand calculator. Use of FORTRAN 95 gives floating point; the programmes could not be executed. It was supposed that the problem could be overcome by writing equation (19) as

$$\exp(x/T_{e1}) - \frac{i_{pj}}{i_{p1}} \exp(x/T_{ej}) + \frac{i_{pj}}{i_{p1}} - 1 = 0, \quad (20)$$

so that  $x = h\nu/k_B$ . Still due to the exponent, there is another problem when the Newton's method [8], was used in FORTRAN coding. Equation (20) has several solutions that closely depend on the input values.

Here, we report most interesting results when the simple statement of Mathematica FindRoot [13] was used (see Appendix A). One notices that the FindRoot statement works by the principle of Newton's method of solving equations. The non-unique solution still persists for either equation (19) or (20). However, Mathematica is able to return a solution. By inputting a value such as  $6.6 \times 10^{-34}$  which is close to the standard value [12], one could get a solution. Once more, it was found that the incorrect  $I_j - i_{pj}$  relation affects the results of equation (19).

We now focus attention on equations (15) and (16) which resulted from the use of RTFE on the heat emitted

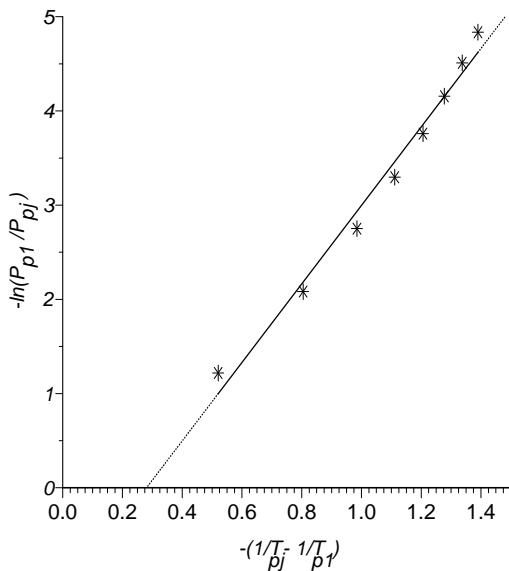
by the filament. With the data in table I, equation (16) becomes

$$\ln\left(\frac{i_{pj}^2 R_{pj}}{i_{p1}^2 R_{p1}}\right) = \frac{h\nu}{k} \left(\frac{1}{T_{pj}} - \frac{1}{T_{p1}}\right). \quad (21)$$

Fig. 4 shows the plot of  $(1/T_{pj} - 1/T_{p1})$  along the horizontal axis against  $\ln(P_{pj}/P_{p1})$  along the vertical axis. The least squares line yielded is

$$\ln\left(\frac{P_{pj}}{P_{p1}}\right) = 4.166409 \left(\frac{1}{T_{pj}} - \frac{1}{T_{p1}}\right) - 1.168903. \quad (22)$$

The value obtained for  $h$  here is  $1.109061 \times 10^{-35}$  Js.



**FIGURE 4.** Plotting of equation (17) with a linear fit to calculated data using the RTFE. Note that  $I_{pj} = i_{pj}^2 R_{pj}/A \equiv P_{pj}/A$  and  $I_{p1} = i_{p1}^2 R_{p1}/A \equiv P_{p1}/A$ , where  $A$  is the filament surface area. See text and table I for further explanation. Equation (22) is the least squares line.

When equation (15) was used the results are satisfactory using the Mathematica statement given in Appendix A. The analysis is in the following. With the data in table I, equation (15) becomes

$$\exp\left(\frac{h\nu}{k_B T_{p1}}\right) - r_{pj} \exp\left(\frac{h\nu}{k_B T_{pj}}\right) + r_{pj} - 1 = 0, \quad (23)$$

where  $r_{pj} \equiv i_{pj}^2 R_{pj} / i_{p1}^2 R_{p1}$  in equation (23).

As an example, if  $j = 3$ , equation (23) is

$$\exp(7.43404 \times 10^{37} x)$$

$$- 8.02535 \exp(4.41127 \times 10^{37} x) + 7.02535 = 0. \quad (24)$$

Mathematica is able to return  $6.62588 \times 10^{-34}$  Js with a starting value of  $6.6279 \times 10^{-34}$  Js. Mathematica returns the same,  $h$ , values for  $j = 2, 3, 4, \dots, 9$ . When the reciprocals of the ratios of intensities are used, *i.e.*, instead of  $I_{pj}/I_{p1}$ , one used  $I_{p1}/I_{pj}$ , by correctly rewriting the resulting equations the same results were obtained.

All procedures above were for the case of yellow filter. Data were collected and similarly analyzed for green and blue filters. See table II.

**TABLE II.** Results obtained for Planck’s constant using the setup of Figs. 1 and 2 for the three filters for the same ambient conditions and with the same numerical experiments explained in the text.

Filter Colour	Filter Wavelength (nm)	Planck’s constant, $h$ (Js) $\times 10^{-34}$
Blue	436.00	$6.62605 \pm 0.00019$
Green	546.00	$6.62588 \pm 0.00019$
Yellow	578.00	$6.62588 \pm 0.00019$

## V. RESULTS AND DISCUSSION

Based on the simple Mathematica FindRoot statement (see Appendix A), our results are given in table 2. The result obtained for the blue filter is of interest for one reason: the range of the multimeters used did not permit more than two data points, and the microammeter used for measurement of photocurrent is analogue. If we used a digital microammeter, or even better a digital nanoammeter, it would be possible to divide the interval into, at least, five or more data points. In deed, under the same conditions of ambient temperature,  $T_0 = 310K$ , the Planck’s constant value,  $h$ , obtained for the yellow and blue filters is the same. It could be observed that the quoted absolute error is tolerable. (table II).

## VI. CONCLUSIONS

Figures 1 and 2 depict the laboratory setup used for the determination of the Planck’s constant,  $h$ . Here, the applied law is the Planck radiation equation. The efforts so far have been directed towards presentation of detailed procedures of numerical experiments. If laboratory precautions are emphasized for didactics, it is now clear that experimental situations abound where numerical precautions are inevitable.

We circumvented systematic errors that affected previous results [4, 9, 10]. It was realized that errors resulted from misleading approximations in the governing equation. It was found, for an example, that the approximations appear to be improved when variables  $T_{pj}$  were used in  $\exp(h\nu/k_B T_{pj}) \gg 1$  instead of variables  $T_{ej}$  in  $\exp(h\nu/k_B T_{ej}) \gg 1$ , where  $j = 1, 2, 3, \dots, 9$ ; 9 being the

number of data points in the laboratory measurements for the case of yellow filter. That is because the emitted temperatures,  $T_{ej}$  are not of equal magnitude with the phototemperatures  $T_{pj}$ . In another words if  $i_{ej} \neq i_{pj}$ , it is reasonable to expect  $T_{ej} \neq T_{pj}$ . Empirically, we find that  $T_{ej} > T_{pj}$ . It would be recalled that  $T_{pj}$  define temperatures calculated using equation (14) which is one of the results of RTFE already explained in the text.  $T_{ej}$  are the temperatures of the emitted heat by the filament, which would consequently be filtered to produce  $T_{pj}$ . It was observed that the filtered currents (i.e., the photocurrents)  $i_{pj}$  would always be less than the emitted currents,  $i_{ej}$ , (i.e.,  $i_{ej} > i_{pj}$ ).

For final analysis, it was found to be corrective by developing and solving equation (15) numerically with a simple Mathematica FindRoot (see Appendix A). The equation has several solutions each depending on the input or starting value. Alongside the correct relation between photointensities,  $I_{pj}$ , and photocurrents,  $i_{pj}$ , which is  $I_{pj} \equiv i_{pj}^2 R_{pj}/A$ ;  $A$  being the surface area of the filament enclosed inside the bulb, we found that (see table II) the recommended value of the Planck's constant,  $h$ , is one of the series of the solutions of equation (15). In particular, if it is assumed that the Planck's constant permissible for applications is in the range of  $6.60 \times 10^{-34}$ Js to  $6.63 \times 10^{-34}$ Js, then, our results for yellow and green filters have absolute error of 0.0029% compared to the standard value, [12], (IOP, 1999/2000). For the blue filter, the absolute error is 0.0003%.

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## APPENDIX A

Essential statement of Mathematica FindRoot are given here as equation (A1). Data for computation are obtained from table I. We use the parameters of equation (24). Answers returned by Mathematica are numbered.

$$\begin{aligned} & E^{\wedge}(c1*x) - ((I2/I1)*(E^{\wedge}(c2*x))) + ((I2/I1) - 1.0) \\ & /. \{c1 \rightarrow 7.434038 * (10^{\wedge}37), c2 \rightarrow 4.411266 * \\ & (10^{\wedge}37), I1 \rightarrow 1.84968 * (10^{\wedge} -13), I2 \rightarrow 1.484433 * \\ & (10^{\wedge} -12)\} \\ & \text{FindRoot}[\% == 0, \{x, 6.6279 * (10^{\wedge} -34), 4.2 * \\ & (10^{\wedge} -34), 7.44 * (10^{\wedge} -34)\}], \end{aligned} \quad (A1)$$

$$\begin{aligned} & 7.02535 - 8.02535 \exp(4.41127 \times 10^{37} x) \\ & + \exp(7.43404 \times 10^{37} x), \end{aligned} \quad (A2)$$

$$\{x \rightarrow 6.62588 \times 10^{-34}\}. \quad (A3)$$

The final answer is given by equation (A3). It should be noticed in the Mathematica statement [13], that  $6.6279 \times 10^{-34}$  Js is the starting value that was chosen and the other two values correspond to an interval within which results are desired. After giving a notice of accuracy Mathematica returns only one value as the answer given by equation (A3). Equation (A2) is the transcendental exponential equation (24) returned by the Mathematica statement.