Filament temperature of low power incandesecent lamps: Stefan-Boltzmann law

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Abstract

An undergraduate experiment using commercially available low power incandescent lamps was performed. The results obtained, for higher temperature of the filament (say above 1000 K), were compared with those calculated using a simple model, based on transfer of electric power predominantly into Planck's radiation channel through Stefan-Boltzmann law. The agreement between the results and the theory was quite satisfactory. Measurement of filament temperature to confirm theoretical results, included in the present work, is expected to give a student more confidence in the theory.

Keywords: Laboratory experiment, radiative heat transfer, radiation detectors.

Resumen

El presente trabajo muestra el desarrollo de un experimento a nivel bachillerato utilizando lámparas incandescentes de baja potencia y disponibles de manera comercial. Los resultados obtenidos a altas temperaturas del filamento (cerca de los 1000K) fueron comparados con los calculados utilizando modelos simples, basándose principalmente en la transferencia de potencia eléctrica predominante en la radiación de canal de Plank a través de la ley de Stefan-Boltzmann. La correspondencia entre los resultados y la teoría fue bastante satisfactoria. Las mediciones de temperatura hechas al filamento confirman los resultados teóricos, los cuales se incluyen en el presente trabajo, se espera que el estudiante adquiera una mayor seguridad respecto a la teoría aprendida.

Palabras clave: Experimento de laboratorio, transferencia de calor por radiación, detectores de radiación.

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I. INTRODUCTION

There are a number of methods for estimating the temperature of the filament of incandescent lamps [1, 2, 3, 4, 5]. These methods are for example (i) the power law between the resistance, R, and temperature, T, of tungsten filament, (ii) the transfer of the input electric power predominantly into Planck's radiative channel through Stefan's law, (iii) exploits the fact that the lifetime of the lamp filament is mostly governed by the rate of thermal evaporation of the metal, (iv) analyses of the radiation emitted by the filament at two well defined wavelengths and (v) study of hysteresis in the current-voltage characteristics in filament lamp.

Incandescent light bulbs, in addition to providing illumination, are useful in the context of teaching physics [2, 4]. In the present work two sets of measurement were made on commercially available low power lamps. These measurements were made on lamp filament: (a) resistance (R) – voltage (V) and (b) temperature (T) – voltage.

An attempt is made to relate these results to those derived on the basis of transfer of the electrical power predominantly into Planck's radiation channel through Stefan-Boltzmann law.

II. EXPERIMENTAL

In the present work three 12-V operated low power (rated at 10-W, 25-W and 35-W) commercial lamps were studied. Current (I) – voltage measurements were performed using a variable dc power supply. Multimeters were used to measure the voltage across the filament and current in the series circuit. The resistance of the filament was obtained using $R = V/I$. The temperature of the filament at different voltage was measured using a Minolta-Land infrared optical pyrometer Cyclops 52. The temperatures were measured for different setting of emissivity, such as 0.3, 0.35, and 0.4. The average value of the emissivity for tungsten filament lamp is close to 0.35

[2]. Therefore, in this work $e = 0.35$ was used. However, the measured temperature values were larger (by less than 2.5%) for e = 0.3 and smaller (by less than 1.5%) for $e = 0.4$ when both compared with those obtained for $e = 0.35$.

III. MODELING

At a given voltage across the filament of the lamp, a steady state is reached when the current (I) passing through the filament is stabilized. In the steady state it is expected that the electrical power input to the lamp is equal to the power lost by the filament through conductive, convective, and radiative processes, such that [2]

$$V^2 / R = K (T - T_0) + e\, \sigma\, A_s\, (T^4 - T_0^4), \qquad (1)$$

where K represents conductive and convective properties of the system. T and T_0 are the temperatures in Kelvin for the filament and the ambient, respectively. e and A_s are the emissivity and surface area of the filament, respectively, and σ is the Stefan-Boltzmann constant.

For higher temperature of the filament (say above 1000 K), it is reasonable to assume that $T^4 >> T_0^4$. Moreover, for low power bulbs [2], such as those used in the present work, the literature indicates that convection and conduction losses are negligible. Hence Eq. (1) may be rewritten as

$$V^2 = e\, \sigma\, A_s\, T^4\, R\, . \qquad (2)$$

Metal resistance increases with temperature. The temperature of tungsten in the range 300 K to 3655 K, can be given in terms of its resistivity (ρ) by the empirical relation, valid in SI units [7]

$$T = 3.05 \times 10^8\, \rho^{0.83}\, . \qquad (3)$$

A power relation similar to this is also given in Ref. [3, 8]. Eq. (3) may be written as

$$T = 3.05 \times 10^8\, [\, (A_c\, R) / L\,]^{0.83}\, , \qquad (4)$$

where A_c and L are the area of cross section and length, respectively, of the filament wire. Substituting this value of T in Eq. (2), we have

$$V^2 = e\, \sigma\, A_s\, R\, \{3.05 \times 10^8\, [\, (A_c\, R) / L\,]^{0.83}\}. \qquad (5)$$

Also, $A_s = 2\pi rL$ and $A_c = \pi r^2$, where r is the radius of the filament wire. The value of $e = 0.35$ (following Clauss *et al.* [2]) and $\sigma = 5.67 \times 10^{-8}$ Wm^{-2} K^{-4} were used in the present work. Thus, Eq. (5) becomes

$$R = B_1\, V^p\, , \qquad (6)$$

where,

$$B_1 = 2.1 \times 10^{-7}\, L^{0.54}\, r^{-1.8}\, , \qquad (6a)$$

and

$$p = 0.46. \qquad (6b)$$

The resistance of the filament at room temperature (R_{RT}) is given by

$$R_{RT} = \rho_{RT}\, (L / \pi r^2), \qquad (6c)$$

where ρ_{RT} is the resistivity of tungsten at room temperature, given by Eq.(3).

Similarly by eliminating R from Eq. (2) and (5), we obtain an expression relating T with V, such as

$$T = B_2\, V^q\, , \qquad (7)$$

where

$$B_2 = 2.4 \times 10^3\, r^{0.19}\, L^{-0.38}\, , \qquad (7a)$$

and

$$q = 0.38. \qquad (7b)$$

In the present work we have measured R and T both as functions of V.

IV. RESULTS AND DISCUSSION

Three unbranded 12–V operated low power lamps were acquired from the local market. The cost of each of the three lamps is given in Table I. The values of the resistance (R_{RT}) of the filaments of the three lamps measured at room temperature are listed in Table I. These were measured using Wheatstone bridge method. The results of the measurement of R as a function of V for the lamps are shown in Figure 1.

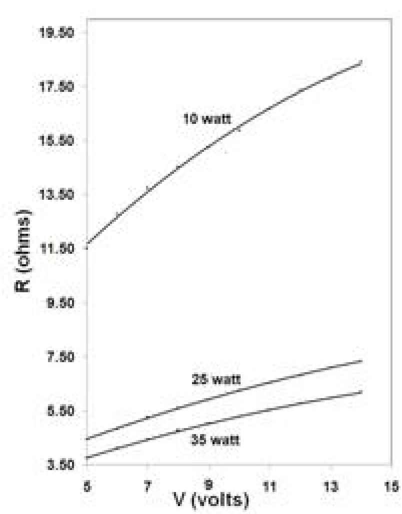

FIGURE 1. Resistance versus voltage curves obtained with 10 W, 25 W and 35 W incandescent lamps.

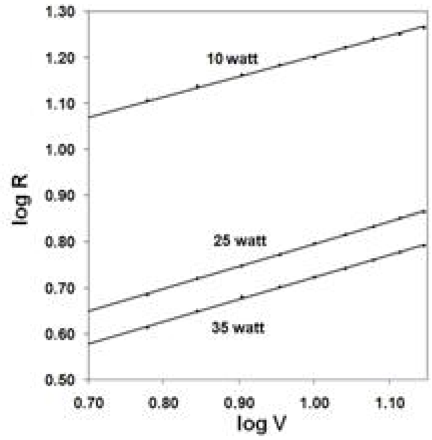

FIGURE 2. Log R curves (based on the data of Fig. 1). The dots represent experimental data and the straight line represents least square fit to the data generated by a computer.

Using the data of figure 1, log R versus log V is plotted in Figure 2. The dots in Figure 2 represent experimental results while straight line is the result of computer generated least square fit to the data. The slope and y-intercept of a straight line (Figure 2) give, respectively, the power p of V and log B_1 (see Eq.6). The values of the exponent p and B_1 thus obtained from Figure 2, are listed in Table I. These values of p are close (within 7%) to the theoretical value of p = 0.46 (Eq.6b). For the measured values of B_1 and R_{RT} (table I), and the value of ρ_{RT} obtained from Eq. (3), Eqs. (6a) and (6c) could be solved simultaneously to obtain the values of L (length) and r (radius) of the filament wire. These parameters for the three lamps are listed in Table I.

The results of the measurement of filament temperature, T, as a function of V for the three lamps are shown in Figure 3.

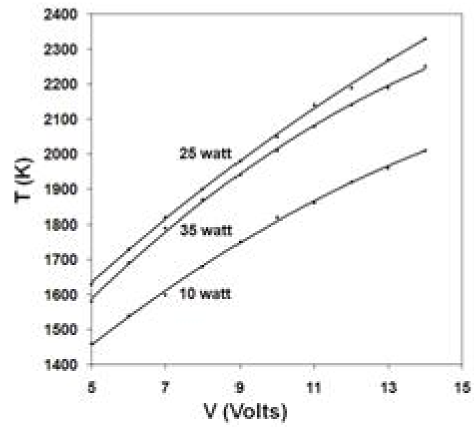

FIGURE 3. Temperature versus voltage curves obtained with 10W, 25 W and 35 W incandescent lamps.

TABLE I. Various parameters of the lamp filaments: Study of R as a function of V.

Rated power of lamp (W)	Cost ($)	*R_{RT} (Ω)	**Measured B_1 (Ω-$V^{-0.46}$)	**Exponent p	***Δp	Length of filament wire L (cm)	Radius of the filament wire r (μm)
10	0.08	1.8	5.8	0.45	2%	9.5	32
25	0.15	0.68	2.0	0.49	7%	17	69
35	0.50	0.50	1.7	0.48	5%	12	68

* R_{RT} is the filament resistance at room temperature, measured by Wheatstone bridge method.
** Determined from plot of *log R* versus *log V* (Figure 2).
*** $\Delta p = \{ [|0.46 - p|] / 0.46 \} \times 100$

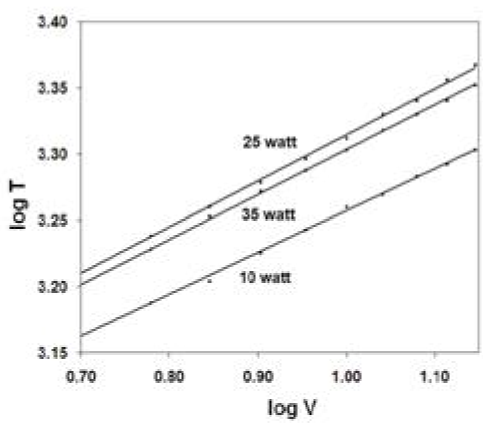

FIGURE 4. Log T versus log V curves (based on the data of figure 3). The dots represent experimental data and the straight line represent least square fit to the data generated by a computer.

Using the data of Figure 3, log T versus log V is plotted in Figure 4. The dots in Figure 4 represent experimental results while straight line is the result of computer generated least square fit to the data. The slope and y-intercept of a straight line (Figure 4) give, respectively, the power q of V and log B_2 (see Eq.7). The values of the exponent q and B_2 thus obtained from Figure 4, are listed in Table II. These values of q are within 16% to the theoretical value of q = 0.38 (Eq. 7b). Using the values of the parameters L and r (Table I), B_2 were calculated from Eq. (7a). These are also listed in Table II. The difference between the calculated and measured values of B_2 was found to be as large as 22% (see Table II).

Simple model used in the present work was based on some assumptions which may introduce errors. The filament was assumed to be uniform cylinder of cross-sectional area A_c and length L. The assumption of uniformity neglects the possibility that a real filament has thin regions due to mechanical processing needed to form the coil as well as evaporation during using which effectively limits the useful lifetime of the lamp [2]. The assumption of a cylinder instead of a coil will overestimate the effective radiating area [6], since the regions on the inside of a coil radiate back and forth, trapping some of the energy and resulting in higher filament temperatures than would be obtained with a straight cylinder. This is more obvious in the 25-W lamp since measured temperature (at given voltage) is larger than that calculated (Table II). On the contrary the measured temperatures were smaller than those calculated for 10–W and 35–W lamps. On examining lamp filaments under a magnifying glass it was observed that spacing between the turns was much smaller in case of 25–W lamp as compared with the corresponding for the 10–W or 35–W lamps. The volumetric thermal expansion of the filament only provides a correction of less than 2 % [5]. Chemical impurities in the tungsten wire may be another factor that have contributions to the observed differences in the measured and calculated values of various parameters (Tables I and II).

TABLE II. Various parameters of the lamp filaments: Study of *T* as a function of *V*.

Rated power of lamp (W)	*Exponent *q*	***Δq*	*Measured B_2 (Ω-K-V$^{-0.38}$)	***Calculated B_2 (Ω-K-V$^{-0.38}$)	****ΔB_2
10	0.32	16%	8.7×10^2	8.2×10^2	6%
25	0.35	8%	9.3×10^2	7.6×10^2	22%
35	0.35	8%	9.1×10^2	8.7×10^2	5%

* Determined from plot of log *T* versus log *V* (Figure 4).
**$\Delta q = \{[|0.38 - q|] / 0.38\} \times 100$.
*** Calculated from Eq. (7a) using the values of *L* and *r* from Table I.
**** $\Delta B_2 = |[(\text{Calculated } B_2 - \text{Measured } B_2) / \text{Calculated } B_2]| \times 100$.

It was reported [5] that emissivity of the filament depends on its temperature. When emissivity of the filament is assumed to be independent of temperature, then we have electric power is proportional to T^4, and this would give

$$R \text{ proportional to } V^p, \text{ where } \quad p = 0, \quad \text{(Eq. 6)}$$

and

$$T \text{ proportional to } V^q, \text{ where } q = 0.38. \quad \text{(Eq. 7)}$$

If we take emissivity to be proportional to *T* (as in Ref. [5]), then we have electric power proportional to T^5, and this would give

$$R \text{ proportional to } V^{0.39},$$

and

$$T \text{ proportional to } V^{0.32}.$$

The average value of the exponent, *p*, is 0.47 (Table I) and this is closer to 0.46 (for emissivity being independent of *T*) rather than 0.39 (the value for emissivity being dependent on *T*). Following above this value of *p* suggests that the emissivity may not depend significantly on temperature. The average value of exponent $q = 0.34$ (*T* versus V^q, table II). This suggests that the emissivity may depend on temperature, though not necessarily a linear dependence. However, the results of *R* versus *V* are less affected as compared with the results of *T* versus *V*, by the uncertainties (outlined above) involved in this work. Therefore, we may suggest that emissivity of the filament is nearly independent of temperature in the present case. It was concluded in Ref. [5] that for tungsten filament whose surface is oxidized its emissivity is independent of temperature. May be this is the case in the present work.

V. CONCLUSIONS

An experiment using low-power incandescent lamps for the verification of Stefan-Boltzmann law was carried out. Two separate sets of measurements were made on the lamps. These included filament resistance versus applied voltage and filament temperature versus applied voltage. The results were compared with those calculated using a simple model. Overall, satisfactory results were obtained.

Such an experiment may be a good addition in student laboratory. Measurement of filament temperature to confirm theoretical results is expected to give a student some confidence in theory. Most of the equipment used is readily available in the laboratory or can be easily acquired from a local market. Some effort is needed to acquire a pyrometer for measuring the filament temperature.

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