

The spring-mass experiment as a step from oscillations to waves: mass and friction issues and their approaches



I. Boscolo¹, R. Loewenstein²

¹Physics Department, Milano University, via Celoria 16, 20133, Italy.

²Istituto Torno, Piazzale Don Milani 1, 20022 Castano Primo Milano, Italy.

E-mail: ilario.boscolo@mi.infn.it

(Received 12 February 2011, accepted 29 June 2011)

Abstract

The spring-mass experiment is a step in a sequence of six increasingly complex practicals from oscillations to waves in which we cover the first year laboratory program. Both free and forced oscillations are investigated. The spring-mass is subsequently loaded with a disk to introduce friction. The succession of steps is: determination of the spring constant both by Hooke's law and **frequency-mass oscillation law**, damping time measurement, resonance and phase curve plots. All cross checks among quantities and laws are done applying compatibility rules. The non negligible mass of the spring and the peculiar physical and teaching problems introduced by friction are discussed. The intriguing waveforms generated in the motions are analyzed. We outline the procedures used through the experiment to improve students' ability to handle equipment, to choose apparatus appropriately, to put theory into action critically and to practice treating data with statistical methods.

Keywords: Theory-practicals in Spring-Mass Education Experiment, Friction and Resonance, Waveforms.

Resumen

El experimento resorte-masa es un paso en una secuencia de seis prácticas cada vez más complejas de oscilaciones a ondas en el que cubrimos el programa del primer año del laboratorio. Ambas oscilaciones libres y forzadas son investigadas. El resorte-masa se carga posteriormente con un disco para introducir la fricción. La sucesión de pasos es: determinación de la constante del resorte tanto por la ley de Hooke y la ley de oscilación frecuencia-masa, reducción del tiempo de medición, y representación de curvas de fase y resonancia. Todos los controles cruzados entre cantidades y leyes se realizan aplicando las normas de compatibilidad. Se discute la masa no despreciable del resorte y los problemas específicos de física y enseñanza derivados de la fricción. Se analizan las formas de onda complejas generadas en los desplazamientos. Se describen en términos generales los procedimientos utilizados en el experimento para mejorar la capacidad de los alumnos para manejar los equipos, para elegir adecuadamente los aparatos, para poner de manera crítica la teoría en práctica, y para practicar el tratamiento de los datos con métodos estadísticos.

Palabras clave: Teoría-prácticas en la educación experimental Masa-Resorte, Fricción y Resonancia, Formas de onda.

PACS:01.40.Di, 01.50.Qb

ISSN 1870-9095

I. INTRODUCTION

The spring-mass oscillator experiment is used worldwide in first year laboratory courses because it is physically and pedagogically rich, and its apparently simple contents lead students to face the gap between theory and practice, to program and master experimental techniques and to treat data statistically. In our course, the spring-mass is the second step in a sequence of six experiments, as follows: the pendulum, vertical oscillations with the spring-mass, spring-mass with two degrees of freedom, horizontal oscillations with two masses and three springs, vibrations in an elastic string, vibrations in a gas tube and the ripple-tank. The idea is to evolve from the single oscillator to waves, given their importance in many fields such as electronics, accelerators, quantum-mechanics, etc. In this

context, our mass and spring experiment is strictly connected to the development towards waves.

The course organization is based on the idea of creating groups of about twenty students at a time in a laboratory, working together as in a research group studying a certain topic. As such, the group head guides the experimental research project: he presents the theory in a few lectures, describes the test strategy, defines steps and, very importantly, creates the occasions for discussions at crucial points. In the laboratory sessions sub-groups of three students are helped by assistants.

Given that our first year students usually do not have a background in practical physics, we focus our attention only on the simplest damped harmonic oscillator during the theory lessons which precede the practical sessions. In the laboratory, students start with the most common spring and

mass apparatus as described in so many textbooks. We try as much as possible to make it behave as a textbook damped simple harmonic oscillator. This means that the spring is assumed massless and friction is assumed velocity dependent. Nevertheless, neither assumption complies with our reality. The discussion about the gap between simple theory and rather more complex reality is postponed to the moment when this is noticed by students in the laboratory sessions. Then we bring up the various oscillating modes, so students notice that the initial mathematical model was inadequately simpler than reality. SHM is indeed far from simple.

The spring force and the weight act in the spring-mass vertical system making the physics somewhat sophisticated, if not very-complex [1, 2]. The real spring-mass behavior cannot be treated as a textbook SHM; many authors discuss how, in practice, we must consider factors such as the torsional force [3], possible standing waves in the spring [4], loaded vertical oscillations [5, 6, 7] and multi-mode oscillations [8, 9], the mass of the spring if non-negligible with respect to the object mass attached to it [2, 8, 9].

The spring-mass system can have a behaviour which is acceptably near a SHM only if proper ratios spring constant/mass, k/m , and spring constant/spring length, k/l , are chosen [5, 9]. In addition, the choice of the mass m is connected to the damping constant γ and also to the resonance curve full width half maximum, FWHM; moreover, it should match the driving oscillator sensitivity.

The main objects of discussion which appear during the lab work are: (1) the relevant action of the spring mass to be considered in the comparison of the two results on the spring constant obtained from the Hooke's law and the frequency-mass oscillation law; (2) the damping of a system whose behavior depends also on the non-linear coupling between longitudinal and transverse oscillations; (3) the motion at the start-up composed by the forced and self oscillations.

The approach to treat and to discuss choices, problems and discrepancies between theory and experiment with students are presented. All choices about the system are discussed with students during the experimental work. Moreover, students have to try to understand and explain several unexpected experimental observations; therefore, they look deeply into the gap between theory and practice and discuss it in small groups and, after that, in plenary sessions.

II. THE TEACHING PLAN, ORGANIZATION AND SETUPS

The theory of free and forced oscillations is presented in about 10 hours of lectures. In the first three we present the theory of free oscillations for the pendulum and springs. We also give guidelines to plan the very first experiments that follows suit. In the other six or seven hours we discuss damped oscillations and guide students towards planning the second spring-mass experiment.

The SHM equations describing the free and forced motion are reported for convenience.

$$\frac{d^2 x(t)}{dt^2} + \frac{c}{m} \frac{dx(t)}{dt} + \frac{k}{m} x(t) = 0, \quad (1)$$

$$\frac{d^2 x(t)}{dt^2} + \frac{c}{m} \frac{dx(t)}{dt} + \frac{k}{m} x(t) = f(t) = \left[\frac{F_0}{m} \sin \omega \right], \quad (2)$$

whose solutions are

$$x(t) = x_0 e^{-\gamma t} \sin(\omega' t) \gamma = \frac{1}{\tau}, \quad (3)$$

$$x(t) = A(\omega) \sin(\omega t + \varphi(\omega)), \quad (4)$$

where

$$\omega'^2 = \frac{k}{m} - \left(\frac{c}{2m} \right)^2 = \omega_0^2 - \gamma^2, \quad (5)$$

$$A(\omega) = \frac{c \omega t}{\sqrt{(\omega_0 - \omega)^2 - \gamma^2}}, \quad (6)$$

$$\tan \varphi = - \frac{2\gamma\omega}{\omega_0^2 - \omega^2}. \quad (7)$$

In these equations, F_0 and ω are amplitude and frequency of the applied force, $\gamma = C/(2m)$ is the damping constant, C is the friction coefficient and φ is the phase difference between the applied force and the position $x(t)$ and m is the mass attached to the spring. The damping time is $\tau = 1/\gamma$. In the above equations, the gravitational force mg is opposed by the spring force $k\ell_0$.

After that, when they come to practicals, students assemble the apparatus and follow their plan after an initial guiding plenary discussion. Before starting each sequence of measurements, we test the apparatus with a few values far apart from each other and compare practical results with theory. Assistance is given to help slower groups keep the pace. At regular intervals data of all groups are collected on the board for statistical treatment, discussion and conclusions. This procedure helps to show how to spot systematic errors and casual mistakes.

In the first part of the experiment students observe and gather data directly, see Fig. 1, in the second part they work on line with electronic data systems, as shown in Fig. 2. Students start with the test of Hooke's law by doing a set of force-distance measures and by doing in succession a linear fit with those data pairs to find the spring constant. Springs are long enough for precise measurement of force and distance. The same springs are used to check the relation $\omega^2 = k/m$. The resonance frequency ω_0 is measured via the period T with a chronometer. Then students do the linear fit of the data pairs (m, T^2) to get the spring constant. Compatibility between the two measured spring constants k is analyzed. Then students make a rough measure of decay time τ and, re-set the system for the resonance experiment section, they take measurements to obtain the two resonance curves as in equations. 6 and 7 above, and obtain results similar to those shown in Fig. 3.

The relation between the FWHM $\Delta\omega$ of the resonance curve and the damping time τ should be checked:

$$\Delta\omega [FWHM] = 2 \cdot \sqrt{3} \gamma \text{ Hz} \rightarrow \Delta\nu \simeq 0.55 \gamma = \frac{0.55}{\tau} \text{ Hz} \quad (8)$$

The apparatus has been kept as simple as possible, just a spring, a weight and a CD-rom to increase air resistance, see Figs. 1 and 2.

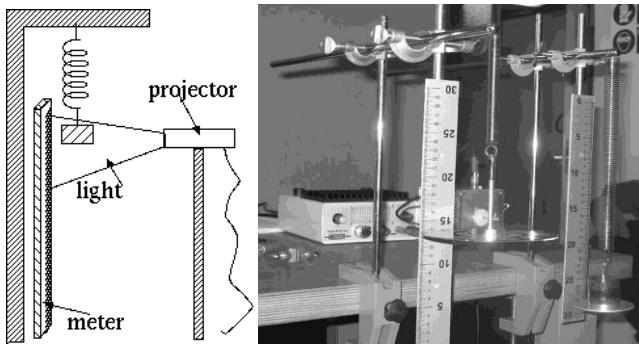


FIGURE 1. Left frame: layout scheme for the measurement of the spring constant k and free oscillation period T as function of different masses and of the damping time. Right frame: photo of the two springs and configurations used in the experiment, the bigger spring is used in the first section, the smaller one is used in the resonance experiment. The choice of a vertical setting makes the system simpler.

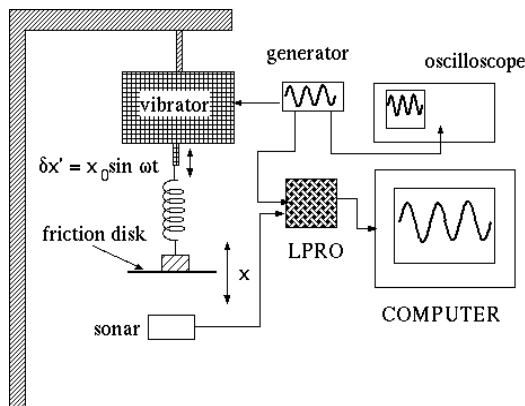


FIGURE 2. Layout scheme for the measurement of the forced oscillation amplitude as function of the force frequency (resonance curve) and of the relative phase between the force and the motion oscillation (phase-curve).

The unpredictable results obtained with the τ measurement are explained in a plenary discussion. In this occasion it is also observed that the heavy springs with the rather small hanging weights are not appropriate to get a wide enough resonance curve. Neither commercial oscillators nor our purpose built sources are sensitive enough to generate frequencies differing only by few MHz. These would be needed to get enough points near the resonance frequency. As a consequence, in order to get more points near the

frequency of resonance, students have to use a lighter spring for forced oscillations. This choice is the focus of the discussion about matching variables such as masses and spring constants to the available apparatus used for generating and detecting oscillations.

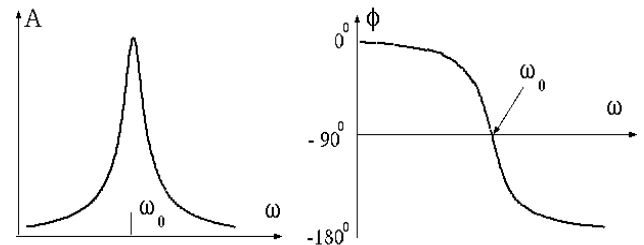


FIGURE 3. Plots of the resonance and phase relations 6 and 7.

For the sake of thoroughness, Eq. 2 is further specified to the apparatus of our experiment shown in Fig. 2.

$$m \frac{d^2x}{dt^2} = -k(x - \delta x') - C \frac{dx}{dt},$$

$$= -k [x - x_0 \sin(\omega t)] - C \frac{dx}{dt}, \quad (9)$$

where $\delta x'$ is the vibrator oscillation amplitude, see Fig. 2.

Using the CD to increase friction is simple and inexpensive but poses questions with damping and resonance which are a challenge to manage with students. We will see that friction does not follow the law $C \cdot \dot{x}$ as assumed in the motion Eqs. 1 and 2.

III. HOOKE'S LAW

The relation to be tested is $\Delta F_i = k \cdot \Delta x_i$. As shown in Fig. 2 the spring is stretched by a mass attached to its end. A ballast mass is put to get a linear graph even with a small spring extension. Students are strongly advised to draw graphs in real time to keep track on data and to avoid parallax errors. Lab work starts by checking the linear extension of the spring for three distant points. Afterwards, systematic data reading takes place. For the linear fit, students transfer the error of the x variable into the y variable by $\sigma_y = k \cdot \sigma_x$, using for previously measured value of k . The confidence level in this test comes out higher than 80 % and compatible with the origin. This is a simple and successful test to practice a straight line fit.

IV. TEST OF $\omega^2 = k/m$

Eq. 5 is approximated at this stage of the experiment as

$$\omega'^2 = k \cdot \frac{1}{m} = \omega_0^2 \rightarrow v'^2 = v_0^2 = \left(\frac{1}{T}\right)^2. \quad (10)$$

For convenience this equation is changed into the following one

$$m = k \cdot \left(\frac{T^2}{4\pi^2} \right). \quad (11)$$

Here k is the spring constant measured in the previous test.

The apparatus is set with tight clamps and short rods to avoid spurious oscillations. The apparatus is tested comparing the values $(m_i \cdot \omega_{0,i}^2)$ for three points set far apart. The three points (m_i, T_i^2) must be aligned and the line must pass through the origin. The minimum mass is determined by students' ability to measure the frequency once the system oscillates regularly.

Students feel baffled when they start to calculate the value of k because the ratios k_i come out un-compatible and the graph does not pass through the origin. The groups are then gathered for a plenary discussion with their data on the board. The data show that the heavier the mass the higher the k value. All the plots show straight lines, as expected, but all of them cross the y -axis at a negative point. We can trust these results because they are similar in all groups. From these first measurements and observations student are forced to conclude that data are not consistent with theory. As a conclusion, we have to revise the theory and the experimental apparatus, but the equipment is simple and works well, so the mistake cannot be there. We have to have a closer look at k , T and m .

The period T is measured with high accuracy, k refers to the same spring of the previous session where it worked perfectly well, so, we are led to conclude that the problem lies in the mass: what is really oscillating in our experiment? Let's start from first principles: Newton's law $F = ma$. The mass m in the equation refers to the mass of the moving body. Are we considering the whole moving mass in our system? No! The mass of the spring is moving as well. OK, we got the problem. But now we observe that the motion of the spring-mass is very complicated: the top part of the spring is at rest while its bottom part moves with the attached object. We may argue that the "motion" (acceleration, velocity) increases linearly from top to bottom. This is too difficult for the students at this stage but we may guess that we can write the total moving mass as the sum of the attached body mass and a fraction of the spring mass, which we call m_{spring} . Eq. 11 can be written as

$$m = k \cdot \left(\frac{T^2}{4\pi^2} \right) - m_{spring}. \quad (12)$$

The equation above explains the negative intersection at the y -axis. The general discussion leads to consider that the spring mass counts for 1/3 (Ref. 14) of its total mass as can be immediately confirmed from the rough graphs obtained so far. So, an estimate for k is calculated and it is closely compatible with the one found in the previous session.

This discussion for students is an example of scientific method in physics: theory - experiment - discrepancy - criticism - new-model - new experimental observation and conclusion.

After the discussion, students test relation 11 (time allows them to take readings only for a few points). The error on the x -variable is transferred to the y -variable, as done in the previous fit.

The confidence level of the fit results $CL \approx 10\%$, the value of the k extracted from the fit is loosely compatible with the k value found by Hooke's law, the line is compatible with the origin, see Fig. 4. The compatibility between the spring constant measured with the two techniques is analysed. The results came out better in groups who chose heavier masses for the test.

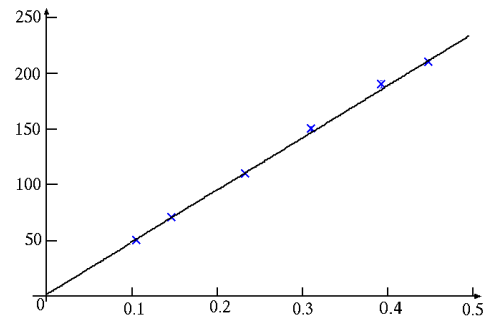


FIGURE 4. Typical graph $m = 468 \cdot T^2 + 2$ fitting the experimental data. We see that there is a small shift between points obtained with lighter and heavier masses.

In the plenary discussion of the different group results it emerged that when using lighter masses students observed an irregular oscillation; initially, the motion was vertical, then it became lateral. It was not easy to get a clean motion. Larger masses gave smoother results, which will happen again later when measuring damping constant γ and its effect on the frequency. Once again the discussion allows students to practice method and critical thinking.

V. FREE OSCILLATION DECAY TIME τ

By definition the decay time τ is the time required for the oscillation amplitude to decay to 1/e of its initial value.

A. Direct measurement of τ

Students start measuring the time with the chronometer and the reducing oscillation amplitude with the meter, see Fig. 2. Different groups choose different masses from 80g to 200 g, and try to produce neat vertical oscillations. The τ values span from 40s to 80s, the relative error is larger than 20%. The results discussed in class lead to: (1) the higher the mass the longer the damping time, (2) that τ value corresponds theoretically to an expected frequency width of resonance, $FWHM = \delta\nu \approx 1/60 \sim 0.017$ Hz. This frequency interval is too narrow for the sensitivity of the generator, 0.02Hz, which will subsequently drive the system. We need a damping time at least five times smaller in order to be able to measure a few points in the upper part of the resonance curve.

By adding a CD we just managed to double its value (judging by the lower decay time). Searching for a new idea we revised the initial equation $\tau = 2m/C$ (Eq. 3) and observed that the higher the mass the longer the damping time. In order to obtain the adequate FWHM, the mass must be very small and as such it will be comparable to the spring mass. We concluded that only a lighter spring would give the required result. Students then take a softer spring and do a rough measurement of: (1) the new spring constant k , (2) the new frequency of resonance and (3) the new τ . The encouraging results with an applied mass $m_{object} \approx 71\text{ g}$ and $m_{spring} \approx 4.8\text{ g}$ are:

$$k = 8 \pm 2\%v \approx 1.67 \mp 2\% \text{ Hz}\tau \sim 12 \mp 2\text{ s.}$$

This leads to the second section of the experiment, which is on-line.

B. Measure of τ using the computer waveform

An accurate measurement of the damping time is done using the graph on the computer screen. A typical waveform with a mass of 73g is shown in Fig. 5.

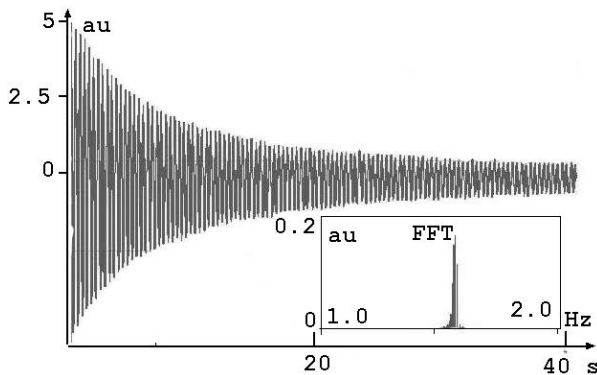


FIGURE 5. Typical recorded damping signal with its FFT analysis. This "well-shaped" waveform is obtained with a very careful initial displacement to start the motion. This must remain perfectly vertical (even with a small deviation other oscillation modes are excited).

The frequency ν can be immediately obtained from the computer waveform using the software FFT (Fast Fourier Transform). The frequency results compatible within one percent error with the result obtained directly using the period.

The measurement of the frequency ω and the decay time τ is done by fitting the waveform with the expected curve $f(t) = a(0) \cdot \exp\left(-\frac{t}{\tau}\right) \cdot \sin(\omega_0 t + \varphi) + b$ putting in the first trial $a(0)$, τ , ν obtained in the above direct measurement. Before this procedure, students are told to get τ from the waveform by measuring the time from the start to $1/e$ of the initial amplitude. The decay time with the waveform method gives much longer values because the oscillation does not decay as fast as we expected, especially towards the end. This second result was completely un-

expected. The fit turns out to be impossible, only partial fits are feasible using parts of the decay pattern. The FIT using the first 10 seconds gives $\tau \sim 12\text{ s}$, the FIT in the range $\Delta t = 0-20\text{ s}$ is $\tau \sim 17\text{ s}$, the FIT in the range $\Delta t = 10-30\text{ s}$ is $\tau \sim 20\text{ s}$, the FIT in the range $\Delta t = 20-60\text{ s}$ is $\tau \sim 30\text{ s}$. The frequencies in the different curve sections differ coherently with the measured τ : longer τ higher frequency. Students observed that the stronger the excitation the shorter the measured τ . The persistence of the oscillation exceeded 100s. The fitting curves showed a tail amplitude thinner than the recorded waveform. The frequency of the waveform was $\omega_0 = 10.50\text{ Hz}$.

The waveforms with 50 and 60g effective masses come out "worse" than that depicted in the above figure, while with 80 and 90g masses the waveform results are even better. To give an idea of the waveform complexity with a small mass, we report the waveform obtained with a hanging mass of 40g in Fig. 6. The second ω' turns out to be about two times higher than the fundamental one.

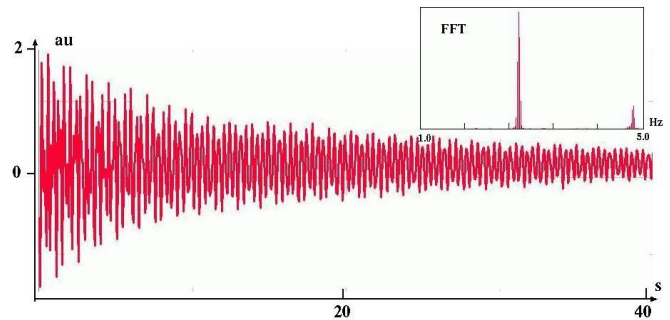


FIGURE 6. Damping signal with its FFT analysis with a mass of 40 g. The FFT signal shows the presence of two frequencies, as suggested by a modulated amplitude.

C. Plenary discussion about characteristics of damped waveforms

The experimental observations indicate that we are facing something much more complicated than initially expected from the motion equation 1: the damping decreases notably at the beginning, then the rate of decay diminishes and ultimately the small oscillations continue for a long time. From this behavior we may guess that friction decreases with smaller amplitudes and, when the amplitude is small enough, there should be a kind of oscillations' feeding given that friction is smaller but not yet negligible.

Several authors have discussed the modes of resonance in which a vertically oscillating spring spontaneously oscillates between spring-bouncing and pendular-swinging [1, 2, 10, 11]. We start observing that the real physical spring-mass system shown in the photo of Fig. 1 cannot have a perfect axial symmetry (the spring and mass hanging systems, the spring deformations at the extremes, the rather irregular mass shape and setting). Then, we observe that there is a coupling (non-linear) between longitudinal and transverse oscillations, due to the "natural" presence of an

axial and transverse force components in the motion. In this connection we observe that the ratio between the transverse and vertical motion frequencies in our setup for the 73g effective mass is

$$\frac{\omega_{long}}{\omega_{trans}} = \sqrt{\frac{k/m}{g/\ell_0}} \sqrt{\frac{8/(7.3 \cdot 10^{-2})}{9.8/0.14}} \approx 1.2.$$

This ratio, not so far from one, suggests a possible energy transfer between the two motions. We would like to stress here that the length ℓ_0 can be approximated constant only when the oscillation amplitude becomes small enough. It is worth remembering that when the ratio between the pendulum and longitudinal frequency is two, Walkinshaw resonance occurs, there is a total energy transfer between the two motions. In Ref. [1, 10] the observation of the autoparametric resonance behavior is reported. In this connection, we may also argue that the decreasing damping shown by the waveform could be due to the energy transfer from the longitudinal to transverse motion. In summary, the damping decrease seems to have two causes, the decrease of the friction coefficient γ with the oscillation amplitude and the feeding of the transverse oscillation. We are not able, at this stage, to separate the two causes. We note that, since friction in the transverse motion is much lower than in the longitudinal one, the transverse motion can feed the longitudinal one at small oscillation amplitude.

We decide to stop discussing the very detailed experimental observations at this point and to continue the discussion after further data about the forced oscillations. Groups were told to hang different masses to allow comparisons.

At the end, we decided to assume as a damping time the value obtained at the first 10s interval, which we call τ_{10} . This because it is reasonable to assume that within that time interval the vertical oscillatory motion is not yet combined with the transverse one, and, the amplitude after that time is reduced by a factor greater than 2.

We want to stress that with heavier masses the τ_{10} is nearer to $\tau_{1/e}$, that is to the time measured at 1/e of the original amplitude.

VI. FORCED OSCILLATION

The second part of the experiment is about resonance and it aims at reproducing and comparing the two theoretical curves of Fig. 3. From the damped waveform analyzed before the two expected analytical curves come out to be

$$y_{lo}(\omega) = \frac{a}{\sqrt{(\omega - \omega_0)^2 + (1/\tau)^2}} = \frac{a}{\sqrt{(\omega - 10.5)^2 + (1/12)^2}}, \quad (13)$$

$$\tan_{ph}(\omega) = -\frac{2\gamma\omega}{\omega_0^2 - \omega^2} = -\frac{0.1666 \omega}{10.50^2 - \omega^2}. \quad (14)$$

The amplitude of the Lorentzian curve will depend on the applied force.

A. Resonance curve measurements

Students assemble the system and set the generator frequency within the frequency interval predicted in the free motion analysis, set the generator amplitude so as to have an oscillation amplitude at resonance near to the maximum of the observed free oscillation. They read the relative oscillation amplitude $a(\omega_i)$ at each frequency ω_i for the resonance curve and the two times t_g and t_s at the crests of the generator and signal sinusoids (for the $\Delta\phi_i = \omega_i \Delta t_i$ atan phase-curve). Students were advised to wait for the oscillation to arrive at a steady state after each change in frequency, typically it would be three times longer than the decay time. In fact, after any change in the system parameters, there is a transitory behavior. The data and the graphs for one set of values (for the system with 70g mass) are shown in Fig. 7. The fitting curve of the resonance amplitude (Lorentzian) is very near the expected one (but with the decay time measured at 10s). This does not occur in the expected phase curve: in Fig 7, we can see that (i) the upper data has a steeper slope than the lower data and (ii) the tail data are not fitted at all by the fitting curve. A better fitting curve has the expression.

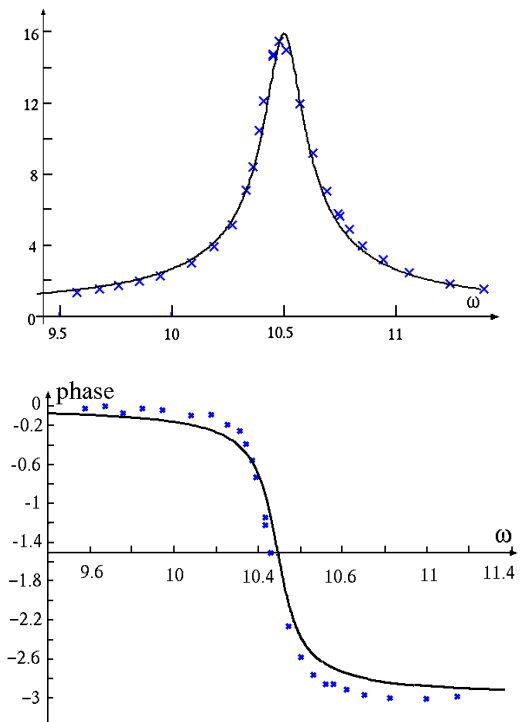


FIGURE 7. The y_{lo} analytical curve matches the data fairly well, as opposed to the analytical atan phase curve. A slightly better atan fit curve leads to $\tau=16s$ instead of 12s. The main observation is that the upper and lower sections of the atan curve (data) have two different slopes: the upper section is much steeper than the lower one. The depicted fit is a kind of compromise between the two, but, anyway, the upper section is worse fitted.

$$\arctan_{ph}(\omega) = -\frac{0.122}{10.49^2 - \omega^2} \Rightarrow \tau = 16.4s.$$

It is worth noticing that a certain amplitude modulation was observed at all forcing frequencies. It appeared more pronounced outside the resonance frequency, but it should simply be due to the increase of the ratio modulation to oscillation amplitude. This observation confirms a similar observation in the free motion.

B. Discussion: Free and forced motion results

In summary, the Lorentzian curve fit is to a good extent compatible with the two motion parameters τ and ω_0 measured from the free motion waveform (assuming the τ_{10} of that waveform). The phase curve resonance is not really satisfactory mostly because the tail data do not fit. Some observations are due: one is that the tail data do not match the Lorentzian curve either; in both cases the tail data would lead to a notably longer τ ; a second observation is that the discrepancy between data and the fitting curve in the atan phase curve looks very large (this happens because atan function is very sensible to small data deviations); another subtle but important observation is that there is a kind of switch in the motion when crossing the resonance frequency, at lower frequencies the slopes of curves and data are steeper than those at the higher frequencies: since in that passage a transition from the in-phase to counter-phase between the force and the motion occurs, we may argue that there is an interference between the vibrator and the spring-mass and it is such that the energy absorption is higher when they are in counter-phase. One more observation is that both resonance and phase tests confirm the friction reduction at reduced oscillation amplitude observed in the free waveform, this indicates that friction depends on the oscillation amplitude and is not directly proportional to velocity as assumed in the model.

The discrepancies observed between expected behaviour and the observed data are mostly due to friction because the initial assumption showed to be wrong. The point is that we have not yet succeeded in creating a relatively simple and inexpensive friction force directly proportional to velocity for this experiment.

Students in the lab do not have time for the large set of data the authors used to support the discussion of the fitting curves in this section. We gathered the data and added them here to look deeper into the matter and to offer suggestions for further investigation.

VII. COMPARING DATA WITH DIFFERENT MASSES

We measured the decay time and repeated the resonance curve using masses from 50g to 90g at 10g steps. Our aim was to investigate how mass and the relevant friction influenced conditions of resonance. Table I and Fig. 8 summarize the results obtained by changing the frequency around the value of resonance.

TABLE I. Values of the decay times obtained from the damped waveform fitting analysis at the time 10s, τ_{10} and at 1/e amplitude reduction, $\tau_{1/e}$, obtained from the Lorentzian curve τ_{10} , and from the phase curve τ_{ph} as function of mass.

mass	τ_{10} s	$\tau_{1/e}$ s	τ_{10} s	τ_{ph} s
50g	10.5	11.8	7.8	8.5
60g	12	12.6	10.2	10.9
70g	12.8	14.4	12.5	15.5
80g	15.5	17.5	15	17.5
90g	17.4	20	18	22

From the table and figure we see that the decay times τ_{10} and τ_{10} begin to converge from a 70g mass (they start overlapping at 80g), while $\tau_{1/e}$ turns out to be on average almost two seconds higher. The system with 50g and 60g shows higher discrepancies between the two τ and also a different slope.

The model for friction $\gamma\dot{x} = (C/m)\dot{x}$ states that the ratio between the τ of two systems is equal to the ratio between the two relative masses. This is verified by the τ listed in the Table 1 with more than 90% accuracy and by the nearly straight line which refers to the forced system in Fig. 8. The reason is that in the forced configuration the system is, in fact, forced to a neat motion, hence its behavior is better represented by the model.

From all these observations, we may guess that the different behavior of the 50g and 60g mass systems and the other higher masses is due to the different coupling between the vertical and transverse motions: the heavier the mass, the neater the oscillation. The free motion has always a coupling between the two motions which makes $\tau_{1/e}$ longer than τ_{10} .

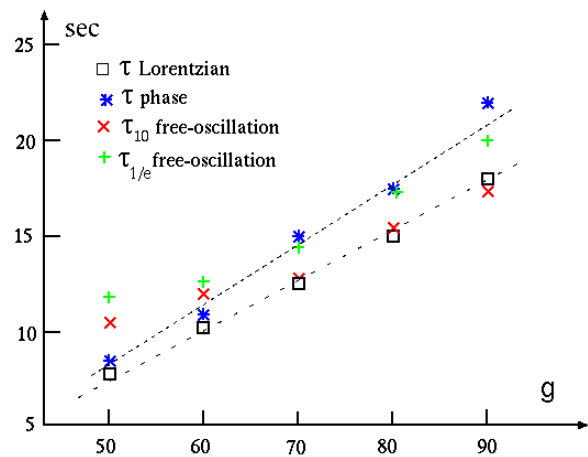


FIGURE 8. Evolution of the relative relaxation times extrapolated from the free-oscillation waveforms, τ_{10} and $\tau_{1/e}$, and from the resonance and phase fitting curves, τ_{10} and τ_{ph} .

VIII. TRANSIENT

Three typical waveforms observed near the resonance frequency are shown in Fig. 9. the smooth amplitude

increase is observed at frequencies very near resonance, the last one is observed at the tail frequency.

In physics books, an amplitude modulated sinusoidal signal implies the sum of two sinusoidal signals having two similar frequencies, which is easily checked with a computer. The phenomenon is called beats (treated in the next programmed experiment).

Looking carefully at our system we suggest that the general mathematical solution of the motion Eq. 2 is

$$x = A \sin(\omega t + \varphi) + Be^{t/\tau} \sin(\omega_0 t), \quad (15)$$

where the two frequencies involved are the vibrator and the self-system frequencies respectively: our mass-spring forced oscillator has two superimposed sinusoids at the start of the excitation, the persistent (particular solution) one and the decaying one (homogeneous solution). In fact, the signal modulation decays in the previously measured damping time.

The first waveform in Fig. 9 occurs when the two particular and homogeneous frequencies are almost equal or strictly equal to the natural frequency: the system drifts towards the maximum amplitude. The second waveform is obtained when the forcing frequency is not far from the natural one: beats die in a time longer than two damping times as obtained in Fig. 5. The last waveform is observed when the forcing frequency is far enough from resonance: the amplitude of the particular solution does not increase with time (it remains with a small amplitude) hence it shows wide beats with the natural vibration. From the modulated waveforms we could measure and thus check the different frequencies. In addition, the systems with a relatively low mass showed a superposition of longitudinal and transverse oscillations.

Careful inspection of the spring action explains both motions because it applies either a longitudinal force or a small torque (due to the helicoidal shape of the spring). The two motions have different frequencies and lead to a modulated waveform. As a final observation we note that the natural frequency is continuously excited because the system is not mechanically isolated from the surroundings.

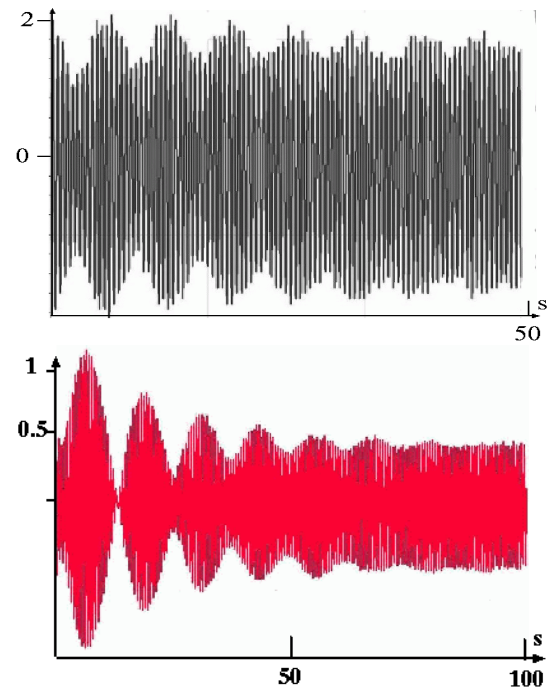
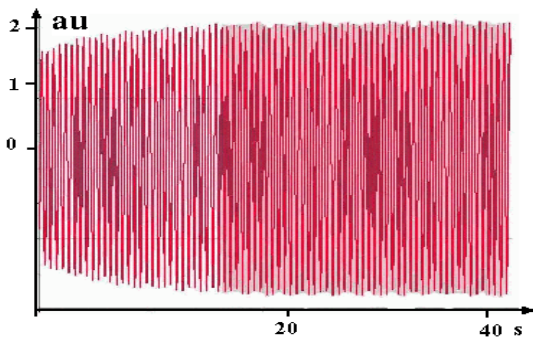


FIGURE 9. The left signal is obtained at resonance frequency: the continuous amplitude increase up to saturation indicates the energy filling into the system. The right signal is obtained with a forcing frequency near resonance: The beat occurs between the free and the forced oscillation, at the stating the amplitudes of the two signals are almost equal, therefore the modulation is complete, afterwards the free oscillation goes to die while the forced one increases up to its maximum. The damping time of the free oscillation comply with its previously measured value.

IX. DISCUSSION AND CONCLUSIONS

The experiment highlights the strong discrepancy between theory and practice as regards the spring mass: in the equation the spring is assumed massless, but it cannot be neglected at all. We have planned the need to change the heavy spring to a lighter one in the course of the experiment, forcing students to realize the necessity of designing a system compatible with the lab instrumentation.

The problem of the mass came up quite naturally while testing $\omega^2 = k/m$. Calculations and graphs lead the way to determine the additional mass due to the real spring operating in the system. The problem of the spring characteristics came up programming the resonance section of the experiment.

A spring-mass system oscillating vertically with a CD to increase air resistance turns out to be quite complex as a lab experiment in a first year physics course because its friction is not directly proportional to velocity. It is easy to set up and rich in Physics but, as a challenge, it is not identical to textbook explanations of damped oscillations and resonance. Therefore it requires careful teaching and strategic organization. The very low friction, $\approx 10^{-3}N$, compared to the 1N spring force, causes that the reaction of the spring-mass on the driving generator is not negligible, furthermore, the work is not constant in the frequency range

of the resonance amplitude and phase curves. The Lorentzian curve shows a left-right dissimetry, while the phase curve shows a top-down dissimetry with respect to the center of the data distribution. We have ascribed this behavior to higher work on the generator when in counter phase whereas the reverse happens when they are in phase.

Other experimental observations related to friction are the non-exponential decay of the damped free-motion amplitude and the shape variation of the two resonance curves. The three curves, the theoretical decaying sinusoid, Lorentzian and atan curves, do not fit at all the experimental data: the decay time τ increases by a factor greater than two when the oscillation amplitude is reduced by a factor two. The cross correlation between the free and forced motion data lead to conclude that there are two contributions to that reduction of friction: (i) the decrease of the oscillation amplitude (reasonably because a variation in the air turbulence with the speed) and (ii) the coupling between the vertical and horizontal motions. This observation indicates that, as the system vibrates, the physics in the middle and at the edges in the resonance curves is different, thus the data should not be treated as homogeneous.

Hence, both the mathematical model for friction and the experimental outcomes are much above students' level at this point. There is a risk of conveying a message of lack of rigour and neatness, which is damaging at this stage. Students must be guided step by step in this experiment through data analysis and plenary discussions.

The relevant problem of the mathematical model for the friction must be stressed at the beginning. It would be very difficult to set the experiment horizontally with simple means and to obtain friction directly proportional to velocity. While observing the computer graphs of modulating amplitude, caused by the particular and homogeneous solutions of the linear differential equation, with a near-delta Dirac function, and when they plot the phase-shift graph, students concentrate on fundamental physics concepts. In addition, many times both free and forced motion waveforms on the computer screen show unexpected complicated shapes, especially if the masses are relatively small with respect to the spring-mass. This happens also when the system is not set with a good axial symmetry and when the excitation is not well centred. In these cases, many modes develop and superimpose. These experimental observations can puzzle and confuse students, causing frustration and consequently a lack of concentration. Teachers have to convince them that a real physics system has always more forces and reactions than idealized textbook situations; moreover, forces and reactions are not completely described by the assumed models. We have tried to prepare a setup where students experience that with patience, rigor and method, laboratory work becomes neater and clearer while bridging the gap between theory and practice.

Throughout the various measurements and cross checks required in this experiment students are obliged to revise the bases of data treatment in depth. With the modern instrumentation students practice organization and time management.

This is, over all, a dense, sophisticated, complex and also a bit "tricky" practical, and therefore rather difficult, more so when it is carried out looking carefully at all conceptual and experimental passages. Students need time to absorb it; we have found that three half-days sometimes are not enough for them to understand all the operations and the whole physics content.

ACKNOWLEDGMENTS

The discussions with the co-teachers M. Fanti and L. Gariboldi have been important for the planning and organization of the teaching process and of the results evaluation. We recognize the determinant contribution of S. La Torre with the construction of the generators tailored to the requests of the experiment.

REFERENCES

- [1] Cayton, T. E., *The laboratory spring-mass oscillator: an example of parametric instability*, Am. J. Phys. **45**, 723-732 (1977).
- [2] Christensen, J., *An improved calculation of the mass for the resonant spring pendulum*, Am. J. Phys. **72**, 818-828 (2004).
- [3] Geballe, R., *Am. Statics and dynamics of a helical spring*, J. Phys. **26**, 287-290 (1958).
- [4] Galloni, E. E. and Kohen, M., *Influence of the mass of the spring on its static and dynamic effects*, Am. J. Phys. **47**, 1076-78 (1979).
- [5] Armstrong, H. L., *The oscillating spring and weight an experiment often misinterpreted*, Am. J. Phys. **37**, 447-449 (1969).
- [6] Edwards, T. W. and Hultsch, R. A., *Mass distribution and frequencies of a vertical spring*, Am. J. Phys. **40**, 445-449 (1972).
- [7] Weinstock, R., *Oscillations of a particle attached to a heavy spring: an application of a Stieltz integral*, Am. J. Phys. **47**, 508-514 (1979).
- [8] Bowen, J. M., *Slinky oscillations and the motion of effective mass*, Am. J. Phys. **50**, 1145-1148 (1982).
- [9] Cushing, J. T., *The spring-mass system revisited*, Am. J. Phys. **52**, 925-932 (1984).
- [10] Olsson, M. G., *Why does a mass on a spring sometimes misbehave?* Am. J. Phys. **44**, 1211-1212 (1976).
- [11] Lai, H. M., *On the recurrence of a resonant spring pendulum*, Am. J. Phys. **52**, 219-223 (1984).
- [12] Boscolo, I., Gariboldi, L. and Loewenstein, R., *The path and the multi-teaching issues in the coupled pendulum and mass-spring experiments*, Lat. Am. J. Phys. Educ. **4**, 40-45 (2010).
- [13] Crawford, F. S., Jr., *Waves, Berkeley Physics Course*, (McGraw Hill book Co., New York, 1968) p. 7.
- [14] Resnick, R. and Halliday, D., *Physics*, (Wiley, New York, 1977).