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PRESENCE OF IMPERFECT DEPOSIT –
CURRENCY SUBSTITUTION**

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THE OPTIMAL INFLATION TAX IN THE PRESENCE OF IMPERFECT DEPOSIT – CURRENCY SUBSTITUTION

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Abstract

During the last decades, technological innovation has generated a major transformation in payment systems, stimulating a widespread use of different forms of electronic money and increasing substitutability between deposits and currency in transactions. A big advantage of deposits is that, unlike currency, they can pay nominal interest on the average balance at a very low cost. As a result, in most developed countries an increasing number of people chose debit cards to make transactions. Despite the huge impact that these cards have had on everyday life, little is known about their consequences for the optimal conduct of monetary policy. This paper contributes to the literature on optimal monetary and fiscal policy by analyzing how the presence of imperfect *deposit-currency substitution* affects inflationary taxation in a public finance framework. The paper presents a model where financial intermediaries supply deposits that can be used to buy goods and services. In order to produce deposits, financial intermediaries must incur a cost. It is shown that if this cost is zero, the optimal inflation tax is zero. However, in the more realistic case in which this cost is positive, the optimal inflation tax is positive whenever there are revenue needs. Furthermore, the higher the cost of producing deposits, the higher is the optimal inflation tax. These results suggest that central banks in countries with less productive financial intermediaries (implying a higher cost of producing deposits), should optimally choose to have higher inflation rates.

Resumen

En las últimas décadas, las invocaciones tecnológicas han generado una gran transformación en los sistemas de pago, estimulado un uso masivo de forma de pago electrónica y aumentando el grado de sustitución entre dinero en efectivo y depósitos. Una gran ventaja de los depósitos es que pueden pagar un interés sobre el balance promedio a un bajo costo. Como resultado, un número creciente de personas eligen utilizar tarjetas de débito (depósitos) como medio de pago. A pesar del tremendo impacto que estas tarjetas han tenido en la vida cotidiana de las personas, poco se sabe sobre las consecuencias que esto tiene sobre el manejo de la política monetaria. Este trabajo contribuye a la literatura sobre política monetaria y fiscal óptima, analizando el impacto que tiene la mayor presencia de sustitutos del dinero en efectivo (como medios de pago) en las decisiones sobre el nivel óptimo de impuesto inflacionario. Se presenta un modelo en que los intermediarios financieros ofrecen depósitos que pueden ser utilizados como medio de pago. De esta forma, los depósitos actúan como un sustituto imperfecto del dinero en efectivo. Producir estos depósitos implica un costo, tanto para los intermediarios financieros como para la sociedad. En el trabajo se demuestra que, si este costo es cero, el impuesto inflacionario óptimo es cero. Sin embargo, si el costo de producir depósitos y el gasto del gobierno son positivos, la tasa de inflación óptima es positiva. Inclusive, la tasa de inflación óptima es una función creciente del gasto del gobierno y el costo de producir depósitos.

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1 Introduction

During the last decades technological innovation has generated a major transformation in payment systems, stimulating a widespread use of different forms of electronic money and increasing substitutability between deposits and currency in transactions. A big advantage of deposits is that, unlike currency, they can pay nominal interest on the average balance at a very low cost. As a result, in most developed countries an increasing number of people chose debit cards to make transactions. Despite the huge impact that these cards have had on everyday life, little is known about their consequences for the optimal conduct of monetary policy. The purpose of this paper is to contribute to the literature on optimal monetary and fiscal policy by analyzing how the introduction of financial intermediaries –institutions that supply deposits that can be used for transactions and pay interest in average balance- affects inflationary taxation in a public finance framework.

It can be argued that the literature on optimal monetary policy started when Friedman (1969) gave the following policy recommendation: ‘...the optimum quantity of money is that it will be attained by a rate of price deflation that makes the nominal rate of interest equal to zero’. This argument followed from first best considerations: the price of any good must be equal to its social cost, and since the social cost of producing money is zero, the central bank should set its price (the nominal interest rate) equal to zero.

This argument didn’t satisfied Phelps, who in Phelps (1973) confronted Friedman’s idea by applying standard Ramsey taxation principles. Phelps (1973) claimed that the so called "Friedman Rule" would be optimal only in the presence of lump-sum taxes, and when lump-sum taxes are not available the government should depart from the zero nominal interest rate rule. Phelps rationale was that money was a good like any other, and consequently should be taxed. Provided that government objective should be to finance its expenditure in the least distorting fashion, the marginal distortion caused by one more unit of revenue should be the same across different goods. An implication of this argument is a strictly positive nominal interest rate.

This reasoning was questioned by Kimbrough (1986 and 1989), who criticized the treatment of money as a final good. In his view, money was an intermediate good (since it is used only for transaction reasons), and as such, it follows from application of standard public finance theory, should not be taxed. Notwithstanding, Guidotti and Végh (1993) emphasized that this is only true if the transaction technology is homogeneous of degree one. Additionally, Chari, Christiano, and Kehoe (1996) showed that the optimality of the Friedman rule holds for transactions costs functions of degree of homogeneity greater than one and Correia and Teles (1996) extended this result

and proved that, as long as the costs of producing real money and collecting taxes are negligible, the Friedman Rule holds for any degree of homogeneity of the transaction technology. However, when the social costs of producing or using real money balances are positive, as is the case of Végh (1989) and Végh (1995), or the costs of collecting taxes are different from zero, as Végh and Guidotti (1996), the Friedman Rule is no longer optimal.

In the present paper I build on this literature. I develop a *deposit-currency substitution* model in a close economy with financial intermediaries. Financial intermediaries supply deposits that can be used to buy goods and services and can pay interest on average balance. The rationale for introducing deposits to analyze optimal monetary policy was best explained by T. G. Bali (2000), who argued that without a theory of banking in which the distinctive roles of currency and deposits are modeled; one could overestimate or underestimate the effects on welfare when government deviates from the Friedman Rule. To tackle my problem, following the work of Brock (1989), I do not specify a cash-in-advance constraint or liquidity in the utility function. Rather, I incorporate banks (financial intermediaries) using a shopping time model that assumes that currency and bank deposits permit agents to reduce the amount of time spent purchasing goods. In the model, there is an implicit cost of producing currency, which is generated by the assumption that it is socially (and privately) costly to produce deposits which are imperfect but close substitutes of currency. This cost will imply that the spread between the nominal interest rate on government bonds and bank deposits is constant and independent of inflation. This result has been found to hold empirically for the case of Italy by Attanasio, Guiso and Jappelli (2002), who show that the average reduction in after-tax nominal interest rates on deposits matches almost exactly the reduction in the after-tax nominal interest rates on short-term government bonds. A positive cost of producing deposits together with a transaction technology that uses currency and deposits as imperfect substitutes generates a distortion in the consumption leisure decision that can be reduced by moving away from the Friedman Rule.

The important result of the paper is to show that when the cost of producing deposits is zero, the optimal inflation tax is zero. However, when the cost of producing deposits is positive and there are revenue needs, it is optimal to deviate from the Friedman Rule and set a positive inflation tax. The intuition is straightforward, when there is a cost to society of producing deposits, a positive labor income tax introduces a distortion between the private and social cost of using deposits to perform transactions, by increasing the implicit effective price of consumption. This creates an incentive for the benevolent central bank/government to lower the labor income tax and increase the nominal interest rate to induce consumers to use more deposits. Furthermore, the optimal inflation tax is an increasing function of the cost of producing deposits and revenue needs. These results suggest that central banks in countries with

less productive financial intermediaries (implying a higher cost of producing deposits), should optimally choose to have higher inflation rates.

Although this paper is very similar to Végh (1989b and 1995), it makes a number of significant contributions. First, Végh's work can help explain why some central banks deviate from the Friedman Rule, but can only be applied to economies that suffer from currency substitution, leaving aside most of the economies in the world. In contrast, my model can be applied to most countries in the world. Second, in order to have a deviation from the Friedman Rule, Végh needs to assume that the foreign interest rate is positive –that is, the foreign central bank also deviates from the Friedman Rule. But, why does the foreign government set a positive interest rate in the first place? My model can not be subject to this criticism. Third and perhaps more important is the issue of relevance. While the key ingredient in Végh's work, *currency substitution*, is decreasing in importance and could be considered a problem of the past, the core of my model, *deposit-currency substitution*, is increasing in importance and may affect the conduct of monetary policy in most countries in the world for years to come.

The model in this paper is not the first to assume that currency and deposits (that resemble debit cards) can be used for transaction purposes. Brock (1989), Kimbrough (1989), Calvo and Végh (1995, 1996), Edwards and Végh (1997) and Marimon, Nicolini and Teles (2003) all share similar characteristics. However, only Brock (1989) and Kimbrough (1989) use a shopping time model. Kimbrough (1989) and Marimon, Nicolini and Teles (2003) find that the Friedman Rule is optimal regardless of the cost of producing demand deposits. The basis for the difference with my model is that Marimon, Nicolini and Teles (2003) assume that currency and deposits are perfect substitutes, therefore in equilibrium only one good is demanded. Since deposits are costly to produce and currency is a free good, the zero nominal interest rate is optimal. Finally, Kimbrough (1989) uses two transaction technologies, one for each asset. Thus, currency can be used to buy 'currency goods', and deposits to buy 'deposit goods', which leads into a clear distinction among assets. As far as I am aware of, my model is the first one to show that the Friedman Rule is not optimal in a model where currency and deposits reduce transaction costs.

The paper proceeds as follows. Section 2 presents the model and solves the government's optimization problem assuming that non-distortionary taxation is available. Under this setup, the Friedman Rule remains optimal and I present it here because it provides some intuition to understand the central part of the paper. Section 3 extends the model to distortionary taxations. Here I show that the Friedman Rule is no longer the optimal policy. Finally, section 4 concludes.

2 The Model

In this section, I analyze the optimal taxation problem. I assume that the economy is populated by infinitely lived consumers who derive utility from the consumption good c_t and leisure l_t . The real money stock is composed of non-interest-bearing real currency m_t and interest-bearing real demand deposits d_t . For simplicity, I assume a single good world. This good is produced using a constant return to scale technology, where one unit of labor (n) produces one unit of good (y): $n_t = y_t$.

Households are assumed to maximize,

$$\sum_{t=0}^{\infty} \beta^t u(c_t, l_t), \quad (1)$$

where $\beta \in (0, 1)$ is the subjective rate of time preference and $u(\cdot)$ is the instantaneous level of utility which is twice-continuously differentiable, with positive and diminishing marginal utilities. c is the consumption of the single good produced in the economy and l is time dedicated to leisure.

Households' maximization problem is subject to two types of constraints. First, they face a dynamic budget constraint. Households hold three assets: domestic currency, deposits and an internationally traded bond whose rate of return is r . They begin period t with a given amount of money and pay a lump sum tax T_t . They can save by accumulating nominal currency M_{t+1} and nominal demand deposits D_{t+1} , and by buying government bonds b_{t+1} . All variables denote quantities held at the beginning of period $t + 1$. Demand deposits are held at financial intermediaries and have a nominal rate of return of $1 + i_t^d$ between t and $t + 1$. The bonds that agents buy in period t , b_{t+1} , are sold at the nominal price p_t , and the nominal rate of return on bonds between t and $t + 1$ is $1 + i_{t+1}$. The gross real rate of return on bonds is, therefore, defined as $1 + r_t = \frac{1+i_t}{1+\pi_t}$, where $1 + \pi_t = \frac{p_t}{p_{t-1}}$ is the inflation rate.

Second, to motivate the demand for currency and deposits, it is assumed that they reduce transaction costs in the form of shopping time. The household is assumed to have an endowment of one unit of time that he can use to work (h), consume leisure (l) or shop (s). Households supply labor

$$n_t = 1 - h_t - s_t. \quad (2)$$

The dynamic budget constraint takes the following form:

$$D_{t+1} + M_{t+1} + p_t b_{t+1} + p_t c_t \leq p_t n_t + p_t (1 + i_t) b_t + (1 + i_t^d) D_t + M_t - T_t.$$

Combining this dynamic budget constraint with restriction (2), initial conditions $M_0 = b_0 = D_0 = 0$ and a No-Ponzi game condition yields the following intertemporal budget constraint:

$$\sum_{t=1}^{\infty} q_t (1 - l_t - s_t) = \sum_{t=1}^{\infty} q_t \left[c_t + i_t m_t + (i_t - i_t^d) d_t + \tau_t \right],$$

where $m_t = \frac{M_t}{p_t}$, $\tau_t = \frac{T_t}{p_t}$ and

$$q_t = \prod_{i=0}^t (1 + r_i)^{-1}.$$

I assume that the households in the economy have access to the technology $s_t = \phi(c_t, m_t, d_t)$. I assumed $\phi(\cdot)$ to be homogeneous of degree 1, so that I can rewrite it as:

$$s_t = c_t \phi \left(\frac{m_t}{c_t}, \frac{d_t}{c_t} \right),$$

where $\frac{m_t}{c_t} = x_t$ are relative real currency balances and $\frac{d_t}{c_t} = z_t$ are relative deposit holdings. It is assumed that $\phi_1, \phi_2 < 0$; $\phi_{11}, \phi_{22}, \phi_{12} > 0$ and $(\phi_{11}\phi_{22} - \phi_{12}^2) > 0$. Currency balances and deposit balances are assumed to be imperfect substitutes. This is an important difference with respect to Marimon et al (2003) who assume perfect substitutability between these assets. Imperfect substitutability may follow from costs associated with transacting with deposit balances.¹ Increased real currency balances or deposit balances produce positive but decreasing reductions in shopping time for a given level of consumption. Finally, we need to assume strict convexity of $\phi \left(\frac{m_t}{c_t}, \frac{d_t}{c_t} \right)$ to assure that the demands for currency and deposit holdings are well behaved.

2.1 Financial Intermediaries

I introduce Financial Intermediaries as in Marimon et al (2003). In the economy there are financial intermediaries. This sector is perfectly competitive, and supplies deposits which are a close substitute for currency. These deposits can be used for electronic payments and are a form of inside money. A financial intermediary offers deposits D_{t+1} at a nominal interest rate i_t^d . The financial intermediation technology is such that the intermediary must pay a

¹ The fact that, in the real world, not all stores accept debit cards or checks as a medium of payment.

real cost, in units of goods,² for the supply of deposits, as a fraction of the real value of the outstanding deposits: θd_t (Where lower-case letters refers to real variables). The financial intermediary invests the total amount deposited, D_{t+1} ; as bonds, b_{t+1} , which pays a nominal interest rate i_t . The cash flow of the financial intermediary in period $t \geq 0$ is:

$$\Pi_t = D_{t+1} - D_t(1 + i_t^d) - p_t b_{t+1} + p_t b_t(1 + i_t) - \theta D_t.$$

Free entry in the financial intermediation sector results in $\Pi_t = 0$; $t \geq 0$; which, given that $D_{t+1} = p_t b_{t+1}$ implies

$$i_t - i_t^d = \theta \quad \forall t \geq 0. \quad (3)$$

Thus, financial intermediaries in equilibrium set a constant spread between the nominal interest rates. This result has been confirmed empirically for the case of Italy by Attanasio, Guiso and Jappelli (2002), who show that there is a constant spread between the nominal interest rates on deposits and the nominal interest rates on short-term government bonds. The consumer takes this spread as given. It is assumed that financial intermediaries are price takers and honor their liabilities.

2.2 Household Behavior

Let λ be a Lagrange multiplier. Using the fact that households take the interest rate differential (θ) as given, the household's problem in Lagrangian form is:

$$L = \sum \beta^t \{u(c_t, l_t) + \lambda q_t [(1 - l_t - c_t \phi(x_t, z_t)) - c_t - i_t m_t - \theta d_t - \tau_t]\}.$$

The first order conditions are:

$$c_t: \quad u_c(t) = \lambda q_t (1 + \phi(x_t, z_t) - x_t \phi_1(t) - z_t \phi_2(t)), \quad (4)$$

$$l_t: \quad u_l(t) = \lambda q_t, \quad (5)$$

$$m_t: \quad \phi_1(x_t, z_t) = -i_t, \quad (6)$$

$$d_t: \quad \phi_2(x_t, z_t) = -\theta. \quad (7)$$

² Modeling this cost in units of labor as is done in Marimon, Nicolini and Teles (2003) and Kimbrough (1989) doesn't change the results. The only thing that matters is that there must be a resource cost to society. In fact, since one unit labor equals one unit of output (consumption good) the difference is just a matter of semantics.

Equation (4) says that the marginal utility of consumption has to be equal to the marginal utility of leisure (5), times the effective price of consumption. The effective price of consumption is equal to the cost of producing the good (one) plus the marginal effect of an extra unit of consumption on the shopping time. Equation (6) says that in an optimum, the cost of holding currency instead of bonds (the lost interest rate (i_t)) has to be equal to the decrease in the disutility of having to spend time shopping generated by holding one more unit of real money balances. Finally, equation (7) implies that in an optimum, the cost of holding demand deposits instead of bonds (the interest rate differential (θ)) has to be equal to the decrease in the disutility of having to spend time shopping generated by holding one more unit of real demand deposits. Remember that from the banks' first order condition (3), we know that the interest rate differential is equal to θ .

Equations (6) and (7) implicitly determine demand functions for currency and demand deposits. It can be shown that the demand functions for currency and demand deposits are well behaved:

$$m = m \left(\underset{(-)}{i}, \underset{(+)}{\theta}, \underset{(+)}{c} \right),$$

$$d = d \left(\underset{(+)}{i}, \underset{(-)}{\theta}, \underset{(+)}{c} \right).$$

2.3 Optimal Monetary Policy

To determine optimal monetary policy, I solve the benevolent government's problem and show that the Friedman Rule is optimal. The government faces a period by period budget constraint:

$$p_t g_t - T_t = M_{t+1} - M_t + p_t b_{t+1} - p_t R_t b_t.$$

It's easy to show that, given that government spending will be assumed to be constant over time, the optimal taxation structure will also be constant over time; that is $i_t = i$ and $\tau_t = \tau \forall t$. Given (g, b) and the target value for the interest rate (i), the lump sum tax (τ) can be chosen to assure that government can finance spending.

The social value of relative real currency and deposit balances is that they reduce the effective price of consumption, by reducing shopping time. To see this, let $\Theta = 1 + \phi(x, z) - x\phi_1 - z\phi_2$ be the effective price of consumption. It follows from the optimality conditions that $c = c(\Theta)$ and $l = l(\Theta)$. Substituting these optimal choices into the utility function yields the indirect utility function:

$$V(\Theta) = u[c(\Theta), l(\Theta)].$$

It can be shown that $\frac{\partial V}{\partial \Theta} < 0$. Thus maximizing $V(\Theta)$ is the same as minimizing Θ . Thus the benevolent government's problem is to minimize the effective price of consumption Θ .

Proposition 1 *When the government has access to Lump Sum taxation, the optimal inflation tax is zero independent of the cost of producing deposits θ . In other words, is optimal for the government to follow the Friedman Rule ($i = 0$).*

Proof. As explained above, given (g) and the value for the interest rate (i), the lump sum tax (τ) can be chosen to assure that government can finance spending. Thus, the benevolent government's problem is to choose (i) to minimize $\Theta = 1 + \phi(x, z) - x\phi_1 - z\phi_2$. Combining with the household's first order conditions, it can be shown that the effective price of consumption (Θ) is strictly increasing in the nominal interest rate, $\frac{\partial \Theta}{\partial i} = \phi_1 \frac{\partial x}{\partial i} + \phi_2 \frac{\partial z}{\partial i} - \phi_1 \frac{\partial x}{\partial i} - \phi_2 \frac{\partial z}{\partial i} + x = x > 0$. Hence, it is optimal to set $i = 0$ to minimize Θ . This proves that the FR is optimal. ■

Intuition: First, note that when the government can use Lump Sum taxation, it can finance any level of spending by correctly setting τ . Second, since this tax is non-distortionary, the private cost of producing deposits (θ) equals the social cost of producing them (θ). Furthermore, the private cost of using real currency balances is the nominal interest rate, where the social cost of using currency balances is zero. Given that the social planner would like to set the marginal utility of using real currency balances and real deposit balances equal to their social marginal cost, it is optimal to set $i = 0$ so that the consumer faces the social cost of both goods.

3 Optimal taxation with distortionary taxes

Given lump sum taxation, Propositions (1) establishes that the Friedman Rule is optimal. The optimal policy is to set the private cost of using the goods (currency and deposits) equal to the social cost of using them. I now ask if the Friedman Rule remains optimal when government revenues must be raised using distortionary taxation and currency and deposits reduce transaction costs.

3.1 Household's Problem

In this section, I modify the shopping time economy from the previous section and introduce a labor income tax. With a labor income tax the household's

problem becomes:

$$L = \sum \beta^t \{u(c_t, l_t) + \lambda q_t [(1 - l_t - c_t \phi(x_t, z_t)) (1 - \tau_t) - c_t - i_t m_t - \theta d_t]\}.$$

Letting $1 + \gamma_t = (1 - \tau_t)^{-1}$, the optimality conditions can be written as:

$$\frac{u_c(t)}{u_l(t)} = \Theta_t, \tag{8}$$

$$\phi_1(x, z) = -i(1 + \gamma), \tag{9}$$

$$\phi_2(x, z) = -\theta(1 + \gamma), \tag{10}$$

where $\Theta = 1 + \gamma + \phi[x(i, \gamma, \theta), z(i, \gamma, \theta)] + i(1 + \gamma)x(i, \gamma, \theta) + \theta(1 + \gamma)z(i, \gamma, \theta)$ and where time subscripts have been omitted for notational simplicity. The interpretation of these equations is the same as for the lump sum taxation case. The variable Θ can be interpreted as the distortion generated by distortionary taxation into the household's consumption-leisure decision or, in other words, the *implicit effective price of consumption*. One important thing to note here, is that the implicit effective price of consumption depends on the nominal interest rate (i), the marginal cost of producing deposits (θ) and the labor income tax rate. This is different from the lump sum taxation case, where the effective price depended only on i and θ .

Conditions (9) and (10) implicitly determine the demands for real currency and real deposit balances: $x = x(i, \gamma)$ and $z = z(i, \gamma)$. Equations (8), (9) and (10) together with the household budget constraint, implicitly define the functions $c_t = c(\Theta_t)$ and $l_t = l(\Theta_t)$. Hence, I can write the household's indirect utility function as $V(\Theta) = V[c(\Theta), l(\Theta)]$. Again, it can be shown that $\frac{\partial V}{\partial \Theta} < 0$. Thus maximizing $V(\Theta)$ is like minimizing Θ .

3.2 Government

The government's period by period budget constraint is now:

$$p_t g_t - p_t \tau_t n_t = M_{t+1} - M_t + p_t b_{t+1} - p_t R_t b_t.$$

Combining this period by period budget constraint with initial conditions $M_0 = b_0 = 0$ and a No-Ponzi game condition, the government's intertemporal

budget constraint becomes:

$$\sum_{t=0}^{\infty} q_t g_t = \sum_{t=0}^{\infty} q_t (\tau_t n_t + i_t m_t).$$

Since we are interested in the stationary equilibrium, it can be shown that for a constant level of government spending (which will be assumed), it is optimal for the government to finance spending with contemporaneous taxation. Thus, we have:

$$g = \tau n + im. \quad (11)$$

By using the economy's feasibility constraint,³ equation (11) can be written as:

$$g = c(\Theta) [\gamma + (1 + \gamma)ix + \gamma\theta z]. \quad (12)$$

3.3 The optimal taxation problem

In this section I derive the optimal taxation structure with labor income tax. It is assumed that the government faces a constant spending path $g_t = g, \forall t$. As I explained above, it can be shown that it is optimal to finance spending from contemporaneous taxes. Therefore, the benevolent government's problem is:

$$\min_{\{\gamma, i\}} \Theta = 1 + \gamma + \phi[x(i, \gamma), z(i, \gamma)] + i(1 + \gamma)x(i, \gamma) + \theta(1 + \gamma)z(i, \gamma),$$

subject to (12). Let $\Omega(i, \gamma) = \gamma + (1 + \gamma)ix + \gamma\theta z$, and let ψ be a Lagrange multiplier on (12). Then, the FOC for the government's problem are:

$$\begin{aligned} i: \quad & \frac{\partial \Theta}{\partial i} \left(1 - \psi \frac{\partial c}{\partial \Theta} \Omega \right) = \psi c \frac{\partial \Omega}{\partial i}, \\ \gamma: \quad & \frac{\partial \Theta}{\partial \gamma} \left(1 - \psi \frac{\partial c}{\partial \Theta} \Omega \right) = \psi c \frac{\partial \Omega}{\partial \gamma}, \end{aligned}$$

and (12).

³ The economy's feasibility constraint is: $n_t = c_t + g_t + \theta d_t$

The FOC can be combined to get:

$$\frac{\frac{\partial \Theta}{\partial i}}{\frac{\partial \Theta}{\partial \gamma}} = \frac{\frac{\partial \Omega}{\partial i}}{\frac{\partial \Omega}{\partial \gamma}}, \quad (13)$$

precisely:

$$\frac{\phi_1 \frac{\partial x}{\partial i} + \phi_2 \frac{\partial z}{\partial i} + (1 + \gamma) x + i(1 + \gamma) \frac{\partial x}{\partial i} + \theta(1 + \gamma) \frac{\partial z}{\partial i}}{1 + \phi_1 \frac{\partial x}{\partial \gamma} + \phi_2 \frac{\partial z}{\partial \gamma} + ix + i(1 + \gamma) \frac{\partial x}{\partial \gamma} + \theta(1 + \gamma) \frac{\partial z}{\partial \gamma} + \theta z} = \frac{(1 + \gamma) x + i(1 + \gamma) \frac{\partial x}{\partial i} + \theta \gamma \frac{\partial z}{\partial i}}{1 + ix + i(1 + \gamma) \frac{\partial x}{\partial \gamma} + \theta \gamma \frac{\partial z}{\partial \gamma} + \theta z}.$$

Combining with the FOC of the household's problem we obtain the following optimality condition:

$$\frac{(1 + \gamma) x}{1 + ix + \theta z} = \frac{(1 + \gamma) x + i(1 + \gamma) \frac{\partial x}{\partial i} + \theta \gamma \frac{\partial z}{\partial i}}{1 + ix + i(1 + \gamma) \frac{\partial x}{\partial \gamma} + \theta \gamma \frac{\partial z}{\partial \gamma} + \theta z}. \quad (14)$$

Equations (12) and (14) implicitly define $\gamma = \gamma(g, \theta)$ and $i = i(g, \theta)$.

The following proposition serves as a building block to understand the intuition of the results that will follow.

Proposition 2 *When government spending (g) is zero, the optimal inflation tax is zero independently of the cost of producing deposits.*

Proof. It is easy to check that, if $g = 0$ and $\theta > 0$, equations (14) and (12) are satisfied if $\gamma = i = 0$, and the solution is unique. In conclusion, the Friedman

Rule is optimal. ■

Intuition: Note that the effective price of consumption is increasing in both γ and i . When there are no revenue needs because government spending is zero, there is no need to collect taxes, so the government can set $\gamma = i = 0$. In some sense, we are back in the problem with lump sum taxation. Hence, it is optimal to set $i = 0$, so that the consumer faces the social cost of both goods (currency and deposits).

The following proposition suggests that for a country with efficient financial markets (where the cost of producing deposits is zero), which I assume to be the case for Germany, it is optimal to follow the Friedman Rule.

Proposition 3 *When government spending (g) is positive and the real cost of producing deposits (θ) is zero, the Friedman Rule is optimal.*

Proof. It is easy to check that, if $g > 0$ and $\theta = 0$ equations (14) and (12) are satisfied if $\gamma > 0$ and $i = 0$. In conclusion, the Friedman Rule is optimal. ■

Intuition: this result is related to Correia and Teles result. If the marginal cost of producing deposits is zero, the implicit costs of producing real domestic currency is zero too and we are back to the world of Correia and Teles. Note that in this case, the private marginal cost of using deposits equals the social marginal cost of using them for any level of γ . Since the social cost of using currency is zero, the best thing the government can do is to set the private cost equal to zero. This can be attained by setting $i = 0$.

In reality, previous works (i.e. Claeys and Vander Venet, 2007; Gambacorta, 2002 and 2007) suggests that the cost of producing deposits is far from zero. The following proposition shows that for such countries it is optimal to deviate from the Friedman Rule and set a positive nominal interest rate.

Proposition 4 *When government spending (g) and the real cost of producing deposits (θ) are positive, the optimal inflation tax is positive.*

Proof. It is easy to check that, if $g > 0$ and $\theta > 0$ equation (14) is not satisfied if $\gamma > 0$ and $i = 0$, while (12) is not satisfied if $\gamma = i = 0$. In conclusion, the

Friedman Rule is not optimal. ■

This Proposition is basically Vegh (1995)'s result for the currency substitution case. To understand the intuition behind this result consider first the situation in which all government spending is financed with a labor income tax ($i = 0$). A positive $\gamma > 0$ implies that the private marginal cost of deposits $\theta(1 + \gamma)$ is higher than the social marginal cost of deposits θ . This implies that the level of relative deposit holdings is too low compared with the social optimum. Thus, the government has an incentive to increase the inflation tax ($i > 0$) and decrease the labor income tax (γ). This would increase the demand for deposit holdings (which reduces transaction costs), at the expense of a lower demand for real currency balances (which increases transaction costs). The total effect in transaction costs of an increase in the inflation tax and a decrease in the labor income tax by the government (moving away from zero) can be divided in two. First, the increase in the interest rate generates a decrease in transaction cost ($\theta(1 + \gamma) \frac{\partial z}{\partial i}$) that is higher than the social cost ($\theta \frac{\partial z}{\partial i}$). Second, the decrease in the labor income tax (γ) reduces transaction costs ($\theta(1 + \gamma) \frac{\partial z}{\partial \gamma}$) by an amount that exceeds its social cost ($\theta \frac{\partial z}{\partial \gamma}$).

Finally, the next proposition shows that the inflation tax is an increasing function of the cost of producing deposits.

Proposition 5 *The optimal inflation tax is an increasing function of the cost of producing deposits around $\theta = 0$ and $g > 0$.*

Proof. We have shown in Proposition 3 that, when $g > 0$ and $\theta = 0$, the Friedman Rule is optimal, thus $i = 0$. Totally differentiate (14) and (12) and

set $\theta = 0$ to obtain $\frac{\partial i(\theta=0, g>0)}{\partial \theta} > 0$. ■

Intuition: when the social cost of producing deposits is zero, the social and private cost of holding currency is the same no matter what is the level of government spending. But as the cost of producing deposits becomes positive, the private cost of holding money is higher than the social cost, and thus is optimal to reduce the labor tax and increase inflationary taxation. Hence, the inflation tax is an increasing function of the cost of producing deposits.

4 Conclusion

In this paper I have investigated the design of monetary and fiscal policy in an economy with imperfect *deposit-currency substitution*. The importance of the paper has been to show that, in an otherwise-standard Ramsey model, the presence of imperfect *deposit-currency substitution* has important consequences for the conduct of optimal monetary policy. In particular, when the cost of producing deposits is positive and there are revenue needs, it is optimal to deviate from the Friedman Rule. Furthermore, the model shows that the less efficient the financial intermediaries are, the higher is the optimal inflation tax. Therefore, the main policy recommendation coming from this investigation is that central banks in countries with less productive financial intermediaries (implying a higher cost of producing deposits), should optimally choose to have higher inflation rates.

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