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# Asymmetric Stochastic Volatility Models and **Multicriteria Decision Methods in Finance**

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## Abstract

In order to make a decision in any given context, it is necessary to have as much information as possible. For this reason, the objective of this paper is to choose a method, the most objectively possible, to establish an order of preferences between different stock index returns using all available statistical and econometrical information.

The TGARCH(1,1) and TA-ARSV(1) are models estimated to obtain the econometrical information. This information is evaluated using discrete multicriteria decision methods such as PROMETHEE Methods, with the aim of obtaining a ranking of preferences between the different Stock Market Indexes in several scenarios. The different scenarios proposed show that the results obtained in the complete ranking of the different financial returns are robust.

**Keywords:**
TGARCH model, TA-ARSV model, Stochastic volatility; Discrete multicriteria methods.

***JEL classification:***
G17, C58, C44.

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## 1. Introduction

In the financial market, especially in the current fluctuating situation, it is advisable for investors to have as much information as possible in order to make decisions that lead them to obtain better results. It is for this reason that we propose combining the results obtained through diverse econometric models concerning the evolution of the returns of different Stock Market Indexes with a methodology based on discrete multi-criteria decisions, as this type of methodology facilitates practical decision making particularly in high risk situations.

The advantage of the discrete multicriteria decision over other types of methodology lies in the possibility of considering simultaneously different criteria or points of view, even if they contradict[1] each other, in order to establish a ranking (from best to worst) of preferences among different alternatives. It is therefore a very useful tool in situations when you have information of a very different nature and need to choose the best alternative. In this specific case, these alternatives will be the returns of the different stock indexes. It is a very versatile methodology as it can be applied not only to financial assets but also to projects, investments, companies, individuals, etc.

The first step to take when analysing this problem is to obtain all the necessary information to compute the pay-off matrix. In our specific case, the results of this matrix will be obtained through assessing the different returns of stock indexes under diverse criteria. These criteria are in conflict because some will maximize and others will minimize. Consequently, the final solution will be a compromise.

Two types of criteria are used, namely statistical and econometrical. The statistical criteria are obtained from the main characteristics of the return series of several stock indexes, such as the mean, the standard deviation, asymmetry and kurtosis of the financial returns. The econometrical criteria are obtained by estimating econometric models such as the threshold generalized autoregressive conditional heteroskedasticity (TGARCH) model proposed by Zakoian (1990), and Glosten *et al.* (1993), the ARSV model and the threshold asymmetric autoregressive stochastic volatility (TA-ARSV) model, proposed by So *et al.* (2002), and developed by García and Mínguez (2009). These models explain the behaviour of the volatility. The importance of volatility lies in the fact that it is a risk measurement noticed by agents in financial markets. Moreover, as volatility is a non-observable variable, it is necessary to propose a variety of econometric models to estimate it.

*[1] These criteria partially contradict each other. If a decision maker chooses one criterion, s/he will choose a specific alternative as the best; if s/he chooses another, then the choice could change. Therefore, the best alternative in each case will depend on the decision maker's own preferences.*

Preference ranking organisation methods for enrichment evaluations (PROMETHEE methods) are the multicriteria decision methods used in this paper to show the order of preference of the daily financial returns of some Stock Market Indexes in different countries. This methodology is of key importance as it helps us to make decisions that not only affect the present but also the future. In this paper we first attain all the statistical and econometrical results to obtain the payoff matrix and, subsequently, apply PROMETHEE methods to establish a ranking of preferences among the returns of several indexes.

It is worth indicating that a sensibility analysis allows us to ascertain how the rankings can change among the different returns when faced with changes in any of the elements of the payoff matrix. This point will not be covered in this paper.

The remainder of the paper is organized as follows: Section 2 shows the main characteristics of the daily financial returns of several Stock Market Indexes. Section 3 illustrates three models used to explain the characteristics of volatility of financial returns. Section 4 shows the PROMETHEE methods used to order the preferences between the different financial returns. Section 5 reports the empirical results for eight daily financial return series and their preference orders depending on the model used to explain the dynamics of volatility and finally, section 6 provides some concluding remarks.

## 2. Characteristics of Daily Financial Returns

The analysis of the main stylized facts of the returns is important as they will be used as criteria in the decision matrix of the program set out in the form of discrete multi-criteria decision making.

The returns ($y_t$) are calculated as follows:

$$y_t = 100\left(\log(X_t) - \log(X_{t-1})\right)$$

where $X_t$ is the index value at time $t$.

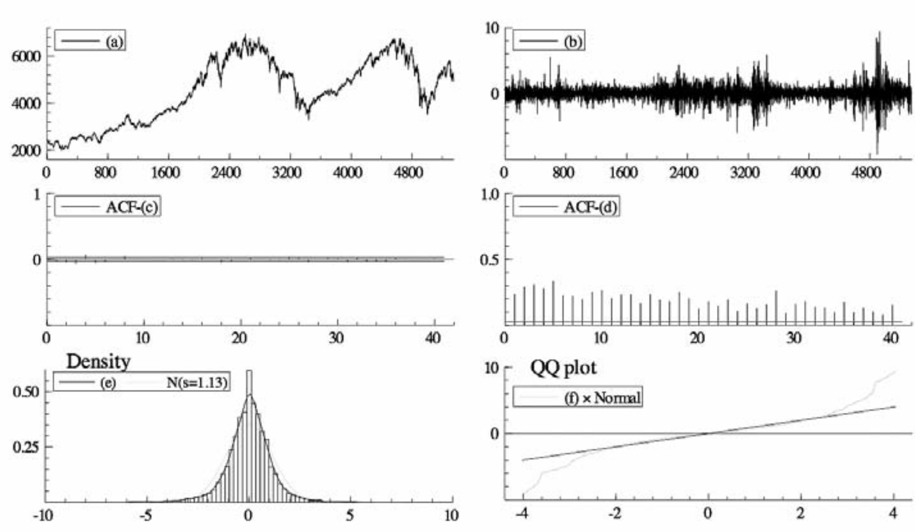
Considering that all the daily financial returns of the Stock Market share the same main characteristics, in this section we will only use the FTSE 100 index[2] as an example to explain the characteristics of daily financial returns series.

[2] *The data for all indexes used in this paper have been obtained from the DataStream Data Base.*

Daily financial returns series have the following main stylized facts:

**1.** Returns fluctuate around a constant small level close to zero, Figure 1(b).

Figure 1. **FTSE 100 Index from 1/1/1990 to 29/06/2010**




*(a):Time plot of FTSE 100 index.*
*(b): FTSE 100 index returns.*
*(c): Correlogram (or autocorrelation function, ACF) of FTSE 100 index returns. 5% significance level.*
*(d): Correlogram (autocorrelation function, ACF) of FTSE 100 index squared returns. 5% significance level.*
*(e): Histogram and estimated density plot of FTSE 100 index. 5% significance level.*
*(f): QQ plot for FTSE 100 index returns.*

**2.** The conditional variance is not constant due to periods of great variability (which coincide with periods in which the variation of stock index returns is larger, Figure 1(b)) following other periods in which there is little variation (which coincide with periods in which the stock index returns do not vary a great deal). This stylized fact is known as volatility clusters.

**3.** The autocorrelation function of returns, Figure 1(c), shows that returns are uncorrelated but not independent because the autocorrelation function of square returns, Figure 1(d), due to the existence of volatility clusters, displays a dependence structure which is shown by way of significant correlations. In the majority of time series, these correlations are positive and decrease slowly to zero; this is known as volatility persistence.

**4.** Returns are not normally distributed because the majority of them are negatively skewed (except Hang Seng, which is positively skewed) and they show kurtosis excess, Figures 1(e) and 1(f) and Table (1).

**Table 1. Descriptive Statistics and Normality Test**

| Indexes | Sample period | Minimum | Maximum | Mean | STD | Skewness | Excess Kurtosis | Normality Test |
|---|---|---|---|---|---|---|---|---|
| CAC 40 | 1/1/1990-29/06/2010 | -9.4715 | 10.595 | 0.0100 | 1.3984 | -0.0088 | 5.0998 | 2234.7* |
| DAX 30 | 1/1/1990-29/06/2010 | -9.8707 | 10.797 | 0.0225 | 1.4572 | -0.1287 | 5.2490 | 2295.9* |
| EUROSTOXX | 1/1/1990-29/06/2010 | -8.2076 | 10.438 | 0.0150 | 1.3484 | -0.0675 | 6.0178 | 2794.0* |
| FTSE 100 | 1/1/1990-29/06/2010 | -9.2656 | 9.3843 | 0.0132 | 1.1333 | -0.1102 | 6.7465 | 3239.9* |
| HANG SENG | 1/1/1990-29/06/2010 | -14.735 | 17.247 | 0.0367 | 1.6710 | 0.0124 | 9.7002 | 5197.8* |
| IBEX35 | 1/1/1990-29/06/2010 | -9.5859 | 13.484 | 0.0208 | 1.3827 | -0.0539 | 6.4485 | 3069.1* |
| S&PCOMP | 1/1/1990-29/06/2010 | -9.3237 | 10.957 | 0.0202 | 1.1556 | -0.2017 | 9.3237 | 4853.9* |
| NIKKEI 225 | 1/1/1990-29/06/2010 | -12.111 | 13.235 | -0.0262 | 1.5306 | -0.0247 | 5.6716 | 2585.4* |

**It is significant at 5% significance level. The Normality test used is the Jarque-Bera test ($H_0$: Normal distribution). The mean is statistically zero for all index returns.*

**5.** The response of volatility is asymmetric when there are different sign shocks in the market. This stylized fact is known as the leverage effect.

These stylized facts show that volatility displays some regularities in its behaviour, making it possible to model volatility trends (see Teräsvirta and Zhao, 2006). In the next section, we show some models used to explain such trends.

## 3. Volatility Models

Volatility and how it behaves over time is a very relevant element within the characteristics of this type of financial time series. The importance of volatility lies in the fact that it is a risk measurement noticed by agents in financial markets. Moreover, as volatility is a non-observable variable, it is necessary to propose several econometric models to estimate it. Two models are usually used to explain the behaviour of volatility (different from implicit volatility in continuous time): on the one hand, the generalized autoregressive conditional heteroskedasticity (GARCH) models proposed by Bollerslev (1986), as a generalization of the ARCH models introduced by Engle (1982), and on the other, the autoregressive stochastic volatility (ARSV) model introduced by Taylor (1986).

Both types of models share similarities, but model volatility in a different way. In the GARCH Model, the conditional variance of returns is a non linear function that depends on past returns and its own past, whereas in the ARSV model, volatility is a different stochastic process from that of returns.

As the volatility of returns has specific characteristics, different types of models are used to explain these stylized facts in the econometric literature. Moreover, in order to explain the leverage effect in volatility, as it does not behave the same when there are shocks of different signs in the market (see Harvey and Shephard, 1996), in this paper we propose three models to explain the dynamics of the volatility of daily financial returns for eight Stock Market Indexes. The models are: the TGARCH, the ARSV[3] and the TA-ARSV.

### 3.1. The TGARCH Model

The threshold generalized heteroskedasticity conditional autoregressive (TGARCH) model aims to detect the asymmetric behaviour of variance. That is, given the returns ($y_{t-1}$) and ($-y_{t-1}$), with values of the same magnitude, the volatility assigned to the negative return is greater than that assigned to the positive return. Once the autocorrelation function and the partial autocorrelation function of returns have been analyzed, our proposal to explain the dynamics of financial return volatility is a TGARCH(1,1) model defined by the following equations:

- The mean equation:

$$y_t=\sigma_t\,\varepsilon_t \quad \varepsilon_t \sim i.i.d.N(0,1) \tag{1}$$

- The conditional variance equation:

$$\sigma_t^2= \alpha_0+\alpha_1\varepsilon_{t-1}^2+\gamma\varepsilon_{t-1}^2 d_{t-1}+\beta_1\sigma_{t-1}^2 \tag{2}$$

Where,

$$d_{t-1}=\begin{cases}1 & \varepsilon_{t-1}<0\\ 0 & \varepsilon_{t-1}\geq 0\end{cases}$$

and, $y_t$ are the daily financial returns; $\sigma_t^2$ represents the conditional variance and it depends, in a linear form, on a constant ($\alpha_0$), the squared innovations ($\varepsilon_{t-1}^2$), the good or bad news in the market ($d_{t-1}$) and its own past (in the last period); $\varepsilon_t$ is a random disturbance in the mean equation (it means that the terms $\varepsilon_t$ are independent with zero mean and unit variance Gaussian distribution). The parameters $\alpha_0$, $\alpha_1$ and $\beta_1$ must verify the following conditions to ensure a positive variance: $\alpha_0>0$, $\alpha_1\geq 0$, $\beta_1\geq 0$.

In this modelling strategy, good news ($\varepsilon_{t-1}>0$) and bad news ($\varepsilon_{t-1}<0$) have different effects on conditional variance. Good news has an impact equal to ($\alpha_1$), while the impact of bad news is equal to ($\alpha_1+\gamma$). If $\gamma$ is significant ($\gamma\neq 0$), then the impact of news is asymmetric. If $\gamma>0$ there is a leverage effect. The TGARCH model seeks to

[3] *The ARSV is not an asymmetric model, but we use it in this paper because it is a nested model of the TA-ARSV and it is necessary to perform a likelihood ratio contrast.*

capture the fact that the risk associated to negative returns is greater than the risk associated to positive returns.

The estimation method used for this model will be maximum likelihood.

### 3.2 The ARSV Model

The process proposed to describe the dynamics of volatility is an ARSV(1), which is defined by the following equations (see also Racicot and Théoret, 2010):

- The mean equation:

$$y_t = \sigma_* exp(0.5 h_t)\varepsilon_t \quad \varepsilon_t \sim i.i.d. N(0,1) \tag{3}$$

- The log-volatility equation:

$$h_t = \phi\, h_{t-1} + \eta_t \qquad \eta_t \sim i.i.d. N(0,\sigma^2_\eta) \tag{4}$$

where, $y_t$ are the returns; $\sigma_*$ is a positive scale factor in the mean equation to avoid including a constant in the log-volatility equation; $\varepsilon_t$ is a random disturbance (white noise) in the mean equation and follows a Normal distribution with a zero mean and a variance of one; $\sigma^2_t$ is the volatility and it is modelled as an exponential function to guarantee it is positive ($\sigma^2_t = exp\, h_t$); $h_t$ is the log-volatility; $\eta_t$ is a white noise process in the log-volatility equation that follows a Normal distribution with a zero mean and variance $\sigma^2_\eta$; the distribution of $\varepsilon_t$ and $\eta_t$ are independent, $E(\varepsilon_t \eta_t)$, $\forall$t,s.

The estimation[4] method for the ARSV model was developed by Durbin and Koopman (1997), and is implemented in the Ox programming language with SsfPack 2.2 program (see Koopman *et al.*, 1999, and Koopman and Hol-Uspensky, 2002).

### 3.3. TA-ARSV(1) Model

The TA-ARSV(1) proposed to describe the dynamics of volatility is defined by the following equations:

- The mean equation:

$$y_t = \sigma_* exp(0.5 h_t)\varepsilon_t \quad \varepsilon_t \sim i.i.d.\ N(0,1) \tag{5}$$

- The log-volatility equation:

$$log(\sigma^2_t) = h_t = (\phi_{11} I_{1t} + \phi_{12} I_{2t}) h_{t-1} + \eta_t; \quad |\phi_{11}|<1; |\phi_{12}|<1; \quad \eta_t \sim i.i.d.\ N(0,\sigma^2_\eta) \tag{6}$$

[4] *The estimation program for the ARSV(1) model has been developed in Ox programming and it can be downloaded for free at www.feweb.vv.nl/koopman/sv*

Therefore, the TA-ARSV(1) model is a generalization of the ARSV(1) model. The modification is based on including the following changes in the log-volatility equation of the ARSV model to obtain the TA-ARSV model and to explain the asymmetric pattern of volatility:

**a)** Two new parameters: $\phi_{11}$ and $\phi_{12}$. The first parameter measures the dynamic effect of positive returns and the latter measures the effect of negative returns on volatility.

**b)** Two observed indicator variables: $I_{1t}$ and $I_{2t}$. These are defined as follows:

$$I_{1t}=\begin{cases}1 & \forall t \text{ when the index return is positive or zero}\\ 0 & \text{in all other cases}\end{cases}$$

$$I_{2t}=\begin{cases}1 & \forall t \text{ when the index return is negative}\\ 0 & \text{in all other cases}\end{cases}$$

The volatility is defined as an exponential function in the TA-ARSV(1) model shown in Equations (5) and (6), thus, this model is not linear. However, the model can be expressed as a linear model by squaring the mean equation and taking logarithms on both sides of Equation (5). As a result, we obtain a model similar to that proposed by Sandmann and Koopman (1998) for models with small or null change in the mean and high dependence on variance. This transformed model can be conveyed in space state form to be estimated[5]. The model expressed in state space form is as follows:

$$\begin{pmatrix}h_{t+1}\\ Y_t\end{pmatrix}=\delta_t+\Phi h_t+u_t$$

where:

$$u_t \sim i.i.d.N(0,\Omega_T);\quad \delta_t=\begin{pmatrix}0\\ log\sigma_*^2\end{pmatrix};\quad \Phi=\begin{pmatrix}\phi_{11}I_{1t}+\phi_{12}I_{2t}\\ 1\end{pmatrix};\quad \Omega_T=\begin{pmatrix}\sigma_\eta^2 & 0\\ 0 & \frac{\pi^2}{2}\end{pmatrix}$$

The likelihood function for a TA-ARSV(1) model, which is a non Gaussian model, is evaluated using the Monte Carlo method approximating the non Gaussian model by importance sampling (see Durbin and Koopman, 1997).

After obtaining the results with these three models, we establish an order of preference between the different indexes using PROMETHEE methods. These methods will be explained in the next section.

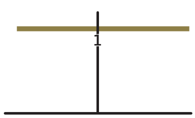
[5] *The estimation program for the TA-ARSV(1) model has been developed by the authors using the Ox 4.1 programming language and SsfPack 2.3.*

## 4. Promethee Methods

PROMETHEE is a multicriteria decision aid method (see Brans *et al.*, 1984, Brans and Vincke, 1985, Goumans and Lygerou, 2000). These methods are based on the principle of pair-wise comparison. They assume that the decision-maker tends to compare each action one-to-one with other actions when there are different evaluation criteria. This method is able to compare the different criteria independently from their measurement units and define priorities among the criteria.

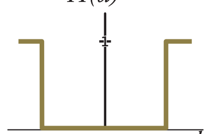
In these methods, the preference function translates the deviation between the evaluations of two actions on a single criterion in terms of a degree of preference. The degree of preference is an increasing function of the deviation: smaller deviations will contribute to weaker degrees of preference and larger ones to stronger degrees of preference. In order to facilitate the association of a preference function to each criterion, the literature has proposed the following six specific forms:

**Usual (No threshold)**



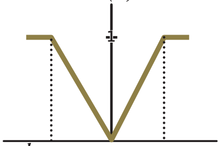

$$H(d)=\begin{cases}0 & d=0\\ 1 & |d|>0\end{cases}$$

**U-Shape ($q$ threshold)**



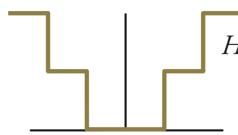

$$H(d)=\begin{cases}0 & |d|\le q\\ 1 & |d|>q\end{cases}$$

**$V$-Shape ($p$-threshold)**



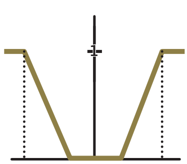

$$H(d)=\begin{cases}\dfrac{|d|}{p} & |d|\le p\\ 1 & |d|>p\end{cases}$$

**Level ($q$ and $p$ thresholds)**



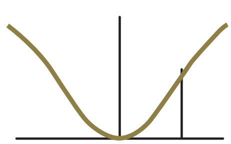

$$H(d)=\begin{cases}0 & |d|\le p\\ \dfrac{1}{2} & q>|d|\le p\\ 1 & |d|>p\end{cases}$$

**Linear ($q$ and $p$ thresholds)**




$$H(d)=\begin{cases}0 & |d|\le p\\ \dfrac{|d|-q}{p-q} & q>|d|\le p\\ 1 & |d|>p\end{cases}$$

**Gaussian ($\sigma$ threshold)**




$$H(d)=1-e^{-\frac{d^2}{2\sigma^2}}$$

The indifference threshold $q$ represents the largest deviation that is considered negligible by the decision-maker. The preference threshold p represents the smallest deviation that is considered decisive by the decision-maker ($p$ cannot be smaller than $q$). The Gaussian threshold $\sigma$ is a middle value that is only used with the Gaussian preference function. To solve the problem it is necessary for each criterion to be assigned a preference function with a weight ($w_i$) that indicates the preference of the decision-maker for the different criteria.

All the information on the problem is summarized in the pay-off matrix. The preference indexes matrix is obtained from the pay-off matrix by systematically comparing each action one-to-one with the others. The preference indexes are calculated as follows:

$$I(a_i, a_j)=\sum_i w_i H_i(d)$$

where, $a_i$, $a_j$ are two different actions; $w_i$ are the normalized weight of each criterion and $H_i(d)$ is the corresponding result for each preference function.

The PROMETHEE I partial ranking is defined as the simultaneous comparisons of the positive flow ($\phi^+$) and negative flow ($\phi^-$) rankings. The positive flow of an action is the degree of preference with which this action is preferred on average over the other actions. The best action is the one that has the largest positive flow. Negative flows are the opposite to positive flows, that is, the degree of preference with which the other actions are preferred on average to that action. Therefore, the best action is the one that has the smallest negative flow. Both positive and negative flows can be used to rank the actions from best to worst and establish an order of preference among the different actions.

When there is a conflict between positive and negative flows, the actions are considered incomparable in the PROMETHEE I ranking and it is necessary to use PROMETHEE II to solve the conflict by using the net flow ($\phi$). These net flows are calculated as following:

$$\phi = \phi^+ - \phi^-$$

## 5. Empirical Evidence

The data analyzed in this section corresponds to eight daily price index returns analyzed in the sample period dating from 1/1/1990 to 29/06/2010. The indexes are the following: France CAC 40 (CAC 40) from Paris; DAX 30 Performance (DAX30) from Frankfurt; EUROSTOXX50 (EUROSTOXX); Financial Time Stock Exchange 100 (FTSE 100) from London; Hang Seng from Hong Kong; Iberia Index

35 (IBEX 35) from Madrid; Standard and Poor's Composite (S&PCOMP) from New York and the NIKKEI 225 from Tokyo. Table 1 summarizes some information about these indexes and their returns.

This section examines two aspects:

**a)** The ability of the different econometric models proposed to explain the dynamics of volatility and the rest of stylized facts for several stock indexes returns; and,

**b)** The order of preference of these returns using the PROMETHEE methods and all the statistical information previously summarized.

**5.1. Estimated Results of the Volatility Models**

The available statistical information of the financial returns analyzed in this paper shows that their mean is constant and statistically zero for all index returns. Also, the majority of financial returns record negative asymmetry, Hang Seng being the only one to display positive asymmetry and, furthermore, all returns show an excess of kurtosis due to, among other facts, the presence of outliers. On the other hand, as it is essential to have as much information as possible so as to choose the best possible alternative, we will also analyze volatility as financial markets consider it. Moreover, volatility is a non-observable variable, which means there are different measuring alternatives.

The analysis of the trend in volatility focuses on two aspects: whether or not the leverage effect exists and persistence in volatility.

The leverage effect could be detected in the TA-ARSV(1) model using a test with a null hypothesis: $\mathrm{H}_0\!:\phi_{11}=\phi_{12}$ (both regimes with equal coefficients) and alternative hypothesis: $\mathrm{H}_1$: $\phi_{11}\neq\phi_{12}$ . This test considers the ARSV(1) model in the null hypothesis, and the TA-ARSV model in the alternative hypothesis.

One suitable possibility of implementing this test is to use the likelihood ratio statistics given by[6] $\lambda=-2\left(ln\left(L^R\right)-ln(L)\right)$. We can apply this ratio because the test nests both models.

If the null hypothesis is not rejected, then there will be no evidence of an asymmetric response on behalf of volatility. In this case, the ARSV(1) model is preferred. On the other hand, if the null hypothesis is rejected, the dynamic effect on volatility in

[6] *$Ln\left(L^R\right)$ is the logarithm of the likelihood function of the ARSV(1) model and $Ln(L)$ is the logarithm of the likelihood function of TA-ARSV(1).*

the case of a negative or positive shock is different. For all daily financial returns series examined in this paper, this dynamic effect is larger when returns are negative because the estimated value of the $\phi_{12}$ parameter is higher than the $\phi_{11}$ parameter, as shown in Table 2. It means that there is some evidence of a leverage effect in all the returns examined.

**Table 2. Persistence and Asymmetry estimated for the TA-ARSV(1), ARSV(1) and TGARCH(1,1) Models and Likelihood Ratio Test**

| | TA-ARSV(1) | | ARSV(1) | | TGARCH(1,1) | |
|---|---|---|---|---|---|---|
| | Estimated parameters | | Estimated parameter | LR(*) | Estimated parameters | |
| **Indexes** | $\phi_{11}$ | $\phi_{12}$ | $\phi$ | $\lambda$ | **Persistence** $(\alpha_1+\beta_1)$ | $\delta$ |
| CAC 40 | 0.966 | 0.998 | 0.984 | 12.92 | 0.9297 | 0.660 |
| | (0.333) | (0.328) | (0.259) | | | (0.204) |
| DAX 30 | 0.972 | 0.991 | 0.984 | 5.46 | 0.9251 | 0.110 |
| | (0.360) | (0.535) | (0.187) | | | (0.021) |
| EUROSTOXX | 0.969 | 0.999 | 0.986 | 52.48 | 0.9233 | 0.1114 |
| | (0.399) | (0.847) | (0.214) | | | (0.021) |
| FTSE 100 | 0.975 | 0.992 | 0.987 | 6.30 | 0.9389 | 0.099 |
| | (0.325) | (0.360) | (0.274) | | | (0.015) |
| HANG SENG | 0.970 | 0.997 | 0.983 | 6.18 | 0.9398 | 0.088 |
| | (0.241) | (0.802) | (0.140) | | | (0.020 |
| IBEX35 | 0.959 | 0.998 | 0.980 | 12.88 | 0.9269 | 0.103 |
| | (0.456) | (0.368) | (0.169) | | | (0.018) |
| S&PCOMP | 0.983 | 0.993 | 0.990 | 4.84 | 0.9319 | 0.116 |
| | (0.521) | (0.354) | (0.240) | | | (0.021) |
| NIKKEI 225 | 0.954 | 0.997 | 0.975 | 6.28 | 0.9280 | 0.115 |
| | (0.175) | (0.327) | (0.151) | | | (0.017) |

*(*) Likelihood Ratio Test (LR). Critical value: 3.84 (5%).*
*The value in parentheses for $\phi_{11}$, $\phi_{12}$, $\phi$ and $\delta$ is the standard error.*

Table 2 shows that TA-ARSV(1) and TGARCH(1,1) detect an asymmetric pattern in all financial returns because the null hypothesis is rejected for these indexes using the TA-ARSV(1) model and, on the other hand, the $\delta$-parameter (which detects asymmetry in the TGARCH(1,1) model) is statistically significant.

Due to all financial returns registering asymmetric patterns, the ARSV(1) cannot correctly explain the dynamics of volatility. As an example of asymmetric volatility, Figures (2) and (3) show the differences between the volatility estimated with the models (TA-ARSV(1)-ARSV(1)) and (TA-ARSV(1)-TGARCH(1,1)) respectively for returns from the FTSE 100 index in sample period analyzed. In Figure 2 reveals that differences between the volatility estimated using both models are small, but when there are some periods with high volatility, the ARSV(1) model overestimates it. In Figure 3, we can observe that if there are periods with high volatility, then volatility estimated with the TGARCH(1,1) model is greater than if estimated using the TA-ARSV(1) model. As a consequence, the TGARCH(1,1) model could be overestimating volatility in these periods.

Figure 2. **FTSE 100 Index Returns and Differences between the Volatility estimated (TA-ARSV(1))-(ARSV(1)). Sample period from 1/1/1990 to 29/06/2010.**

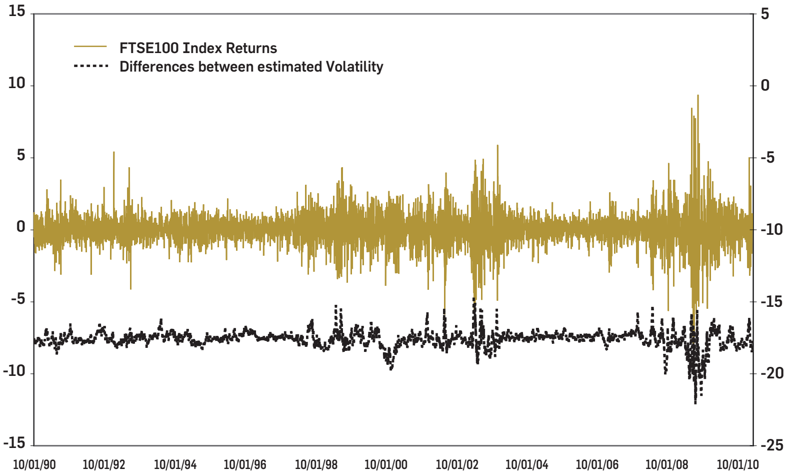
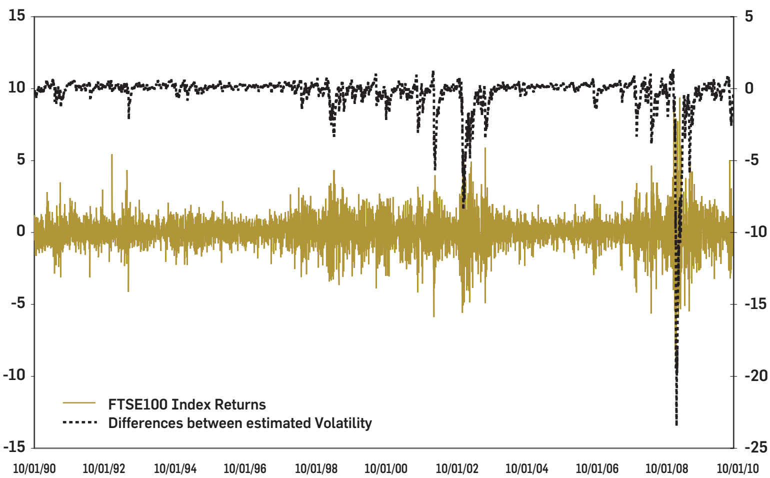



Figure 3. **FTSE 100 Index Returns and Differences between the Volatility estimated (TA-ARSV(1))-(TGARCH(1,1)). Sample Period from 1/1/1990 to 29/06/2010.**



The estimated parameter, ($\phi$), measures the persistence of volatility in the ARSV(1) model and its estimated value is included between the values of the estimated parameters in the TA-ARSV(1) model, $\phi_{11}$ and $\phi_{12}$ (which measure the estimated persistence in each regime), see Table 2. The persistence estimated by the sum of the parameters ($\alpha_1$+$\beta_1$) in the TGARCH(1,1) model for all indexes is always lower than the $\phi_{11}$ and $\phi_{12}$ parameters. This could imply that TGARCH(1,1) underestimates the persistence of volatility.

Moreover, estimated persistence is high and close to one, but in no case does it reach unit value. Therefore, all estimated processes are stationary. Persistence is significant because it indicates how volatility will behave in a certain period depending on the situation in the previous period.

Once we have analyzed the main results from the econometric models, we will proceed to establish an order of preference among the results using PROMETHEE methods.

**5.2. Order of Preference of Daily Financial Returns**

We use the PROMETHEE methods to establish an order of preference for the daily financial index returns analyzed in previous sections. These are: CAC 40, DAX 30, EUROSTOXX, FTSE 100, HANG SENG, IBEX 35, S&PCOMP and NIKKEI 225. These returns are evaluated using several criteria, some of which are based on the descriptive statistics of the returns and others on the results of the estimation of volatility with the asymmetric models. The main criteria, related to the objective statistical information gathered, are the following: the mean, the standard error (STD), skewness and the kurtosis of the returns, the mean and the standard error of the estimated volatility and the estimated persistence of volatility with the TGARCH and TA-ARSV models. The criteria minimized are STD returns and STD volatility; the rest of the criteria are maximized.

We propose several scenarios to analyse the robustness of the results. In the first one we include the estimation of the TGARCH(1,1) model and in the second the estimation of the TA-ARSV(1) model. In each of these scenarios we have obtained an order of preference among the returns in two cases. The first is when we assume that all the criteria have the same importance and, therefore, all weights are the same (in this case we assume they are equal to one), see Figueira and Roy (2002). The second, when the criteria have different weights. In this case we use a subjective valuation which consists of applying more weight to the mean and standard deviation of volatility because volatility is a measurement of risk in financial markets. It is important to point out that there are other subjective and objective valuations depending on the decision maker.

Every criterion is evaluated by the most adequate generalized criteria. We have assigned their corresponding thresholds in accordance with the evaluations of each action; see Tables 3A and 3B for the pay-off matrix of scenario I (when we use the results obtained with the TGARCH model) and Tables 4A and 4B for the pay-off matrix of scenario II (when we used the results obtained with the TA-ARSV model).

**Table 3A. Evaluation for Scenario I (including Estimation of TGARCH Model)**

| | Mean Returns | STD Returns | Mean Volatility | STD Volatility | Skewness | Persistence | Kurtosis |
|---|---|---|---|---|---|---|---|
| CAC 40 | 0.0100 | 1.3984 | 0.0262 | 1.5307 | -0.0088 | 0.9297 | 5.0998 |
| DAX 30 | 0.0225 | 1.4572 | 0.0208 | 1.3828 | -0.1287 | 0.9251 | 5.2490 |
| EUROSTOXX | 0.0150 | 1.3484 | 0.0157 | 1.3485 | -0.0675 | 0.9233 | 6.0178 |
| FTSE 100 | 0.0132 | 1.1333 | 0.0224 | 1.4572 | -0.1102 | 0.9389 | 6.7465 |
| HANG SENG | 0.0367 | 1.6710 | 0.0367 | 1.6711 | 0.0124 | 0.9398 | 9.7002 |
| IBEX35 | 0.0208 | 1.3827 | 0.0101 | 1.3985 | -0.0539 | 0.9269 | 6.4485 |
| S&PCOMP | 0.0202 | 1.1556 | 0.0202 | 1.1556 | -0.2017 | 0.9319 | 9.3237 |
| NIKKEI 225 | -0.0262 | 1.5306 | 0.0132 | 1.1333 | -0.0247 | 0.9280 | 5.6716 |

**Table 3B. Preferences for Scenario I**

| | Mean Returns | STD Returns | Mean Volatility | STD Volatility | Skewness | Persistence | Kurtosis |
|---|---|---|---|---|---|---|---|
| Function Type | V-Shape | Usual | V-Shape | Usual | Linear | Usual | Usual |
| Minimized | False | True | False | True | False | False | False |
| $p$ | 0.02 | - | 1 | - | 1 | - | - |
| $q$ | - | - | - | - | 0.05 | - | - |
| Equal Weights | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Different Weights | 1 | 1 | 2 | 2 | 1 | 1 | 1 |

**Table 4A. Evaluation for Scenario II (including Estimation of TA-ARSV Model)**

| | Mean Returns | STD Returns | Mean Volatility | STD Volatility | Skewness | Persistence | Kurtosis |
|---|---|---|---|---|---|---|---|
| CAC 40 | 0.0100 | 1.3984 | 1.2377 | 0.5469 | -0.0088 | 0.9822 | 5.0998 |
| DAX 30 | 0.0225 | 1.4572 | 1.2560 | 0.6252 | -0.1287 | 0.9819 | 5.2490 |
| EUROSTOXX | 0.0150 | 1.3484 | 1.1415 | 0.6266 | -0.0675 | 0.9841 | 6.0178 |
| FTSE 100 | 0.0132 | 1.1333 | 0.9780 | 0.4947 | -0.1102 | 0.9838 | 6.7465 |
| HANG SENG | 0.0367 | 1.6710 | 1.4240 | 0.7236 | 0.0124 | 0.9842 | 9.7002 |
| IBEX35 | 0.0208 | 1.3827 | 1.1980 | 0.5937 | -0.0539 | 0.9790 | 6.4485 |
| S&PCOMP | 0.0202 | 1.1556 | 0.9813 | 0.5575 | -0.2017 | 0.9886 | 9.3237 |
| NIKKEI 225 | -0.0262 | 1.5306 | 1.3605 | 0.5634 | -0.0247 | 0.9759 | 5.6716 |

### Table 4B. Preferences for Scenario II

| | Mean Returns | STD Returns | Mean Volatility | STD Volatility | Skewness | Persistence | Kurtosis |
|---|---|---|---|---|---|---|---|
| Function Type | V-Shape | Usual | V-Shape | Usual | Linear | Usual | Usual |
| Minimized | False | True | False | True | False | False | False |
| $p$ | 0.01 | - | 0.05 | - | 1 | - | - |
| $q$ | - | - | - | - | 0.1 | - | - |
| Equal Weights | 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| Different Weights | 1 | 1 | 2 | 2 | 1 | 1 | 1 |

The partial ranking shown with PROMETHEE I is based on strongly established preferences. As a consequence, not all the financial returns of the different indexes can be compared one-to-one with the others. For scenario I, see Figure 4 (with equal weights) where NIKKEI 225 and DAX 30 are incomparable and Figure 5 (with different weights), where EUROSTOXX and IBEX 35 are incomparable with HANG SENG and NIKKEI 225. Table 3C shows the result of the positive and negative flows together with their corresponding ranking. The rankings shown in Figures 4 and 5 emphasize that the best financial returns are S&PCOMP and FTSE 100; the worst is the CAC 40. Financial returns cannot be compared to each other when there is not an arrow between them.

### Figure 4. Partial Ranking (PROMETHEE I) for Scenario I (TGARCH) with Equal Weights


### Figure 5. Partial Ranking (PROMETHEE I) for Scenario I (TGARCH) with Different Weights

## Table 3C. **Ranking for Scenario I**

| | WITH EQUAL WEIGHTS | | | | WITH DIFFERENT WEIGHTS | | | |
|---|---|---|---|---|---|---|---|---|
| | Phi Plus ($\phi^+$) | Phi Minus ($\phi^-$) | Phi Net ($\phi$) | Ranking ($\phi$) | Phi Plus ($\phi^+$) | Phi Minus ($\phi^-$) | Phi Net ($\phi$) | Ranking ($\phi$) |
| CAC 40 | 0.1907 | 0.4713 | -0.2807 | 8 | 0.1650 | 0.4620 | -0.2969 | 8 |
| DAX 30 | 0.2185 | 0.4285 | -0.2101 | 7 | 0.2338 | 0.3813 | -0.1475 | 7 |
| EUROSTOXX | 0.2949 | 0.3472 | -0.0523 | 5 | 0.3088 | 0.3025 | 0.0063 | 3 |
| FTSE 100 | 0.4334 | 0.2138 | 0.2196 | 2 | 0.3693 | 0.2459 | 0.1234 | 2 |
| HANG SENG | 0.4256 | 0.2857 | 0.1399 | 3 | 0.3330 | 0.3333 | -0.0003 | 4 |
| IBEX35 | 0.3138 | 0.3261 | -0.0123 | 4 | 0.2917 | 0.3185 | -0.0268 | 5 |
| S&PCOMP | 0.5131 | 0.1369 | 0.3762 | 1 | 0.4947 | 0.1228 | 0.3719 | 1 |
| NIKKEI 225 | 0.2700 | 0.4503 | -0.1802 | 6 | 0.3212 | 0.3512 | -0.0300 | 6 |

Therefore, the problem that arises when PROMETHEE I is used is that the preference ranking among the returns when the positive flows are obtained ($\phi^+$) do not coincide with the result recorded when the negative flows are calculated, ($\phi^-$). Hence, incomparabilities arise among the returns due to the fact that two different criteria are being used to rank the returns: one based on how the returns of an index are dominated by other alternative returns when they are evaluated under different criteria ($\phi^-$). In order to resolve these incomparabilities and be able to establish an order of preference from best to worst, it is necessary to turn to PROMETHEE II. This method uses decision criteria based on the order of preference among returns that is obtained from net flows ($\phi$), calculated as the difference between positive and negative flows.

The complete ranking shown with PROMETHEE II for scenario I indicates that all the financial returns are ranked from best to worst, leaving no incomparability of actions. In this case we can assert that the best Stock Market Index returns are S&PCOMP and FTSE 100 and the worst is the CAC 40, see Phi Net in Table 3C and Figures 6 (with equal weights) and 7 (with different weights).

## Figure 6. **Complete Ranking (PROMETHEE II) for Scenario I (TGARCH) with Equal Weights**

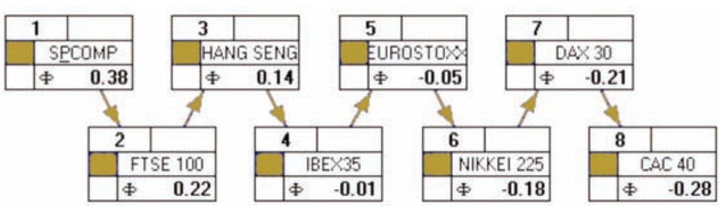
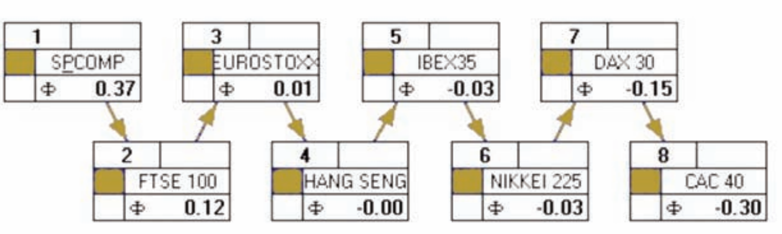


Figure 7. **Complete Ranking (PROMETHEE II) for Scenario I (TGARCH) with Different Weights**




For scenario II, which includes the results of the model TA-ARSV estimation, PROMETHE I shows the best Stock Market returns are S&PCOMP and FTSE 100, and the worst NIKKEI 225, see Table 4C and Figures 8 and 9 (with equal and different weights respectively). In this case, as in scenario I, there are incomparable returns too. However, highlighting incomparable returns is interesting for the decision-maker because it usually emphasizes returns with quite different profiles. PROMETHEE II shows the complete ranking between the returns; this information is more straightforward and is easier to use than the PROMETHEE I partial ranking. In our case, S&PCOMP and FTSE 100 are the best and the worst is NIKKEI 225, see Table 4C and Figures 10 and 11 (with equal and different weights respectively).

Figure 8. **Partial Ranking (PROMETHEE I) for Scenario II (TA-ARSV) with Equal Weights**

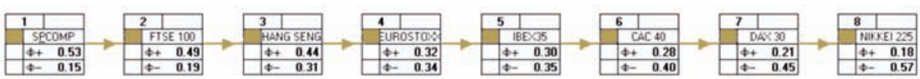
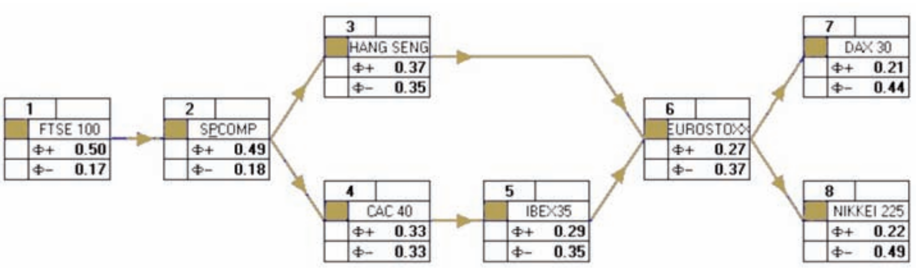


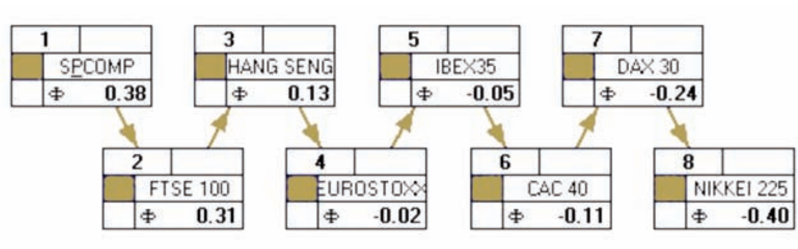

Figure 9. **Partial Ranking (PROMETHEE I) for Scenario II (TA-ARSV) with Different Weights**



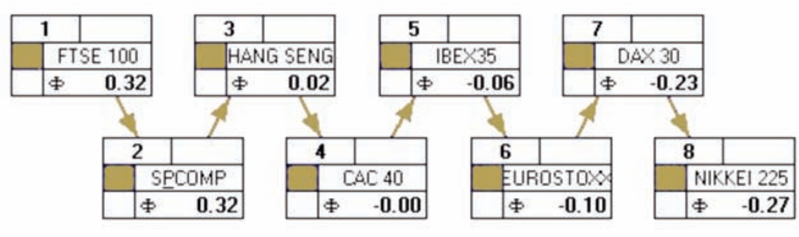
### Table 4C. **Ranking for Scenario II**

| | WITH EQUAL WEIGHTS | | | | WITH DIFFERENT WEIGHTS | | | |
|---|---|---|---|---|---|---|---|---|
| | Phi Plus ($\phi^{+}$) | Phi Minus ($\phi^{-}$) | Phi Net ($\phi$) | Ranking ($\phi$) | Phi Plus ($\phi^{+}$) | Phi Minus ($\phi^{-}$) | Phi Net ($\phi$) | Ranking ($\phi$) |
| CAC 40 | 0.2845 | 0.3961 | -0.1117 | 6 | 0.3268 | 0.3292 | -0.0023 | 4 |
| DAX 30 | 0.2128 | 0.4540 | -0.2413 | 7 | 0.2090 | 0.4368 | -0.2278 | 7 |
| EUROSTOXX | 0.3217 | 0.3415 | -0.0198 | 4 | 0.2712 | 0.3730 | -0.1018 | 6 |
| FTSE 100 | 0.4940 | 0.1880 | 0.3060 | 2 | 0.4953 | 0.1740 | 0.3213 | 1 |
| HANG SENG | 0.4396 | 0.3061 | 0.1335 | 3 | 0.3707 | 0.3492 | 0.0215 | 3 |
| IBEX35 | 0.3034 | 0.3547 | -0.0513 | 5 | 0.2915 | 0.3471 | -0.0557 | 5 |
| S&PCOMP | 0.5331 | 0.1513 | 0.3818 | 1 | 0.4941 | 0.1769 | 0.3172 | 2 |
| NIKKEI 225 | 0.1755 | 0.5727 | -0.3972 | 8 | 0.2217 | 0.4941 | -0.2724 | 8 |

### Figure 10. **Complete Ranking (PROMETHEE II) for Scenario II (TA-ARSV) with Equal Weights**




### Figure 11. **Complete Ranking (PROMETHEE II) for Scenario II (TA-ARSV) with different weights**




The TGARCH model is more sensitive than the TA-ARSV model to the outliers in the sample period. This could be one reason why the order of preference in both scenarios could change. It is important to emphasize this fact because the best returns (S&PCOMP and FTSE 100) coincide in the different scenarios analyzed.

These results are original because, in our knowledge, it is the first time that the PROMETHEE methods have been used to establish a ranking of preference among different financial returns using the results obtained from estimating the TGARCH(1,1) and TA-ARSV(1) models as criteria.

## 6. Concluding Remarks

This paper focuses on two main points: firstly, all the available statistical information about financial returns is summarized, together with the stylized facts, through statistical measures and econometric models; secondly, this information is used to establish an order of preference among the financial returns of eight stock indexes. Volatility is an important variable in the behaviour of financial returns. One of the variables considered is the volatility which is a measure of risk in the financial market although it is also non-observable. We have used two asymmetric methods to estimate it: the TGARCH and the TA-ARSV. Both models allow us to obtain estimations of volatility and some of its stylized facts in the sample period, such as the leverage effect and the persistence of volatility for several daily stock index returns.

The leverage effect is present in all the indexes analyzed. For this reason it would not be correct to use the symmetric ARSV model because volatility could be underestimated or overestimated depending on the sign of the returns. However, until recently symmetric volatility models have been the most commonly used in the stochastic volatility literature.

As a consequence, persistence and the leverage effect are explained better by an asymmetric model. Furthermore, the models estimated are stationary in covariance in all the indexes analyzed.

When there are periods with high volatility, then the volatility estimated with a TGARCH model is greater than with a TA-ARSV model because the former is more sensitive to the outliers in the sample period analyzed and might overestimate it as a result.

Once volatility has been estimated with the appropriate model, we use all the information obtained and apply the PROMETHEE methods to establish an order of preference for the eight daily stock index returns as it is necessary to consider every piece of information to choose the best returns at each time particularly now in times of crisis.

In the different scenarios proposed we have been able to confirm that the partial ranking of the alternatives (PROMETHEE I) reveals that there are returns that cannot be compared. This is due to the fact that, in this order of preference, two flows are used which collect different information: positive and negative flows. The direct consequence of this is that incomparabilities emerge among the returns: positive flows show when one alternative is preferred over another when evaluated under all the criteria, whereas negative flows show how one alternative can be dominated by another. However, these incomparabilities have been solved by using a complete ranking (PROMETHEE II), which only uses net flows. At this point, we can choose the best alternatives.

With the aim of carrying out a robust analysis of the results, we have proposed different scenarios for diverse asymmetric volatility models. First we assigned the same weight to all the criteria and, afterwards, different weights are assigned to the mean and standard deviation of volatility (these criteria have doubled their weighting). Hence, the best returns in the different scenarios analyzed were those of S&PCOMP and FTSE 100 indexes, while CAC 40 recorded the worst in the first scenario and NIKKEI 225 in second one. Consequently, this methodology allows us to obtain robust results when analyzing a variety of possibilities.

We can conclude by saying that this methodology is a system that allows building financial rankings based on statistical information. Moreover, it would permit us to expand on any other type of information available for the decision maker. It is consequently a useful tool for decision making.

## References


- Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity, *Journal of Econometrics,* **31,** pp. 307-327.

- Brans, J.P., Mareschal, B. and Vincke, P.H. (1984). PROMETHEE: a new family of outranking methods in multicriteria analysis, in Brans, J.P. (Ed.), *Operational Research'84,* North-Holland, Amsterdam, pp. 477-490.

- Brans, J.P. and Vincke, P.H. (1985). A preference ranking organization method, the PROMETHEE method, *Management Science,* **31,** pp. 647-656.

- Doornik, J.A. (2001). *An object-oriented matrix programming language using Ox.* Timberlake Consultants Ltd., London, http://timberlake.co.uk.

- Durbin, J. A. and Koopman, K. J. (1997). Monte Carlo Maximum Likelihood Estimation for Non Gaussian State Space Models, *Biometrika,* **84,** pp. 669-684.
- Engle, R.F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of UK inflation, *Econometrica,* **50,** pp. 987-1007.
- Figueira, J. and Roy, B. (2002). Determining the weights of criteria in the ELECTRE type methods with a revised Simos1' procedure, *European Journal of Operational Research,* **139,** pp. 317-326.
- García, M. C and Mínguez, R. (2009). Estimation of Asymmetric Stochastic Volatility Models for Stock-Exchange Index Returns, *International Advances in Economic Research,* **15,** pp. 71-87.
- Glosten, L., Ragannthan, R. and Runkle, D. (1993). On the relation between the expected value and the volatility of the nominal excess returns on stocks, *Journal of finance,* **48,** pp. 1779-1801.
- Goumans, M. and Lygerou, V. (2000). An extension of the PROMETHEE method for decision making in fuzzy environment: Ranking of alternative energy exploitation projects, *European Journal of Operational Research,* **123**(3), 606-613.
- Harvey, A. C. and Shephard, N. G. (1996). Estimation of an asymmetric stochastic volatility model for asset returns, *Journal of Business and Economic Statistics,* **14,** pp. 429-434.
- Koopman, S. J., Shephard, N. and Doornik, J. A. (1999). Statistical algorithms for models in state space using SsfPack 2.2, *Econometrics Journal,* **2,** pp. 1-55.
- Koopman, S. J. and Hol-Uspensky, E. A. (2002). The Stochastic Volatility in Mean Model: Empirical evidence from international stock markets, *Journal of Applied Econometrics,* **17**(6), pp. 667-689.
- Racicot, F. E. and Théoret, R. (2010). Forecasting Stochastic Volatility Using the Kalman Filter: An Application to Canadian Interest Rates and Price-Earnings, *AEstimatio,* **1,** pp. 28-47.
- So, M. K. J., Li, W. K. and Lam, K. (2002). A threshold stochastic volatility model, *Journal of forecasting,* **21,** 473-500.
- Sandmann, G. and Koopman, S. J. (1998). Estimation of stochastic volatility models via Monte Carlo Maximum Likelihood, *Journal of Econometrics,* **87,** pp. 271–301.
- Teräsvirta, T. and Zhao, Z. (2006). Stylized facts of return series robust estimates and three popular models of volatility. Working Paper, Stockholm School of Economics, Stockholm.
- Taylor, S.J. (1986). *Modelling Financial Time Series,* John Wiley and Sons Ltd, New York.
- Zakoian, J. (1990). Threshold heteroskedastic models, *Journal of Economic and Control,* **18,** pp. 931-995.