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“Nobody Can Sit There”: Two Perspectives on how Mathematics Problems in Context Mediate Group Problem Solving Discussions

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"Nobody Can Sit There": Two Perspectives on how Mathematics Problems in Context Mediate Group

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Abstract

This study examines how a group of bilingual ninth grade algebra students discussed two word problems stated in terms of "real life" contexts. Using a lens of mediated action (Wertsch, 1998), the analysis reveals two distinct ways that the problem contexts influenced the group's mathematical reasoning. In one problem, the problem context afforded particular ways of interpreting the given inscriptions, which had benefits as well as costs. In the other problem, the unfamiliar story and terminology appeared the hinder the group's mathematical reasoning. These two forms of context mediation are discussed in light of current research on the use of real life problems in mathematics education.

Keywords: mathematical discussions, mediation, word problems, algebra, group discussions

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Solving mathematics story problems set in an imaginary context is a common experience for school children. Prior research suggests that giving students mathematics problems set in a familiar context can promote their problem solving success by increasing their motivation and drawing upon their expertise from outside of school (Baranes, Perry, & Stigler, 1989; Moschkovich, 2002; National Council of Teachers of Mathematics, 1989, 2000). However, the semantic complexities and non-mathematical considerations that can arise when solving problems with real life context may also obscure mathematical relationships, leading students away from providing the expected responses (Boaler, 1993; Gerofsky, 1996; Martiniello, 2008; Walkerdine, 1988).

This paper examines two ways that the "real life" contexts of word problems entered bilingual students' discussions of mathematics story problems. Using the lens of mediated action (Vygotsky, 1978; Wertsch, 1991, 1998), the analysis examines how drawing upon "real life" contexts given in the mathematics problems both facilitated and hindered the group's problem solving efforts. In particular, detailed analysis of the students' talk during problem-based group discussions shows how the given problem contexts had a complex interaction with the reasoning and resources that students drew upon while solving two non-routine mathematical problems. This analysis shows how the problem contexts mediated the students' problem solving discussions in at least two distinct ways. This observation leads to a discussion that can deepen the mathematics education community's understanding of the affordances and constraints of solving mathematics problems stated in terms of "real life" context.

The primary data for this paper are drawn from a study of how high school algebra students learned and reasoned about rates during group discussions. The data are used to develop and to illustrate a theoretical connection with the notion of mediated action. While the larger study investigated the relationship between group interactions and students' mathematics learning, this paper focuses on the narrower question, How might adding "real life context" to a mathematics problem afford and constrain students' group problem solving?

The theoretical framework below introduces mediated action

(Wertsch, 1991, 1998) from cultural historical activity theory. Then a brief literature review examines results from previous studies on how students solve mathematics problems stated with real life context. Next, a case study of how one group of students solved problems in context is introduced to develop a distinction between two influences of "problem context" that may be conflated. Finally, the discussion returns to the theoretical and practical considerations that arise from the data and theory presented in this paper.

Theoretical Framework

The overarching study, from which this paper was drawn, was rooted in a sociocultural approach to teaching and learning mathematics (Forman, 1996; Moschkovich, 2004). The central concept from sociocultural studies that is used here is mediation. Mediated action has been used since Vygotsky and colleagues argued that human thinking and goal-directed actions are inseparable from the cultural tools employed to reach goals (Luria, 1979; Vygotsky, 1978, 1986). Language is one essential tool for thought, and Vygotsky (1986) argued that children's use of "egocentric speech" was evidence that children internalize a socially shared tool for thinking. However, the mediation of cultural tools in human thinking and action goes beyond the use of language in verbal thought. Wertsch (1991, 1998) showed how human actions, such as pole vaulting and doing mathematical calculations, are also instances of mediated action. He argued that goal-directed actions, for example, performing multi-digit multiplication, cannot be analyzed without accounting for the mediation of cultural tools (e.g., decimal numbers, algorithms, & calculators) used to reach those goals.

Wertsch's (1998) example of multidigit multiplication is helpful for illuminating how doing calculations is a form of mediated action that is deeply shaped by the use of cultural tools. Most adults who have learned a standard multiplication algorithm in school could compute 343 × 822 = 281 946 without the aid of a calculator. However, the multiplication algorithm, and even the decimal number system used to represent the numbers, are culturally-developed meditational tools. The affordances of these tools is made visible when the problem is stated in a different way, for example, CCCXLIII × DCCCXXII.

Numerous mathematics education researchers who use a semiotic perspective have drawn on this framework of mediated action to analyze children's mathematical activity (e.g., Radford, 2001; Radford, Bardini, & Sabena, 2007; Walkerdine, 1988). One important insight from these studies has been that mathematical activities of all kinds have semiotic entanglements—the notion that mathematics can happen independently of human language and sign systems is a myth. Moreover, mathematics itself is transformed as humans' semiotic resources expand. For example Hegedus and Moreno (2011) have argued that new digital technologies are transforming the very nature of what is called mathematics, what constitutes mathematical activity, as well as the possibilities for mathematics teaching and learning. This paper follows in this tradition, but rather than analyzing high technology and digital media, it focuses on how the stories and hypothetical situations given in word problems mediated students' mathematical activity. Wertsch's (1991, 1998) notion of mediation provides a framework to analyze how the imagined problem context shaped students' group discussions and influenced the mathematical conclusions they reached.

This paper is drawn from a larger study of how students learned key concepts in algebra by engaging in discussions with a small group of peers. In the larger study, learning was considered as a process of appropriating and using culturally shared tools for reasoning (Forman, 1996; Moschkovich, 2004; Rogoff, 1990). For example, in the algebra classrooms where this study was situated, the students appropriated ways of reasoning about the slope of linear functions by focusing on the "rise" and the "run" between two points on the line.

The definition of "problem context" for this analysis captures one meaning of "context." This paper focuses on the context as the "cover story" in word problems (Gerofsky, 1996). In particular, the problem context is defined as the characters, objects, and relationships introduced in the problem statement. There are many other possible meanings for "context" in studies of learning. For example, the school setting, and the social composition of the group of students working together are also part of the problem-solving context. Of course, this choice of focus on problem contexts only addresses part of the overarching situation when students work together as a group to solve mathematics problems. However, this choice was necessary in order to make this analysis manageable, and to allow for a detailed explication of how seemingly inconsequential details of mathematics word problems can shape students' discussions.

Prior Research

Stating problems with stories has been part of mathematics since antiquity (Gerofsky, 1996; Schoenfeld, 1992). Oftentimes the stories that appear in word problems are unrealistic, and the stories are carefully constructed to require the use of a recently learned algorithm (Schoenfeld, 1992). Part of a student's task when solving such problems is to learn to attend to certain features in the problem, and to recognize what quantities and operations should be combined to produce the desire result. Another important skill that students develop through solving problems is learning when it is permissible to ignore the context.

Despite the fact that the problem context in word problems is often regarded as superfluous, there is some evidence that the context within which a mathematics problem is presented influences children's solutions and their mathematical success. Researchers who focus on the interaction between "everyday" and "academic" mathematics have found that many people and students can do certain mathematical tasks in everyday settings that they find impossible when given as a school mathematics problem (Brenner, 1998; Carraher & Schliemann, 2002; Moschkovich, 2002; Saxe, 1995). For example, in one landmark study, Carraher, Carraher, and Schliemann (1985) discovered that children who sold food on the streets in Brazil were quite adept at doing the arithmetic necessary to make change in commercial transactions, but these children could not do the "same" calculations in decontextualized form with paper and pencil. Carraher, Carraher, and Schliemann argued that changing the problem context from a selling problem to a school problem also changed the arithmetic resources that children used to do their calculations. Therefore, the statement of problems in context cued the children's problem solving choices and success in each condition.

This work has been followed by multiple studies probing the affordances of using real life contexts as a tool for teaching school mathematics (Boaler, 1993; Brenner, 1998; Civil, 2002; Civil & Andrade, 2002; Gerofsky, 1996; Greer, 1997; Moschkovich & Brenner, 2002; Saxe, 1995). For example Brenner (1998) studied the affordances of using coins to teach decimal numbers. Many elementary school curricula in the US have used coins to represent the decimal number system and place value concepts. Brenner conducted an ethnographic study of how Hawaiian children used money, and she found that the way Hawaiian children used money outside of school did not match the way that coins were used to teach mathematical concepts in school. One specific mismatch was that the children treated a quarter (25 cents) as the basic unit of money in their purchases, while the school curriculum treated the penny as the basic unit (Brenner, 1998). One curriculum-focused response to this work has been a push to use more realistic "real life" contexts in mathematics curricula, which focus on building meaningful connections between important mathematical concepts and the real life context of school children's lives (Boaler, 1993; Greer, 1997; National Council of Teachers of Mathematics, 1989, 2000).

A related body of research from the cognitive framework has also examined the costs and benefits of adding context and personalizing mathematics word problems (e.g., Koedinger & Nathan, 2004; Walkington & Maull, 2010). As with the findings from the socioculturally focused research, these studies have shown that some forms of context can help students reach correct solutions, especially when a problem is stated in a way that draws on students' resources and that motivates the student to persevere. In particular, Koedinger and Nathan found that students were able to solve more complex problems when they were stated as stories, as opposed to bare algebraic equations. In sum these studies indicate that, under felicitous conditions, adding context can aid students' mathematical problem solving.

However, there are important limitations inherent to the use of context. One critical issue is that the use of contextualized problems might interfere with the intended mathematics curriculum. While students may draw on certain aspects of the problem context as an aid to solve complex word problems, students are also expected to know when to ignore real life considerations in order to solve a problem using the intended mathematical algorithm. Both Gerofsky (1996) and Walkerdine (1988) observed this can be problematic for children, especially those who are not aware of the game, or whose out of school language practices are not congruent with the use of langauge(s) in school. At times, using too much knowledge about the context can actually result in students giving wrong answers.

The issue of adding context to word problems can take on an added layer of complexity for students who are learning the language of instruction (Abedi & Lord, 2001; Martiniello, 2008). Since contexts for mathematics problems are usually stated in the form of a written story, adding context might also add unnecessary linguistic complexity to mathematics problems. This linguistic interference can, in turn, obscure the mathematical proficiency of students who are learning the language of instruction and result in educators making incorrect inferences about students' mathematical knowledge. For example, Martiniello investigated how English Learners (ELs) performed on a state-wide mathematics assessment in the US that included some problems stated as stories. She found that ELs did worse than would be expected by their mathematical proficiency on questions that used unfamiliar terminology. In follow up interviews with selected students, she found that although the students knew the mathematical concept being assessed (e.g., using the counting principle to compute combinations), they were unable to answer questions on these topics because of the language used to state these questions.

In light of the complexity of findings in prior research, it may be too simplistic to ask whether adding context to mathematics problems will help or hinder students' performance. A more apposite question may be to ask how adding context to a problem might influence student reasoning. This study sets out to add to the literature by addressing this open question.

Data & Methods

The data for this paper are drawn from a study of how bilingual ninth grade algebra students learned to reason with linear functions. Specifically the study examined how the students generalized from data and reasoned about the relationships between the rate of change and slope of linear functions through engaging in small group discussions with their peers. Data collection in the larger study traced the reasoning of two groups of four students each in one algebra class across six weeks of class meetings. The data collected included video recorded observations of classroom interactions, as well as video and written work from a series of focused group problem solving sessions recorded outside of class. The in-class observations provided data on the naturalistic setting of the classroom and the types of reasoning the students did there. The out of class group discussions sessions provided more focused data on how the students reasoned through tasks, and how their reasoning developed across the six week data collection time period. This analysis focuses on the students' out-of-class discussions.

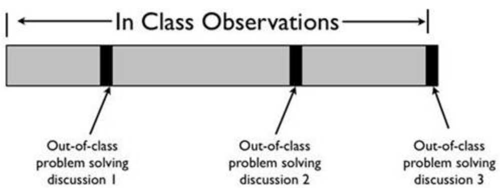

Figure 1. **Timeline of data collection**

The Setting and Study Design

The class that was observed was a bilingual (English-Spanish) algebra class taught by a highly regarded, experienced, and qualified teacher. The school population was over 90% Latino/a, and 77% of the students qualified for a free or reduce priced lunch, indicating that they were from low socioeconomic backgrounds. Thirty five percent of the school population was classified as English Learners. In the focal classroom, about one-third of the students were recent immigrants from Latin America who were learning English (the language of instruction), while other students were bilingual. The bilingual students, who were classified by the school as proficient in English, were either born in the United States but grew up in Spanish-speaking households, or they had immigrated to the United States and learned English several years before this study. The data collection time period coincided with a six week classroom unit when the students were learning to graph lines, write linear equations, and interpret linear functions. The students' mathematics curriculum highlighted the use of applied problem solving and using real-life data to learn these concepts (Fendel, Resek, & Alper, 1996).

In order to document changes in the students' reasoning about linear functions across time, each group solved a set of three tasks at the start, middle and end of the data collection time period. These group problem-solving discussions took place outside of class, and the students were instructed to discuss each problem as a group, come to consensus, and then write one shared answer for their group. The problems that the students solved together were specially chosen to highlight key conceptual relationships, such as the relationship between the slope of a linear function and the rate of change of the dependent variable (two of the problems that will be discussed in more detail in this paper appear in the Appendix). In order to make valid inferences based on these students' reasoning, these problems were drawn from assessments used in prior research on student reasoning about generalization, rates, and linear functions. The problems were similar to the type of problems the students solved in class, but they were chosen to highlight key conceptual relationships.

These group discussions were video recorded and transcribed with a focus on capturing the propositional content of the students' talk. Copies of each group's agreed-upon written answers, as well as their scratch work, were collected. Finally, the author was present and recorded field notes during each discussion, but he did not intervene in the group discussions.

Focal Group

This case study analysis focuses on how one of the two groups used the given problem contexts as a resource while reasoning through the tasks. This group was selected for several reasons. First, the focal group engaged in extensive discussions of the problems, while the other group tended to have shorter discussions with less dialogue about each problem. Second, this group included four bilingual students who were all classified as "Fully English Proficient," which decreases (but does not eliminate) the probability that the students' level of proficiency in English would interfere with their mathematical reasoning on these questions written in English. Finally, this group made the most references to the problem context in their discussions, so its discussions help illuminate the theoretical issues discussed in this paper. Although the focus of this analysis is on one group of students, both of the groups in the study did refer to the real life contexts of the problems, and they drew upon mathematical reasoning, as they worked through these problems.

The focal group consisted of two boys, Mateo and Jaime, and two girls, Krystal and Susanna. Mateo and Krystal were immigrants from Spanish-speaking countries, and all four of the students reported speaking both Spanish and English outside of school. The group members primarily spoke English when working in their group, and they were given all in-class assignments in English. The teacher selected this group to participate in this study by assembling groups of students who she thought would work well together, and who represented a range of prior achievement in her class. Mateo and Krystal had relatively high grades, while Jaime and Susanna had relatively low mathematics grades.

This analysis focuses on the transcripts and written work produced during three out-of-class problem-solving sessions among the focal group. Additionally, the videos and field notes were used as a resource throughout the analysis process to clarify meanings in the transcripts and in the students' written work. The group solved three problems about rate and accumulation across eight weeks, repeating each problem at least twice. Although the students solved the same problem twice, they did not simply recall their answers to each question from their first solution. For example, the group engaged in three sustained discussions of a problem called Hexagon Desks, using over ten minutes to discuss the problem *each time* they attempted it.

Tasks

This analysis focuses primarily the group's discussion of the problem Hexagon Desks. The group's discussion of a second problem called The Tortoise and the Hare is also presented to highlight contrasts in how the problem context can mediate students' mathematical problem solving.

Hexagon Desks was adapted from a released eighth grade item from the National Assessment of Educational Progress, and variations of this problem have appeared in numerous other forms in research and curricula. Hexagon Desks focused on using multiple representations to explore a linear relationship between the length of a chain of hexagon shaped desks and the number of students who could sit at the chain of desks. The Tortoise and the Hare was adapted from previous research on student interpretations of motion graphs, and it asked students to interpret two velocity-time graphs plotted on the same axes. In each task, the sequence of the questions was designed to elicit how students reasoned about rates in relation to slope (on Hexagon Desks) and accumulation (on The Tortoise and the Hare).

This analysis focuses on the mathematical content of the students' discussions, treating the group discussion as the unit of analysis. The students' talk during the discussion is treated as evidence of the group's reasoning. The design of the discussion sessions, requiring the group to agree on an answer, provided a rich source of talk because the students were forced to reconcile differences and come to an agreement before writing their final, agreed upon answer.

Analysis

The data analysis followed three steps. First, the students' talk was transcribed with a focus on capturing the propositional content of the students' talk. A total of 180 minutes of group talk was transcribed (though not all of that time was dedicated to talking about mathematics). Second, the transcripts were divided into segments corresponding with the students' talk about each part of the problems. For example, one segment included all of the group's talk about question two from Hexagon Desks. The third stage of analysis was identifying segments where the students made reference to the problem context, either implicitly or explicitly. All references that the students made to the problem context were catalogued. Finally, all of the references to context were coded according to whether referring to the context helped, hindered, or had a neutral effect on the students' problem solving success. Problem solving success was measured by whether the group was eventually able to provide correct answers to the questions on each task.

Findings

Written Responses

In total, the group's written responses to Hexagon Desks task showed some development as well as a fair amount of consistency across the three times they attempted the problem. Each time the group attempted this problem, they successfully completed the table in question one which asked them to show how many people could sit around a row of three, four, five, six and seven hexagon desks arranged in a row. The only exception was that the group answered "31" in the last row of the table during Discussion 2 because they added five rather than four to 26, the previous value. On question 2, the students also successfully found the number of students who could sit at a row of 100 desks (402) each time they solved this problem. The group also successfully solved question three, and they derived an equation for how many people could sit at n desks (total = $4n + 2$), during all three discussions of this problem. Question four required the students to graph a set of points, and, as expected, the group succeeded in this task each time they discussed it. The group skipped question five, which required computing the slope of a line, during their first discussion. However, during their second and third discussions, after slope had been taught in their class, the students successfully found the slope, and they wrote it in fraction form as 4/1. The group skipped question six, which asked them to interpret the slope of a linear function in terms of the problem, each time they solved Hexagon Desks. Finally, during their second and third discussions of this task, the group successfully solved question seven, which required them to generalize this relationship for octagons.

The group produced only one set of written responses to The Tortoise and the Hare because they ran out of time when the problem was given to them a second time. They successfully answered questions one through three on the task, but their written answers had incorrect units, indicating that they struggled to reason with the velocity as a quantity on the y-axis. This response pattern makes sense because reasoning about intensive quantities is more challenging for students than reasoning about extensive quantities (Schwartz, 1988). On questions four through ten, the students agreed on incorrect answers, but they also showed signs of confusion for these questions. These relatively difficult questions required the students to reason about intervals (rather than points), and to work backwards and find the distance traveled as the product of velocity and time. Finally, in addition to their mathematical struggles on this problem, it appeared that the words "tortoise" and "hare" were unfamiliar to two of the group members.

The Mediation of the Problem Context in Discussions

While the students referred to the imagined story in each problem when discussing their solutions, their reliance on the context, and their relative success by using the given contexts, revealed two distinct ways that problems in context can mediate students' mathematical reasoning. One influence of the problem context was that it drew the students' attention to reason about the given problem in particular ways related to the story. This was evidenced by the students' use of examples and terminology indicating that they were drawing on the context as a resource. By imagining how many students could sit at a row of desks, the context on Hexagon Desks may have helped the group reason through this problem. On the converse side, the other evidence of the mediation of context was through the evident confusion on the part of the students with unfamiliar vocabulary and an unrealistic situation in The Tortoise and the Hare.

For both Hexagon Desks and The Tortoise and the Hare, the imaginary context did not always help the group reach a correct response, nor did it necessarily hinder their progress. However, the influence of the problem context, and the mediation of the imagined situation differed in important ways. One way the context mediated the group's reasoning on Hexagon Desks was that the story allowed the students to reason about perimeter of a chain of hexagons by imagining a person sitting at each external segment on the figure. In this case, the imaginary context promoted a particular way of looking at the given inscriptions. A second way the context mediated the group's reasoning was evident in The Tortoise and the Hare. For this problem, both the challenging vocabulary (tortoise and hare) as well as the implausible story appeared to hinder the group's mathematical reasoning because it distracted from their mathematical focus. Below I illustrate how the problem context mediated the group's discussions on both problems.

The Context as a Resource on Hexagon Desks

The students did not appear to have any struggles imagining the given context in Hexagon Desks, which might indicate that the idea of pushing desks together was relatively familiar. The students' familiarity with the story in Hexagon Desks is affirmed by the contrast with how they talked about the unfamiliar and unrealistic context from The Tortoise and the Hare (see below). At key points early in their discussions of this problem the group members did use the story about seating students around desks as a resource for their mathematical reasoning. The majority of the group's references to the story occurred as they completed the table in question one. In particular, as the group members filled in the "number of students" column in the table, they discussed whether any students could sit at the spaces represented by vertical segments in the diagrams. They agreed that "nobody" could sit at the segments where two desks meet. They also noted that "somebody" could sit at the two vertical segments at the ends of the row of hexagon desks.

Excerpts 1 and 2 below contain two instances where the group referred to the context as they solved this problem. Excerpt 1 is from the group's first discussion of this problem while Excerpt 2 is from their third discussion of the problem. In both cases they pointed to the chain of hexagons given on their paper as they made reference to the story about seating students at a row of hexagon shaped desks. Figure 2 illustrates where Mateo and his group mates were pointing as they used the words "somebody" and "nobody" to reason about this problem.

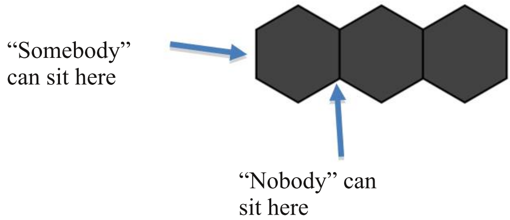

Figure 2. **How the students used "somebody" and "nobody" in relation to the Hexagon Desks problem.**

Excerpt 1 is from the group's first discussion of the problem. The three group members who were present that day, Mateo, Krystal, and Jaime, each appeared to use slightly different methods for solving the problem. Krystal drew chains of hexagons and counted sides. Mateo appeared to be coordinating the image and the story about seating students around desks to devise pattern. Jaime derived a recursive rule for the pattern, noting that the first term was six, and subsequent terms were four more than the previous terms.

In the following excerpts, clarifying comments are enclosed in double parenthesis. Square brackets are used to show the start of overlapping talk. The students used some Spanish words in Excerpt 1 and translations are in double parentheses with quotations immediately after the terms in Spanish.

Excerpt 1
The group discusses the table in Hexagon Desks during Discussion 1

1. Mateo It's gonna be eighteen ((referring to row 4 of the table))
2. Krystal Huh?
3. Mateo It's gonna be eighteen because you know [nobody's

gonna sit
4. Krystal [but how does that work
5. Jaime Oh it's some pattern
6. Krystal I know its a [pattern but
7. Mateo [no it- [[you
8. Jaime [[its going by six y luego (("and then")) by [four
9. Krystal: [four five ((continues counting silently))
10. Mateo six plus four is ten. Ten plus for is fourteen
11. Krystal ((speaking louder)) Twenty twenty one twenty two twenty three twenty four. I was right
12. Mateo No but watch
13. Jaime xxx
14. Krystal Watch one two three four five six seven eight nine ten eleven twelve thirteen fourteen fifteen sixteen seventeen eighteen nineteen twenty twenty one twenty two three ((short pause)) damn it I was wrong its twenty [t-
15. Mateo [but no cause you add another one and nobody's gonna be sitting on that one ((pointing at Krystal's paper))
16. Krystal I know but right now it's like yeah it works

Krystal was attempting to count the perimeter of a chain of five hexagons, but she appeared to get lost while counting to 22 in lines 11 and 14. Mateo then made an explicit reference to the problem context in line 15 to explain why Krystal's answers were wrong. After Mateo's comment, Krystal took up his idea and verified that it agreed with the numerical patterns she observed for this problem. Therefore, Except 1 illustrates how the story appeared to help the students agree on the correct solutions to question 1 from Hexagon Desks.

Excerpt 2 is from this group's third discussion of this problem. During their first discussion they agreed that each row of the "number of students" column in the table should be four more to the previous row, giving answers of 18, 22, 26, and 30. However, during their second discussion (which occurred between the discussions in Excerpts 1 and 2), the group agreed upon a slightly different rule: each row was four more than the previous row, except for the last row, which was five more than the previous. Thus they incorrectly wrote 31 rather than 30 in the final row during their second discussion of this problem. For their third time working through this problem, the group returned to the question of whether to “add four” or “add five” to each row in the table.

Prior to the exchange in Excerpt 2, the group agreed that the “number of students” in each row of the table should be four more than the previous row. However, Mateo also argued that the number of students in the *last* row of the table should again be 31—*five* more than in the previous row—because “somebody can sit here [on the last vertical spot].” Krystal disagreed, and Excerpt 2 shows the start of the ensuing discussion. Ultimately this group agreed that the final answer for the last row of the table should be 30 (i.e., four more than the previous row), but only after several minutes of discussion. The group came to consensus after Krystal drew out a chain of seven hexagons and counted the perimeter of all seven of them.

Excerpt 2
The completes the table in Hexagon Desks during Discussion 3

1. Mateo You just add four to all of them
2. Krystal Then you add four and then you add like two ((possibly referring to the general rule, perimeter = 4n+2))
3. Mateo The last one is five
4. Krystal What?
5. Mateo See six plus four is ten plus four is fourteen and the last one you just add five that's all these ((pointing to end of chain))
6. Susana Why add five?
7. Krystal Really? yeah
8. Mateo ‘Cause it that's the last one and somebody can sit on this ((points at the end of the chain of hexagons))

The question of whether “somebody” or “nobody” can sit in a particular spot in the imagined chain of hexagon desks illustrates one way the problem context and everyday language mediated the mathematical discussion among this group of students. This use of the context both afforded and constrained the students’ reasoning. The affordance was that by imagining students sitting at a desk, they were able to reason about the perimeter of the chain of hexagons. However, imagining people sitting at the desks also introduced a subtle problem. While it is true that “nobody” can sit at the intersection of two desks in the Hexagon Desks problem, the word *nobody* is slightly problematic because *two* spaces are removed each time two desks are pushed together. The intersection of two desks removes two sides from the available seating, so the net change in the number of seats (i.e. the perimeter) is (new perimeter) = (previous perimeter) + 6 − 2. Mateo’s use of “nobody” may have obscured this relationship because “nobody” does not quantify how many people cannot sit at an intersection. This issue may help explain why the group debated whether to “add four” spaces or “add five” new spaces with each desk, even though they quickly identified the numerical pattern “add four” in the first few rows of the table.

For the remaining problems in Hexagon Desks, the group appeared to shift in their reasoning and in their reliance on the context. While they relied on the images of chains of hexagons to answer questions two and three on the task, they made relatively few references back to the problem context. For example, they did not comment on the absurdity of making a row of 100 desks. Moreover, the group also skipped question six, which explicitly asked the students to make a connection between the problem context and the slope of the linear function (defined on the natural numbers) that models this situation.

The Interference of Context on The Tortoise and The Hare

While the prosaic problem context in Hexagon Desks appeared to be familiar to the students, the more whimsical context in The Tortoise and the Hare was not. In this case, the story did not serve as a resource for the students; in fact, the unfamiliar context may have detracted from the students’ mathematical reasoning. The students’ distraction illustrates a second, clearly unhelpful, way that problem contexts can mediate students’ mathematical problem solving.

None of the students indicated that they had heard the fable of the tortoise and the hare as they discussed this question (though knowing the fable was not necessary to solve this problem). The students' lack of familiarity with the context resulted in a qualitatively different form of mediation of the problem context. First, the students used some time to discuss the meaning of the unfamiliar terms "tortoise" and "hare." In Excerpt 3 below, Jaime struggled with pronunciation while reading "tortoise," and Krystal asked her group mates what a tortoise was. Likewise Mateo corrected Krystal's use of the word "bunny" for "hare" and the students discussed the Spanish and English words for tortoise and hare. Second, the students' few attempts to use the context as a resource to reason about the mathematics were unsuccessful. For example, in line 12 of Excerpt 3, Krystal appeared to try to draw on her knowledge of rabbits to reason about the plausibility of the hare's graph. Unfortunately, it is not clear whether Krystal's erroneous reasoning is a result of misunderstanding the context or of misunderstanding the graph.

Excerpt 3
The group reads the problem statement in The Tortoise and The Hare

1. Jaime ((reading the problem)) One day tom the tortoise ((struggles with pronunciation of tortoise))
2. Krystal Tor tus a ((sounding out the word, pronounces incorrectly))
3. Mateo Tortoise ((Pronounces correctly))
4. Jaime Tortoise and Harold the hare race ((pause)) ran a race. Tom got a running start but they both ran across the starting line at the same moment when the times said zero seconds. They ran along a straight road for ten seconds and the graph below shows Tom and Harold's velocity during the ten second race.
5. Krystal So oh this is tortoise the ((pointing at image))
6. Mateo It's Tom is a tortoise
7. Krystal What's a tortoise?
8. Mateo Its a [turtle
9. Krystal [A turtle?
10. Mateo Yeah but uh bigger [[xxx
11. Jaime [[Who was running fas

12.	Krystal	This isn't possible for a bunny ((traces pencil along the inverted V shape on the graph))
13.	Jaime	haha
14.	Mateo	No this is a turtle
15.	Krystal	Yeah, this is a bunny ((again makes the inverted V)) like faster and then stopped
16.	Mateo	No that's a hare. Hare is bigger skinny
17.	Krystal	So it's better than a bunny
18.	Mateo	Yeah

After discussing the unfamiliar terms *tortoise* and *hare*, the students were able to answer questions 1-3 by reading specific values on the graph. However, the units in the students' written answers were incorrect (e.g., they wrote "At 2 seconds Tom ran 1 second faster than Harold" in response to question 1), and the group struggled to make sense of the units throughout the remainder of the problem. They skipped questions 5, 9, and 10, and their written answers to questions 4, 6, 7, and 8 were incorrect.

In terms of the mediation of problem context, the most striking contrast between the group's discussion of Hexagon Desks in Excerpts 1 and 2 and their discussion of The Tortoise and The Hare in Excerpt 3 was that the context and terminology used in Hexagon Desks was readily accessible to the students while the context and terminology in The Tortoise and the Hare was not. Moreover, the story in Hexagon Desks was more "real life" than the story in The Tortoise and the Hare. In Excerpt 3, the mediation of everyday language was most clearly evidenced by the students' lack of knowledge of the vocabulary used in the problem. Of course, knowing the terms tortoise and hare is not actually required to solve this problem, but that does not mean that the students were not distracted by these terms. In this sense there is a parallel with assessment items considered by Martiniello (2008) where she showed that unfamiliar terminology, even if it is unrelated to the mathematics content, can distract students during problem solving. In a strange twist, successfully solving The Tortoise and the Hare required some extra knowledge—the knowledge that the context was meant to be ignored.

Discussion

Distinctions between Two Ways Problem Contexts Mediated Discussions

This analysis highlights two distinct ways that incorporating real life contexts in school mathematics problems might mediate students' mathematical problem solving. One way that context can mediate students' reasoning is through invoking particular semiotic resources and ways of reasoning. This form of mediation was evident in the group's reasoning on Hexagon Desks, where they discussed the perimeter of chains of hexagons by asking whether "somebody" could sit at particular locations in the diagrams. The second type of context-related mediation addressed in this paper was that using unfamiliar contexts and terminology in the statement of mathematics problems might interfere with students' mathematical reasoning. This type of problem context mediation was evident in the group's discussion of the Tortoise and the Hare, where some students in the group were unfamiliar with both the fable of the tortoise and the hare, as well as the meaning of the words *tortoise* and *hare*. While the group also struggled with the mathematical concepts in this problem, there is evidence that the peculiar story occupied some of their attention.

Together, Hexagon Desks and The Tortoise and the Hare illustrate how the mediation of problem contexts in students' joint problem solving can operate on different levels. The interference of the problem context in the Tortoise and the Hare was readily apparent to both the students and the researcher. Because the story and the vocabulary in the Tortoise and the Hare were unfamiliar to the students, the students exerted some effort to make sense of the story and the characters, even though knowing the story did not necessarily help solve the graph analysis task. The students' efforts to understand the story suggest that they were unfamiliar with the genre of school mathematics word problems, and the fact that the context often can—and at times must—be ignored while solving the math problem (Gerofsky, 1996). This form of mediation illustrates one way that cultural and linguistic bias enters into school mathematics tasks. As previous research has noted, when the task is an assessment, one result is that some students may suffer linguistic discrimination on assessments (Abedi & Lord, 2001; Martiniello, 2008). Conversely, Abedi and Martiniello's research has also shown that with some minor linguistic adjustments, tasks can be made more comprehensible for language minority students.

The students' reasoning on Hexagon Desks shows a different, and more subtle way in which problem context can mediate students' problem solving. In this case, the students used the context to interpret both the numerical pattern in the table and the images of chains of hexagons. This shows an affordance of the context. However, the students' discussion also revealed that the language and metaphors used to reason about students sitting around desks may have introduced some ambiguity in the students' mathematical problem solving (see, e.g., the problems with the term "nobody" addressed in the Findings). The students' focus on whether "somebody" or "nobody" could sit at different spaces around the chain of desks was not inevitable. One might imagine a different situation where students were asked a similar question framed by a story about coloring the outside edges of a chain of one, two, three, and more hexagons. In such a case the mathematical pattern would be similar, but the students would likely draw on a different semiotic resources for problem solving.

Research Implications

This paper illustrates how Wertsch's (1991, 1998) framework of mediated action can be used to rethink the influence of story problem contexts on students' mathematical reasoning. While prior studies have used genre analysis, situated learning, and cognitive frameworks to examine facets of mathematics story problems, the mediated action framework helps illuminate how very subtle changes in mathematical story problems may effect significant changes in students' reasoning. By framing the story of Hexagon Desks in terms of students sitting around a row of desks, the students were "primed" to imagine bodies arranged in physical space, and the traces of this way of thinking were evident in the students' talk. This is closely tied to Wertsch's (and Vygotsky's) fundamental contention that human activity must be analyzed as a system, and the agent taking action to achieve a goal cannot be considered without also considering the meditational means used to achieve those goals.

For mathematics education researchers, this analysis is a reminder that we must be careful to account for the meditational means when we analyze students' mathematical reasoning. One possible follow up to this analysis would be to examine how changing the story in word problems corresponds to changes in the resources students draw upon for their mathematical reasoning. Researchers in the field of educational assessment pilot test several versions of mathematics test items with an eye toward avoiding cultural and linguistic discrimination. This consideration is critical for the development of fair tests (American Educational Research Association, American Psychological Association, & National Council on Measurement in Education, 1999). However, in the field of mathematics education, mathematics problem contexts are often treated as if they were transparent. Analyses like the one presented here, reveal that the context is highly salient for students, even if experts (such as educational researchers) know that problem solvers are supposed to ignore the story and focus on the mathematical relationships. Wertsch's notion of mediated action provides a unique way to understand why some "equivalent" problems are more difficult than others.

One caveat is that this analysis should not lead to the conclusion that experts always ignore the problem context and novices are confused by contextualized problems because they do attempt to reason based on the context. The actual differences between experts and novices may be subtler than this dichotomy. For example, ethnographic studies have shown that physicists in the university setting often draw upon multiple metaphors and imagine themselves in problem spaces as they skillfully solve abstract problems (Ochs, Jacoby, & Gonzales, 1994).

Practical Implications

This case study analysis does not include enough data to support definitive recommendations for teaching. However, this case does provide grist for examining how mathematics problems are used in the service of teaching and learning mathematics. Traditionally in the school curriculum, problems have been written to require the use of particular solution methods, and the problem context was secondary to the targeted mathematical technique (Gerofsky, 1996; Schoenfeld, 1992). With mathematics education reform, curriculum designers sought to use more "real life" problems in mathematics texts (Boaler, 1993; National Council of Teachers of Mathematics, 1989, 2000). This case study, together with Wertsch's (1991, 1998) notion of mediated action, indicates one reason why educators should be cautious about the use of problems in context. While "real life" applications may be motivating for students, they also invite students to use alternative semiotic systems for reasoning through problems, which may result in the students providing unexpected answers. One amusing instance of this occurred when piloting items for this study. When one group of students was asked how many hexagon desks would be required to seat 44 students, the students responded 10. When the students were asked to explain their reasoning, they said that, although there would be 42 spaces at a row of ten desks, the two extra students could squeeze in somewhere on the side. In this case the students provided an answer that was incorrect from a mathematical perspective, but which would be practical in the real-life situation. Teachers may need to be aware of potential conflicts like this.

Conclusion

The brief analysis here shows how adding "real life" context to a mathematics problem can constrain students' mathematical problem solving while also providing some affordances. There is little doubt the fanciful context and new vocabulary in The Tortoise and the Hare interrupted the students' focus on the mathematical problem. The mediation of language and the problem context on Hexagon Desks is subtler. While the problem context appeared to be familiar (or did not warrant comment), the students' use of terms like "somebody" and "nobody" indicated that imagining the context mediated the group's mathematical reasoning. In the students' discussions of Hexagon Desks, Wertsch's notion of mediation can help explain the group's responses to this task. While we cannot know for sure, it is interesting to consider whether these students would have been more successful reasoning through a problem about a chain of hexagons rather than a chain of desks.

This study is limited by the fact that the two problems differ in terms of their mathematical difficulty. In addition to using unfamiliar terms, the Tortoise and Hare required students to interpret a velocity graph. Nonetheless, the distinction between two forms of mediation—the subtle influence of everyday language and the more overt issue of unfamiliar language—can help researchers, curriculum designers, and teachers as we consider what tasks to use for instruction and assessment.

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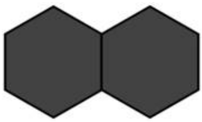
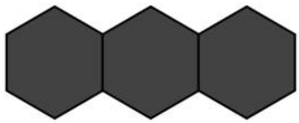
Appendix

Hexagon Desks

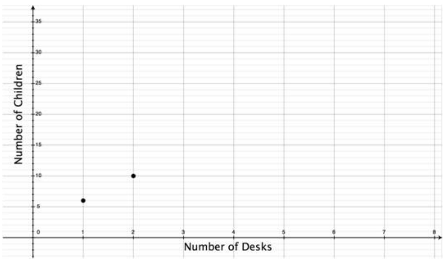
Ms. West wants to know how many students can sit around a row of hexagon shaped desks.

If one desk is by itself then six students can sit around it.	If two desks are pushed together, then 10 students can sit at the table.	If three desks are pushed together in a row as shown below, then 14 students can sit together.
		

1. Fill in the following table for the number of students who can sit together for the number of desks pushed together in a row:

Number of Hexagon Desks	Number of Students

2. Imagine that 100 of the hexagon desks were pushed together in a row. How many students could sit around that row of desks? Show the work you used to find that solution.
3. If *n* hexagon shaped desks, were pushed together, then how many students could sit at the row of desks? Give your answer as a formula in terms of *n*.
4. Use the table you made in problem 1 to draw a graph showing the number of children who could sit at a row of desks

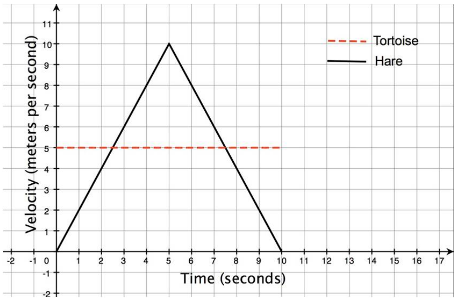


5. If you connect the dots between points in the graph to make a line, what is the slope of that line? How do you know?
6. What is the meaning of the slope of the line *in terms of the problem about children sitting at desks*? Explain your answer in terms of the problem and using words and ideas that you know from math class.
7. What if *n* octagon-shaped desks were pushed together? How would this problem be different? How would it be the same? Explain your answer in as much detail as possible (you may use equations, tables, graphs, words, etc.).

The Tortoise and the Hare

One day Tom the Tortoise and Harold the Hare ran a race. Tom got a running start, but they both ran across the starting line at the same moment when the timer said 0 seconds. They ran along a straight road for 10 seconds and the graph below shows Tom and Harold's velocity (speed) during the 10-second race:

A drawing of Tom and Harold's race



This graph shows the velocity (speed in meters per second) of Tom the tortoise and Harold the hare during the race.

1. Who was running faster at $t = 2$ seconds? How do you know?
2. Who was running faster at $t = 4$ seconds? How do you know?
3. During what time periods was Tom running faster? How do you know?
4. At what time (or times) were Tom and Harold in the same location during the race? How do you know?
5. At what time (or times) were Tom and Harold running at the same speed during the race? How do you know?
6. How far did Tom run?
7. How far did Harold run?
8. Who was ahead after 5 seconds?
9. Did Tom or Harold run backwards at some point during the race? How do you know?
10. Who won the race? How do you know?

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