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A new equation to predict the footings settlement on sand based on the finite element method

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Abstract

In the design of shallow foundations normally used in rural buildings on sand, the settlement criterion is more critical than the bearing capacity of the soil. Likewise, it has also been found that widely used methods to estimate the footings settlement on sand, generates differences with the observed in full scale field tests. The aim of this study was to find a new equation based on finite element method (FEM) easy to apply to estimate the footings settlement on sand. This new equation considers the effect of the depth of the founding level, the footing breadth, the equivalent soil stiffness, the Poisson's ratio and the net increase in the effective stress on settlement values. To obtain this equation, a three-dimensional finite element model was generated and subsequently validated using actual footings settlement measured during field tests. The results of settlement predictions extracted from this method are slightly better than those obtained by other methods, but this equation has the advantage of being easier and faster to apply, which implies a savings in computation time.

Additional key words: granular soils; rural buildings; shallow foundations; vertical displacement.

Resumen

Nueva ecuación para estimar el asiento de zapatas sobre arenas basado en el método de elementos finitos

En el diseño de las cimentaciones superficiales normalmente utilizadas en las construcciones rurales sobre arena, el criterio de asientos es más crítico que el de capacidad de carga del suelo. Igualmente se ha encontrado que los métodos más utilizados para estimar el asiento de zapatas sobre arenas generan diferencias con los observados en ensayos de campo a escala real. El objetivo de este estudio fue encontrar una nueva ecuación, basada en el método de elementos finitos (MEF) fácil de aplicar, para estimar el asiento de zapatas sobre arena. Esta nueva ecuación considera el efecto de la profundidad de cimentación, la anchura de la zapata, la rigidez equivalente del suelo, el coeficiente de Poisson y el incremento neto de la presión efectiva sobre los valores del asiento. Para obtener esta ecuación, se generó un modelo tridimensional de elementos finitos y se validó posteriormente utilizando los asientos reales de zapatas medidos durante ensayos de campo. Los resultados de las predicciones del asiento extraídos de esta formulación son ligeramente mejores que los obtenidos por otros métodos, pero esta formulación tiene la ventaja de ser más fácil y rápida de aplicar, lo que implica un ahorro en el tiempo de cálculo con el ordenador.

Palabras clave adicionales: cimentaciones superficiales; construcciones rurales; desplazamientos verticales; suelos granulares.

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Abbreviations used: **Nomenclature:** *ARE* (average relative errors); B, D (footing breadth and depth of founding level); E_f, E_s (footing and soil stiffness at founding level); E_{sz}, E_{sinc} (soil stiffness at a certain depth and increase of this stiffness per unit of depth); FEM (Finite Element Method); f_q (soil stiffness influence factor on settlement for any net increase in effective stress); f_s (soil stiffness influence factor on settlement for any breath); I_B (influence factor of footing breath on settlement); I_D (influence factor of depth of founding level on settlement); I_q (influence factor of net increase in effective stress on settlement); N_{av} (average SPT blow count). **Greek letters:** α, β (soil stiffness influence on δ_s); δ, δ_s (settlement in depth and in surface); δ_A (settlement according to the equation proposed); δ_B, $\delta_{1.2}$ (settlement for a certain B and for B = 1.2 m); δ_{BB} (settlement according to the method of Burland & Burbidge, 1985); δ_{FEM} (settlement according to FEM); δ_{SF} (settlement according to Steinbrenner´s modified by Fox method); δ_m (measured settlement in the field test); δ_{MP} (settlement according to the method of Mayne & Poulos, 1999); δ_q, δ_{100} (settlement for any net increase in effective stress and for an increase equal to 100 kPa); λ (soil stiffness influence on δ).

Introduction

In the design of shallow foundations normally used in the rural buildings on sand, the settlement criterion is more critical than the bearing capacity of soils. On sandy soils, the elastic settlement is the most relevant result to be considered (Shin & Das, 2011).

Most of the available methods to calculate the settlement value can be classified in two categories (Shin & Das, 2011):

1. Empirical or semi-empirical methods based on observed settlements of structures and full scale prototypes. These values are correlated with the results of *in-situ* test data to quantify, in an indirect way the soil parameters, which can be useful to predict the settlement values. Among these procedures is the Burland & Burbidge's method (Burland & Burbidge, 1985), which provides more reasonable estimation of settlement in shallow foundations on sands, however it is difficult to determine the overconsolidation ratio and the preconsolidation pressure from field exploration (Shin & Das, 2011). Sivakugan & Johnson (2004) established that, although this method is a substantially improved technique to estimate settlement, its results are conservative.

2. Methods based on theoretical relationships derived from the theory of elasticity, which has the advantage of considering three-dimensional deformation of soil and simplify its behaviour, considering it as elastic material. The expressions for settlement predictions contain the term of equivalent soil stiffness, which introduces some uncertainty in it. Among these procedures is the methods proposed by Steinbrenner modified by Fox (Das, 2006) and more recently, by Mayne & Poulos (1999), which appears to give better elastic settlement predictions than the above theoretical method (Das & Sivakugan, 2007).

However, soil is far from an elastic, homogeneous and isotropic material, as these methods consider, which generates differences between observed and predicted settlement. These discrepancies can be attributed to the complexity of the proposed formulations and the current inability to estimate a reliable modulus of elasticity of the soil. These require reviewing and upgrading traditional foundation design procedures and tools, using new experimental and theoretical findings.

In this study a three-dimensional nonlinear finite element analysis (Comodromos *et al.,* 2009; Eid *et al.*, 2009; Li & Zhang, 2009), of an axially, vertical, centred, loaded square footing on sand, has been performed. The interaction soil-footing is taken into account (Breysse *et al.,* 2005) and the soil is considered as an elastic-perfectly plastic anisotropic material (Mabrouki *et al.*, 2010; Loukidis & Salgado, 2011; Oh & Vanapalli, 2011), which implies a better approach to actual soil behaviour.

The aim of this study is to find a new equation fairly simple to use based on finite element method (FEM), which allows estimating the footing settlement on sand.

Material and methods

Development and verification of finite element model

In this study, the commercial finite element program ANSYS v.10 was used. Both the footing and soil were modelled using twenty-noded 3D solid element SOLID95 (Moaveni, 2008).

Fig. 1 shows a finite element mesh of a sandy soil submitted to an axially vertical, centred, loaded, 1.8-m-breadth square footing and the surrounding landfill surcharge. The lateral and bottom boundaries of the mesh were located 6 m horizontally and 10 m vertically from the centre of the footing base, and their movements were restricted in perpendicular directions.

The meshing on the soil took place gradually, from the limits of the soil model where the larger elements

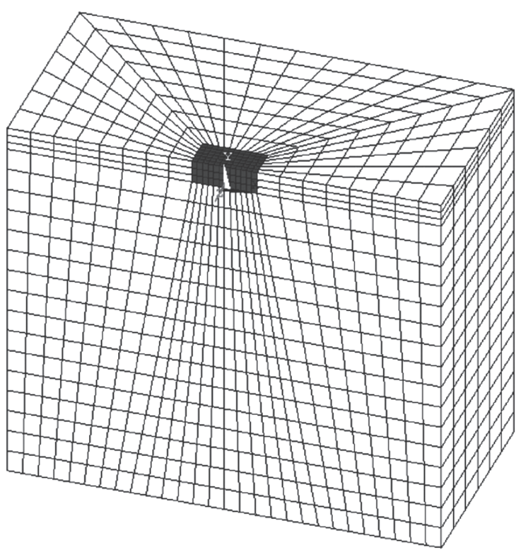

Figure 1. Finite element mesh for footing field test.

were placed with 1 m long, until reaching the minimum size of 0.15 m just around the footing, where the most accurate results are needed.

The symmetric nature of the foundation allowed the generation of one half of the model only, reducing the computational effort (Moaveni, 2008).

The models were built in two steps: one corresponding to the generation of the initial state and to the simulation of digging the excavation and the other corresponding to the formwork installation and the footing loading.

The footings were considered to be rigid and rough, as it most often is in reality and were modelled as elastic with a much greater stiffness than the soil (footing stiffness [E_f] = 3 × 10^7 kPa, unit weight of concrete [γ_f] = 25 kN m^{-3}, Poisson's ratio of concrete [v_f] = 0.2). The soils were modelled with a non-associated flow rule (Loukidis *et al.*, 2008) and an anisotropic behaviour, which led to variable stiffness with depth, following a linear Drucker-Prager yield criterion, which assumes that soil is an elastic–perfectly plastic material.

Although the soils are not linearly elastic and perfectly plastic for the entire range of load applied (Ti *et al.*, 2009), along with the effects of stress and strain level on soil stiffness, in this study they has been considered as an elastic-perfectly plastic material. The advantages of the Drucker-Prager model applied to foundations cases like the one analysed here, in which the nonlinearity of the load-settlement responses are not large owing to the relatively small width of footings used on medium dense to dense sands (Loukidis *et al.*, 2008), are remarkable. This makes it a favourable option as soil model.

The interface between the footing and the surrounding soil was modelled using a rigid-to-flexible, face-to-face, eight-noded element without thickness or stiffness, compound by an TARGE170 element beneath the footing and a CONTA174 element over the soil surface (Potts & Zdravkovic, 1999), which introduces some nonlinearity in model (Moaveni, 2008).

The global stiffness matrix of this model was solved by an iterative process known as the Newton-Raphson iterative method, which makes small successive load increments until the difference between the applied load and the obtained ones by solving the governing equation systems of the model is lower than a reference value, which is sufficiently small to assume that the analysis has converged, giving the solution. This method allows reducing computation time (Potts & Zdravkovic, 1999).

To verify the finite element model adopted, finite element analysis of five cases of actual footings test, published by Burland & Burbidge (1985) over sandy soils, was carried out.

Table 1 shows the data from these five field tests, where B is the footing breadth, D is the depth of the founding level and q_N is the net increase in the effective stress. Soils compactness, according to the criteria proposed by Terzaghi *et al.* (1996), were obtained from penetration test data (N_{av}). The parameters of the soils obtained by Tiznado & Rodriguez-Roa (2011) in their works were used as representative of medium-dense sandy soils, while the parameters proposed by Al-Shayea & Mohib (2011) were chosen as representative of dense sandy soils. Table 1 also gathers the geotechnical parameters used in each type of soil, where γ is the unit weight of the dry soil, E_s is the equivalent soil stiffness at the founding level, c is the cohesion value, v is Poisson's ratio, $Ø$ is the effective internal friction angle and Ψ is the dilatancy angle. To account for the variation in soil properties with depth, the equivalent soil stiffness was assumed to increase linearly according to $E_{sz} = E_s + E_{sinc}\,(z\text{-}z_0)$, where E_{sinc} is the increase of the soil stiffness per unit of depth z and z_0 is the founding level.

Table 1. Analyses of Burland & Burbidge (1985)'s case records and soils geotechnical parameters used in model verification

Field test data					Compactness	Source	γ (kN m^{-3})	E_s (MPa)	E_{sinc} (kPa m^{-1})	c (kPa)	v	$Ø$ (º)	Ψ (º)
Case	B (m)	D (m)	q_N (kPa)	N_{av} (b ft^{-1})									
44/M1	1.2	0.6	150	28	medium-dense	Tiznado & Rodriguez-Roa (2011)	17	40	400	0	0.2	37	7
44/M3	1.2	0.6	150	45	dense	Al-Shayea & Mohib (2011)	18	125	65	0	0.43	38.33	8
44/P1	1.5	0.6	150	35	dense	Al-Shayea & Mohib (2011)	18	125	65	0	0.43	38.33	8
44/P2	1.5	0.6	150	50	dense	Al-Shayea & Mohib (2011)	18	125	65	0	0.43	38.33	8
58/B	1.5	1.2	77	15	medium-dense	Tiznado & Rodriguez-Roa (2011)	17	40	400	0	0.2	37	7

Development of a new method to predict the footings settlement on sand

The finite element model has been applied to nine footings in depth and three footings in surface with different sizes, submitted to four different loads, resting on 18 types of sandy soils, yielding 864 settlement values, which has allowed developing a mathematical model that estimates the settlement values of a square footing submitted to centred loads.

Table 2 shows the geotechnical parameters of the 18 soils used in this study and the reference used to set their values. To analyse the influence of E_s, v, D, q_N and B on settlement (δ), three different footing breadths, 1.2, 1.5 and 1.8 m, and four different footing dephts, 0, 0.45, 0.65 and 0.85 m, were used in the analysis. The footing thickness was equal to the depth of the foundation, except for a depth of 0, where 0.45 m was used.

To perform the model, a non-associated flow rule ($\O \neq \Psi$) and anisotropic behaviour of the soils were considered. Pore pressure was neglected.

The net increases in effective stress tested in these soils were equal to 100, 150, 200 and 250 kPa. The applied stress was always lower than one-third of the ultimate soil pressure. Thus, the load-settlement curve studied was linear, where E_s at founding level, remains as a constant value.

Results and discussion

Development and verification of finite element model

Table 3 shows the measured settlement in footing field tests (Burland & Burbidge, 1985) (δ_m), the predicted ones by FEM (δ_{FEM}), the obtained through the method of Mayne & Poulos (1999) (δ_{MP}) and the Steinbrenner's modified by Fox method (Das, 2006) (δ_{SF}).

From settlement values gathered in Table 3, a statistic analysis of relative error values of predicted settlements by different methods, compared with the measured ones, has been performed [*e.g.* $(\delta_{FEM} - \delta_m) / \delta_m$]. The average and the variance of these relative errors, lead to conclude that the predicted results from the finite element model here proposed (average relative error ARE = 30.5%), reveal a good agreement with the settlement obtained by other analytical methods (AREs of 34.4% and 26.5% for Mayne & Poulos and Steinbrenner & Fox methods, respectively). Therefore this model is as good predictor of measured settlement as

Table 2. Soils geotechnical parameters used to develop the new method

Soil / Source	γ (kN m^{-3})	E_s (MPa)	E_{sinc} (kPa m^{-1})	ν	Ø (º)	Ψ (º)
Medium dense sand (Peng *et al.*, 2010)	16	12	1,000	0.25 0.28 0.30	35	5
Medium dense sand (Peng *et al.*, 2010)	16	25	688	0.25 0.28 0.30	35	5
Medium dense sand (Tiznado & Rodriguez-Roa, 2011)	16	34	400	0.25 0.28 0.30	36	6
Medium dense sand (Tiznado & Rodriguez-Roa, 2011)	17	42	400	0.25 0.28 0.30	38	8
Dense sand (Loukidis & Salgado, 2011)	20	80	190	0.30 0.35 0.37	40	10
Dense sand (Al-Shayea & Mohib, 2011)	18	130	65	0.35 0.40 0.45	40	10

Table 3. Settlement prediction by different methods and statistic analysis of average relative errors (ARE)

Case	Settlements			
	δ_m (mm)	δ_{MP} (mm)	δ_{SF} (mm)	δ_{FEM} (mm)
44/M1	1.3	3.32	3.00	3.27
44/M3	0.6	0.92	0.91	0.89
44/P1	2.1	1.13	1.16	1.10
44/P2	1.0	1.13	1.16	1.10
58/B	2.1	2.01	1.66	1.90
ARE		34.4%	26.5%	30.5%
Variance		0.59	0.47	0.58

the others widely used methods and so the validity of this finite element model is verified.

Development of a new method to predict the footings settlement on sand

Tables 4, 5 and 6 show predicted settlements from foundations analyses on each type of soils, depending on the footing breadth.

Taking into consideration the δ values obtained from foundation analyses, a equation to predict these settlements is proposed:

$$\delta = \delta_s \cdot I_D \cdot I_q \cdot I_B \qquad [1]$$

where δ is the footing settlement in depth, δ_s is the settlement values at the surface, and I_D, I_q and I_B represent the influence of D, q_N and B on δ, respectively.

Settlement values at the surface δ_s

Keeping in mind the δ_s values given in Table 4, which correspond to a surface square footing with B equal to 1.2 m and q_N equal to 100 kPa, a linear regression analysis of these values depending on v, was performed to obtain six different straight lines, one for each type of soil. These lines correspond to the general Eq. [2], where the α and β parameters, that define each line, are described in Fig. 2, for each type of soil analysed. The coefficient of determination (R^2) in all cases was greater than or equal to 0.99.

$$\delta_s = \alpha + \beta \cdot v \qquad [2]$$

where the α and β parameters represent the influence of E_s on δ_s.

Influence factor I_D

From δ values collected in Tables 4, 5 and 6, for each B, E_s, v and q_N, it is possible to appreciate that the settle-

Table 4. Settlement from foundations analyses for B = 1.2 m (mm)

q_N (kPa)	D (m)	E_s = 12 MPa			E_s = 25 MPa			E_s = 34 MPa			E_s = 42 MPa			E_s = 80 MPa			E_s = 130 MPa		
		ν 0.25	ν 0.28	ν 0.30	ν 0.25	ν 0.28	ν 0.30	ν 0.25	ν 0.28	ν 0.30	ν 0.25	ν 0.28	ν 0.30	ν 0.30	ν 0.35	ν 0.37	ν 0.35	ν 0.40	ν 0.45
100	0.00	6.21	5.69	5.39	3.31	3.09	2.87	2.47	2.28	2.17	1.89	1.77	1.70	0.96	0.88	0.85	0.58	0.53	0.47
	0.45	5.71	5.25	4.99	3.07	2.86	2.67	2.28	2.11	2.02	1.76	1.65	1.59	0.90	0.83	0.80	0.55	0.50	0.45
	0.65	5.52	5.09	4.84	2.97	2.78	2.59	2.21	2.05	1.96	1.71	1.61	1.55	0.87	0.81	0.78	0.54	0.49	0.44
	0.85	5.34	4.92	4.69	2.88	2.69	2.51	2.14	1.98	1.90	1.65	1.56	1.51	0.85	0.79	0.76	0.52	0.48	0.43
150	0.00	10.09	9.25	8.73	5.43	5.06	4.68	4.02	3.70	3.50	3.02	2.81	2.68	1.49	1.36	1.31	0.89	0.81	0.72
	0.45	9.31	8.51	8.08	5.03	4.70	4.33	3.72	3.43	3.26	2.80	2.62	2.51	1.40	1.27	1.24	0.85	0.77	0.69
	0.65	9.02	8.23	7.84	4.88	4.57	4.20	3.61	3.32	3.17	2.71	2.55	2.44	1.36	1.24	1.21	0.83	0.76	0.68
	0.85	8.73	7.95	7.60	4.73	4.43	4.07	3.50	3.22	3.08	2.63	2.48	2.38	1.33	1.21	1.18	0.81	0.74	0.66
200	0.00	14.24	13.04	12.29	7.66	7.14	6.63	5.66	5.22	4.91	4.24	3.93	3.74	2.05	1.85	1.78	1.22	1.09	0.98
	0.45	13.10	12.03	11.42	7.07	6.64	6.16	5.22	4.85	4.56	3.94	3.67	3.49	1.93	1.74	1.67	1.16	1.04	0.94
	0.65	12.67	11.66	11.09	6.85	6.44	5.98	5.06	4.71	4.43	3.83	3.56	3.39	1.89	1.69	1.63	1.13	1.02	0.92
	0.85	12.24	11.28	10.76	6.63	6.25	5.81	4.90	4.57	4.30	3.71	3.46	3.30	1.84	1.65	1.59	1.11	1.00	0.90
250	0.00	18.62	16.89	16.10	8.72	8.37	8.02	7.41	6.80	6.48	5.50	5.09	4.84	2.63	2.37	2.27	1.56	1.40	1.26
	0.45	17.12	15.59	14.91	8.10	7.75	7.45	6.86	6.32	6.00	5.09	4.55	4.53	2.46	2.23	2.14	1.48	1.34	1.21
	0.65	16.56	15.10	14.46	7.87	7.52	7.24	6.66	6.14	5.82	4.94	4.62	4.41	2.40	2.17	2.09	1.45	1.31	1.19
	0.85	16.01	14.61	14.01	7.64	7.28	7.02	6.45	5.95	5.64	4.79	4.48	4.29	2.33	2.11	2.04	1.42	1.29	1.17

Table 5. Settlement from foundations analyses for B = 1.5 m (mm)

q_N (kPa)	D (m)	E_s = 12 MPa			E_s = 25 MPa			E_s = 34 MPa			E_s = 42 MPa			E_s = 80 MPa			E_s = 130 MPa		
		ν 0.25	ν 0.28	ν 0.30	ν 0.25	ν 0.28	ν 0.30	ν 0.25	ν 0.28	ν 0.30	ν 0.25	ν 0.28	ν 0.30	ν 0.30	ν 0.35	ν 0.37	ν 0.35	ν 0.40	ν 0.45
100	0.00	7.34	6.71	6.34	4.03	3.75	3.48	3.04	2.79	2.66	2.33	2.18	2.09	1.19	1.08	1.04	0.71	0.65	0.57
	0.45	6.87	6.29	5.95	3.73	3.47	3.22	2.82	2.58	2.47	2.20	2.06	1.98	1.13	1.03	0.99	0.68	0.63	0.55
	0.65	6.68	6.13	5.76	3.62	3.37	3.13	2.73	2.51	2.40	2.15	2.02	1.94	1.10	1.01	0.98	0.67	0.62	0.54
	0.85	6.49	5.96	5.60	3.51	3.26	3.03	2.65	2.43	2.33	2.10	1.97	1.90	1.08	0.98	0.96	0.66	0.61	0.53
150	0.00	11.96	11.00	10.39	6.52	6.07	5.62	4.94	4.55	4.27	3.71	3.47	3.30	1.83	1.64	1.61	1.09	0.99	0.88
	0.45	11.22	10.29	9.77	6.03	5.64	5.22	4.58	4.21	3.97	3.49	3.28	3.13	1.74	1.56	1.54	1.04	0.95	0.85
	0.65	10.93	10.01	9.53	5.85	5.48	5.07	4.44	4.09	3.85	3.41	3.21	3.06	1.71	1.52	1.51	1.02	0.94	0.84
	0.85	10.63	9.73	9.28	5.67	5.32	4.92	4.30	3.96	3.74	3.32	3.13	2.99	1.67	1.49	1.48	1.01	0.92	0.82
200	0.00	16.80	15.39	14.50	9.27	8.64	8.02	6.91	6.37	5.99	5.17	4.79	4.56	2.52	2.28	2.19	1.49	1.33	1.20
	0.45	15.72	14.43	13.68	8.58	8.03	7.46	6.39	5.92	5.55	4.88	4.54	4.32	2.41	2.17	2.08	1.43	1.28	1.15
	0.65	15.29	14.06	13.35	8.32	7.80	7.24	6.20	5.75	5.38	4.76	4.43	4.22	2.36	2.12	2.04	1.40	1.26	1.14
	0.85	14.86	13.67	13.02	8.07	7.57	7.02	6.01	5.58	5.21	4.64	4.33	4.12	2.31	2.07	2.00	1.38	1.24	1.12
250	0.00	21.86	19.99	19.00	10.46	10.04	9.62	9.04	8.30	7.91	6.71	6.28	5.92	3.26	2.91	2.79	1.90	1.72	1.51
	0.45	20.45	18.75	17.87	9.69	9.30	8.94	8.40	7.71	7.32	6.31	5.94	5.62	3.09	2.77	2.66	1.83	1.66	1.46
	0.65	19.89	18.26	17.42	9.40	9.02	8.69	8.16	7.49	7.10	6.15	5.81	5.49	3.03	2.71	2.61	1.80	1.64	1.44
	0.85	19.33	17.76	16.97	9.11	8.74	8.43	7.92	7.26	6.88	5.99	5.67	5.37	2.96	2.65	2.56	1.76	1.61	1.41

Table 6. Settlement from foundations analyses for B = 1.8 m (mm)

q_N (kPa)	D (m)	E_s = 12 MPa			E_s = 25 MPa			E_s = 34 MPa			E_s = 42 MPa			E_s = 80 MPa			E_s = 130 MPa		
		ν 0.25	ν 0.28	ν 0.30	ν 0.25	ν 0.28	ν 0.30	ν 0.25	ν 0.28	ν 0.30	ν 0.25	ν 0.28	ν 0.30	ν 0.30	ν 0.35	ν 0.37	ν 0.35	ν 0.40	ν 0.45
100	0.00	8.22	7.51	7.10	4.63	4.31	4.00	3.54	3.25	3.09	2.74	2.55	2.44	1.40	1.28	1.24	0.84	0.76	0.67
	0.45	7.77	7.12	6.75	4.29	3.99	3.72	3.28	3.02	2.87	2.61	2.43	2.33	1.34	1.23	1.19	0.81	0.74	0.65
	0.65	7.59	6.96	6.60	4.16	3.87	3.61	3.18	2.93	2.79	2.54	2.39	2.29	1.31	1.21	1.17	0.80	0.73	0.64
	0.85	7.42	6.81	6.46	4.03	3.75	3.50	3.08	2.85	2.71	2.49	2.34	2.25	1.29	1.19	1.15	0.79	0.72	0.63
150	0.00	13.49	12.17	11.61	7.55	7.03	6.51	5.75	5.29	4.97	4.35	4.03	3.86	2.18	1.98	1.91	1.28	1.17	1.03
	0.45	12.79	11.51	11.06	6.99	6.54	6.05	5.34	4.92	4.62	4.14	3.85	3.69	2.09	1.89	1.84	1.24	1.13	1.00
	0.65	12.50	11.24	10.83	6.78	6.35	5.87	5.19	4.78	4.49	4.06	3.77	3.63	2.05	1.86	1.81	1.22	1.12	0.99
	0.85	12.23	10.98	10.61	6.57	6.16	5.70	5.03	4.63	4.35	3.98	3.70	3.56	2.02	1.83	1.79	1.20	1.10	0.97
200	0.00	18.94	17.34	16.35	10.65	9.92	9.22	8.09	7.41	6.97	6.11	5.66	5.39	2.97	2.68	2.58	1.77	1.58	1.42
	0.45	17.91	16.44	15.53	9.86	9.19	8.56	7.52	6.86	6.48	5.81	5.40	5.14	2.85	2.57	2.48	1.71	1.53	1.38
	0.65	17.49	16.07	15.20	9.56	8.91	8.32	7.30	6.66	6.29	5.70	5.30	5.04	2.80	2.53	2.44	1.69	1.51	1.36
	0.85	17.09	15.72	14.88	9.27	8.64	8.07	7.09	6.45	6.11	5.58	5.20	4.94	2.75	2.48	2.40	1.67	1.49	1.35
250	0.00	24.51	22.40	21.41	12.12	11.63	11.15	10.5	9.66	9.20	7.92	7.35	6.98	3.81	3.45	3.29	2.26	2.03	1.78
	0.45	23.18	21.23	20.30	11.23	10.74	11.32	9.74	8.94	8.52	7.54	7.02	6.68	3.65	3.31	3.16	2.19	1.97	1.73
	0.65	22.64	20.76	19.84	10.89	10.46	10.01	9.45	8.67	8.26	7.39	6.88	6.56	3.58	3.25	3.11	2.15	1.95	1.71
	0.85	22.12	20.30	19.41	10.55	10.06	9.70	9.16	8.40	8.01	7.24	6.75	6.44	3.51	3.19	3.06	2.12	1.92	1.68

ment decreases hyperbolically with D, which implies that increasing D leads to a greater difference between δ and δ_s. The main reason for this observation is the lateral movement of soil particles under the footing edge while it settles, which is more difficult in depth because of the overloading of lateral landfill surrounding the footing.

Tables 4, 5 and 6 show that increasing E_s leads to a greater uniformity in the recorded δ for the different D tested, which means that there are very few differences between δ and δ_s. The reason for this is that stiffer soils resist better lateral movement of soil particles under the footing edge while it settles. Likewise,

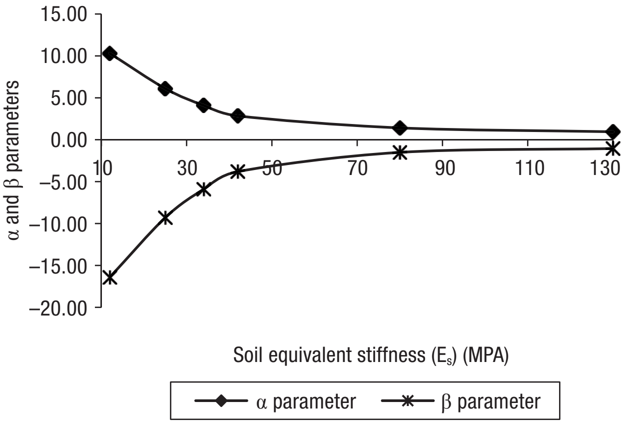

Figure 2. Values of α and β with regard to E_s.

an increase in Poisson's ratio, within the range considered in this study, leads to stiffer soils and lower δ.

To consider the influence of D on δ, the ratios of δ_s and δ corresponding to the different B, E_s, v, D and q_N applied, were calculated. The values of these ratios remain within a narrow range in each type of soil and for each B, q_N and D considered, independent of v used (variance lower than 0.01). As a result, it is possible to set an average ratio independent of v for each one of these cases, which appear in Table 7.

From these average ratios, eighteen linear regression analyses depending on D/B, were performed, which correspond to the different B and E_s analysed, for each one of q_N applied. The coefficient of determination (R^2), in all analysed cases, was greater or equal to 0.99. The mathematical fit of this linear regression analysis corresponds to the following equation:

$$\frac{\delta_s}{\delta} = 1 + \lambda \cdot \left(\frac{D}{B} \right) \qquad [3]$$

where λ is the rate of decrease of δ with D/B, shown in Table 7.

Because the λ values obtained on each E_s, for the different B and q_N applied, are within a narrow range (variance < 0.006), this rate can be considered as a constant value for each type of soil, independent of B and q_N used. Therefore, it is possible to establish an average rate ($\lambda_{average}$) for all possible combinations of B and q_N used on each type of soil. This method relies on the fact that the errors in predicting δ through

Table 7. Average ratio between δ_s and δ and λ values

q_N (kPa)	D (m)	D/B	B = 1.2 m						B = 1.5 m						B = 1.8 m					
			E_s (MPa)						E_s (MPa)						E_s (MPa)					
			12	25	34	42	80	130	12	25	34	42	80	130	12	25	34	42	80	130
100	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.45	0.30	1.08	1.08	1.08	1.07	1.06	1.05	1.07	1.07	1.07	1.06	1.05	1.04	1.06	1.05	1.06	1.05	1.04	1.03
	0.65	0.43	1.12	1.12	1.11	1.10	1.09	1.07	1.10	1.11	1.10	1.08	1.07	1.05	1.08	1.08	1.08	1.07	1.06	1.05
	0.85	0.57	1.16	1.15	1.15	1.13	1.12	1.09	1.13	1.13	1.10	1.11	1.09	1.07	1.10	1.10	1.10	1.09	1.08	1.06
		λ	0.22	0.21	0.21	0.19	0.17	0.13	0.22	0.21	0.20	0.19	0.17	0.13	0.22	0.22	0.20	0.19	0.17	0.13
150	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.45	0.30	1.07	1.07	1.07	1.06	1.06	1.04	1.06	1.07	1.07	1.06	1.06	1.04	1.05	1.05	1.04	1.04	1.04	1.03
	0.65	0.43	1.12	1.11	1.11	1.11	1.10	1.08	1.09	1.10	1.09	1.08	1.06	1.06	1.08	1.08	1.07	1.07	1.06	1.05
	0.85	0.57	1.16	1.15	1.14	1.14	1.11	1.09	1.14	1.14	1.13	1.11	1.10	1.07	1.11	1.11	1.11	1.10	1.08	1.06
		λ	0.22	0.21	0.20	0.19	0.17	0.13	0.22	0.21	0.20	0.19	0.17	0.13	0.22	0.21	0.21	0.19	0.17	0.13
200	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.45	0.30	1.08	1.08	1.08	1.07	1.07	1.05	1.07	1.07	1.06	1.05	1.05	1.04	1.06	1.06	1.05	1.05	1.04	1.03
	0.65	0.43	1.12	1.11	1.10	1.09	1.09	1.07	1.09	1.08	1.08	1.08	1.08	1.05	1.08	1.07	1.08	1.07	1.06	1.05
	0.85	0.57	1.15	1.15	1.15	1.15	1.11	1.09	1.12	1.12	1.12	1.12	1.09	1.07	1.10	1.10	1.09	1.09	1.08	1.06
		λ	0.22	0.21	0.21	0.19	0.17	0.13	0.22	0.21	0.21	0.19	0.17	0.13	0.22	0.20	0.20	0.19	0.17	0.13
250	0.00	0.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00
	0.45	0.30	1.08	1.08	1.08	1.07	1.05	1.04	1.07	1.07	1.06	1.06	1.04	1.04	1.06	1.05	1.06	1.05	1.04	1.03
	0.65	0.43	1.12	1.11	1.11	1.10	1.10	1.07	1.09	1.08	1.08	1.08	1.07	1.05	1.08	1.08	1.07	1.07	1.06	1.05
	0.85	0.57	1.17	1.15	1.14	1.14	1.12	1.10	1.13	1.12	1.12	1.11	1.11	1.07	1.10	1.09	1.09	1.09	1.08	1.06
		λ	0.22	0.21	0.20	0.19	0.17	0.13	0.22	0.21	0.21	0.19	0.17	0.13	0.22	0.21	0.20	0.19	0.17	0.13

Eq. [3] considering $\lambda_{average}$, compared with those obtained by model analyses, are lower than 0.7%. Fig. 3 describes the $\lambda_{average}$ values for each type of soil, which represent the influence of E_s on δ.

Finally, the I_D factor corresponds to the following equation:

$$I_D = \frac{1}{1+\lambda_{average}\cdot\left(\frac{D}{B}\right)} \qquad [4]$$

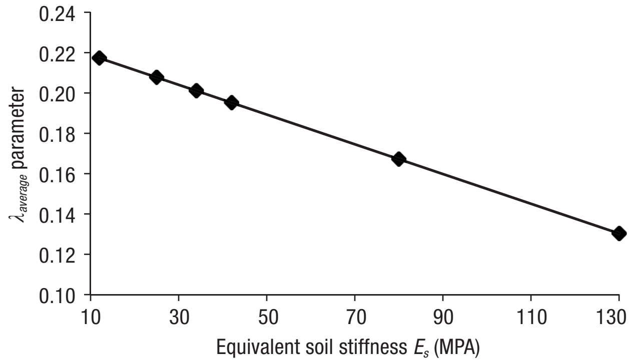

Figure 3. Values of $\lambda_{average}$ with regard to E_s.

Influence factor I_q

As can be seen in Tables 4, 5 and 6, when a certain footing size is considered, the increase in q_N leads to greater δ. From these settlement values, it is possible to compare the ratio of the pair of δ for any q_N applied (δ_q) with those obtained for q_N equal to 100 kPa (δ_{100}).

The values of these ratios, for surface footings, reveal a good agreement among them for any v used in each type of soil, B and q_N considered. Moreover, because the λ parameter hardly changes for any D studied in each soil, these ratios are close to the ones between δ_s, regardless of the D tested (variance < 0.01). In this sense, considering a q_N, the obtained ratios for each type of soil, are also very similar to each other regardless of the size of footing studied (variance < 0.01). As a result, it is possible to consider an average constant value of these ratios in each type of soil and for each q_N used, independent of v, D and B used. This decision also relies on the fact that errors in predicting δ_q, considering these average ratios, are lower than 1.8% with regard to the obtained by finite element analyses. The values of these average ratios for each type of soil, depending on $q_N - P_a$, where P_a is the atmospheric pressure (100 kPa), are shown in Table 8.

The mathematical fit of these average ratios obtained by a linear regression analysis depending on $q_N - P_a$ leads to the generation of six different straight lines, one for each type of soil, corresponding to Eq. [5]. The coefficient of determination (R^2) in all cases was greater than or equal to 0.99.

$$\left(\frac{\delta_q}{\delta_{100}}\right)_{average} = 1 + f_q\cdot\left(q_N - P_a\right) \qquad [5]$$

where f_q defines the slope of the straight lines for each soil analysed and represents the influence of E_s on δ for any q_N used, in units of kilopascal. Fig. 4 outlines the f_q parameter curve versus E_s.

Thus, the I_q factor behaves following the Eq. [6]:

$$I_q = 1 + f_q\cdot(q_N - P_a) \qquad [6]$$

Influence factor I_B

Tables 4, 5 and 6 describe that when a constant value of q_N is considered, an increase in footing size leads to increased settlements on each type of soil.

The ratios of the pairs of δ for a certain B (δ_B) compared with those obtained when B is equal to 1.2 m ($\delta_{1.2}$) were calculated for each v, D and q_N used in each E_s studied. These ratios are very similar each other (variance < 0.01), which allows to set an average value for this ratio in each type of soil and for each footing size, because the errors in predicting δ_B through this average ratio, with regard to the obtained by finite ele-

Table 8. Average ratios between δ_q and δ_{100} in each type of soil

(q_N-P_a) (kPa)	E_s = 12 (MPa)	E_s = 25 (MPa)	E_s = 34 (MPa)	E_s = 42 (MPa)	E_s = 80 (MPa)	E_s = 130 (MPa)
0	1.00	1.00	1.00	1.00	1.00	1.00
50	1.63	1.63	1.62	1.59	1.54	1.53
100	2.30	2.31	2.28	2.21	2.11	2.09
150	2.99	2.98	2.98	2.87	2.70	2.67

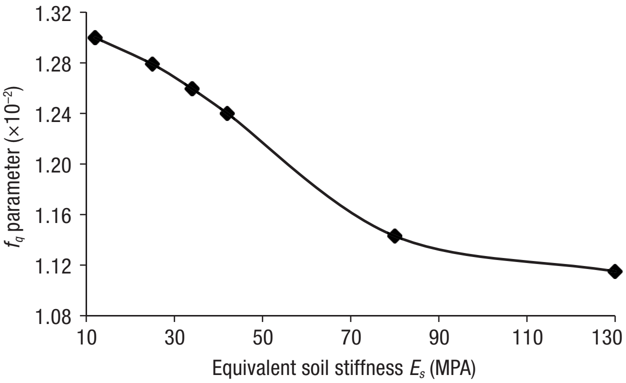

Figure 4. Values of f_q with regard to E_s.

ment analyses, are lower than 1.8%. Table 9 shows these average ratios.

The mathematical fit of these ratios depending on B by an exponential function, corresponds to the equation of Terzaghi *et al.* (1996):

$$\left(\frac{\delta_B}{\delta_{1.2}}\right)_{average} = \left(\frac{2B}{B+1.2}\right)^{f_S} \qquad [7]$$

where the exponent f_s represents the influence of E_s on δ for any B analysed and can be calculated by the Eq. [8], for B greater than 1.2 m:

$$f_s = \frac{\log\left(\left(\frac{\delta_B}{\delta_{1.2}}\right)_{average}\right)}{\log\left(\frac{2B}{B+1.2}\right)} \qquad [8]$$

Table 9 gathers the f_s values obtained from Eq. [8] which are very close for each E_s analysed (variance < 0.007). This means that δ_B calculated through this parameter, hardly change. Thus, it is possible to consider an average value of f_s for each type of soil, because the errors in predicting δ_B using this average parameter, compared with the obtained from finite element analyses, are lower than 2%. Fig. 5 shows the average f_s values depending on E_s.

Therefore, the I_B factor corresponds to the Eq. [9]:

$$I_B = \left(\frac{2B}{B+1.2}\right)^{f_S} \qquad [9]$$

Model validation

After analysing all of the factors involved in calculating the footing settlement on different sandy soils, it is possible to replace the Eqs. [2], [4], [6] and [9] in the main Eq. [1] to obtain the following:

$$\delta = \frac{(\alpha+\beta\cdot\nu)\cdot\left(1+f_q\cdot(q_N-P_a)\right)}{1+\lambda\cdot\left(\frac{D}{B}\right)}\cdot\left(\frac{2B}{B+1.2}\right)^{f_S} \qquad [10]$$

To verify the applicability of this new equation, numerical solutions of the five foundation field tests cases (Burland & Burbidge, 1985) used to verify the model generated in this study, were calculated (δ_A) and subsequently compared with the measured settlement in field tests and with the obtained ones through the methods proposed by Burland & Burbidge (1985) (δ_{BB}), Mayne & Poulos (1999) and Steinbrenner modified by Fox (Das, 2006).

Table 10 shows the settlement obtained through this new Eq. [10], the measured ones in footings field tests and the calculated ones through the analytical methods. Likewise, the Table 10 gathers the average and the variance of relative error values of predicted settlements by different methods, compared with the measured ones [*e.g.* $(\delta_A - \delta_m) / \delta_m$]. This statistic analysis shows that the analytical method of Burland & Burbidge (1985) overestimates the results. On the other hand, the new Eq. [10] predicts the settlement extracted from field

Table 9. Values of f_s parameter for each type of soil and footing breadth

B (m)		**E_s 12 MPa**	**E_s 25 MPa**	**E_s 34 MPa**	**E_s 42 MPa**	**E_s 80 MPa**	**E_s 130 MPa**
1.2	$(\delta_B/\delta_{1.2})_{average}$	1.00	1.00	1.00	1.00	1.00	1.00
1.5	$(\delta_B/\delta_{1.2})_{average}$	1.18	1.21	1.22	1.23	1.23	1.22
	f_s	1.58	1.78	1.92	1.94	1.95	1.88
1.8	$(\delta_B/\delta_{1.2})_{average}$	1.33	1.39	1.43	1.44	1.45	1.44
	f_s	1.54	1.81	1.95	2.01	2.05	1.99

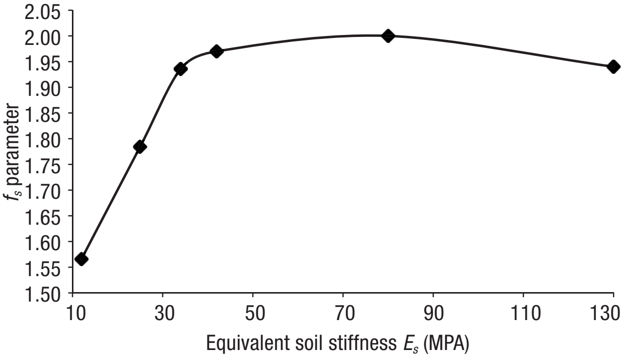

Figure 5. Values of f_s with regard to E_s.

Table 10. Predicted settlement using the new equation [10] versus other methods and statistic analysis of average relative errors (ARE)

Case	δ_m (mm)	δ_{MP} (mm)	δ_{SF} (mm)	δ_{BB} (mm)	δ_A (mm)
44/M1	1.3	3.32	3.00	2.60	3.30
44/M3	0.6	0.92	0.91	1.30	0.91
44/P1	2.1	1.13	1.16	2.20	1.14
44/P2	1.0	1.13	1.16	1.30	1.14
58/B	2.1	2.01	1.66	3.90	1.72
ARE		34.2%	26.5%	67.4%	31.1%
Variance		0.59	0.47	0.23	0.58

tests cases, with an accuracy (ARE = 31.1%) similar to the methods proposed by Mayne & Poulos (1999) and Steinbrenner modified by Fox (Das, 2006) (AREs of 34.2% and 26.5% respectively), which leads to confirm the validity of the new equation to predict the settlement at the centre of the base of an axially vertical, centred, loaded square footing (B = 1.2 to 1.8 m), resting on dense to medium-dense drained sand.

However, this new Eq. [10] has the advantage of the speed and ease in its application; since unlike the other analytical methods used, it is possible to obtain, in a direct way, the values of the influence parameters, function of the equivalent soil stiffness (E_s) that define the new equation, which implies a savings in computation time.

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