RESEARCH ARTICLE

AESTIMATIO, THE IEB INTERNATIONAL JOURNAL OF FINANCE, 2013. **6**: 76-95
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Are Economic Fundamentals unable to explain current European Benchmark Yields? Empirical Evidence from a Continuous Time Affine Term Structure Model

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RECEIVED: 12 JULY 2012
ACCEPTED: 9 JANUARY 2013

Abstract

The results in this paper show that current European benchmark yields can be explained, with a high degree of accuracy, by using an affine term structure (ATS) model with the following four state variables: (i) the EU unemployment rate, (ii) the EU production price index, (iii) the ECB monetary aggregate M3 index and, (iv) the EU consumer confidence index. In fact, the present calculations accounts for the EONIA rate from Dec 1999 to Jan 2011 remarkably well. Furthermore, German government bonds with maturities ranging from 3 month to 30 years are observed to be reproduced fairly well, too. Additionally, the predictive capability of the ATS model is also analysed. It is found that the parameters connecting bond-yields with state-variables do not change with time so rapidly. As a consequence, the values that bond yields may have in the future can be calculated with an accuracy that solely depends on the precision one may achieve in predicting the state variables. Finally, the results presented in this paper show that current yield curve levels are satisfactorily explained by economic fundamentals, and cast a doubt on press headlines pointing the current yields to speculative effects of market participants acting irrationally.

Keywords:

European Benchmark Yields, Affine Term Structure Model, Financial simulation.

JEL classification:

B22, C58, E27, G17.

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¿Son los Fundamentos Económicos incapaces de explicar los Tipos de Referencia Europeos actuales? **Evidencia Empírica basada en un Modelo Afín, continuo en el Tiempo, de la Estructura de los Tipos de Interés**

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Resumen

Los resultados de este trabajo muestran que los movimientos actuales de la curva de tipos Europea de referencia pueden ser muy bien explicados a partir de un modelo afín de estructura de tipos de interés, continuo en el tiempo, y usando como variable de estados: (i) la tasa de desempleo en la Union Europea, (ii) el índice de precios de la producción de la Union Europea, (iii) el índice del agregado monetario M3 del Banco Central Europeo y (iv) el índice de confianza del consumidor de la Unión Europea. De hecho, estos cálculos reproducen con gran exactitud el índice medio del tipo del euro a un día (EONIA) a lo largo de un período de tiempo que va desde diciembre de 1999 hasta enero de 2011. Además, a lo largo del mismo período, se reproduce con bastante exactitud el comportamiento varios bonos del gobierno alemán con tiempos de madurez que van desde 3 meses hasta 30 años. Así mismo, se estudia la capacidad predictiva de este modelo. Los resultados de este análisis muestran que los parámetros que unen el rendimiento de los bonos alemanes con las variables de estado no cambian muy rápido con el tiempo. En consecuencia, se puede afirmar que la exactitud con la que puede predecir los bonos, depende solamente de la exactitud con la que se pueda predecir las variables de estado. Finalmente, los resultados presentados en este trabajo muestran que los niveles actuales de la curva de rendimiento se explican satisfactoriamente por los fundamentos económicos, y arrojan una duda sobre titulares de prensa que señalan los rendimientos actuales a efectos especulativos de los participantes del mercado que actúan irracionalmente.

Palabras clave:

Tipos de referencia Europeos, modelo afín de estructura de tipos de interés, simulación financiera.

1. Introduction

Fear, as any other human emotion, is part of the market economy. However, this does not mean that in even during such apparently irrational times, market behaviour cannot be accounted for by econometric models. In fact, according to the results in this paper, current, admittedly low yields seem to be explained by Euro-zone macroeconomic variables fairly well. It turns out, therefore, that observed yields are still being described by a state space vector of macroeconomic variables. In the present study, we apply the continuous time affine term structure (CT-ATS) model to analyse Euro-zone yield benchmark data. A novel approach, indeed, since most empirical works so far have limited themselves to US data, test Fed policy and Taylor's (1993) rules, as in Christiano *et al.* (1999), Cogley and Sargent (2001, 2002) Sims (1999) and Sims and Zha (2002), Piazzesi (2001), Cochrane and Piazzesi (2005), and Evans and Marshall (1998, 2001).

Similarly, other authors have concentrated themselves in testing expectations hypothesis as in Mankiw and Miron (1986), or used yield curve models to identify central bank latent targets as in Piazzesi (2001, 2002). Likewise Barr and Campbell (1997) and Campbell and Viceira (2001) also apply the ATS models to determine the correlation of real, short term yields with inflation and risk premium. Similar papers on inflation and risk premium with index linked bonds are seen in Buraschi and Jilsov (2005), and Campbell and Shiller (1991) with U.K. data and using two-factor models.

A different perspective is offered by Ang and Piazzesi (2003), who studied the role of macro variables upon yields by looking at the out-of-sample forecasts. The works by Aït-Sahalia (2002) and Aït-Sahalia and Kimmel (2002) present multifactor models which use closed form likelihood expansions, though the emphasis is given more to the methodology itself rather than to the state variables. Ang et al. (2006) estimate a three-factor model based on a short rate, term spread and GDP growth, but, again, mostly limited to US data and the short rate is not treated endogenously as we do in the present paper. This is an important difference though, since by entering the short rate as an explanatory variable one certainly improves fitting, but it reduces the predictive power of the model. Backus, Foresi, and Telmer (1998 and 1996) present a discrete approach of bond pricing mostly on US data and applied it to analyse the forward premium anomaly in foreign exchange prices.

In general, little has been done with Euro-zone data using the affine term structure models. Recently, Jakas (2011 and 2012) resorted to ordinary last-square (OLS) regressions and an affine discrete approach for testing Vasicek (1977) and Cox-Ingersoll-Ross (1985) stochastic processes under a multifactor setup. Moreover, most publications analyse Euro-zone data from the inflation and inflation-risk premium perspective, or the data is limited to a particular European country. Looking into the works from Hördahl and Oreste (2010) they mainly confine their work to a joint model of macroeconomic and term structure dynamics to estimate inflation-risk premia. While it is true that they look not only to US but also to Euro-zone data, their objective, however, is not to model the European benchmark term structure but the inflation-risk premia. Other celebrated ECB working papers such as in Hördahl, Tristani and Vestin (2007) show that micro-founded dynamic stochastic general equilibrium models with nominal rigidities can be successful in describing the most relevant features of bond yields, however, the work is based all on US data from the Federal Bank of Saint Louis. E.g. to be more precise they use PCECC96 for consumption and PCECTPI for prices. Amisano and Tristani (2007) focus on inflation not on term structure. Hördahl, Tristani and Vestin (2004) limit their work to German yields ignoring calibrating these with European aggregated macroeconomic data and, therefore, without analysing it from a real Euro-zone perspective.

It is the purpose of this paper to use the ATS model to link the fields of macroeconomics and mathematical finance from a practitioner's point of view. We understand that the yield curve is an integral part of a network of macroeconomic variables. Moreover, most of the empirical work does not provide much discussion about the state variables used in those models. In fact, most of the empirical works show a rather poor performance from a practical standpoint. This is so, mainly because even if the models are robust, theoretically speaking, they performed poorly on the empirical arena, and not all state variables fit the models so well. In contrast, in this work we have identified state variables which show that ATS models can perform better and, as a consequence, this paper opens the opportunity for better forecasts.

In this paper we adhere to the assumption that risk-free government bonds satisfy the Duffie-Kan (1996) class of affine models and we also stick strictly to the methodology published in Cochrane (2006) and Piazzesi (2010). In this point, however, we would like to make it clear that our purpose is not to develop a new model for calculating bonds dynamics, but to simply calibrate the benchmark curve to observed macroeconomic data and show that low yields are explained by those data when applying this model.

We define the European Yield curve benchmark as the European Overnight Index Average (EONIA) for the short rate, and the rest of the curve comprises the Euribor 3 and 6 months, and the 2, 5, 10, 15, 20 and 30 year German government yields. Furthermore, we use the following state vectors: (i) unemployment rate, (ii) consumer confidence, (iii) money aggregate ECB M3, and (iv) the production price index. We show that these state variables reproduce the above mentioned benchmark yield curves remarkably well.

2. The Model

In this section we will briefly outline the basics equations used in the present paper. As was already mentioned, we will closely follow the approach in Piazzesi (2010) and Cochrane (2005). Therefore, we assume that the bond price is given by the approximation,

$$P(N,t) = exp\left[A(N) - B^T(N) \cdot x\right] , \tag{1}$$

where t denotes time, N the bond maturity and x is vector of the *state variables*. Similarly, A and B are functions of maturity obeying the boundary conditions $A(0)=0$ and $B^T(0)=[0...0]$.

As is customary, the bond yield i.e. $y(N,t)$, is obtained from Eq. (1) using the equality,

$$y(N,t) = -\frac{ln\ P(N,t)}{N} = \frac{\sum_j B_j(N) x_j - A(N)}{N} , \tag{2}$$

where ln denotes natural logarithm. Similarly, the risk-free interest or so-called short rate — which in our case is the EONIA — , is assumed to be a linear function of the state variables,

$$r = \delta_0 + \sum_j \delta_j x_j . \tag{3}$$

where δ_0 and δ_j are parameters in the model which, as is explained in the Appendix, are obtained from a linear regression of state variables to the observed risk-free interest data.

As proposed by Piazzesi (2010) and Cochrane (2005), functions A and B can be obtained using the so-called *expectation approach*. Accordingly, the expected change of the bond price with an infinitesimal change of time dt is given by the equation,

$$E_t\left(\frac{dP|_N}{P}\right) - \left(\frac{1}{P}\frac{\partial P}{\partial N} + r\right) dt = -E_t\left(\frac{dP|_N}{P}\frac{d\Lambda}{\Lambda}\right), \tag{4}$$

where Λ is the discount factor, $dP|_N$ denotes differentiation at constant maturity and $E_t(x)$ represents the expectation operator (see Cochrane, 2005). Furthermore, the equations governing the time evolution of the state variables and that of the discount factor are given by the expressions,

$$dx_i = \sum_j \phi_{i,j} \left(\bar{x}_j - x_j\right) dt + \sum_j S_{i,j} \left|\alpha_j + \beta_j^T x\right|^{1/2} dz_j , \tag{5}$$

and,

$$\frac{d\Lambda}{\Lambda}=-r\,dt-\sum_j b_{\Lambda,j}\left|\alpha_j+\beta_j^T x\right|^{1/2} dz_j \ , \tag{6}$$

where $\overline{x}_j$ is the average value of the state variable x_j over the pertinent period of time and, dz_j is a (0,1)-normal distributed random variable. Similarly, $\beta_j^T x = \sum_k \beta_{j,k} x_k$ and $\phi_{i,j}$, $S_{i,j}$, α_j and $\beta_{j,k}$ are additional parameters entering the model.

By introducing expression (1) into Eq.(4) and using Eqs.(5,6) as well as well-known properties of the random variables (see the Appendix), one may arrive to equations,

$$\frac{dA}{dN}=-\delta_0-\sum_{k,j} B_k \phi_{k,j}\overline{x}_j+\sum_j \alpha_j\left(b_{\Lambda,j}+\frac{1}{2}\sum_k B_k S_{k,j}\right)\sum_k B_k S_{k,j} \ , \tag{7a}$$

and,

$$\frac{dB_i}{dN}=\delta_i-\sum_j\left[B_j\phi_{j,i}+\beta_{j,i}\left(b_{\Lambda,j}+\frac{1}{2}\sum_k B_k S_{k,j}\right)\sum_m B_m S_{k,j}\right]. \tag{7b}$$

This system of ordinary differential equations can be solved by numerical means and so, using Eq.(2), the bond yields are readily obtained.

Although a much more detailed description of the procedure used to calculate the bond yields are produced in the Appendix, for the time being, suffices it to say that, in the first place, the values of δ_0, δ_i , $\phi_{i,j}$, $S_{i,j}$, α_j and $\beta_{j,k}$ are obtained by least-square-error fittings of predictions in Eqs.(3,5) to available data. Secondly, the values of $b_{\Lambda,j}$ are calculated by resorting, again, to last-square-fitting of Eq.(2) to observed yields.

It must be noticed that, contrary to the assumptions in the previous paper by Vasicek (1977) and Cox-Ingersoll-Ross (1985), where either α, or β are assumed to be zero, in the present paper α and β are treated as free, fitting parameters and therefore both can be different from zero. In addition to that, we have opted for assuming that $S_{i,j}=\delta_{i,j}$, where $\delta_{i,j}$ is the Kroenecker's delta function.

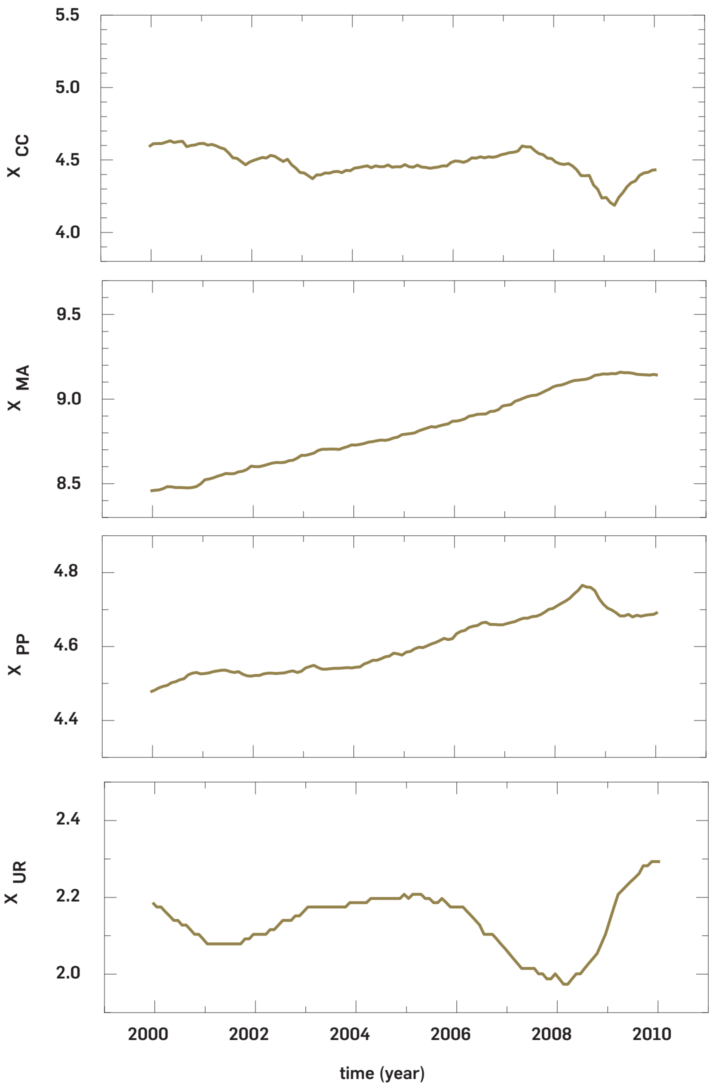
The results of numerically calculating the various expressions derived above are presented and discussed in the following section.

3. Results and Discussion

3.1. Yield Calculations

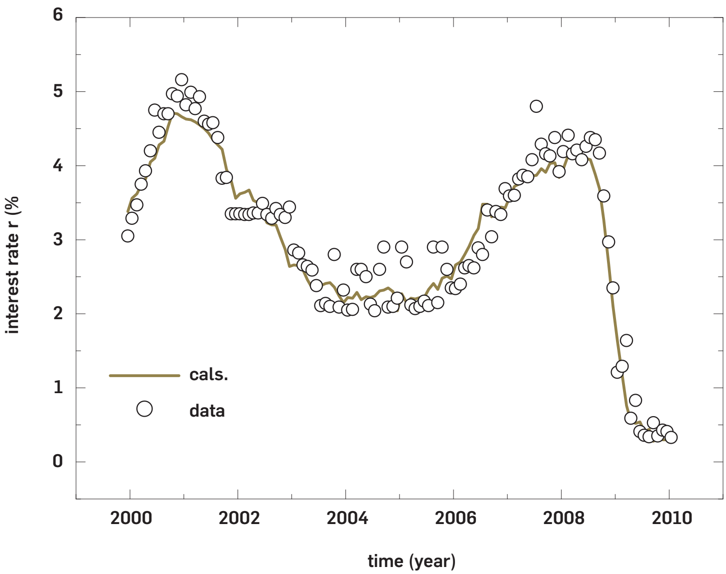
As was already mentioned, in this paper we analyse the behaviour of the EONIA, 3-and 6-month Euribor rate as well as those of the 2, 5, 10, 15, 20 and 30-year German government bonds. Likewise, as state variables we used the natural logarithm of the Euro-zone unemployment rate, the Euro zone production price index, the ECB monetary aggregate M3 index, and the EU consumer confidence index, which will be denoted as x_{UR} , x_{PP} , x_{MA} and x_{CC} , respectively. Notice that the EU consumer confidence index exhibits negative values which complicate the use of natural logarithms. In order to overcome this we have simply added 100 to the observed values before taking logarithm. The data, which are published on a monthly basis, span a period of time ranging from December 1999 to January 2010. The aspects exhibited by the state variables are plotted in Figure 1.

■ Figure 1. **State Variables Used in This Paper, Namely, the Natural Logarithms of the Unemployment Rate (x_{UR}), Production Price Index (x_{PP}), Monetary Aggregate (x_{MA}), and Consumer Confidence (x_{CC})**



In the first place, we calculate the parameters connecting the EONIA rate r and the aforementioned state variables as indicated in Eq.(3). After minimizing expression (A1) we find that the δ_i-coefficient so obtained, reproduce the observed free-risk data with an accuracy of the order of 4% relative error, per point.

Figure 2. Risk-free Interest or Short Rate r (EONIA) As a Function of Time (Starting on Dec. 1999)



Open circles denote data, whereas the continuous line stands for the approximation in Eq.(3) using four state variables (see text).

Such an agreement becomes evident in Figure 2, where expression (3) appears to reproduce the main features of the EONIA rate remarkably well all over the entire range of time. Only a handful of points however deviate from theoretical curve, indicating that the majority of shocks in the EONIA are fairly well described by the chosen state variables, which confirms the results published in Jakas (2011 and 2012). The values of the coefficients in Eq.(3) that were used to plot the theoretical results in Figure 2 are listed in Table 1. Notice that the linear coefficients are multiplied by the mean-value of the corresponding state variable in order to compensate for the differences arising from the different absolute values of the state variables.

Table 1. Fitting Coefficients in Eq.(3)

δ_0	$\delta_1 \bar{x}_{UR}$	$\delta_2 \bar{x}_{PP}$	$\delta_3 \bar{x}_{MA}$	$\delta_4 \bar{x}_{CC}$
0.13	-0.20	0.63	-0.61	0.068

Linear coefficients, i.e. δ_i $i=1,\ldots,4$, multiplied by the mean-values of the state variables, i.e. $\bar{x}_{UR}$, $\bar{x}_{PP}$, $\bar{x}_{MA}$ and $\bar{x}_{CC}$.

According to the results in Table 1, it turns out that the risk-free rate is more sensitive to Euro zone production price and ECB monetary aggregate M3 indices rather than to unemployment rate, and shows nearly no sensitiveness to consumer confidence. Similarly, r appears to be negatively correlated with the monetary aggregate M3 whereas, r and the production price index is positively correlated. While r and unemployment show a slightly negative correlation.

Table 2. Value of the Parameters Entering the Present Calculations.

	$j \diagdown i$	1	2	3	4
$\bar{x}_i$	-	2.13	4.6	8.81	4.48
$\phi_{i,j}\,\bar{x}_i$	1	1.9	2.0	0.95	2.2
	2	0.94	4.7	0.67	0.53
	3	0.23	0.35	1.2	0.66
	4	1.1	0.54	1.3	3.3
$\sqrt{\lvert\alpha_i+\beta_i^T\bar{x}\rvert}$	-	0.035	0.029	0.026	0.058
$\beta_{j,i}\,\bar{x}_i \times 10^4$	1	0.95	1.4	6.1	1.8
	2	2.4	0.95	1.9	0.42
	3	4.6	0.85	0.81	0.16
	4	12.0	4.7	7.7	6.1
$\alpha_i \times 10^4$	-	1.9	2.6	0.46	3.4
$b_{\Lambda,i}\sqrt{\lvert\alpha_i+\beta_i^T\bar{x}\rvert}$	-	-0.022	-1.9	1.9	1.3

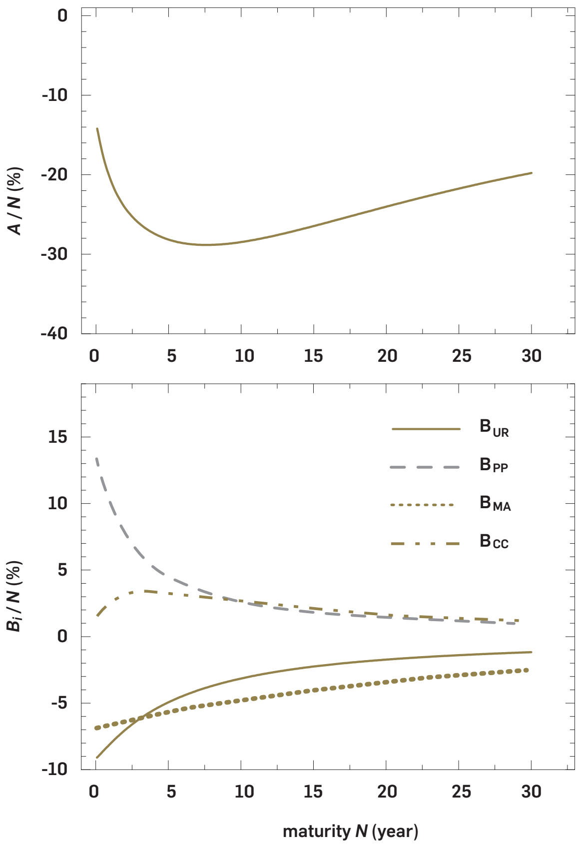
Excepting $\bar{x}_i$, which is directly obtained from data, all other figures are the result of the minimizing procedure described in text. Indices relate to the state variables as follows: 1: EU unemployment, 2: EU production price index, 3: ECB M3 and 4: EU consumer confidence.

In Table 2 we list the values of the most relevant parameters obtained after performing minimization of expressions (A1-A4) and calculating the coefficients A and B according to the procedure described in the Appendix. As we have already done in Table 1, the differences arising from the different absolute values of the state variables are somehow removed by multiplying $\phi_{i,j}$ by $\bar{x}_j$, $\beta_{i,j}$ by $\bar{x}_j$, and $b_{\Lambda,j}$ by $\sqrt{\lvert\alpha_i+\beta_i^T\bar{x}\rvert}$.

One can observe that the off-diagonal elements in matrix "$\phi_{i,j}\bar{x}_j$" are not small compared with those in the diagonal. This indicates that state variables are certainly interrelated. By observing that the values exhibited by $\beta_{i,j}\bar{x}_j$ and α_i are all comparable, we can unambiguously conclude that the case analysed in this paper does not seem to fit on any of the cases described in Vasicek (1977) and Cox-Ingersoll-Ross (1985). Similarly, the results for $b_{\Lambda,j}\sqrt{\lvert\alpha_i+\beta_i^T x\rvert}$ clearly show that the random shocks in the unemployment rate does have little or nearly no significant impact on shocking the bond yields.

It must be mentioned however that the results in Table 2 are presented for the sole purpose of recording the results of the present calculations. However, one should be cautious about interpreting them since, according to our own experience some of these results may change, depending on the initial guess used in the minimization procedure.

Figure 3. Coefficients A/N and B_i/N in Eq.(2), Obtained by Fitting to Euribor and German Government Bonds (See Text).

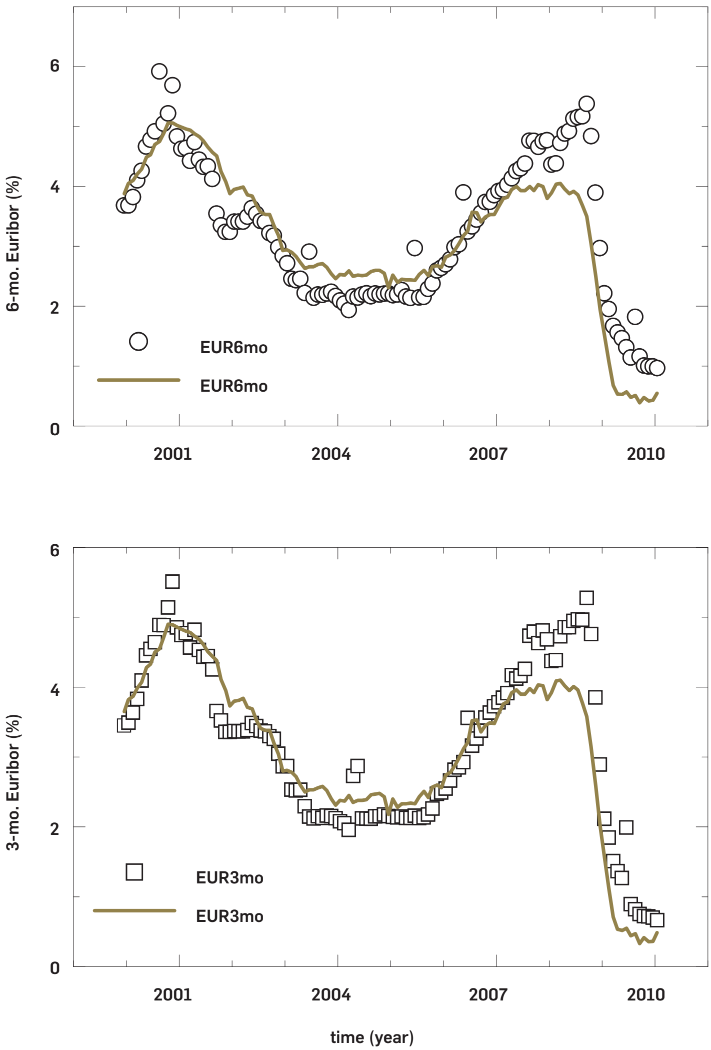


It must be noticed that, in the legend of the lower figure, indices of coefficients B_i were replaced by abbreviation used for the corresponding state variables.

Once all the previous parameters are calculated one could obtain the bonds yields using the expression in Eq.(2). We start by plotting the results of calculating the three- and six-month Euribor rate. The results, which are plotted in Figure 4, show a remarkable agreement with data all over the whole range of time used in this paper. Our numerical calculations deviate from data with a mean relative error of the order of fifteen percent. It must be mentioned though, that this is not at all an unexpected result since, in the light of the good agreement already found for the EONIA rate in

Figure 2, these two, short-maturity bonds should be also well-described by the present model. This is so, because at small N-values, A/N and B_i/N are strongly dominated by the EONIA terms, namely δ_i.

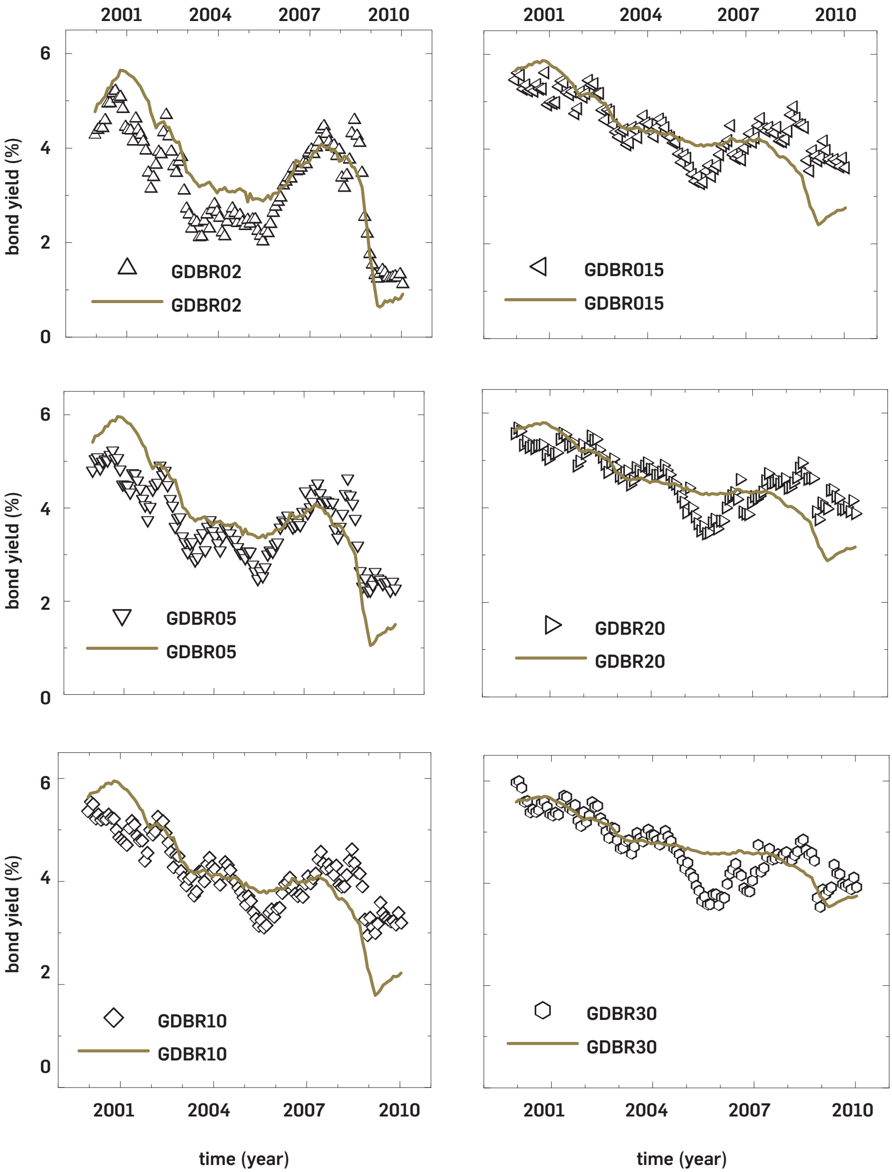
Figure 4. Three- and Six-month Euribor Rate.



Data appear as open symbols, whereas continuous lines denote theoretical approximation given by Eq.(2).

Remarkably however, a fairly good agreement is found between data and calculated yields for longer maturity bonds as those in Figure 5. There, one can see that two-, five-, ten, fifteen, twenty and thirty-years maturity German Government bonds appear to compare with calculations fairly well. According to the results of our numerical calculations, these bonds are reproduced within relative errors of the order of ten percent.

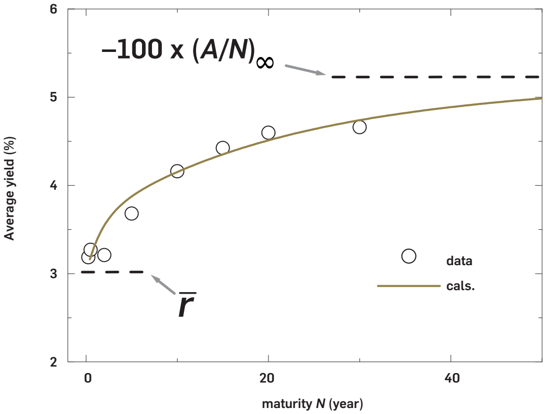
Figure 5. Two-, Five-, Ten-, Fifteen-, Twenty- and Thirty-year German Government Bonds.



Data are denoted as open symbols, and the results of calculating Eq.(2) appear as continuous lines.

Another interesting aspect of the present calculations is depicted in Figure 6. It shows the mean-value of the Euribor and German Bonds taken over the period of time spanned by the used data, i.e. form Dec. 1999 to Jan 2010, and then plotted as a function of maturity N. The results show that, as is expected, the yield should increases with increasing maturity though, at rates which are larger for short N's and becoming nearly constant as maturity reaches values larger than, say, fifteen years.

Figure 6. **Average Bonds Yield As a Function of Maturity.**



Data appear as open circles, and calculations are denoted as a continuous line. Theoretical limiting values (see text), i.e. $\bar{r}$ (3.1%) *and* $-100\times(A/N)_\infty$ (4.9%) *are indicated by dashed lines.*

Interestingly enough, these average yields should have limiting values which can be readily obtained from Eqs.(7a,b). For small maturities, one can readily verify that $dA/dN \cong -\delta_0$ and $dB_i/dN \cong \delta_i$, therefore, $A(N) \cong -\delta_0 N$ and $B_i(N) \cong \delta_i N$. As a consequence, according to Eq.(2) one has $y(0,t_n) = \sum_j \delta_j x_j^{(n)} + \delta_0$, which after taking average over time and taking into account Eq.(3) we may write $\bar{y}(0) = \sum_j \delta_j \bar{x}_j + \delta_0 = \bar{r}_{CAL}$, where $\bar{r}_{CAL}$ stands for the mean-value of the EONIA rate calculated using Eq.(3).

For a sufficiently large N, one has $dB_i/dN \cong 0$. This implies that $B_i(N) = k_1$ and, as a consequence, $dA/dN \cong k_2$, where k_1 and k_2 are two constants. Therefore $A(N) \propto k_2 N$ and, accordingly, as N approaches infinity, the leading term in calculating the average bond yield will be $-A(N)/N$, which can be denoted as $-(A/N)_\infty$.

As one can see in Figure 6, the two limiting values compare remarkably well with the average yields of the bonds used in this paper. This result also agrees with the known fact that bond yields approach asymptotically a constant value with an increase of maturity. Similarly, since $-A(N)/N \gg B_i(N)/N$ for a large N, it means that, within the approximations in this paper, large maturity bonds must be less sensitive to the state variables an therefore, they will look nearly constant over time as one can observe in Figure 5.

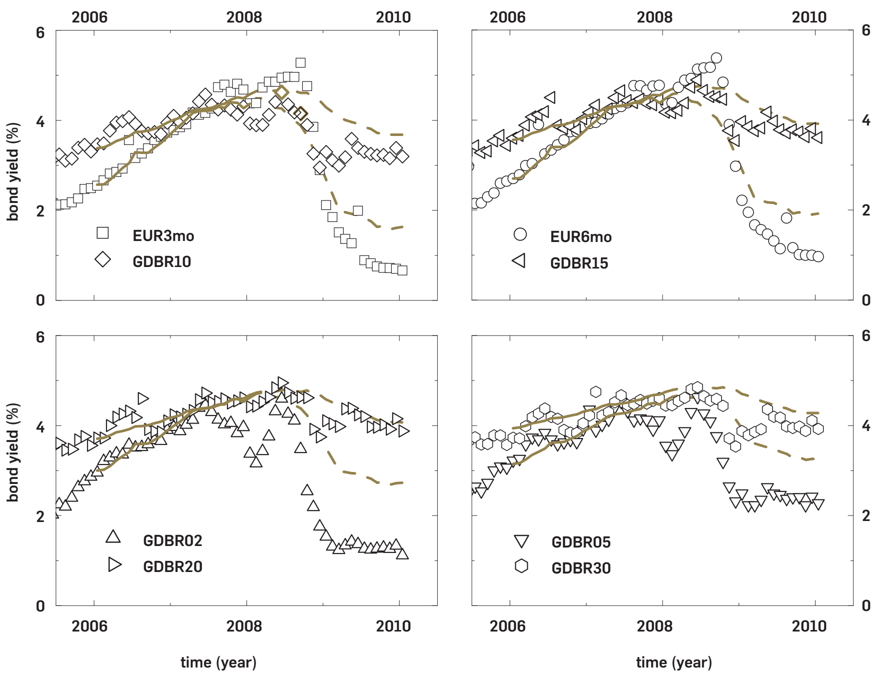
3.2. The Predictive Power of the ATS-model

According to the approximation given by expression in Eq.(2), the bond yields depend on the coefficients A and B and the state variable x. As we have already seen, x must be evaluated at the time bonds are evaluated, i.e. t_n, whereas A and B are calculated from our knowledge of x over a certain range of time within the near past. In other words,

x provides all information relative to current state of the economy whereas A and B carry information about the state variables averaged out over the near past. Having said that, one may legitimately ask oneself: how actual must be the data used in calculating A and B ? In this subsection, we calculate the bond yields by using A and B obtained with previous state variables (excluding most recent ones), and only x assumes the present value.

The results of calculating the bonds yields using this approach is plotted in Figure 7. We call them *predicted values,* although these are not strictly speaking predictions since state variables are still the real ones, but the coefficients A and B are calculated using data ranging from 48 to 24 months older than the most recent ones, i.e. those of the 122nd month.

Figure 7. Predicting Euribor and German Bonds Yields Values Using Current State Variables, but A and B Coefficients Are Calculated Using Data Ranging from 48 to 24 Months Older Than Newest.



Data are indicated by open symbols, predicted values are denoted as dashed lines and calculated yields within fitting range appear as continuous lines.

As one can see, the so-called predicted values compare remarkably well with data. Perhaps the worst agreement is observed for the 3 and 6 months Euribor rates and that of the two-year German bond. To some an extend, this is not at all unexpected since, as we have already stated, small-maturities bonds are more sensitive to current state variables than larger ones.

4. Policy Implications

From an asset pricing perspective, the present results are perfectly in line with the theory. For example looking at parameter B_i/N from Figure 3, implies that an increase in unemployment results in a decrease in the short rate with a rather steepening effect along the yield curve, as the front end of the yield curve decreases faster than the long end. This is mainly explained by the Taylor's rule (1993) on central bank policy as well as by modern asset pricing theory. From a Taylor rule perspective, an increase in unemployment implies a fall in output and a fall in inflationary risks, which would lead to a fall in the policy rate. From the asset pricing theory side, however, an increase in unemployment produces an increase in the expected future aggregate marginal utility growth and hence it will cause a reduction in the risk-free rate. In a similar fashion, it is possible to explain the effects of changes in the EU consumer confidence index in the yield curve. Since a decrease in this index represents an increase in aggregate marginal utility growth with the subsequent drop of the risk-free rate and, from a Taylor rule standpoint, it can be interpreted as a decrease in the inflationary risk. Likewise, when observing that an increase in the production price index results in an increase in the risk free rate, one can see it as the expected central bank policy reaction, according to Taylor rule, against an expected increase of the inflation. Finally, the monetary aggregate shows that an increase in the monetary aggregate would result in a fall in the short rate, in line with the classical Investment-Saving/ Liquidity-preference Money-supply (IS/LM) framework.

From a portfolio management perspective, the results in this paper show that EONIA, Euribor and the German government yields are at their lowest levels when unemployment is high, consumer confidence is low, production price index is low and lending is tight. A scenario that can be compensated by a central bank policy aimed at increasing the monetary aggregate to stimulate growth. On the contrary, yields are at their highest levels when unemployment is low, consumer confidence is high, production price index is high and monetary policy is lax, and partially compensated by central bank policy which tightens monetary aggregates and so, controlling inflation. When yields are at their highest levels the yield curves are flat and the representative investor will have the incentive to short the front end of the curve and take long positions in the long end of the yield curve. Alternatively, in times when yields are at their lowest levels the curve is at its steepest and so, a representative investor will have the incentive to take long positions in short maturity risk-free assets and short the long end. Either way, the gains in the front end are expected to more than offset losses in the long end. In addition, from a risk management perspective the positions in the long end act as a hedge against downside risks stemming from unfavourable and unexpected yield curve movements.

From a sovereign debt policy perspective, the previous analysis works differently, in a way that largely depends on the debt-roll over schedule. If yields are at their lowest levels, governments which enjoy the risk-free status should roll-over maturing bonds with longer maturities in order to ensure that the roll-over of debt does not happen in times when yields are high. In addition, in times when yields are high, which will coincide with a booming economy, governments — whose issuances enjoy a risk-free status — will have the chance to redeem short term issuances at lower prices and hence reduce the size of their total debt outstanding. By doing so they would generate capacity to increase indebtedness for the rainy days, thus in times when consumption growth is low and government's fiscal countercyclical engagement is desired.

5. Conclusions and Final Remarks

This paper shows that the affine term structure model performs very well as long as the selected space variables have significant explanatory power over the short rate, which, in this case, is the European Overnight Index Average (EONIA). Specifically, this paper also shows that the state variables that can be used for this purpose are: (i) the EU unemployment, (ii) EU production price index, (iii) monetary aggregate ECB M3, and (iv) the EU consumer confidence index. These variables accounts for the EONIA rate over a period of time ranging from Dec 1999 to Jan 2011 remarkably well. In addition to that, the proposed states variables not only are observed to work well during times of financial stability, but they also are seen to perform very well during periods of extended financial distress. We see that our calculations work perform better on the front end of the yield curve rather than on the long end. This may seem to be the case because front end yields are more sensitive to the state variables, whereas this dissipates as maturities become larger.

We have seen that yields are high in times when unemployment is low, consumer confidence, M3 and the price levels are high. During times of boom this yield curve exhibits a flat shape, with front end yields almost as high as the 30 year bonds and during times of recessions, the yield curve shows a steeper shape with long term yields exhibiting greater spreads versus short maturity bond yields. Our findings are thus in line with modern asset pricing theory, providing evidences in favour of both Taylor's (1993) central bank policy rule and classical IS/LM models.

Finally, results in this paper lead us to conclude that current yield curve levels are indeed explained by current economic fundamentals and that its behaviour is in line with economic theory. This refutes press headlines pointing to speculative effects of market participants acting irrationally.

Acknowledgements

Thanks are due to Prof-Dr. Ashok Kaul from Saarland University for the valuable guidance and advice.

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Appendix. Bond Yields Calculations

The scheme used in this paper to obtain the bond yields can be summarized as follows: **(i)** Following Eq.(3), δ_0 and δ_j are obtained by minimizing the mean square error (MSE),

$$T_1 = \sum_{n,} \left(r^{(n)} - \delta_0 - \sum_j \delta_j x_j^{(n)} \right)^2 . \tag{A1}$$

This is carried out by the *amoeba* routine from Press *et al.* (1996), which can minimize a N-dimension function without using derivatives. It could be interesting to explore the use of so-called variance reduction techniques for bond pricing in affine models, as seen in for example in Rostan and Rostan (2012).

(ii) Next, we calculate $\phi_{i,j}$'s by using Eq.(5) and assuming $dz_i^{(n)} = 0$. Therefore, we have to minimize the expression,

$$T_2 = \sum_{n,i} \left[dx_i^{(n)} - \sum_j \phi_{i,j} \left(x_j - x_j^{(n)} \right) dt \right]^2 . \qquad \text{(A2)}$$

(iii) Having obtained $\phi_{i,j}$, we calculate α_j and $\beta_{j,k}$ by using Wiener's condition $\sum_n \left(dz_i^{(n)} \right)^2 \cong N dt$, where N denotes the number of time-steps spanned by used data. Furthermore, since $S_{i,j} = \delta_{i,j}$, α_j and $\beta_{j,k}$ resulted from minimizing the expression,

$$T_3 = \sum_{n,i} \left\{ \frac{\left[dx_i^{(n)} - \sum_j \phi_{i,j} \left(x_j - x_j^{(n)} \right) dt \right]^2}{\left| \alpha_i + \beta_i \, x^{(n)} \right|} - N dt \right\}^2 . \qquad \text{(A3)}$$

(iiii) Finally, the $b_{\Lambda,j}$'s are calculated by finding the minimum of the expression,

$$T_4 = \sum_{k,n} \left(y^{(n)}(N_k) - \sum_j \frac{B_j(N_k)}{N_k} x_j^{(n)} - \frac{A(N_k)}{N_k} \right)^2 . \qquad \text{(A4)}$$

To this end, Eqs.(7a,b) are integrated along maturity by using the routine ODEINT of the Numerical Recipes package in Press et al. (1996). Obviously, such integration has to be carried out several times during the minimization of T_4, which is performed by the *amoeba* routine.