AUDITING THE NUMERACY DEMANDS OF THE MIDDLE YEARS CURRICULUM

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The National Numeracy Review Report *recognized that numeracy development requires an across the curriculum commitment. To explore the nature of this commitment we conducted a numeracy audit of the South Australian Middle Years curriculum, using a numeracy model that incorporates mathematical knowledge, dispositions, tools, contexts, and a critical orientation. All learning areas in the published curriculum were found to have distinctive numeracy demands. The audit should encourage teachers to promote numeracy in even richer ways in the curriculum they enact with students.*

Keywords: Critical thinking; Curriculum development; Numeracy

Inspección de las demandas de la alfabetización matemática en el currículo de la educación secundaria obligatoria

El National Numeracy Review Report *reconoció que el desarrollo de la alfabetización matemática requiere un compromiso a través del currículo. Para explorar la naturaleza de este compromiso realizamos una inspección de la alfabetización matemática en el currículum de los últimos años de la educación primeria y los primeros de la educación secundaria en el sur de Australia. Utilizamos un modelo de alfabetización matemática que incorpora conocimiento matemático, disposiciones, herramientas, contextos y una orientación crítica. Encontramos que todas las áreas de aprendizaje en el currículo oficial presentan exigencias específicas de alfabetización matemática. Estos resultados deberían animar a los profesores a promover la alfabetización matemática de manera cada vez más rica en el currículo que implementan con los estudiantes.*

Términos clave: Alfabetización matemática; Desarrollo del currículo; Pensamiento crítico

Numeracy, a term used in many English speaking countries such as the UK, Canada, South Africa, Australia, and New Zealand, was originally defined as the mirror image of literacy, but involving quantitative thinking (Ministry of Educa-

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tion, 1959). Another early definition (Cockcroft, 1982) described "being numerate" as possessing an at-hominess with numbers and an ability to use mathematical skills to cope confidently with the practical demands of everyday life. In the United States of America and elsewhere it is more common to speak of *quantitative literacy* or *mathematical literacy*. Steen (2001) described quantitative literacy as the capacity to deal with quantitative aspects of life, and proposed that its elements included: (a) confidence with mathematics, (b) appreciation of the nature and history of mathematics and its significance for understanding issues in the public realm, (c) logical thinking and decision-making, (d) use of mathematics to solve practical everyday problems in different contexts, (e) number sense and symbol sense, (f) reasoning with data, and (g) the ability to draw on a range of prerequisite mathematical knowledge and tools. The PISA program developed by the Organisation for Economic Cooperation and Development (OECD) similarly defines mathematical literacy more broadly than earlier descriptions of numeracy as:

> *An individual's capacity to identify and understand the role mathematics plays in the world, to make well-founded judgments, and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen.* (OECD, 2004, p. 15)

Steen (2001) maintains that, for numeracy to be useful to students, it must be learned in multiple contexts and in all school subjects, not just mathematics. Although this is a challenging notion, a recent review of numeracy education undertaken by the Australian government concurred, recommending:

> *That all systems and schools recognize that, while mathematics can be taught in the context of mathematics lessons, the development of numeracy requires experience in the use of mathematics beyond the mathematics classroom, and hence requires an across the curriculum commitment.* (Human Capital Working Group, Council of Australian Governments, 2008, p. 7)

What such an "across the curriculum commitment" might look like is the subject of this paper. In 2009 we conducted a yearlong study that investigated approaches to help teachers plan and implement numeracy strategies across all learning areas in the South Australian school curriculum in years 6-9. Before beginning our work with teachers, we conducted an audit of the published curriculum framework (Department of Education and Children's Services [DECS], 2005). This paper addresses the question that guided our audit: What are the inherent numeracy demands of each of the seven learning areas (other than mathematics)? We begin by presenting the numeracy model we developed to synthesize and extend previous work in this area. This is followed by an outline of the audit methodology. A sample evaluation of one learning area demonstrates how we conducted the audit. Findings from the entire audit are then summarized, with a discussion of implications for teachers.

NUMERACY MODEL

In Australia, educators and policy makers have embraced a broad interpretation of numeracy similar to the OECD definition of mathematical literacy. For example, the report of a 1997 national numeracy conference proposed: "To be numerate is to use mathematics effectively to meet the general demands of life at home, in paid work, and for participation in community and civic life" (Australian Association of Mathematics Teachers, 1997, p. 15). This definition became widely accepted in Australia and formed the basis for much numeracy-related research and curriculum development.

Recently, however, Goos (2007) argued that a description of numeracy for new times needs to better acknowledge the rapidly evolving nature of knowledge, work, and technology. She developed the model shown in Figure 1 to represent the multi-faceted nature of numeracy in the 21st century. This model was designed to capture the richness of current definitions of numeracy while introducing a greater emphasis on tools as mediators of mathematical thinking and action. While the model was intended to be readily accessible to teachers as an instrument for planning and reflection, its development was also informed by relevant research, as outlined below.

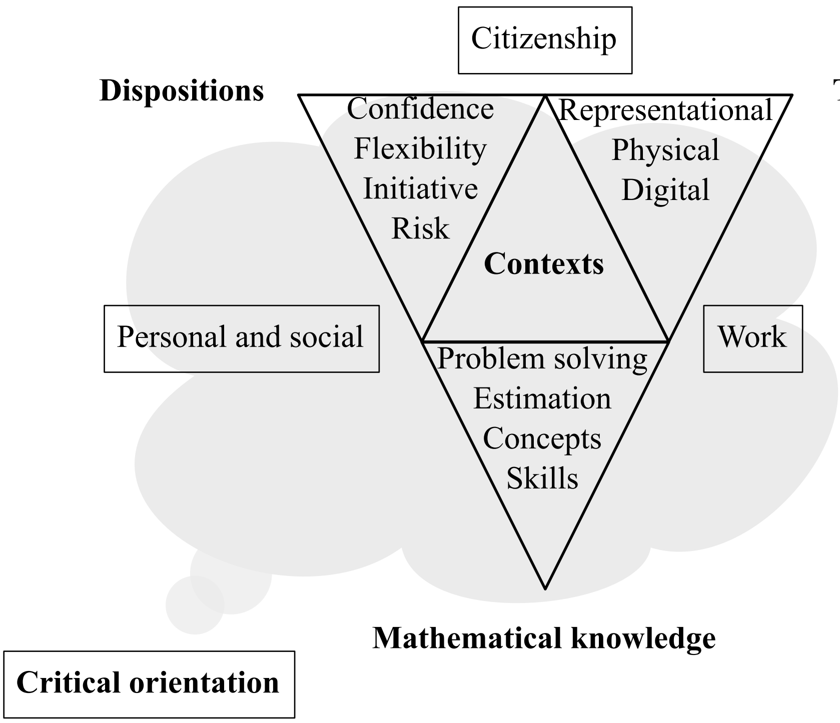

Figure 1. A model for numeracy in the 21st century

A numerate person requires mathematical knowledge. Internationally, there seems to be agreement that all children are entitled to democratic access to powerful mathematical ideas so that they have the knowledge, skills, and understanding to become educated citizens (Malloy, 2002). In a numeracy context, mathematical knowledge includes not only concepts and skills, but also problem solving strategies and the ability to make sensible estimations (Zevenbergen, 2004).

A numerate person has positive dispositions—a willingness and confidence to engage with tasks, independently and in collaboration with others, and apply their mathematical knowledge flexibly and adaptively—. Affective issues have long been held to play a central role in mathematics learning and teaching (McLeod, 1992), and the importance of developing positive attitudes towards mathematics is emphasized in national and international curriculum documents (e.g., National Council of Teachers of Mathematics, 2000; National Curriculum Board, 2009).

Being numerate involves using tools. Sfard and McClain (2002) discuss ways in which symbolic tools and other specially designed artifacts "enable, mediate, and shape mathematical thinking" (p. 154). In school and workplace contexts, tools may be representational (symbol systems, graphs, maps, diagrams, drawings, tables, ready reckoners), physical (models, measuring instruments), and digital (computers, software, calculators, internet) (Noss, Hoyles, & Pozzi, 2000; Zevenbergen, 2004).

Because numeracy is about using mathematics to act in and on the world, people need to be numerate in a range of contexts (Steen, 2001). For example, a numerate person can organize their finances, make decisions affecting their personal health, and engage in leisure activities that require numeracy knowledge. All kinds of occupations require numeracy, and many examples of work-related numeracy are specific to the particular work context (Noss et al., 2000). Informed and critical citizens need to be numerate citizens. Almost every public issue depends on data, projections, and the kind of systematic thinking that is at the heart of numeracy. Different curriculum contexts also have distinctive numeracy demands, so that students need to be numerate across the range of contexts in which their learning takes place at school (Steen, 2001).

This model is grounded in a critical orientation to numeracy since numerate people not only know and use efficient methods, they also evaluate the reasonableness of the results obtained and are aware of appropriate and inappropriate uses of mathematical thinking to analyze situations and draw conclusions. In an increasingly complex and information drenched society, numerate citizens need to decide how to evaluate quantitative, spatial or probabilistic information used to support claims made in the media or other contexts. They also need to recognize how mathematical information and practices can be used to persuade, manipulate, disadvantage or shape opinions about social or political issues (Frankenstein, 2001).

Evaluating the Numeracy Demands of Learning Areas

We were commissioned to conduct a numeracy audit of the *South Australian Curriculum, Standards and Accountability (SACSA) Framework* (DECS, 2005). The curriculum scope is organized around learning areas, each of which is defined by strands comprising key ideas that increase in complexity through the years of schooling and standards that represent expectations of learners. The audit evaluated the numeracy demands of the arts, design and technology, English, health and physical education, languages, science, and society and environment learning areas for the middle years (years 6 to 9), as represented by the relevant curriculum scope and standards statements provided in the SACSA framework.

Numeracy demands of each learning area were evaluated by reference to the elements of the numeracy model in Figure 1. Mathematical knowledge demands were examined by assessing the extent to which the target learning area drew on the five strands of the mathematics learning area of the SACSA framework: (a) exploring, analyzing, and modeling data; (b) measurement; (c) number; (d) patterns and algebraic reasoning; and (e) spatial sense and geometric reasoning.

Numeracy Demands: Society and Environment

A sample evaluation of the numeracy demands of the society and environment learning area illustrates how we carried out the audit of the whole curriculum.

Mathematical Knowledge

Table 1 maps numeracy learning demands in society and environment onto the mathematics strands of the SACSA framework. Shading is used to indicate the level of numeracy learning demands.

Table 1
Mathematical Knowledge Demands within Strands of the Society and Environment Learning Area

	Mathematics strands				
Society and environment strands	Exploring, analyzing, and modeling data	Measurement	Number	Patterns and algebraic reasoning	Spatial sense and geometric reasoning
Time, continuity, and change					
Place, space, and environment					
Societies and cultures					
Social systems					

Note. Unshaded: low level; light shading: moderate level; dark shading: high level.

The need to make decisions and adopt positions based on evidence and take a data driven approach to argumentation means the mathematical strands of exploring, analyzing and modeling data, measurement, and spatial sense and geometric reasoning are particularly relevant. Students can demonstrate changes to aspects of the environment, for example, the rate of deforestation in developing countries, through reference to available data, or consider the advantages of different architectural designs for built environments in hot dry climates.

Contexts

According to the SACSA framework, the study of society and environment aims to assist learners to understand the processes that lead to change in the world and, in doing so, to empower them to act in the shaping of the society and environment into which they will grow. The framework outlines a wide range of contexts that align with students' current and future lives at school, work, and in the community:

> *The complexities and contradictions arising from rapidly changing technologies; unequal distribution of wealth and power; global interdependence; the dynamic nature of social, economic, political and ecological systems; the changing nature of work, and social practices around paid and unpaid work; and the need for increasingly sustainable social and environmental management practices bring challenges to people in all*

> *societies. The concepts and processes employed in society and environment enable learners to think clearly about current issues confronting them and their world.* (DECS, 2005, p. 291)

Dispositions
Students are expected to develop dispositions enabling them "to be active citizens who can make informed and reasoned decisions and act on these" (DECS, 2005, p. 291). While there is an obvious connection between this aim and the need for mathematically valid approaches to collecting and analyzing data, teachers should not assume that the study of society and environment would automatically ensure that students develop positive dispositions towards mathematics. Explicit attention also needs to be given to development of mathematical confidence in using appropriate techniques for dealing with problems in real life contexts.

Tools
The use of tools to collect and then analyze the information necessary for a critical approach to decision making is vital to this learning area. These tools include representational, physical, and digital tools such as:

- maps and charts for identifying the characteristics of a specific environment (e.g., contours, the paths and interconnectedness of river systems);
- plans for built environments;
- instruments for measuring location and position (e.g., GPS systems and surveying tools);
- on-line data sources (e.g., archival records of rainfall in specific catchment areas); and
- digital tools such as spreadsheets or software applications developed specifically for the analysis and representation of data.

Critical Orientation
As the ultimate goal of learning through society and environment is to enable students to participate as ethical, active and informed citizens, a critical orientation to viewing information and an analytical approach to the interpretation of data must be embedded within this learning area.

Summary of Numeracy Audit Findings

We present our findings for the seven learning areas organized in the five strands included in the model for numeracy.

Mathematical Knowledge
Table 2 synthesizes the mathematical knowledge requirements of the seven learning areas apart from mathematics itself. The synthesis was carried out by

combining the mappings of numeracy learning demands of each strand in each learning area onto the mathematics strands. For each of the latter mappings, scores of 2 were allocated for strands with high numeracy demands (dark shading), 1 for strands with moderate numeracy demands (light shading), and 0 for strands with low numeracy demands (un-shaded). These scores were then tallied, by mathematics strand, for each learning area. The total scores for science strands were scaled by a factor of 0.75, since science has four strands whereas the other learning areas have only three strands each. This procedure resulted in each cell of Table 2 having a score between 0 and 6. No shading of cells represents low numeracy demand (score of 0-1), light shading represents moderate numeracy demand (score of 2-4), and dark shading high numeracy demand (score of 5-6).

Table 2
Mathematical Knowledge Demands within the Learning Areas of the SACSA Framework

	Mathematics strands				
Learning area	Exploring, analyzing, and modeling data	Measurement	Number	Patterns and algebraic reasoning	Spatial sense and geometric reasoning
Arts					
Design and technology					
English					
Health and physical education					
Languages					
Science					
Society and environment					

Note. Unshaded: low level; light shading: moderate level; dark shading: high level.

The level of numeracy demand is highest for design and technology, science, and the arts; moderate for society and environment and health and physical education; and lowest for English and languages. Despite these differences, however, it is important to recognize that all learning areas have distinctive numeracy demands in relation to the type of mathematical knowledge required by students in order to demonstrate successful learning. Teachers are ultimately responsible for enacting the curriculum in their classrooms, and they can therefore exploit numeracy learning opportunities in the learning areas beyond those implied by the published curriculum.

The strands of mathematical knowledge are also represented to different degrees in the learning areas. Exploring, analyzing and modeling data is most strongly represented in the intended curriculum, followed by measurement, number, and spatial sense and geometric reasoning, with the strand of patterns and algebraic reasoning the least strongly represented. It is perhaps not surprising that algebra, as an element of numeracy knowledge, appears to be under-represented in the curriculum as it may be thought that algebraic ideas are abstract and have little connection with real world contexts or learning areas other than mathematics. Nevertheless, it is worth emphasizing the potential connection between algebraic reasoning and modeling with data since exploration of patterns and generality in the middle years of schooling can begin with an empirical focus on data collection and analysis.

Contexts

The range of numeracy learning contexts highlighted in the numeracy model of Figure 1 is well represented across the learning areas. Each emphasizes the value of connecting students' learning to real life contexts that are meaningful for them, whether this involves personal interests, family and community life, leisure pursuits, the physical environment, vocations and careers, diverse cultures, or social, economic, and political systems.

Dispositions

Throughout the SACSA framework there is evidence of a desire to develop positive dispositions such as perseverance, confidence, resilience, willingness to take risks and show initiative, respect for cultural diversity, and commitment to ecological sustainability. These are admirable goals, but we would argue that dispositions towards learning in one discipline do not automatically transfer to another discipline: It is possible, for example, for students to feel confident about their learning in the arts but not in mathematics and not in relation to numeracy more generally. Teachers need to be aware of the damaging effects of negative mathematical dispositions, to look for opportunities to successfully engage their students with the numeracy demands of their learning area, and to make explicit to students the positive dispositions that are helping them to achieve this success.

Tools

Representational, physical, and digital tools are used across all learning areas. Some of these are specific to the discipline while others are more generically useful. Graphs, diagrams, tables, maps, and plans are commonly used in many learning areas, as are measuring instruments, both physical and digital. There is also a strong emphasis on digital tools, software, and web resources. Thus all learning areas have specific numeracy demands in relation to accurate and intelligent use of tools to represent and analyze ideas. Students need to become proficient with the tools of each learning area, but they also need to be aware that some tools are used in more than one learning area and to be flexible in applying tools in different curricular contexts. For example, students may come to believe that mathematics and society and environment promote different approaches to reading and creating maps, or that science and health and physical education promote different ways of creating graphs that show relationships between variables. Teachers in these learning areas need to be aware of any differences in techniques and terminologies associated with the use of these representational tools and to draw students' attention to important similarities between underlying concepts.

Critical Orientation

The SACSA framework emphasizes developing a critical orientation in students across all learning areas. Such an orientation cannot be fully enabled without numeracy knowledge, dispositions, and tools, nor can it be convincingly enacted unless learning takes place in a range of real life contexts. Conversely, being numerate requires adopting a critical stance in order to question, compare, analyze, and consider alternatives. The numeracy demands inherent in the learning areas should facilitate development of this critical orientation.

Conclusion

The audit of the SACSA framework for the middle years found that all learning areas have distinctive numeracy demands in relation to mathematical knowledge, contexts, dispositions, tools, and development of a critical orientation. While the audit was necessarily restricted to the published curriculum framework, by highlighting the ubiquitous nature of numeracy throughout this intended curriculum it has opened a window of opportunity for teachers to promote numeracy in even richer ways in the curriculum that they enact with students in their classrooms. Developing an appreciation of the numeracy demands of the intended curriculum, as outlined in this audit, may help teachers become more attuned to the numeracy demands (and opportunities) presented by any learning task in which their students are engaged.

At the time of writing this paper the first Draft Consultation version of the Australian curriculum in mathematics K-10 had just been released for comment by the mathematics education community. Numeracy is one of the "general capabilities" to be specifically covered in the curriculum as a fundamental responsibility of mathematics and for application in other learning areas. It remains to be seen as to how successfully the final version of the curriculum achieves this aim through its content statements and achievement standards.

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