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The Art of Earth Measuring: Overlapping Scientific Styles

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ABSTRACT

The aim of this paper is to point out significant and meaningful overlapping between several styles of scientific thinking, as they were proposed by Crombie (1981) and discussed by Hacking (1985; 2009). This paper is divided in four sections. First, I examine an interpretation made by Barnes (2004) about the incompatibility among scientific styles. As explained by its author, this interpretation denies any possibility of similarities between styles of scientific reasoning. In opposition, the following sections of this paper include explanations of relevant characteristics of Geometry, as stated in Euclid's Elements, which are also present in other three scientific styles: modeling, experimental exploration and taxonomic style (and some of the shared characteristics are even used to define such styles). By stressing out these characteristics I argue that these four styles discussed by Hacking are not so different from each other, in fact, they overlap and may even be abridged into one foundational style: geometric.

KEYWORDS
Scientific styles, Geometry, modeling, measuring, taxonomy.

RESUMEN

El objetivo de este artículo consiste en explicar traslapes importantes y significativos entre varios estilos de pensamiento científico, tal como fueron propuestos por Crombie (1981) y discutidos por Hacking (1985; 2009). El presente artículo está dividido en cuatro secciones. Primero, se examina la interpretación de Barnes (2004) sobre la incompatibilidad entre diversos estilos. Según este autor, no existe posibilidad alguna de encontrar similitudes en los estilos de razonamiento científico. En oposición a esta interpretación, las siguientes secciones explican características relevantes de la Geometría, tal como fueron plasmadas en Los Elementos de Euclides, y que están presentes en otros tres estilos: de modelación, de experimentación y taxonómico (las características compartidas incluso definen algunos de estos estilos). Destacar estas características me permite argumentar que estos cuatro estilos discutidos por Hacking no son tan diferentes entre sí, de hecho, se traslapan e incluso pueden ser condensados en un estilo fundacional: el geométrico.

PALABRAS CLAVES
Estilos científicos, Geometría, modelar, medir, taxonomía.

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Styles of scientific thinking & doing

Hacking (1992; 2009) explains that he got the idea of a small manifold of distinct styles of scientific thinking from the historian of science A. C. Crombie (1981). He also states that a particular style of scientific reasoning involves a particular way of thinking, but also a specific kind of argumentation and a way of talking and showing. According to this author, "knowing is doing" and in consequence, the notion of style can be applied to Science as well as to Arts and Art crafts.

Hacking (2009) also asserts that we have many different cognitive abilities, and that human history runs on many paths (Cfr. p. 4). Not surprisingly, this author claims, that there are many ways to conduct scientific research:

- Mathematicians construct deductive proofs (among other things).
- We make theoretical models of aspects of nature in order to understand them or to alter them.
- Laboratory sciences demand not just "experiment", but also the building of apparatus to elicit, and, often, to create phenomena.
- Taxonomists classify living things according to principles of hierarchic structure, although what those principles are, continue to be matters of intense debate.
- Decision under uncertainty, thinking in probabilities, is yet another distinct style of scientific thinking.
- There may also be a genetic way of understanding, most successful in such evolutionary theories as Darwin's theory

> of natural selection, tried out in enterprises as diverse as Freudian analysis and Marxist historiography.

According to Barnes (2004), there are no similarities at all between different styles of scientific reasoning. This author states that Hacking's work is a continuation of Kuhn's historical, anti-rationalist project: "[Hacking] thinks that the history of Western science is characterized by a distinct set of non-overlapping styles of scientific reasoning (of which he names six: mathematics, statistical analysis, theoretical modeling, the experimental method, taxonomy, and genetic development). They are non-overlapping because as for Kuhn there is no rational comparison and means for integration; they are their own Gestalt worlds" (p. 4). Barnes asserts that there cannot be any overlapping, let alone integration of styles, because of two features that separate each style. First, styles determine the very criteria of evaluation by which they are judged. Second, styles bring into being the subject matter they claim to study.

Barnes uses two explanations of Hacking in order to reinforce his argument. First, on the evaluation criteria: "[…] the very candidates for truth and falsehood have no existence independent of the styles of reasoning that settle what it is to be true and false in their domain" (Hacking, 1985, p. 146). Second, on the subject matter he quotes a list that includes "new types of: objects; evidence; sentences; new ways of bringing a candidate for truth and falsehood; laws, or at any rate modalities; and possibilities" (Hacking, 2002, p. 189). The consequence of having both features is that each style is "self-authenticating". By bringing with them their own criteria of assessment and objects of investigation, scientific styles become resistant to external criticism. Each style is based on its own vocabulary, logic, practice and way of argumentation and explanation.

Barnes remarks the following idea from Hacking: given that each scientific style includes its own evaluation criteria, the replacement of a particular style by another is not due to an augmented rationality in any of them. The emergence of a style and its posterior replacement depend upon historically contingent reasons rather than rational ones.

Against the arguments of Barnes, I will point out similarities among different styles of reasoning. I will argue that these similarities are strong enough to create meaningful overlapping between the following styles:

- Geometric style or mathematics.
- Taxonomic style or ordering of variety by comparison and taxonomy.
- Analogical modeling or hypothetical construction of analogical models.
- Experimental exploration or deployment of experiment both to control postulation and to explore by observation and measurement.

Before proceeding with the discussion of particular styles, it is important to acknowledge a general caution made by Hacking (2009): "Styles of scientific thinking are not sciences or scientific disciplines, and they are not mutually exclusive. Most modern sciences use most of Crombie's styles of scientific thinking" (p. 39).

In other words, scientific styles are not supposed to be particular sciences. Different styles may be deployed in any science. This is a very important statement that has been underestimated by scholars. If taxonomical style were a way of reasoning belonging only to Biology (*i.e.*, Linnaeus' classification) and cares only about living beings, then a different taxonomical style of reasoning is used in Chemistry (*i.e.*, Mendeleev's classification; *e.g.*, Scerri, 2007), and it should be considered as a completely distinctive style. But then again, a new whole variety of taxonomic styles should be added, for example astronomical (*i.e.*, Hertzsprung-Russell classification; *e.g.*, Nielsen, 1964) and so on. Thus, we are forced to make a logical decision, either we have a huge multiplicity of taxonomic styles or there is only one kind of taxonomical reasoning that is being used in Biology, Chemistry and Astronomy (we cannot choose both scenarios). The parsimonious choice is to have only one style applied into several disciplines but even Hacking (2009, p. 4) emphasizes only the classification of living things when explaining the taxonomical style.

There is also one important corollary derived from the former observation. If there is one general taxonomical style, which is used in several disciplines, then it does not care for specific objects. A taxonomical way of reasoning can be used to classify living organisms, as well as chemical elements and stars in heavens. This contradicts a major argument supporting non-overlapping, self-vindicating scientific ways of thinking, which is the following:

> Styles of scientific thinking introduce their own distinct class of objects. Think of the abstract mathematical objects ("Platonist"), of the unobservable theoretical entities at the centre of the recent debates about scientific realism, or of systematic biology with its taxa. Each style is specific to it its own domain, but only because it introduces the objects peculiar to that domain. (Hacking, 2009, p. 22).

If scientific styles of reasoning are *not* sciences, but general ways of thinking that can be deployed in any science, then particular objects cannot characterize such styles. That is, if taxonomic style is used in many disciplines, it cannot be characterized by "systematic biology with its taxa" and also be characterized by a myriad of *newly objectified* chemical elements, stars and others. Either we have a multitude of styles each applied to newly introduced objects (biological, chemical and astronomical styles) or we have only one general style applied to a multitude of different objects (living organisms, chemical elements and stars).

If we accept that there is one taxonomic style, which can be applied into Biology, Chemistry and Astronomy, then we can validly ask if there are other disciplines that also use this very same style of reasoning. In particular, we can validly investigate if classical Geometry involves the formalization of a complete and rigorous taxonomy.

Modern sciences, as Hacking (2009, p. 7) admits, use several scientific styles. And I will argue that even classic Geometry, as stated 300 years before Christ in *The Elements* by Euclid of Alexandria, is not the "crystallization" of one particular style but the combination of several. And I will dare to advance a working hypothesis: Geometry may be regarded as a foundational scientific style that was later applied to other fields of human interest.

The art of earth measuring

The very name of the geometric style reveals a major overlap with the analogical modeling style. Geometry literally means "the art of Earth measuring". The development of geometric way of reasoning responded to the practical necessities of land measurement and building construction. In order to fulfill such necessities, geometric style provided simple and useful models, which worked by analogy to physical objects.

Carson and Rowlands (2005) explain several developmental events in Geometry. Three of such events will suffice to relate this discipline with the analogical modeling style. The first event is the solution of problems implied by land measurement: practical measurement, map-making, reconstruction of fields. Geometric objects such as *Point*, *Line* and *Plane* are analogical models of stakes, ropes and fields.

> The Egyptian achievement of measurement, applied to the earth, and to the building of monumental structures was mirrored by similar developments wherever early agricultural settlements occurred. Knotted ropes, records, and procedures were invented and refined to assist in these tasks. A technology of measurement then yielded mathematical curiosities that stimulated mathematical imagination. The multiplication of length across two or more dimensions became the concepts of area and volume, and so a

> consciousness was built up around the idea of quantifying space and spatial relationships. These same abilities were adapted to the building of irrigation systems, city street grids, the pyramids and other monumental structures. As these various concepts and practices became represented on paper, they established the practical basis for an evolving abstract geometry. (Carson & Rowlands, 2005, p. 3).

The second event that these authors mention is the development and formalization of levels of abstraction. The starting point is an actual object, then a model emerges, followed by literal drawings, abstract drawing, personal concepts, authorized concepts and Platonic ideals.

> Abstraction and imagination are prevalent throughout prehistory, as evidenced by mythology, hybrid beasts, superstition, poetry, art, and other activities. But in classical geometry it becomes a formalized topic of conversation, is brought under the governance of well-defined rules, and becomes therefore a self conscious and deliberate activity. Plato devised the heuristic of a metaphysical realm in which ideas became more real, even, than physical objects. (Carson & Rowlands, 2005, p. 4).

The third event is the shift from an aesthetic shape to a mathematical concept, and the use of the acquired concept to solve some practical problem.

> A goat, tethered to a stake in a field, will eat the grass until it has created a circle [in the long run, i.e., hypothetically]. The definition of a circle is implicit in that scene. A circle enters human consciousness as a shape, an aesthetic object. In mathematics it becomes redefined as the points on a plane equidistant from a fixed point. Every radius is of equal length. This is the definition one has to work with in order to yield mathematically useful insights in problem solving. (Carson & Rowlands, 2005, p. 4).

Oddly enough, Hacking (2009) does not take into account these three developmental events in Geometry, and only focuses on mathematical proof: "For me, Greek geometry is a matter of proof, not postulates. Whether we focus on postulates or proof, mathematical reasoning and the ability to do it is something all of us recognize, even if some of us are good at it and others are not. We know when something demands mathematics." (p. 13).

But some of the mathematical proofs, as stated in *The Elements*, are nothing but detailed logical guides for solving practical problems implied by land measurement and building construction. It is not hard to see that lines and circles are models, and some of the mathematical proofs are detailed construction guides using *any* given linear o circular model (*i.e.*, guides to be implemented in *any practical problem* where such models could be applied). Let's take for example the very first proposition of *Book I* (Euclid, 300 BC, as translated by Fitzpatrick, 2008,

- *"Proposition 1*. To construct an equilateral triangle on a given finite straight-line". (p. 8)
- The very last sentence of the mathematical proof of this proposition is revealing, it shows that the proof is nothing but a detailed guideline for solving the stated problem: "Thus, the triangle *ABC* is equilateral, and *has been constructed* on the given finite straight-line *AB*. (Which is) the very thing it was required to do". (p. 8) (emphasis added).
- Furthermore, the entire *Book IV* is devoted to the "construction of rectilinear figures in and around circles". (p. 109)

There are other developmental events in Geometry that reveal similarities with the modeling style, such as the use of imagination and inventiveness, in imposing own creative ideas into problem spaces. However, the three events already discussed in this section clearly reveal the overlap between geometric and modeling styles: in order to solve practical problems related to land measurement and building construction, geometric style created models that work by analogy. Such models were formalized and brought into governance by well-defined rules; the entire framework, models and rules, was then applied (and still is) into problem solving.

The activity of modeling is not randomly present in the development of Geometry, but the very same objects that this discipline studies are analogical models, which are handled and constructed under rigorous rules. In this sense, I dare to propose that the modeling style is already contained or embedded in the geometric one.

Measuring the earth

The very name of the geometric style also reveals overlapping with the experimental style, which involves exploration by *observation and measurement*. When discussing the topic of "measurement", Hacking (2009) recognizes that, "Nevertheless, a primary use of measurement is in planning buildings for habitation, worship, or protection. As soon as you start making rigid dwelling places for family, for priests, to keep out your enemies, or for the afterlife, you have got to do some measuring. So we may guess that the first sustained need for measurement was from builders." (pp. 95-96).

Nevertheless, Hacking never relates measurement with Geometry. But this connection is pretty obvious. As explained in the previous section, Geometry involves the rigorous construction and manipulation of analogical models. And the main purpose of this discipline is clearly stated in its name: earth *measuring*. But also, the purpose of Geometry involves a major refinement or, as Hacking would say, "crystallization" of measuring, which is the rigorous *manipulation* of earth: building construction.

The notion of equivalence involved in Geometry, as explained by Carson and Rowlands (2005), is the main essence of any measurement task. Such notion involves the mastering of concepts such as equality, congruence and commensurability. Equivalence enables separate objects to be equal, similar, the same, homologous, but also longer, smaller, bigger, and etcetera. As a truly fundamental notion in Geometry, the presence of this notion is overwhelming in Euclid's masterpiece. Furthermore, the entire *Book XII* of *The Elements* is devoted to measurement problems. Let's see some examples (Euclid, 300 BC, as translated by Fitzpatrick, 2008):

- *Book I. "Proposition 2.* To place a straight-line equal to a given straight-line at a given point (as an extremity)". (p. 8)
- *Book V. "Definition 1.* A magnitude is a part of another magnitude, the lesser of the greater, when it measures the greater." (p. 130)
- *Book XII. "Proposition 1.* Similar polygons (inscribed) in circles are to one another as the squares on the diameters (of the circles)." (p. 472)
- *Book XIII.* "Proposition 1. If a straight-line is cut in extreme and mean ratio then the square on the greater piece, added to half of the whole, is five times the square on the half." (p. 506)

The notion of equivalence is remarkably important because it also relates Geometry to the so-called crystallization of the modeling style. According to Hacking (2009): "The Galilean style is the crystallization of what Crombie called the style of hypothetical modeling. Better to say that it is a definitive crystallization of what Crombie called (c) the hypothetical construction of analogical models." (p. 42). And the Galilean style is: "essentially the mode of modern mathematical physics; from this point of view, the Newtonian style can be seen as a highly advanced and very much refined development of the Galilean" (p. 12).

In my view, the strongest refutation to Hacking´s former assertions can be found in the very own writings of Galileo Galilei. One of his most famous quotations reveals that, Galileo, did not see himself as the founder or "crystallizer" of a completely new scientific style but merely as a student of Geometry:

> Philosophy is written in this grand book —I mean the universe— which stands continually open to our gaze, but it cannot be understood unless one first learns to comprehend the language in which it is written. *It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures, without which it is humanly impossible to understand a single word of it*; without these, one is wandering about in a dark labyrinth. (Galileo, 1623, as translated by Popkin, 1966, p. 65; emphasis added).

Geometry involves measuring, modeling and rigorous manipulation (building construction). The notion of equivalence, specific examples of which can be found in Euclid's work and Galileo's assertion, is sufficient evidence of meaningful overlapping between the geometric and the modeling style (along with its so-called "Galilean crystallization"). However, some authors might be tempted to argue further differences stressing out the importance of "crystallizations". But such argumentations fail to see that the notion of equivalence and other relevant geometric notions, all of them in Euclid's *Elements*, are at the main core of further scientific advancements.

The notion of equivalence can be applied to many different activities and environments. More importantly, when this notion is taken from land measurement and building construction problems and applied into general human settings and activities, a quantitative depiction of such settings and activities becomes possible. Moreover, a cognitive transformation takes place:

> The Greeks achieved a conceptual mapping of the physical world with respect to its quantitative dimensions. Time and space were established quantitatively. From these conceptual templates, other dimensions were reflected upon using the notion of gradations of change. From the merchant's expectation that dissimilar items can be brought into a numerical comparison of value, there emerges a conceptual mapping of the physical world with respect to its quantitative dimensions. This quantification in the marketplace predisposed Greek philosophers to thinking quantitatively in other domains of understanding as well. Through music, and

> aesthetics, the notion that the world is a mathematical construct was reinforced, as was the belief that there was a natural, mathematically determined, standard of beauty, order, and reason. (Carson & Rowlands, 2005, p. 5).

Secondly, as a derivation of Geometry, Archimedes interpreted physical actions mathematically, and vice versa: "In the twilight years of classical geometry, Archimedes foreshadows developments that will be taken up fifteen centuries later as geometry and algebra are first applied to simple machines and mechanical systems" (Carson & Rowlands, 2005, p. 7).

It is important to remark that the very foundational stones for the Newtonian style can also be found in classical Geometry, for example, the concept of Infinity:

> Some of the first serious mathematical reflections on the notion of infinity arise in the study of geometry. Classical Greek thinkers reflected on the infinitely small and the infinitely large. The paradoxes of Zeno were an attempt to show that the use of indivisibles leads to contradiction and to point up the inability of a static formalism to manage dynamic concepts. Euclid's parallel postulate concedes the inability to predict what happens to space at an infinite distance. These efforts to approach the topic are tentative and thoughtful; they open up an inquiry without resolving it. (Carson & Rowlands, 2005, p. 6).

Another foundational stone of the Newtonian style can be found in Euclid's *Elements*; Book XII includes an explanation of the method of *exhaustion* (also used by Archimedes). This method, along with the discussions about Infinity, constitutes early steps toward infinitesimal calculus (later formalized by Leibnitz and Newton).

Greeks never thought that there could be infinite steps in this procedure. For them there was always a tiny piece, which was not exhausted, even if this piece could be made arbitrarily small. Thus the Greeks did not have to deal with actual infinity, this procedure requires only its potential existence. It is very similar to how we compute limits today. In our *e-d* definition of limit we do not use any infinitesimals, we always compute with the finite quantities *e* and *d*… From this it follows that at the turn of the 3rd century B.C., there was an established theory based on the ideas of mathematical analysis. (Cernekova, 2008, p. 29).

Also, some propositions and their mathematical proofs, included in *The Elements*, can be seen as mental experiments, specially the ones that involve the concept of Infinity. For example (Euclid, 300 BC, as translated by Fitzpatrick, 2008):

- *Book X."Proposition 115.* An infinite (series) of irrational (straight-lines) can be created from a medial (straight-line), and none of them is the same as any of the preceding (straight-lines)." (p. 422)

Thus, geometric style involves analogical modeling for solving practical problems of land measurement and building construction. It requires the development and formalization of levels of abstraction, as well as realization of mental experiments. Its application is not restricted to practical problems related to surveying and construction; geometric style can also be applied to physical actions and mechanical systems, as Archimedes did. Starting from classical Geometry, this style of scientific thinking included complex notions that later became the foundational basis for mathematical calculus, developed by Newton and Leibnitz.

Given all these properties of the geometric style, a working hypothesis may be advanced. Geometry was originally developed in order to effectively measure and manipulate land (building construction is a very rigorous and effective way to manipulate land). But later Archimedes applied this style of thinking to practical problems of physical actions and mechanical systems. Thus, it might be possible to show that the so-called Galilean and Newtonian "crystallizations" were nothing more than new applications of the geometric style of thinking. In the case of Galileo, this style was applied to the study of heavens (in plural) and allowed him to effectively measure motions of projectiles and stars. In the development of calculus, by Newton and Leibnitz, geometric style was applied to the study of fluids, originally gasses and liquids. Even the "creation of vacuum" by Boyle, that so much impressed Hacking and many others, might be regarded not as the "creation of a new phenomenon" but merely as an effective manipulation of gasses (with the aid of an air-pump).

In this sense, it is relevant to remark that the distinction between modeling and experimental styles is not clear at all, not even for Crombie (1994): "The particular intellectual and artistic ambience of early modern Europe came to make (3) *the method of hypothetical modeling* a characteristically effective scientific combination of theoretical and experimental exploration." (p. 1087).

Hacking (2009) realized that Crombie's claim amalgamated two scientific styles but then, he embarked on an argumentative quest to keep them apart: "Whether he was fully conscious of it or not, Crombie here speaks of the *combination* of the methods of two different styles. I shall try to keep him to his original conception of distinct styles." (p. 99; italics in the original).

Why did Hacking ignore the later statement of Crombie and decided to defend Crombie's first depiction of six distinct styles? Because Hacking realized that, if two styles could be merged together, a sensible and very relevant doubt would arise: maybe other styles could also be effectively combined. And a reasonable doubt would soon be casted upon the entire setting of distinct, self-authenticating, non-overlapping six scientific styles:

> It is a very good reminder, to connect hypothetical modeling with architectural models and hence with measurement. We see that Crombie's distinction between his second and third styles curiously melts away, as is indicated in the single sentence I have just quoted. Does this mean that his catalogue of six distinct styles is just a sham? I think not. I believe that two crystallizations restore *the sacred six to their right relationships*. (Hacking, 2009, p. 100; emphasis added).

Therefore, divisions among the original six styles must be kept *sacred* according to Hacking; such divisions must be defended as a genuine dogma of faith against reasonable doubts. This sort of dogmatic vindication can also be noticed elsewhere, Hacking (2009) uses the following argument to deter the addition of new styles: "A form of Occam's maxim provides a good rule of thumb: the list of styles of scientific thinking should not be enlarged beyond necessity" (p. 12). However, it is worth noticing that Hacking does not follow Occam's maxim when confronted to the possibility of reducing the original list (Hacking argument is incoherent when guarding his dogmatic six styles). In other words, if Occam's razor is such a "good rule of thumb", then it is relevant to explore the possibility of this list being "enlarged beyond necessity" since its very first formulation.

Classifying earth measuring models

As mentioned in the first section of this paper, taxonomical way of thinking is present in many scientific disciplines. For example, in Biology (*i.e.*, Linnaeus), Chemistry (*i.e.*, Mendeleev) and Astronomy (*i.e.*, Hertzsprung-Russell). Mathematics is not the exception. Taxonomical way of thinking is overwhelmingly present in Mathematics; trained mathematicians are constantly classifying numbers, functions and many other objects. However, some scholars might be surprised to find that taxonomical thinking is fundamental to Mathematics:

> Taxonomic reasoning must seem wholly removed from mathematics—until you reflect that some of the most profound theorems are about classification, say the exhaustive classification of the finite groups. Such theorems go back to the five regular solids that so impressed Plato and his heirs. (Hacking, 2009, p. 7).

There is no need to reflect on "most profound theorems" as Hacking erroneously suggests, taxonomical tasks are performed in everyday Mathematics. Moreover, classical Geometry involves

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the construction and application of a very rigorous and precise taxonomy.

Once again, the notion of equivalence is outstandingly important. This notion refers to the mastering of concepts such as equality, congruence and commensurability. “What it means for separate objects to be equal, similar, the same, homologous, etc. A concept that arises first with arithmetic, in geometry imagination grasps visual or conceptual patterns and compares them for relatedness, equivalence, similarity, etc. Equivalence requires various shades of meaning, since it does not always mean identical. Making these distinctions explicit and precise advances the state of mathematics and sharpens the powers of cognition” (Carson & Rowlands, 2005, p. 5)

Thus, the notion of equivalence makes possible the construction of taxonomies. In Euclid’s *Elements*, very precise taxonomical rules are stated. Furthermore, along its many books, an accurate and meticulous taxonomy is depicted. Geometric style is not only concerned with constructing analogical models but also on deriving relevant characteristics of those models. Such characteristics conform a taxonomical framework and all of the objects studied by Geometry are carefully inserted into this taxonomy. We all know the taxonomy of triangles as stated in *Book I* (Euclid, 300 BC, as translated by Fitzpatrick, 2008):

- “Definition 20. And of the trilateral figures: an *equilateral* triangle is that having three equal sides, an *isosceles* (triangle) that having only two equal sides, and a *scalene* (triangle) that having three unequal sides.” (p. 6)
- “Definition 21. And further of the trilateral figures: a *right-angled* triangle is that having a right angle, an *obtuse-angled* (triangle) that having an obtuse angle, and an *acute-angled* (triangle) that having three acute angles.” (p. 7)

All along *The Elements*, comparison rules are established and used to construct a general taxonomy. Every geometric object mentioned and constructed in *The Elements* is also carefully inserted in this general taxonomy. Here are some examples (Euclid, 300 BC, as translated by Fitzpatrick, 2008):

- All the *Definitions* in *Book I* are taxonomical rules (it is important to understand that nomenclature rules *always* conform a taxonomy). *E.g.*, "*Definition 22*. And of the quadrilateral figures: a *square* is that which is right-angled and equilateral, a *rectangle* that which is right-angled but not equilateral, a *rhombus* that which is equilateral but not right-angled, and a *rhomboid* that having opposite sides and angles equal to one another which is neither right-angled nor equilateral. And let quadrilateral figures besides these be called *trapezia*." (p. 7)
- *Postulates* and *Common notions*, also in *Book I*, include equivalence and comparison rules. *E.g.*, "*Common notion 1*. Things equal to the same thing are also equal to one another." (p. 7)
- Propositions in *Book I* incorporate taxonomic results. *E.g.*, "*Proposition 38*. Triangles which are on equal bases and between the same parallels are equal to one another)." (p. 39)
- *Book II* establishes geometric equivalences between different algebraic identities. *E.g.*, "*Proposition 4.* If a straight-line is cut at random then the square on the whole (straight-line) is equal to the (sum of the) squares on the pieces of the straight-line), and twice the rectangle contained by the pieces." (p. 52)
- Whole *Book VI* deals with problems of similarities and proportional relations among geometric figures. This book includes fundamental theorems of similarity. *E.g.*, "*Proposition 19.* Similar triangles are to one another in the squared ratio of (their) corresponding sides. [...] *Corollary.* So it is clear, from this, that if three straight-lines are proportional, then as the first is to the third, so the figure (described) on

the first (is) to the similar, and similarly described, (figure) on the second. (Which is) the very thing it was required to show." (p. 175, 176)

- Taxonomic rules for numbers, based on their geometric characteristics, can be found in *Book VII*. *E.g.*, *"Definition 16.* And when two numbers multiplying one another make some (other number) then the (number so) created is called *plane*, and its sides (are) the numbers which multiply one another." (p. 194)
- *Book X* is devoted entirely to the taxonomy of incommensurable magnitudes (*i.e.*, irrational numbers).
- Other books from *The Elements* also include taxonomic rules and outcomes. *E.g.*, *Book V* discusses abstract proportions; *Book XII* deals with measurement by using notions of proportionality.

Therefore, Geometry involves the study of a wide diversity of objects (*e.g.*, lines, angles, figures, solids, numbers), including all of their possible variations. A very important feature of studying such objects is ordering them by comparison, which is to say, ordering them according to a specific taxonomy.

Moreover, understanding geometric objects as analogical models reveals the usefulness of the taxonomy described in *The Elements*. For example, given *any* triangular shape imposed as a model over *any* land field or building, it will be enough to measure few relevant characteristics in order to classify this triangle, and thanks to its classification it will be possible to derive all of its other characteristics without actually measuring them (in many cases, it may be practically impossible to measure all of the characteristics of interest). The key idea to be noticed here is that this example is applicable to *any* given triangle under *any* imaginable circumstances, so it becomes evident that the great power of applicability of Geometry arises from its taxonomical rules.

It is not that Geometry uses sometimes a taxonomical style of thinking, but taxonomical rules are the very fabric upon which Geometry is built upon. In other words, it is not the case that taxonomical ideas were included in *The Elements* and crystallized two thousand years later with Biology. Geometric style involves a very rigorous and advanced taxonomy, and this taxonomy is fundamental for the entire discipline of Geometry.

Furthermore, I dare to propose that the taxonomic style is embedded in the geometric one. And the use of a taxonomic style in other disciplines, like in Biology and Chemistry, is not due to historically contingent conditions that allowed the emergence of a new style of reasoning. Rather it seems that, such historically contingent conditions allowed the application of the geometric style, and its embedded taxonomical thinking, upon new subject topics like living organisms, chemical elements and stars.

Discussion

In this paper I have shown relevant overlapping between four styles of scientific reasoning: geometric, modeling, experimental and taxonomical. Given the importance of the overlapped characteristics, which can be regarded as foundational basis for all the discussed styles, I have also proposed that these four styles are not distinct and all of them are embedded in classical Geometry (as portrayed in Euclid's *Elements*).

As I explained in the third section of this paper, effectively combining several styles of scientific thinking might be regarded as an advancement of Crombie's later insight (cfr. Crombie 1994, p. 1087). It can also be seen as a reasonable application of Occam's maxim, which Hacking did not dare to apply to his "sacred" list of scientific styles (cfr. Hacking, 2009, p. 100).

There are many lines of future investigation deriving from this paper. The first one, as I have already stated, is that it is possible to show, through detailed research and accurate understanding of Geometry, that this discipline is not merely one among several scientific styles, but a foundational way of thinking. Moreover,

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that it was later applied to other fields of study, and then produced scientific advancements that are now regarded as a "crystallization" of the modeling, experimental and taxonomical styles.

A second line of future investigation is the existence of possible overlaps between Geometry and the other two remaining styles. There are some obvious connections as the formalization of probabilistic thinking through a system of axioms and mathematical proofs. But there are some other overlapping areas that demand a good understanding of mathematics, as the study of the rigorous taxonomy constructed via probability distributions.

From the advancement of those two lines of investigation, a more general and ambitious hypothesis might be pursued. And that would be rebutting the idea of specific beginnings, in human history, of distinct scientific styles, as Hacking (2009) believes: "But scientific styles are themselves the product of cultural innovation and evolution. Much of this has happened in the Mediterranean regions—North Africa, West Asia and Greece— and later in Europe. *Each has a beginning in history*, which sometimes exists chiefly in the form of legend, and each has its own trajectory of development" (emphasis added; p. 48).

This later and more general line of investigation would be sustained by the fact that a geometric style of thinking seems universal. Geometry, as a discipline of study, can be found in all ancient human cultures:

> The origins of geometry are very ancient (it is probably the oldest branch of mathematics) with several ancient cultures (including Indian, Babylonian, Egyptian, and Chinese, as well as Greek) developing a form of geometry suited to the relationships between lengths, areas, and volumes of physical objects. In these ancient times, geometry was used in the measure of land (or, as we would say today, surveying) and in the construction of religious and cultural artifacts. Examples include the Hindu Vedas, thought to have been composed between 4000 BCE to 3100 BCE, the ancient Egyptian pyramids, Celtic knots, and many more examples. (Jones, 2002, p. 122).

If it can be proved that Geometry is the keystone for all others supposedly distinct scientific styles, then all human cultures have (or had) the potential to develop other scientific disciplines. That is, the potential to apply mathematical reasoning to new subject matters. And the realization of such potential would depend on historically contingent reasons, which might have furthered or prevented such independent applications. "The story is told that the Greek philosopher Aristippus and some friends were shipwrecked on what appeared to be a deserted island near Rhodes. The company was downcast at its ill fortune when Aristippus noticed some geometric diagrams drawn on the beach sand. He told his companions: Be of good cheer, I see traces of civilized man" (Kline, 1981, p. 73).

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