A Fuzzy Group Prioritization Method for Deriving Weights and its Software Implementation

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***Abstract* — Several Multi-Criteria Decision Making (MCDM) methods involve pairwise comparisons to obtain the preferences of decision makers (DMs). This paper proposes a fuzzy group prioritization method for deriving group priorities/weights from fuzzy pairwise comparison matrices. The proposed method extends the Fuzzy Preferences Programming Method (FPP) by considering the different importance weights of multiple DMs. The elements of the group pairwise comparison matrices are presented as fuzzy numbers rather than exact numerical values, in order to model the uncertainty and imprecision in the DMs' judgments. Unlike the known fuzzy prioritization techniques, the proposed method is able to derive crisp weights from incomplete and fuzzy set of comparison judgments and does not require additional aggregation procedures. A prototype of a decision tool is developed to assist DMs to implement the proposed method for solving fuzzy group prioritization problems in MATLAB. Detailed numerical examples are used to illustrate the proposed approach.**

***Keywords* — Fuzzy Non-linear Programming, Fuzzy Preferences Programming Method, Multiple Criteria Decision Making, Triangular Fuzzy Numbers.**

I. Introduction

THERE are various techniques for deriving priorities/weights for decision elements (e.g. attributes/criteria) from a decision maker (DM) or group of DMs, some of which are reviewed by Choo and Wedley [1] and Ittersum et al. [2]. Most techniques are based on either direct weighting or on pairwise comparison. In direct weighting, the DM is directly asked to give values between 0 and 1 to each decision element to assign their importance. Some methods for deriving attributes/criteria weights by direct assigning techniques are: the Simple Multi-Attribute Rating Technique (SMART) [3], SWING weighting methods [4], and SMART Exploiting Ranks (SMARTER) [5].

When the DM or the group of DMs are unable to directly assign decision elements' weights, the Pairwise Comparison (PC) method proposed in [6] can be used.

Psychological experiments have shown that weight derivation from PC is much more accurate than direct weighting [7]. Therefore, the PC methods are often used as an intermediate step in many MCDM methods, as Analytic Hierarchy Process (AHP) [7], Analytic Network Process (ANP) [8], PROMETHEE [9], and Evidential Reasoning (ER) [10].

The PC methods require construction of Pairwise Comparisons Judgment Matrices (PCJMs). In order to construct a PCJM, the DM is asked to compare pairwisely any two decision elements and provide a numerical/linguistic judgment for their relative importance. Thus, the DM gives a set of ratio judgments to indicate the strength of his/her preferences, which are structured in a reciprocal PCJM. Then, the weights or priority vectors of the decision elements can be derived from the PCJM by applying some prioritization methods.

There are numerous Pairwise Comparisons Prioritization Methods (PCPMs), such as the Eigenvector Method [7], the Direct Least Squares Method [11], the rank-ordering method [7], the Logarithmic Least Square Method [12], and the Fuzzy Programming Method [13]. Choo and Wedley [1] summarised and analysed 18 PCPMs for deriving a priority vector from PCJMs. They discussed that no method performs best in all situations and no method dominates the other methods.

However, in many practical cases, in the process of prioritization the DMs are unable to provide crisp values for comparison ratios. A natural way to deal with the uncertainty and imprecision in the DMs' judgments is to apply the fuzzy set theory [14] and to represent the uncertain DMs' judgments as fuzzy numbers. Thus, Fuzzy PCJMs can be constructed and used to derive the priority vectors by applying some Fuzzy PCPMs. Such methods are proposed by Laarhoven and Pedrycz's [15], Buckley [14], Chang [16] and Mikhailov [17], and applied for group decision making.

The existing fuzzy PCPMs have some drawbacks. They require an additional defuzzification procedure to convert fuzzy weights into crisp (non-fuzzy) weights. However, different defuzzification procedures will often give different solutions [17].

The linear and non-linear versions of the Fuzzy Preference Programming (FPP) method [17] do not require such defuzzification procedures, but their modifications for group decision making situations assume that all the DMs have the same weight of importance. However, in real group decision making problems, sometimes some experts are more

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experienced than others [18-19]. Therefore, the final results should be influenced by the degree of importance of each DM.

In order to overcome some of the limitations of the group FPP method, a new group version of the FPP method is proposed by introducing importance weights of DMs in order to derive weights for decision elements in group decision problems. The proposed method has some attractive features. It does not require any aggregation procedures. It does not require a defuzzification procedure. It derives crisp priorities/weights from an incomplete set of fuzzy judgments and incomplete fuzzy PCJMs. Moreover, the proposed method considers the DMs weights.

For applying the proposed method and solving prioritisation problems, a Non-Linear FPP Solver is developed based on the Optimization Toolbox of MATLAB, in order to overcome the complexity of programming. This decision tool is demonstrated by solving a few numerical examples.

The remainder of this paper is organised as follows. In Section II, representation of the fuzzy group prioritization problem is briefly explained. Then, the proposed method is presented in Section III and illustrated by numerical examples in section IV. The developed Non-Linear FPP Solver is presented in section V, followed by conclusions.

II. Representation of the Fuzzy Group Prioritization Problem

Consider a group of K DMs ($DM_k, k = 1,2,..., K$) that evaluate n elements $\boldsymbol{E_1,..E_n}$ (in MCDM, these elements could be clusters, criteria, sub-criteria or alternatives). With respect to some fixed preference scales, each DM assesses the relative importance of any two elements (E_i, E_j) $(i, j = 1,2,..,n)$ by providing a ratio judgment a_{ijk}, specifying by how much E_i is preferred/not preferred to E_j.

In a fuzzy environment, suppose that each DM provides a set of y fuzzy comparison judgements $A^k = \{\tilde{a}_{ijk}\}$, $y \le n(n-1)/2$, where $i = 1,2,.., n-1$, $j \succ i$, $j = 2,3,.....n$, $k = 1,2,.., K$ and those judgments are represented as Triangular Fuzzy Numbers (TFNs) $\tilde{a}_{ijk} = (l_{ijk}, m_{ijk}, u_{ijk})$, where l_{ijk}, m_{ijk} and u_{ijk} are the lower bound, the mode and the upper bound, respectively. Fig. 1 shows the TFN $\tilde{a}_{ijk} = (l_{ijk}, m_{ijk}, u_{ijk})$.

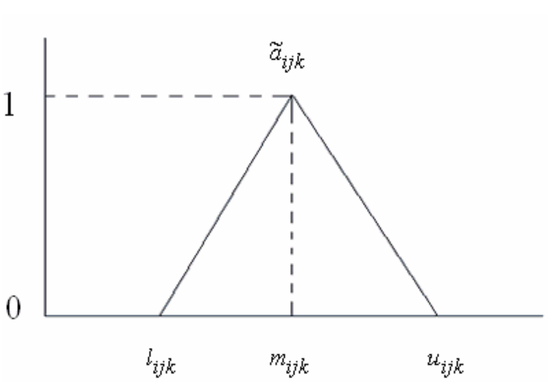

Fig. 1. Triangular Fuzzy Number $\tilde{a}_{ijk} = (l_{ijk}, m_{ijk}, u_{ijk})$

The set A^k can be used to form a Fuzzy PCJM of the form (1):

$$A^k = \begin{bmatrix} (1,1,1) & (l_{12k}, m_{12k}, u_{12k}) & .... & (l_{1jk}, m_{1jk}, u_{1jk}) \\ (l_{21k}, m_{21k}, u_{21k}) & (1,1,1) & .... & (l_{2jk}, m_{2jk}, u_{2jk}) \\ ... & ... & .... & ... \\ (l_{i1k}, m_{i1k}, u_{i1k}) & (l_{i2k}, m_{i2k}, u_{i2k}) & .... & (1,1,1) \end{bmatrix} \quad (1)$$

Then, the fuzzy group prioritisation problem is to determine a crisp priority vector (crisp weights) $w = (w_1, w_2, ..., w_n)^T$ from all A^k, $k = 1,2,\ldots,K$, which represents the relative importance of the n elements.

III. Group Fuzzy Preference Programming Method

The non-linear FPP method [17] derives a priority vector $w = (w_1, w_2, ..., w_n)^T$, which satisfies:

$$l_{ij} \tilde{\le} w_i / w_j \tilde{\le} u_{ij} \quad (2)$$

where $\tilde{\le}$ denotes 'fuzzy less or equal to'. If M is the overall number of fuzzy group comparison judgments, then $2M$ fuzzy constraints of the type (3) are obtained.

$$\begin{aligned} -w_i + w_j l_{ij} &\tilde{\le} 0 \\ w_i - w_j u_{ij} &\tilde{\le} 0 \end{aligned} \quad (3)$$

For each fuzzy judgment, a membership function, which represents the DMs' satisfaction with different crisp solution ratios, is introduced:

$$\mu_{ij}(w_i/w_j) = \begin{cases} \dfrac{(w_i/w_j) - l_{ij}}{m_{ij} - l_{ij}}, & w_i/w_j \le m_{ij} \\ \dfrac{u_{ij} - (w_i/w_j)}{u_{ij} - m_{ij}}, & w_i/w_j \ge m_{ij} \end{cases} \quad (4)$$

The solution to the prioritization problem by the FPP method is based on two assumptions. The first, requires the existence of a *non-empty fuzzy feasible area* $\tilde{P}$ on the $(n-1)$ dimensional simplex Q^{n-1},

$$Q^{n-1} = \{(w_1, w_2, ..., w_n), w_i \succ 0, \sum_{i=1}^{n} w_i = 1\} \quad (5)$$

The fuzzy feasible area $\tilde{P}$ is defined as an intersection of the membership functions (4). The membership function of the fuzzy feasible area $\tilde{P}$ is given by:

$$\mu_{\tilde{P}}(w) = [Min\{\mu_1(w), \mu_2(w), \ldots \mu_{2M}(w)\} \backslash \sum_{i=1}^{n} w_i = 1] \tag{6}$$

The second assumption identifies a selection rule, which determines a priority vector, having the highest degree of membership in the aggregated membership function (6). Thus, there is *a maximizing solution* w^* (a crisp priority vector) that has a maximum degree of membership λ^* in $\tilde{P}$, such that :

$$\lambda^* = \mu_{\tilde{P}}(w^*) = Max[Min\{\mu_1(w), \ldots, \mu_{2M}(w)\} \backslash \sum_{i=1}^{n} w_i = 1] \tag{7}$$

A new decision variable λ is introduced which measures the maximum degree of membership in the fuzzy feasible area $\tilde{P}$. Then, the optimization problem (7) is represented as

$$\begin{aligned} & Max \quad \lambda \\ & s.t. \\ & \lambda \le \mu_{ij}(w) \\ & \sum_{i=1}^{n} w_i = 1, \quad w_i \succ 0, \\ & i = 1,2,\ldots, n, \quad j = 1,2,..n, \quad j \succ i \end{aligned} \tag{8}$$

The above max-min optimization problem (8) is transformed into the following non-linear optimization problem:

$$\begin{aligned} & Max \quad \lambda \\ & s.t. \\ & (m_{ij} - l_{ij})\lambda w_j - w_i + l_{ij} w_j \le 0 \\ & (u_{ij} - m_{ij})\lambda w_j + w_i - u_{ij} w_j \le 0 \\ & i = 1,2,\ldots n-1; \quad j = 2,3,..n \;\; ; \; j \succ i \; ; \\ & \textstyle\sum_{i=1}^{n} w_i = 1 \; ; \quad w_i \succ 0; \quad i = 1,2,\ldots, n \end{aligned} \tag{9}$$

The non-linear FPP method can be extended for solving group prioritization problems. Mikhailov *et al.* [20] proposed a Weighted FPP method to the fuzzy group prioritization problem by introducing the importance weights of DMs. However, the Weighted FPP method requires an additional aggregation technique to obtain the priority vector at different α - thresholds. Consequently, this process is time consuming, due to several computation steps needed for applying the α - threshold concept. Therefore, this paper modified the non-linear FPP method [17], which can derive crisp weights without using α - threshold and by introducing the DMs' importance weights.

When we have a group of K DMs, the problem is to derive a crisp priority vector, such that priority ratios w_i / w_j are approximately within the scope of the initial fuzzy judgments a_{ijk} provided by those DMs, i.e.

$$l_{ijk} \mathrel{\tilde{\le}} w_i / w_j \mathrel{\tilde{\le}} u_{ijk} \tag{10}$$

The ratios w_i / w_j can also express the satisfaction of the DMs, as the ratios explain how similar the crisp solutions are close to the initial judgments from the DMs.

The inequality (10) can be represented as two single-side fuzzy constraints of the type (3):

$$\begin{aligned} & R_q{}^k W \mathrel{\tilde{\le}} 0, \\ & k = 1,.., K \;,\; q = 1,2,..2M_k \end{aligned} \tag{11}$$

The degree of the DMs' satisfaction can be measured by a membership function with respect to the unknown ratio w_i / w_j :

$$\mu_q{}^k (R_q{}^k W) = \begin{cases} \dfrac{\left(w_i^k / w_j^k\right) - l_{ijk}}{m_{ijk} - l_{ijk}} & , w_i^k / w_j^k \le m_{ijk} \\ \dfrac{u_{ijk} - \left(w_i^k / w_j^k\right)}{u_{ijk} - m_{ijk}} & , w_i^k / w_j^k \ge m_{ijk} \end{cases} \tag{12}$$

We can define K fuzzy feasible areas, $\tilde{P}_k$, as an intersection of the membership functions (12) corresponding to the k -th DMs' fuzzy judgments and define the group fuzzy feasible area $\tilde{P} = \bigcap \tilde{P}_k$.

By introducing a new decision variable λ_k , which measures the maximum degree of membership of a given priority vector in the fuzzy feasible area $\tilde{P}_k$, we can formulate a max-min optimisation problem of the type (8), which can be represented into:

$$\begin{aligned} & Max \quad \lambda_k \\ & s.t. \\ & \lambda_k \le \mu_q{}^k (R_q{}^k W) \\ & \sum_{i=1}^{n} w_i = 1; \quad w_i \succ 0, \\ & i = 1,2,\ldots,n; \quad k = 1,.., K; \quad q = 1,2,..2M_k \end{aligned} \tag{13}$$

For introducing the DMs' importance weights, let us define I_k as the importance weight of the $DM_k; k = 1,2,\ldots, K$. For aggregating all individual models of type (13) into a single group model, a weighted additive goal-programming (WAGP) model [21] is applied.

The WAGP model transforms the multi-objective decision making problem to a single objective problem. Therefore, it can be used to combine all individual models (13) into a new single model by taking into account the DMs' importance weights.

The WAGP model considers the different importance weights of goals and constraints and is formulated as:

$$\begin{aligned} & \mu_D(x) = \textstyle\sum_{s=1}^{p} \alpha_s \mu_{z_s}(x) + \sum_{r=1}^{h} \beta_r \mu_{g_r}(x) \\ & \textstyle\sum_{s=1}^{p} \alpha_s + \sum_{r=1}^{h} \beta_r = 1 \end{aligned} \tag{14}$$

Where:

μ_{z_S} are membership functions for the p–th fuzzy goal z_S, $s = 1,2,\dots p$;

μ_{g_r} are membership functions of the h-th fuzzy constraints $g_r, r = 1,2,..h$;

x is the vector of decision variables;

α_s are weighting coefficients that show the relative important of the fuzzy goals;

β_r are weighting coefficients that show the relative important of the fuzzy constraints.

A single objective model in WAGP is the maximisation of the weighted sum of the membership functions μ_{z_S} and μ_{g_r}. By introducing new decision variables λ_s and γ_r, the model (14) can be transformed into a crisp single objective model, as follows:

$$
\begin{aligned}
& Max \ \ \textstyle\sum_{s=1}^{p}\alpha_s\lambda_s + \sum_{r=1}^{h}\beta_r\gamma_r \\
& s.t. \\
& \lambda_s \le \mu_{z_s}(x), \qquad s = 1,2,\dots p \\
& \gamma_r \le \mu_{g_r}(x), \qquad r = 1,2,\dots h \\
& \textstyle\sum_{s=1}^{p}\alpha_s + \sum_{r=1}^{h}\beta_r = 1 \\
& \lambda_s, \gamma_r \in [0,1]; \qquad \alpha_s, \beta_r \ge 0
\end{aligned}
$$
(15)

In order to derive a group model, where the DMs have different importance weights, we exploit the similarity between the models (13) and (15). However, the non-linear FPP model (13) does not deal with fuzzy goals; it just represents the non-linear fuzzy constraints. Thus, by taking into account the specific form of $R_q^{\ k}W \,\tilde{\le}\, 0$ and introducing the importance weights of the DMs, the problem can be further presented into a non-linear program by utilising the WAGP model as:

$$
\begin{aligned}
& Max \ \ Z = \textstyle\sum_{k=1}^{K} I_k\lambda_k \\
& s.t. \\
& (m_{ijk} - l_{ijk})\lambda_k w_j - w_i + l_{ijk}w_j \le 0 \\
& (u_{ijk} - m_{ijk})\lambda_k w_j + w_i - u_{ijk}w_j \le 0 \\
& i = 1,2,\dots n-1; \quad j = 2,3,..n \ ; \\
& j \succ i \ ; \ k = 1,2,\dots K; \\
& \textstyle\sum_{i=1}^{n} w_i = 1 \ ; \quad w_i \succ 0; \qquad i = 1,2,\dots, n
\end{aligned} \tag{16}
$$

Where the decision variable λ_k measures the degree of the DM's satisfaction with the final priority vector $w = (w_1, w_2, \dots\dots, w_n)^T$; I_k denotes the importance weight of the k-th DM, $k = 1,2,..K$.

In (16), the value of Z can be considered as a consistency index, as it measures the overall consistency of the initial set of fuzzy judgments. When the set of fuzzy judgments is consistent, the optimal value of Z is greater or equal to one. For the inconsistent fuzzy judgments, the maximum value of Z takes a value less than one.

For solving the non-linear optimization problem (16), an appropriate numerical method should be employed. In this paper, the solution is obtained by using MATLAB Optimization Toolbox and a Non-linear FPP solver is developed to solve the prioritization problem.

IV. Illustrative Examples

The first example illustrates the solution to the fuzzy group prioritization problem for obtaining a priority vector and a final group ranking. The second example demonstrates how the importance weights of DMs influence the final group ranking.

A. Example 1

This example is given to illustrate the proposed method and also the solution by using the Non-linear FPP Solver.

We consider the example in [20], where three DMs ($K = 3$) assess three elements ($n = 3$) and the importance weights of DMs are given as: $I_1 = 0.3$; $I_2 = 0.2$; $I_3 = 0.5$.

The DMs provide an incomplete set of five fuzzy judgments, presented as TFNs:

DM 1: $a_{121} = (1,2,3)$; $a_{131} = (2,3,4)$.

DM 2: $a_{122} = (1.5,2.5,3.5)$; $a_{132} = (3,4,5)$.

DM 3: $a_{123} = (2,3,4)$.

The group fuzzy prioritization problem is to derive a crisp priority vector $w = (w_1, w_2, w_3)^T$ that approximately satisfies the following fuzzy constraints:

For DM 1: $1 \,\tilde{\le}\, w_1/w_2 \,\tilde{\le}\, 3$; $2 \,\tilde{\le}\, w_1/w_3 \,\tilde{\le}\, 4$.

For DM 2: $1.5 \,\tilde{\le}\, w_1/w_2 \,\tilde{\le}\, 3.5$; $3 \,\tilde{\le}\, w_1/w_3 \,\tilde{\le}\, 5$.

For DM 3: $2 \,\tilde{\le}\, w_1/w_2 \,\tilde{\le}\, 4$.

Using the above data and the non-linear model (16), the following formulation is obtained:

$$
\begin{aligned}
&Max\ Z = 0.3\lambda_1 + 0.2\lambda_2 + 0.5\lambda_3 \\
&s.t. \\
&\lambda_1 w_2 - w_1 + w_2 \le 0 \\
&\lambda_1 w_2 + w_1 - 3w_2 \le 0 \\
&\lambda_1 w_3 - w_1 + 2w_3 \le 0 \\
&\lambda_1 w_3 + w_1 - 4w_3 \le 0 \\
&\lambda_2 w_2 - w_1 + 1.5w_2 \le 0 \\
&\lambda_2 w_2 + w_1 - 3.5w_2 \le 0 \\
&\lambda_2 w_3 - w_1 + 3w_3 \le 0 \\
&\lambda_2 w_3 + w_1 - 5w_3 \le 0 \\
&\lambda_3 w_2 - w_1 + 2w_2 \le 0 \\
&\lambda_3 w_2 + w_1 - 4w_2 \le 0 \\
&w_1 + w_2 + w_3 = 1 \\
&w_1 \ge 0, \quad w_2 \ge 0, \quad w_3 \ge 0
\end{aligned} \tag{17}
$$

Regarding the judgments of this example, the results have been conducted by the Non-Linear FFP Solver. The solution to the non-linear problem (17) is:

$w_1 = 0.621$, $w_2 = 0.212$, $w_3 = 0.167$.

This solution can be compared with the crisp results from the example in [20] as shown in Table I. We may observe that we have the same final ranking $w_1 \succ w_2 \succ w_3$, from applying the two different prioritization methods. However, the Weighted FPP method [20] applies an aggregation procedure for obtaining the crisp vector from different values of priorities at different α - threshold. While, the proposed non-linear group FPP method does not require an additional aggregation procedure.

If the third DM, who has the highest important weight, provides a new fuzzy comparison judgment $a_{323} = (1,2,3)$, which means that the third element is about two times more important than the second element, the weights obtained by using the proposed Non-Linear FFP method are: $w_1 = 0.538$, $w_2 = 0.170$, $w_3 = 0.292$ and the final ranking is $w_1 \succ w_3 \succ w_2$. Consequently, it can be observed that the third DM's judgments strongly influence the final ranking.

However, if the importance weight of the third DM is lower than the first two DMs' weights, then the new fuzzy comparison judgment does not change the final ranking. Thus, we can notice the significance of introducing importance weights of the DMs to the fuzzy group prioritization problem.

TABLE I
RESULTS FROM THE TWO PRIORITIZATION METHODS

Methods	w_1	w_2	w_3
Weighted FPP method [a]	0.615	0.205	0.179
Non-linear FPP method [b]	0.623	0.216	0.161

[a] The method proposed in [16] with applying α - threshold.
[b] The method proposed in this paper without applying α - threshold.

The computation time of the proposed method has been investigated by using the Non-Linear FFP Solver. It was found that the group non-linear FFP method performs significantly faster compared to the Weighted FPP [20] with different α - thresholds ($\alpha = 0, 0.2, 0.5, 0.8, 1$), as seen in Fig. 2.

We can conclude that the average computation time (Minutes) for the Weighted FPP method highly increases as the number of decision elements n increases, compared with the proposed method. Hence, these results showed that the method proposed in this paper is more efficient, with respect to the computation time. Therefore, the proposed method in this paper demands less computation time than the Weighted FPP method [20].

B. Example 2

This example shows that the importance weights of the DMs influence the final group ranking.

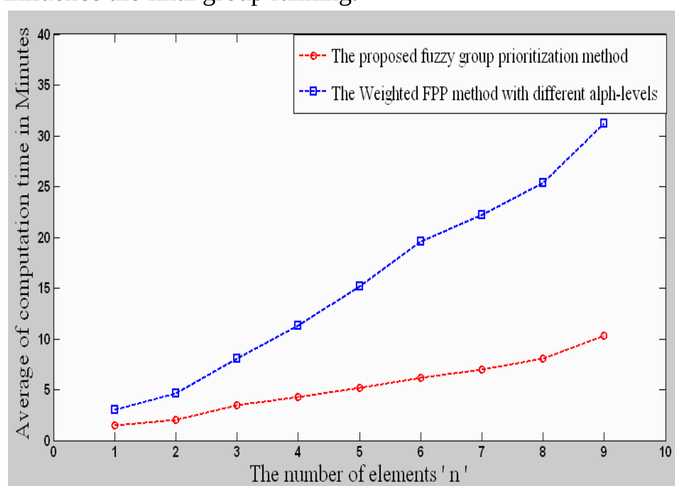

Fig. 2. Average Computation Time (Minutes)

Consider that two DMs ($K = 2$) assess three criteria ($n = 3$). The DMs provide an incomplete set of four fuzzy judgments ($m = 4$) presented as TFN:

DM 1: $a_{121} = (1,2,3)$; $a_{131} = (2,3,4)$.

DM 2: $a_{212} = (3,4,5)$; $a_{312} = (2,3,4)$.

Two situations are investigated when both DMs have the following different weights:

1. $I_1 = 0.2$, $I_2 = 0.8$
2. $I_1 = 0.8$, $I_2 = 0.2$

For both situations, the final rankings for both individual DMs are shown in Tables II and III respectively. The final group rankings are also shown in Tables II and III (the third row of each table). The results are obtained by using the Non-Linear FFP Solver. Each final group ranking is obtained by solving a non-linear program of type (15), which includes eight non-linear inequality constraints corresponding to the given DMs' fuzzy comparison judgements.

It can be observed from Tables II and III that the final group ranking tends to be the individual ranking of the DM who has the highest importance weights. In more detail, it can be seen from Table II that the judgements of the second DM with the highest importance weight ($I_2 = 0.8$) influence, more strongly, the final group ranking. On the other hand, the final group ranking in Table III is dependent on the first DM, who has the highest importance weight ($I_1 = 0.8$).

From examples 1 and 2, we can observe the importance of introducing importance weights of the DMs to the fuzzy group prioritisation problem. It is seen that the final group ranking depends on the DMs' importance weights.

V. Software Implementation Using MATLAB

MATLAB is a numerical computing environment, which allows matrix manipulations, plotting of functions and data, implementation of algorithms, creation of user interfaces, and interfacing with programs written in other languages, including

TABLE II

Individual and Group Results ($I_1 = 0.2$, $I_2 = 0.8$)

DMs	w_1	w_2	w_3	Final ranking
DM 1	0.545	0.273	0.182	$w_1 \succ w_2 \succ w_3$
DM 2	0.117	0.530	0.353	$w_2 \succ w_3 \succ w_1$
Group	0.117	0.529	0.354	$w_2 \succ w_3 \succ w_1$

TABLE III

Individual and Group Results ($I_1 = 0.8$, $I_2 = 0.2$)

DMs	w_1	w_2	w_3	Final ranking
DM 1	0.545	0.272	0.181	$w_1 \succ w_2 \succ w_3$
DM 2	0.117	0.530	0.353	$w_2 \succ w_3 \succ w_1$
Group	0.402	0.397	0.201	$w_1 \succ w_2 \succ w_3$

C, C++, Java, etc. [22]. This development environment includes many functions for statistics, optimization, and numeric data integration and filtering [23].

In this paper, we use the Optimization Toolbox and the Graphical User Interface (GUI) of MATLAB as the development tools for implementing the proposed group non-linear FPP method, because these tools provide powerful numerical functions, optimisation procedures, good visualisation capabilities and programming interfaces.

Essentially, there are three steps for programming and developing the Non-Linear FFP solver:

Step 1: Coding the model into the system. A number of functions are available in the Optimization Toolbox-MATLAB to solve the non-linear programming problem. In our prototype, the optimisation problem is solved using the sequential quadratic programming procedure [19].

Step 2: Creating a basic user interface. In this step, the interface is designed, so that it can run in the MATLAB command window. The aim of this user interface is to obtain the input information from the DMs.

Step 3: Developing the system based on the GUI functions. In this step, the MATLAB GUI functions are employed to develop a more user-friendly system.

Regarding the given data in example 1, the input information which should be acquired includes the total number of decision elements, the names of these elements and the total number of DMs, as shown in Fig.3. Then, the pairwise judgments for each DM can be entered by the user, as illustrated in Fig. 4. According to example 1, the fuzzy judgments for the DM 1 are illustrated in Fig. 4. Thus, the main feature in the developed interface is that the user can input the fuzzy judgments into the system directly and easily.

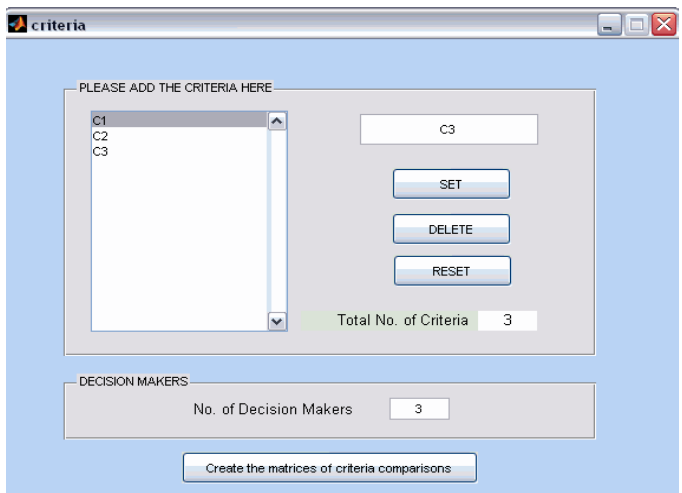

Fig. 3. The criteria setting window

Fig. 4. The fuzzy comparison judgments window for the DM 1

However, if the user is unable to provide fuzzy comparison judgments between two elements, then he/she can click on the '**Missing Data**' button and the system temporarily puts -1 for this comparison. The negative value is not a true judgment in the real world; it just indicates that those elements should not be included in the further calculations. For instance, in the given example, the judgment a_{231} is missing for DM1 and it is recorded as $(-1,-1,-1)$ in Fig. 4.

After entering the fuzzy judgments from all DMs, the user can set the DMs' importance weights into the system. According to the given data in example 1, the importance weights of the three DMs are entered, as shown in Fig. 5.

Finally, the Solver finds the optimal solution and visualises it graphically – Fig. 6.

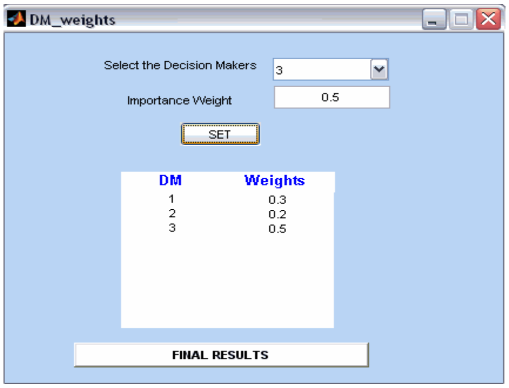

Fig. 5. The DMs' importance weights window

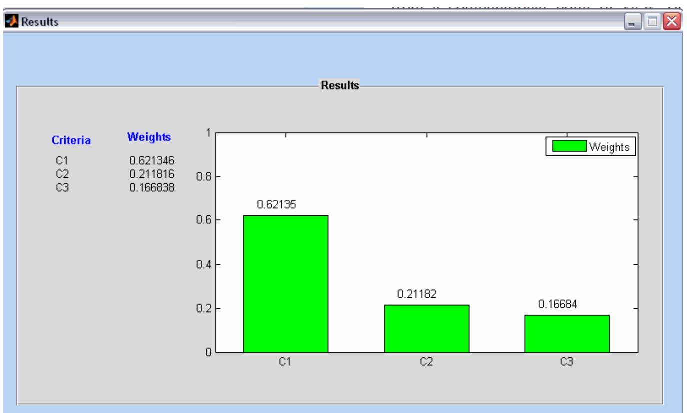

Fig. 6. The results from the Non-Linear FFP Solver

VI. CONCLUSIONS

This paper proposes a new method for solving fuzzy group prioritisation problems. The non-linear FPP is modified for group decision making by introducing DMs' importance weights. The proposed method derives crisp priorities/weights from a set of fuzzy judgements and it does not require defuzzification procedures. Moreover, the proposed method is capable of deriving crisp priorities from an incomplete set of DMs' fuzzy pairwise comparison judgments. Comparing with the Weighted FPP method, the proposed method is efficient from a computational point of view. Hence, the proposed method is a promising and attractive alternative method to existing fuzzy group prioritisation methods.

Another contribution of this study is the development of a Non-Linear FPP Solver for solving group prioritisation problems, which provides a user-friendly and efficient way to obtain the group priorities.

Future work includes presenting the importance weights for the DMs as fuzzy numbers, not just as crisp numbers, in order to model the uncertain importance weights of DMs. Moreover, we would like to incorporate the proposed method into other MCDM methods such as the Fuzzy Analytic Hierarchy Process, the Fuzzy Analytic Network Process and the Evidential Reasoning approach for complex decision problem analysis.

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