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Logical and Linguistic Games from Peirce to Grice to Hintikka

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RESUMEN

La concepción de la lógica y del pragmaticismo de Peirce es casi la de la semántica de teoría de juegos de Hintikka, incluida la idea de jugadores "simulados en nuestra imaginación". Peirce estaba interesado por la lógica como teoría de la acción estratégica, habitual, convencional y normativa. Más tarde, Grice erigiría su teoría de la conversación sobre fondo peirceano. Pero la cooperación es una propiedad de los juegos de construcción de modelos y un elemento fundamental del método de Peirce. La construcción de modelos cooperativa conlleva el mismo constructo teórico que el de los juegos semánticos estrictamente competitivos. Ambos tipos de juegos, los semánticos y los de construcción de modelos, constituyen las dos caras de una misma moneda conceptual. Los principios generales que gobiernan las prácticas matemáticas están relacionados con las actividades de construcción de modelos.

PALABRAS CLAVE: *semántica de teoría de juegos, pragmaticismo, significado estratégico, construcción de modelos, principio cooperativo, descubrimiento matemático*.

ABSTRACT

Peirce's conception of logic and pragmaticism is virtually that of Hintikka's game-theoretical semantics (GTS), including the idea of players "feigned in our make-believe". Peirce was interested in logic as a theory of normative, conventional, habitual and strategic action. Later Grice erected his theory of conversation on Peircean background. But cooperation is a property of model-building games and an integral part of Peirce's method. Cooperative model-building resorts to the same theoretical construct as the strictly competitive semantic games do. The two kinds of games, the semantic and the model-construction games, are two sides of the same conceptual coin. General principles governing mathematical practices are related to model-building activities.

KEYWORDS: *GTS, Pragmaticism, Strategic Meaning, Model Building, Cooperative Principle, Mathematical Discovery*.

I

Jaakko Hintikka is the undisputed modern architect of *game-theoretical semantics* (GTS), but the spirit and often even the letter of that theory date back to polymath Charles Peirce and his groundbreaking discoveries in the logical studies of the late 19^{th} and the early 20^{th}-century. Peirce took logic to concern the questions of *semeiotic*, the theory of signs, as well as *pragmaticism*, the theory of meaning. His normative conception of the method of logic was not predominantly concerned with formal or deductive aspects of symbolic logic but with the significations of signs involved in that method, that is, with the semantics and pragmatics of game-theoretic conceptualisations [see Hilpinen (1982); Hintikka (1996); Pietarinen (2006, 2011c)].

What is the game-theoretic approach? First, we need the players, the participants of semantic games. According to Peirce, in logic such parties are "feigned in our make-believe" [MS 280, p. 29, 1906]. Peirce's logic is not meant to serve as a formal calculus or pasigraphy, but to be an "aid in the analysis of reasoning" for which certain procedures of "imaginary" parties are called for [MS 1589].[1] Thus players are *theoretical constructs* rather than actual agents partaking in real conversational situations. They are introduced in order to articulate the conceptual workings of one's logical systems. Hand (1989) used to ask, who plays semantical games? In Peirce's writings – some of the most important of which are still unpublished to date – we are able to gather some significant answers to that question.

Second, in Peirce's wide notion of logic, there is room for many kinds of games and moves that the players entertain. We find not only actions pertaining to the *semantics* of logic but also the *assertoric*, *definitory*, *interrogative*, *deductive* and *model-building* moves. However, it is worth acknowledging that he did not always keep these different moves separate from one another. It is through Hintikka's work that we have learned to appreciate the differences as well as the interdependencies of different kinds of moves in our quest for understanding the conceptual underpinnings of logical theories [Hintikka (1995)].

As mentioned, Peirce's logic is grounded on the idea of contemplating there being two "make believe" players. In Peirce's terms, there is the *Graphist* (the Utterer) who "scribes the graphs and proposes modifications to them", and there is the *Interpreter* (the Grapheus) who "authorizes the modifications" [MS 492, 1903]. This somewhat peculiar terminology refers to the diagrammatic logic of Existential Graphs which he began developing in the 1890s and which involves the activities of scribing logical assertions in their diagrammatic forms [Pietarinen (2011a)]. Generally, he designates the parties as the Utterer and the Interpreter. Sometimes he even used the terms the Attacker and the Defender, as the case is in dialogue logics. Just as with one's favourite children Peirce coined a plethora of names, and we also find him talking about the "assertor and critic", "concurrent and antagonist", "speaker and hearer", "addressor and addressee", "Artifex of Nature and Interpreter of Nature", "symboliser and thinker", "scribe and user", "affirmer and denier", "compeller and resister", "agent and patient", "Me and Against-Me", and so forth. And just as is the case with a large family, these names may easily be confused with one another.

The fact that the players have opposing roles is nevertheless clear from his characterisations. The logical graphs scribed by the Graphist are true, Peirce writes, as "the truth of the true consists in his being satisfied with it" [MS 280, p. 29]. This refutes [Hodges (2001)], who claimed that since Peirce did not distinguish the opposite roles of the players he could not have had real semantic games for his logic. This is just ignorance: Peirce is very clear that the Graphist is the supporter of assertions. Ending with true atomic graphs – in Peirce's terms the "indecomposable" elements resembling the notions of abstract algebra – amounts to a win for the Graphist while ending with false ones amounts to a win for the Interpreter. All this is quite evident in his writings [Pietarinen (2006)].

Furthermore, the truth of the whole graph agrees with the existence of a winning strategy for the Graphist, which in Peirce's terms is the being of a *habit of action*. Hintikka (1998) rightly pointed out that Peirce lacked the central notion of game theory, that of a strategy. However, habits of action are for Peirce "generalizing tendencies" which are of "a tolerably stable nature" [MS 280, p. 32]. (The idea of stableness re-emerged in von Neumann's work on game theory, among others.) Likewise, falsity is the existence of such habits for the Interpreter. What else than a game-theoretic solution concept could Peirce have been thinking of here?

II

Peirce developed a theory of the meaning of "intellectual concepts and signs", which he named *pragmaticism*. Semantic and pragmatic considerations go hand in hand in it. Besides an early attempt at the semantics and model-theory for logic, pragmaticism is concerned with strategic aspects of meaning. It has also turned out to also reach beyond a mere theory of meaning of assertions and to imply a pragmatist philosophy of mathematics [Pietarinen (2010)] as well as a general methodology for the social sciences and economics [Pietarinen (2013a)].

Influenced by the publication of the Buchler's *The Philosophy of Peirce* (1940), (1956, 2nd edition) [Pietarinen (2013b)], Paul Grice began working on a theory of meaning. His proposal might more appropriately speaking be termed a theory of *signs* or *signification* for reasons I explain later. In the course of his research Grice came to outline some maxims of conversation as well as the principle of cooperation [Grice (1989)]. They were intended to do largely the same job as what Peirce's pragmaticism had done [Pietarinen (2004)]. Grice's maxims, and especially the maxim of relevance and which Grice terms the maxim of relation, have later on been shown to involve strategic aspects of reasoning about information: agents may aim at maximising linguistic information while minimising the processing effort required in getting at that relevant information [Pietarinen (2005)]. Over the years, Hintikka (1987) has urged logicians and natural language semanticists to develop theories of *strategic meaning* over and above of what is accomplished by the *abstract* (material) *meaning* alone. Some of Grice's maxims have to date indeed been shown to be naturally cast in the terms of strategic interactions [Hintikka (1986); Benz et al. (2005); Clark (2010); Pietarinen (2007a)].

Grice's own attempt remained rather one-sided, however, as it largely ignored the role of the interpreter in deciding on the interpretations of what is relevant in conversational situations [Pietarinen (2004)]. Another shortcoming in Grice's work was pointed out by [Hintikka (1986)], who showed that dialogical models of communicative strategies provide better basis for theories of discourse than Grice's own analyses in which discourse is taken to proceed utterance-by-utterance, neglecting the all-important properties that transpire at the level of entire dialogues such as coherence of assertions.

In Peirce's diagrammatic logic we begin to find some awareness of the importance of strategic aspects of semantic and pragmatic theories. What Peirce terms the *sheet of assertion* represents everything that is well understood to be taken for granted between the two parties.[2] He is interested in *truthful assertions* much more than in the nature of propositions. In some of the late writings he even suggests that the sheet is one of *affirmation* rather than *assertion*, because "whatever state of things you represent on this page, you will be understood to affirm as existing somewhere, or, at least, consistently to make believe to affirm" [MS 650, 1910]. In another place he calls it the sheet of *assent*.[3] At any event, the emphasis here is on assertions that are binding. Utterers are responsible for what they state, scribe or assert. Thus on the sheet only true assertions may be scribed.[4]

Peirce took the notion of the *universe of discourse* essential to any feasible method of logic: "The different points of this sheet shall represent the different possible states of a certain individual subject, it being well-understood between the drawer and the interpreter of the diagram what this subject is. Let this subject be termed the *Universe of Discourse*" [MS 479, 1903]. He conceived its role in logic in predominantly two ways. First, there is the contextualisation: players gain collateral observation and experience by virtue of which communication becomes possible. The "common ground" – again an idea widespread in contemporary pragmatics dating back to Peirce – is built from an endless series of "common familiar knowledge" [MS 614].[5] In interpreting non-logical constants the boundaries of language need to be fixed. We observe that these activities correspond to *definitory moves* of the game.

These activities are followed by model building. Peirce describes it in terms of *collaborative* activities of the Graphist and the Grapheus, the "author of the universe of discourse" [MS 450, 1903]: the Graphist "proposes modifications to the graphs" and the Grapheus "creates the universe" and comes to decide the truth of atomic expressions. The Grapheus does this by either "authorizing" or "refuting" the actions proposed by the Graphist [MS 492]. Interestingly, he held that there is no competition in the description of such activities. The common aim is to agree first on the relevant aspects of the system to be modelled and its properties.

Concerning the parties undertaking the scribing, authorizing and interpreting logical assertions, Peirce assumed that the minds of the Graphist and the Grapheus (and the minds really are not so much the human minds as what he terms the "quasi-minds", anything that can produce and process signs) should be able to control the process as well as to develop the habits of action**.** This is in line with the idea that the players ought to share the essential ingredients of an intelligent mind:

> Now nothing can be controlled that cannot be observed while it is in action. It is therefore requisite that both minds but especially the Graphist-mind should have a power of self-observation. Moreover, control supposes a capacity in that which is to be controlled of acting in accordance with definite general tendencies of a tolerably stable nature, which implies a reality in this governing principle. But these *habits*, so to call them, must be capable of being modified according to some ideal in the mind of the controlling agent; and this controlling agent is to be the very same as the agent controlled; the control extending even to the modes of control themselves, since we suppose that the interpreter-mind under the guidance of the Graphist-mind discusses the rationale of logic itself. Taking all these factors into account, we should come to the same conclusion that common-sense would have jumped to at the outset; namely, that the Graphist-mind and interpreter-mind must have all the characters of personal intellects possessed of moral natures [MS 280].

Here the strategic aspects of reasoning of the two agents – stable general tendencies, modifications of habits of actions, and the meta-logical principle of self-control – are beginning to see the daylight [see Pietarinen (2011b)]. Peirce's note that the parties, though created in our make-believe, nevertheless share "all the characters of personal intellects", which assumes that they are capable not only of controlling their own reasoning but also of entertaining normative ideals in their "quasi-minds".[6]

Peirce describes the interaction between the two parties as follows: "The *grapheus* communicates to the *graphist* from time to time his determinations in regard to the character of the universe. Each such communication *authorizes* the graphist to express it" [MS 492, 1903]. This is consonant with the idea of *interrogating* Nature. The commonplace idea of putting questions to nature is here put in the outfit of communication concerning the determinations of some fact or a law that authorises the interpreter, who perhaps is the scientist, to assert the content of that determination in terms of his or her favourite system of representation. The overall methodological value of thinking in the terms of 'putting questions to Nature' was familiar to Peirce though he did not go on to systematise the idea beyond what is expressed in these couple of pages. Hintikka (2007) presents a comprehensive theory of interrogative games.

Interestingly, though, Peirce continues the previous passage by stating that "an authorization once given is irrevocable: this constitutes the universe to be perfectly *definite*". Being perfectly definite and perfectly determinate are not the same thing, however: "Should the graphist risk an assertion without authorization, he must hope to receive an authorization later; for what never will be authorized is forbidden*:* this constitutes the universe to be perfectly *determinate*" [MS 492]. If it so happens that a modification needs to be made to the asserted graph, it has to be made according to the illative permissions of the system. The modifications that may be made to the assertions once scribed on the sheet of assertion proceed by way of the given sound rules of transformation, that is, they describe the *deductive moves* of the given logic.

Peirce then refers to the Graphist as the one who does all the scribing:

> In our make believe, two parties are feigned to be concerned in all scribing of graphs; the one called the *Graphist*, the other the *interpreter*. Although the sheet that is actually employed may be quite small, we make believe that the so-called sheet of assertion is only a particular region or area of an immense surface, namely that it is the field of 'distinct vision' of the interpreter. It is only the Graphist who has the power to scribe a graph, and the graphs that he scribes are true, because the truth of the true consists in his being satisfied with it. The interpreter, for his part, has the power, with more or less effort, to move the graph-instances over the sheet, out of his field of distinct vision or into it if they are not quite out of his sight [MS 280].

What is the reasonable interpretation of the key idea of what Peirce is attempting to illustrate here? It is a little hard to see what kinds of moves at the end are involved here, as his way of setting up the numerous conventions for his systems of logic is far from customary compared to logics of the past century, but my suggestion is that at the end, these processes strive to describe *model-construction* over and above other kinds of moves. The former operate by way of the Utterer (the Graphist) putting forth an assertion by scribing it on the sheet of assertion, followed by the Interpreter either refuting (that is, moving the instance thus asserted out of the field of distinct vision) or accepting it (that is, keeping it in the field of such vision).[7]

III

Tableaux methods are well-known examples of model-building processes in which we search for counterexamples to propositions. Creating the tableaux is to draw certain logical pictures. To check the consistency of the assertion is to perform a satisfiability check, which means building a model for it. Peirce's notion comes in terms of the Grapheus "being satisfied with" the graphs the Graphist scribes. Such methods introduce a competitive element in that a set of assertions having a model is tantamount to the existence of a winning strategy for the Graphist (or the Builder or the Proponent), while the negation of an assertion is tantamount to the existence of the winning strategy for the Grapheus (or the Critic or the Opponent). What the Grapheus is doing is to search for counterexamples that would demonstrate the invalidity of the initial assertions.

Peirce's games evoke the Graphist, the Utterer of the assertion, to propose modifications to the initially blank sheet of assertion on which logical graphs come to be scribed. As Peirce aptly recognizes, any one graph represents "one possible state of the universe" [CP 4.431]. Thus its model exists. The Grapheus, on the other hand, determines the characters of the universe as he pleases. This brings to mind how the interpretations of the underlying language become determined. What Peirce is aiming at with these descriptions is thus not far from how the processes of building models may be accounted for.

As a technical point, these processes leave no room for partial interpretations, since "the blank of the blank sheet…as expressing that the universe, in [a] process of creation by the grapheus, is perfectly definite and entirely determinate" [CP 4.431; cf. MS 492, 1903].[8] Later, in 1909, Peirce came to present a number of systems of triadic logic in which the requirement of determinateness is given up.

As the earliest fully articulated proposal of its kind, the Scottish Book from 1930s by Ulam proposed that forcing (Banach-Mazur games) applies to model building. Such games are ones of teamwork and cooperation. Peirce's idea already mentioned is that the Graphist and the Grapheus "collaborate" [CP 4.552] in the building of what he calls a "Pheme", that is, a model for the assertions of the system [CP 4.538; CP 4.552]. In such a game, the Graphist "proposes modifications to the graphs" while the Grapheus "creates the universe" and decides upon its "determinations". These determinations are the interpretations by way of authorizing or refuting the actions of the Graphist [CP 4.538; CP 4.552]. The model-building proceeds by way of mutual consent. Another way of looking at what Peirce is striving to articulate here is to take these activities to represent what much later were named as the 'cut-and-choose' strategies in game theory and in stability theory.

After the building phase that has to do with scribing tentative graphs subject to criticism, acceptance or refutation, competitive *semantic games* on the accepted assertions commence. Secondly, then, logical graphs are interpreted by sequences of competitive plays between the Graphist who proposed the assertion that any graph thus created represents, and the Grapheus, now playing the role of the Interpreter or Nature, who had created the universe and has an antagonist aim. We might initially think of this as an *experimentation* resembling the age-old idea of putting questions to Nature. The experimentation concerns one state of the universe at a time and proceeds by an interchange between the Graphist and the Grapheus. However, since the universe is determinate, the Grapheus may not change his mind about the authorisations or selections. More appropriately speaking, then, these games perform semantic rather than interrogative or experimentative moves, that is, moves constitutive of the notion of material truth.

IV

What I proceed to enquire next is the question concerning the relationship between the two predominant types of logical games. Are the two games, the model construction and the semantic one, two different kinds of games after all? As a perceptive reader may already have surmised, Peirce did not quite keep them separate from one another. Was this in fact a conscious choice or were his ideas concerning the two kinds of activities not clear enough to be thus differentiated?

It turns out that these two logical activities in fact are two sides of the same conceptual coin, which suggests that the former might in fact have been the case in Peirce's mind. For we can show that, for all sentences S of a language L (say, first-order or modal) there exists a winning strategy for the utterer in the model-construction game B(S,L) if and only if there exists a winning strategy for him/it in the semantic game G(M,S) in a model M.

To see this, let the model set (Hintikka set) be defined as a set of plays of the model-construction game B. From the winning strategies of the model-construction games for the utterer we can then construe a model set that guarantees the existence of witness individuals and witness predicates in the semantic game G correlated with a sentence S. Conversely, if there exists a winning strategy for the utterer in the semantic game correlated with S, we can construe a model set from which we get a model for S by playing through the positions allowed by the model set in question. Given the rules of the semantic game, we can seek and find individuals and predicates precisely in those cases in which we can construe the models that make these selections possible according to the rules of the model building.

Here we have a method to examine what the models are that are permitted by the rules of the semantic game. We construe models for logical constants, interpreted by semantic games, by model-construction games. These games then permit us to see which models a sentence in a semantic game has. As soon as we have found out the models for the sentences containing these constants, we can read off the rules of the semantic game.

In the light of this correspondence, what model building and playing semantic games accomplish amounts largely to one and the same conceptual situation. The fact that the former activities are cooperative and the latter competitive does not affect this correspondence. If you cannot beat them, try to join them. If you cannot join them, try to beat them. After all, building models and semantically evaluating the assertions share the same overall goal of finding out the meaning of logical and intellectual concepts and purports.

This result has several implications as to the nature of the pragmatist theory of meaning, as well as to the overall methodological approach Peirce wanted to present to us. Let me next illustrate some of these implications in relation to linguistic and mathematical methodologies.

V

Peirce insists that "the two [players] must come to an agreement of convention" [MS 280] about what constitutes the universe of discourse. The sheet of assertion "represents the state of mind of the interpreter" [MS 280]. At the same time, it contains everything that is well understood to be taken for granted between the two parties. The parties have formed the common ground concerning it. Grice's preferred term was the "common-ground status" of discourse particles and assertions that "conventionally commit the speaker to [their] acceptance" [Grice (1989), p. 65]. In contrast to Stalnaker (1978), Grice is in agreement with Peirce's thesis concerning common knowledge not depending upon propositional presuppositions but upon conventional and habitual manners in which we become acquainted with the objects of discourse. Peirce adds that the universe itself may be indefinitely extendible, but the domains that the players' various moves are concerned with must be limited to those areas that are within "the field of distinct vision" of the interpreter [MS 280].

How to interpret his idea in the sense of model building processes? Perhaps one way of looking at this is that the players' activities can constrain the possible classes of models. This could be in order to attain the intended models and the descriptive completeness of the mathematical axiomatic system in question, for example.[9] Thus the games that Peirce is proposing here are not 'only' games but can grow deadly serious, at least as far as the foundations of mathematics are concerned. I do not claim to be able to conclusively find out from Peirce's own prose whether something like this was what he envisioned, or even how that idea might at the end be best implemented, but however it happens, such constrains are internal to the processes of construing the models. Since we cannot quantify outside the structures of the models, such constraints are not the usual axiomatic tools of trade. They do not pertain to what can be captured by our system of the language of logic in any obvious sense of such language, as the quantification employed therein does not reach beyond the classes of structures of models.

What I nonetheless do like to propose as a consequence of the possibility that players' activities may provide closure conditions constraining the admissible classes of models is that the general principles governing mathematical practices come to be related to the game-theoretic model-building activities. And therefore, in the first place, they pertain to the qualitative features of mathematical discovery.

And here Hintikka again comes into play with his 1993 proposal of the *extremalist programme* in the philosophy of mathematics [Hintikka (1993)]. The extremalist programme takes the key principles governing model constructions for first-order logic to be the principles of parsimony and plenitude (such as minimality/Archimedean axiom and maximality/Hilbert-completeness). We may see this project as related both to Peirce's and Grice's insights concerning the strategic nature of the meaning of assertions of logic and discourse. It is also part and parcel of abstract model theory. Mathematical reasoning is not limited solely to deductive mechanisms. And certainly it is not for Peirce, who took the ampliative, especially the abductive, modes of reasoning to be indispensable in creative mathematical discovery [see Pietarinen (2010)].[10]

Thus he would not have felt comfortable with only having elementary (first-order) languages at his disposal whenever analysing logically how to arrive at important mathematical conceptualisations or what the significations of such conceptualisations are. And indeed he did not, as most of the expressions and analyses of mathematical notions took place within his *higher-order diagrammatic logic of potentials*, not within the elementary confines of the beta part of the method of Existential Graphs [see Pietarinen (2013c)]. As elementary logic is insufficient in uniquely characterising the intended models, suffering the loss of deductive completeness (yes, Peirce even remarked in [MS 478] that it is impossible to draw up a complete set of transformation rules for these higher-order gamma graphs) did not worry him much.[11]

VI

In a similar vein, Grice's theory of meaning is at bottom concerned with *non-Bayesian abductive reasoning* concerning speakers' intentions. Grice's ideas come to a sharper focus when analysed in the context of the discovery of intentions. I will comment on Grice's theory, in actual fact a theory of signs and signification, as the last point here. His theory is grounded on the principle of cooperation. But cooperation is a property of the logical activities in the model-building games. Yet as we just observed, in view of the correspondence result cooperative model-building resorts to the same theoretical constructs of make-believe agents as the competitive semantic activities do.

In order to understand Grice's insights, it therefore appears insufficient to explicate the more or less contingent maxims of conversation in terms of one 'super-maxim' of the cooperative principle. Contra [Benz et al. (2005)], it seems not right to align the principle of cooperation with cooperative game theory or Lewis's signalling of Stenius's gardening games, either [Lewis (1969), Stenius (1967)]. Such attempts do not go to the heart of the philosophical question about the nature of intentions in a given theoretical context. What is to be needed is an account according to which intentions may be seen as part and parcel of the instruments of the game-theoretic toolbox of strategic interactions and not as something hardwired in the structure of language or to be explained reductively in some non-intentional terms.

I can only provide a brief suggestion as to how one might attain something like this. Grice's theory could be put under sharper theoretical focus by seeing it as related to *incomplete-information games*. Incomplete information is not the same thing as *imperfect* information, although there is the well-known Harsanyi translation according to which the lack of knowledge of other players' payoffs, and typically also of one's own payoffs, is translated into the contexts of imperfect information concerning past histories of the game [Harsanyi (1995)]. But this is a reductive argument. The incompleteness in unreduced incomplete-information games concerns players' uncertainty about the *types* of the players partaking in strategic activities. These types are normally given by chance moves by a fictitious player according to prior probability distributions. The resulting games are commonplace in theories of economics and strategic decision making whenever the exact identity or the mathematical structure of the games is unknown.

Particular kinds of conversational contexts may be seen to correspond to such chance moves with pre-defined probability distributions by the player generating them. Chance moves reveal the information about players' types. Initially such information is not public. Now let the intentions come into view in the form of the state descriptions of agents, distributed in these initial chance moves. In this fashion, the rest of Grice's theory may find its home in the context of incomplete-information games. For example, the first part of the maxim of quantity (the economy of information) aims at turning private conversational information to public, because mutual beliefs about players' intentions ought to maximise the number of true (or rational) beliefs.[12] As the types of the players become common knowledge as the game goes on, intentions are also partly revealed and recognised and beliefs concerning utterer's meaning can be formed in the minds of the interpreters.

I do not venture to suggest that Peirce had much inkling of the kinds of moves corresponding to incomplete-information games. And in all likelihood he would not have accepted them as they typically are presented in game-theoretic literature. However, and here is a link back to Peirce: the Gricean process of inferring the intentions of the speaker assumes the capacities for abductive reasoning from the hearer of the assertions. But abductive reasoning cannot be inference to the best explanation. Therefore, it does not concern Bayesian reasoning either, which is merely about rational belief revision and update. Neither Grice nor Peirce were interested in rational belief update. They wanted to understand the nature of hypothesis generation, and for Grice this means generation in the context of the discovery of conversational intentions, so to speak. The Bayesian approach, in which players begin with different priors about the game, is not the right way to implement Grice's ideas. It is unlikely to work in practice, either, because in the context of real discovery there may be no priors to work with in the first place. Moreover, as it is well known Peirce was an outspoken critic of any kind of Bayesian method in explaining the real nature of scientific discovery.

However, we face an immediate puzzle as the incomplete-information games are routinely couched in Bayesian terms. This is not the end of the story, however. A non-Bayesian approach is available, although such a theory is much less known and less developed than its Bayesian cousin. Many-player repetitive games of incomplete information have been investigated in [Megiddo (1980)], for example.

Such non-Bayesian repetitive games might fit the Gricean bill better in other respects, too, as it is by suitable repetitions that the common knowledge concerning the universe of discourse and the common ground that is required of the players in conversational games is guaranteed.

VII

Peirce managed to investigate, in one way or another, a surprisingly rich variety of logical cum game-theoretic activities in the course of developing his pragmatist theory of meaning. Among them we find model-building and semantic moves, as well as deductive, interrogative, assertoric and definitory moves. He did not seem to have chance (probabilistic) moves, however, which is only natural in the light of the fact that such methods lose their validity in the context of abductive reasoning.

Furthermore, the players in these contexts are theoretical, make-believe constructs, and thus do not turn his original pragmatist theory into any of those constructivist variants according to which our actual truth-seeking practices of verification or falsification, or the epistemic contexts of scientific inquiry, could alter the concept of truth.

Peirce also highlighted the role of the common ground that the players are required to possess in the relevant theoretical contexts from logical to linguistic and even to mathematical [MS 614; Pietarinen (2006)].

The general theory of methods of discovery was named by Peirce "methodeutic" or "speculative rhetoric", and abductive reasoning plays a key part in it. It brings the semantic and the pragmatic under the same overall rubric [Pietarinen (2007a)]. It would do violence to his vision to separate the two. In particular, competition versus cooperation cannot be appealed to in trying to distinguish semantic from pragmatic phenomena. Yet that is what largely happened in the subsequent science of linguistics.

In relation to mathematical discovery in Hintikka's sense, interesting questions in philosophy of mathematics are those concerning the expressivity and meaning of mathematical propositions in so far as they are captured in certain suitable logical conceptualisations. My observations concerning strategic aspects of meaning suggest that linguistic and mathematical theorising are linked through the normative conception of logic, and that there is a path from Peirce to Grice to Hintikka marking out those links. Methodeutic, which Peirce once laid out as the field of study concerned with the search for the method of methods, may clarify how to pin down the processes by which a mathematician can arrive at non-logical axiomatisations. Hintikka's work on game-theoretical semantics and other types of games of logic and inquiry has both continued this venerable tradition as well as pushed it into genuinely new directions.

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NOTES

[1] In a letter to E.V. Huntington in 1904 Peirce reveals: "I have twice devoted my utmost energy and entire devotion day and night for several months to endeavoring to get a pure mathematical view of logic; and whatever else I may have learned from these experiences, I have learned (what I knew well enough before) that my genius does not lie in that direction."

[2] MS 280 (1906): "the sheet at the outset represented all the truth that was well-understood between Graphist and interpreter to be taken for granted."

[3] "The surface…shall be called our sheet of assent, and every proposition the regular expression of which the imaginary graphist, with the concurrence of the interpreter shall at any time place upon that sheet of assent, shall be understood to be mutually agreed to, by them both, as representing the truth upon the universe of discourse. …It renders the graphist responsible for the truth of the proposition he writes" [MS 450, 1903].

[4] To represent something that is not the case one adds a new logical operation, namely a cut, which strikes such denied assertions out from the sheet by making an

incision around them. If the area of the cut is empty, this gives rise to "an impossible state of things" [MS 478], the pseudograph.

[5] MS S 28 (1903): "No matter how surprising to reason a fact may be, yet once it is actually experienced, that closes all questioning. It is necessary, therefore, to imagine that graphist and interpreter should have, should understand one another to have, a common basis of already existing experience, and that they should agree that the sheet of assertion together with all that is scribed upon it should relate to a definite object of this common experience."

[6] Pietarinen (2011b) discusses further the implications of such normative-strategic dimensions of Peirce's logic.

[7] MS 280 (1906): "It is on this part of the immense surface that the Graphist almost always prefers to scribe new graphs, on the sheet of assertion, in the interpreter's field of distinct vision, which means of his mental vision of his attention; and although they soon steal away, fatigued by the glare, yet at the very first the newborn graphs have a strong attraction toward the sheet of assertion so that if, as seldom but sometimes happens, the Graphist scribes a new graph elsewhere, it almost leaps to that focus."

[8] According to one of Peirce's initial conventions for the logic of Existential Graphs, "If we assume that the universe of discourse is the creation of some mind, as it obviously is when men dispute for example about the sanity of Hamlet, and as…it must be wherever necessary reasoning is admissible, and as…we virtually assume it to be whenever we reason at all, then we shall have to take account of this author of the universe of discourse who may be called the *grapheus*. The universe of discourse, it is to be understood, is in every respect determinate and definite, so that every possible assertion about it is either true or false, while no assertion about it is both true and false" [MS 450, 1903].

[9] E.V. Huntington, with whom Peirce exchanged several letters, may be credited with the introduction of the notion of categoricity in 1904. The term is attributed to Peirce's student John Dewey by Oswald Veblen.

[10] Peirce's observations about the nature of mathematics are pertinent to contemporary philosophy of mathematics [Pietarinen (2010)]: "It is an error to make mathematics consist exclusively in the tracing out of necessary consequences. For the framing of the hypothesis of the two-way spread of imaginary quantity, and the hypothesis of Riemann surfaces, were certainly mathematical achievements. Mathematics is, therefore, the study of the substance of hypotheses, or mental creations, with a view to the drawing of necessary conclusions" [NEM 4:268, 1896].

[11] And by the Gödelian-type reflection principle, anything expressible in the elementary language of set theory concerning the universe of all sets, and which by reflection equally concerns any of its parts, is also expressible in non-elementary languages with the corresponding reflection property.

[12] Clark (2012) has recently argued that this part of the maxim of quantity may emerge from the pooling equilibrium.

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